$$g_i(x) = \log p(x|\omega_i) + \log P(\omega_i)$$

eq 23 $$p(x) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)\right)$$

$$p(x|\omega_i) \sim N(\mu_i, \Sigma_i)$$

$$g_i(x) = -\frac{1}{2}(x-\mu_i)^T \Sigma_i^{-1}(x-\mu_i) - \frac{d}{2}\log(2\pi)$$

$$-\frac{1}{2}\log|\Sigma_i| + \log P(\omega_i) \qquad \text{eq 31}$$

If $\Sigma_i = \sigma_i^2 I$ $\Rightarrow$ $|\Sigma_i| = \sigma_i^{2d}$

$$\Sigma_i^{-1} = \frac{1}{\sigma_i^2} I$$

$$g_i(x) = -\frac{1}{2\sigma_i^2}\|x-\mu_i\|^2 + \log P(\omega_i) \qquad \text{eq 32}$$

Assign x to the category of nearest mean.

min-distance classifier

template-matching classifier

$$g_i(x) = \frac{-1}{2\sigma^2}\left(x^T x - x^T \mu_i - \mu_i^T x + \mu_i^T \mu_i\right) + \log P(\omega_i)$$

$$= \frac{-1}{2\sigma^2}\left(\underbrace{x^T x}_{\text{same } \forall i} - 2\mu_i^T x + \mu_i^T \mu_i\right) + \log P(\omega_i)$$

$$g_i(x) = -\frac{x^T x}{2\sigma^2} + \frac{1}{\sigma^2}\mu_i^T x + \frac{\mu_i^T \mu_i}{2\sigma^2} + \log P(\omega_i)$$

$$\tilde{g}_i(x) = w_i^T x + w_{i0} \qquad \text{eq } 34$$

$$w_i = \frac{1}{\sigma^2}\mu_i \qquad w_{i0} = \frac{\mu_i^T \mu_i}{2\sigma^2} + \log P(\omega_i) \qquad \text{eq } 36.$$

$$g_i(x) = g_j(x)$$

$$w_i^T x + w_{i0} = w_j^T x + w_{j0}$$

$$(w_i - w_j)^T x + w_{i0} - w_{j0} = 0. \quad *$$

$$(w_i - w_j)^T \left\{ x + \frac{(w_{i0} - w_{j0})(w_i - w_j)}{\|w_i - w_j\|^2} \right\} = 0$$

Since

$$\Rightarrow w_i - w_j = \frac{1}{\sigma^2}(\mu_i - \mu_j)$$

From eq *

$$(w_i - w_j)^T x + \frac{\mu_i^T \mu_i}{2\sigma^2} - \frac{\mu_j^T \mu_j}{2\sigma^2} + \log P(\omega_i) - \log P(\omega_j)$$

$$= 0.$$

Now $w_i - w_j = \frac{1}{\sigma^2}(\mu_i - \mu_j)$

$$\frac{1}{\sigma^2}(\mu_i - \mu_j)^T x + \frac{1}{2\sigma^2}(\mu_i^T \mu_i - \mu_j^T \mu_j) + \log\left(\frac{P_i}{P_j}\right) = 0.$$

$$+ \frac{(\mu_i - \mu_j)^T(\mu_i + \mu_j)}{2\sigma^2} + \ldots = 0$$

$$\cancel{(\mu_i - \mu_j)^T\left[x + \frac{1}{2}(\mu_i + \mu_j) + \sigma^2 \log\left(\frac{P_i}{P_j}\right)\right] = 0}$$

$$(\mu_i - \mu_j)^T x + \frac{1}{2}(\mu_i - \mu_j)^T(\mu_i + \mu_j) + \sigma^2 \log\left(\frac{P_i}{P_j}\right) = 0$$

$$\Rightarrow (\mu_i - \mu_j)^T \left[x + \frac{\mu_i + \mu_j}{2} + \frac{\sigma^2}{\|\mu_i - \mu_j\|^2} \log\left(\frac{P_i}{P_j}\right)(\mu_i - \mu_j)\right] = 0$$

$$w^T(x - x_0) = 0$$

w/ $w = \ldots$ & $x_0 = -\frac{(\mu_i + \mu_j)}{2} - \frac{\sigma^2}{\| \quad \|^2} \log\left(\frac{P_i}{P_j}\right)(\mu_i - \mu_j)$

1

$$p(x|\omega_i) = \frac{1}{\pi b}\,\frac{1}{1+\left(\frac{x-a_i}{b}\right)^2} \qquad i=1,2$$

$$P(\omega_1) = P(\omega_2)$$

Then $P(\omega_1|x) = ?$ & $P(\omega_2|x) = ?$

By Bayes rule

$$P(\omega_1|x) = \frac{p(x|\omega_1)P(\omega_1)}{\sum_{j=1}^{2} p(x|\omega_j)P(\omega_j)} = \frac{p(x|\omega_1)P(\omega_1)}{p(x|\omega_1) + p(x|\omega_2)}$$

Simi $$P(\omega_2|x) = \frac{p(x|\omega_2)P(\omega_2)}{p(x|\omega_1) + p(x|\omega_2)}$$

So $P(\omega_1|x) = P(\omega_2|x)$ iff

$$p(x|\omega_1) = p(x|\omega_2)$$
$$\parallel$$
$$\cancel{\frac{1}{\pi b}}\,\frac{1}{1+\left(\frac{x-a_1}{b}\right)^2} = \cancel{\frac{1}{\pi b}}\,\frac{1}{1+\left(\frac{x-a_2}{b}\right)^2}$$

$$\left(\frac{x-a_1}{b}\right)^2 = \left(\frac{x-a_2}{b}\right)^2$$

$$\Rightarrow \left|\frac{x-a_1}{b}\right| = \left|\frac{x-a_2}{b}\right| \Rightarrow \begin{cases} \frac{x-a_1}{b} = \frac{x-a_2}{b} & \text{no solution exists} \\ \text{or} \\ \frac{x-a_1}{b} = -\left(\frac{x-a_2}{b}\right) \Rightarrow 2x = a_1+a_2 \\ \Rightarrow x = \frac{a_1+a_2}{2} \end{cases}$$

$a_1 = 3, \; a_2 = 5, \; b = 1.$

Assuming $P(\omega_1) = P(\omega_2)$

$$P(\omega_1|x) = \frac{p(x|\omega_1)P(\omega_1)}{p(x|\omega_1) + p(x|\omega_2)} = \frac{\dfrac{1}{1+\left(\frac{x-3}{1}\right)^2}}{\dfrac{1}{1+\left(\frac{x-3}{1}\right)^2} + \dfrac{1}{1+\left(\frac{x-5}{1}\right)^2}}$$

$$= \frac{1}{1 + \dfrac{1+(x-3)^2}{1+(x-5)^2}}$$

$$= \frac{1+(x-5)^2}{1+(x-5)^2 + 1 + (x-3)^2} = \frac{1+(x-5)^2}{2 + (x-5)^2 + (x-3)^2}$$

(2)

$P(\text{error}) = ?$

$$= \int_{-\infty}^{+\infty} P(\text{error}|x)\, p(x)\, dx$$

w/ $P(\text{error}|x) = \begin{cases} P(\omega_1|x) & \text{we decide } \omega_2 \\ P(\omega_2|x) & \text{we decide } \omega_1. \end{cases}$

$$= \begin{cases} \dfrac{p(x|\omega_1)P(\omega_1)}{p(x)} \\ \dfrac{p(x|\omega_2)P(\omega_2)}{p(x)} \end{cases}$$

$\therefore \quad P(\text{error}) = P(\omega)\displaystyle\int_{-\infty}^{L} p(x|\omega_1)\,dx + P(\omega)\int_{L}^{+\infty} p(x|\omega_2)\,dx$

$$\frac{dP(\text{error})}{dL} = P(\omega)\left\{ p(L|\omega_1) - p(L|\omega_2) \right\} = 0$$

$\rightarrow \quad p(L|\omega_1) = p(L|\omega_2)$

$$\frac{1}{\pi b}\,\frac{1}{1+\left(\frac{x-a_1}{b}\right)^2} = \frac{1}{\pi b}\,\frac{1}{} \quad ? \text{ whats wrong?}$$

(3) By pg 13

$$P(\text{error}) = \int_{-\infty}^{+\infty} P(\text{error}, x) \cdot dx$$

$$= \int_{-\infty}^{+\infty} P(\text{error} \mid x)\, p(x)\, dx$$

w/ $P(\text{error} \mid x) = \begin{cases} P(\omega_1 \mid x) & \text{if we decide } \omega_2 \\ P(\omega_2 \mid x) & \text{if we decide } \omega_1. \end{cases}$

$\Rightarrow$ ~~P(error)~~ decide ω_1 if $x > \theta$

$$P(\text{error}) = \int_{-\infty}^{\theta} P(\omega_2 \mid x)\, p(x)\, dx + \int_{\theta}^{+\infty} P(\omega_1 \mid x)\, p(x)\, dx$$

$$= \int_{-\infty}^{\theta} \frac{p(x \mid \omega_2) P(\omega_2)}{p(x)} \cdot p(x)\, dx + \int_{\theta}^{+\infty} \frac{p(x \mid \omega_1) P(\omega_1)}{p(x)} \cdot p(x)\, dx$$

$= \ldots$

$$\frac{dP(\text{error}, \theta)}{d\theta} = P(\omega_2)\, p(\theta \mid \omega_2) - P(\omega_1)\, p(\theta, \ldots$$

def θ unique?

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(4)

$$\sum_{i=1}^{c} P(\omega_i|x) = 1$$

$$1 = \sum_{i=1}^{c} P(\omega_i|x) \leq P(\omega_{max}|x) \cdot c \rightarrow P(\omega_{max}|x) \geq 1/c$$

Minimum error rate decision rule

$$\cancel{P(\alpha_i|x)} = \sum_{j=1}^{c} \cancel{\lambda(\alpha_i|\omega_j) P(\omega_j|x)}$$

$$= \cancel{\sum_{j \neq i} P(\omega_j|x)} \quad 1 -$$

$$P(\text{error}) = \sum_{i \neq j} P(x \in R_i|\omega_j) = \sum_{i \neq j} P(x \in R_i|\omega_j) P(\omega_j)$$

?

If $P(\omega_{max}|x) \geq 1/c$

$P(error) \leq 1 - 1/c = \frac{c-1}{c}$

When would $P(error) = \frac{c-1}{c}$? Uniform distribution of c

$1 - \frac{1}{c}$ disjoint pos bins of equal ~~~~ size.

(5) By cauchys inequality:

$$(a \mp b)^2 \geq 0$$

$$a^2 \mp 2ab + b^2 \geq 0$$

$$\frac{a^2+b^2}{2} \geq ab$$

$$\therefore \sqrt{ab} \leq \frac{1}{2}(a+b)$$

$$\sqrt{ab} = \sqrt{\min(a,b)^2 \frac{ab}{\min(a,b)^2}}$$

$$= |\min(a,b)| \sqrt{\frac{a}{\min(a,b)} \frac{b}{\min(a,b)}} > |\min(a,b)|$$

know $a > 0$ $b > 0$

or $a < 0$ $b < 0$

$$\frac{a}{\min(a,b)} > 1$$

$$\frac{b}{\min(a,b)} > 1$$

Since we are told that $a, b \geq 0$

$\sqrt{ab} > \min(a,b)$ Don't see how to do next part...