

$$\frac{1}{2} \frac{d^2 \phi}{dx^2} = \frac{-g}{\epsilon_0} (\delta(x-x') - \delta(x+x'))$$

↑
change +g at
 x'

↑
change $-g$
at $-x'$

$$\frac{1}{2} \frac{d^2 \phi}{dx^2} = \frac{-g(x)}{\epsilon_0}$$

~~$\phi(x)$~~ + $\phi(x+L) = \phi(x)$

Then $\phi(x) = A + Bx$ in each region: $-\frac{L}{2} \leq x \leq -x'$

$$-x' \leq x \leq x'$$

+ ~~$\phi(x)$~~

+ $\phi(x)$ is continuous at x' & $-x'$ i.e. $x' \leq x \leq \frac{L}{2}$

+ ~~$\phi(x)$~~ $\lim_{x \rightarrow x'^-} \phi(x) = \lim_{x \rightarrow x'^+} \phi(x)$ ~~same for $\phi(x)$~~ eqs #1 & #2

$\lim_{x \rightarrow x'^-} \phi(x) = \lim_{x \rightarrow -x'^+} \phi(x)$ ~~same for $\phi(x)$~~ eqs #2 & #3

+ $\phi(\frac{L}{2}) = \phi(-\frac{L}{2})$ ϕ is periodic of period L .

Then $\phi_I = A_I + B_I x$ $-\frac{L}{2} \leq x \leq -x'$

$\phi_{II} = A_{II} + B_{II} x$ $-x' \leq x \leq +x'$

$\phi_{III} = A_{III} + B_{III} x$ $x' \leq x \leq \frac{L}{2}$

+ by integrating * over $-x' - \delta$ to $+x' + \delta$
 + $x' - \delta$ to $+x' + \delta$ $\delta > 0$

we get

$$\frac{d\phi}{dx}(+x' + \delta) - \frac{d\phi}{dx}(-x' - \delta) = -\frac{\rho}{\epsilon_0}(-1) \quad \text{eq \# 3} \quad \checkmark$$

$$\frac{d\phi}{dx}(+x' + \delta) - \frac{d\phi}{dx}(x' - \delta) = -\frac{\rho}{\epsilon_0}(+1) \quad \text{eq \# 4} \quad \checkmark$$

so eqs \#1 + \#2 require

$$A_I + B_I(-x') = A_{II} + B_{II}(x') \quad \checkmark \checkmark$$

$$A_{II} + B_{II}(x') = A_{III} + B_{III}(x') \quad \checkmark \checkmark$$

eqs \#3 + \#4 require

$$B_{II} - B_I = +\frac{\rho}{\epsilon_0} \quad \checkmark \checkmark$$

$$B_{III} - B_{II} = -\frac{\rho}{\epsilon_0} \quad \checkmark \checkmark$$

the periodicity eq $\phi(\frac{L}{2}) = \phi(-\frac{L}{2})$ requires

$$A_I + B_I \left(-\frac{L}{2}\right) = A_{III} + B_{III} \left(\frac{L}{2}\right) \quad \checkmark \checkmark$$

Now have 5 eqs + 6 unknowns

$$A_I - x' B_I - A_{II} + x' B_{II} = 0 \quad \checkmark$$

$$A_{II} + x' B_{II} - A_{III} - x' B_{III} = 0 \quad \checkmark$$

$$-B_I + B_{II} = \frac{q}{\epsilon_0} \quad \checkmark$$

$$-B_{II} + B_{III} = -\frac{q}{\epsilon_0} \quad \checkmark$$

$$A_I - \frac{L}{2} B_I - A_{III} - \frac{L}{2} B_{III} = 0 \quad \checkmark$$

performing gaussian elimination on $A_I, B_I, \dots; A_{III}, B_{III}$ gives

$$\begin{bmatrix} 1 & -x' & -1 & x' & 0 & 0 \\ 0 & 0 & 1 & x' & -1 & -x' \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 \\ 1 & -\frac{L}{2} & 0 & 0 & -1 & -\frac{L}{2} \end{bmatrix} \begin{bmatrix} A_I \\ B_I \\ A_{II} \\ B_{II} \\ A_{III} \\ B_{III} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ q/\epsilon_0 \\ -q/\epsilon_0 \\ 0 \end{bmatrix} \quad \checkmark$$

Switch rows # ~~1~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ ~~6~~ ~~7~~ ~~8~~ ~~9~~ ~~10~~ ~~11~~ ~~12~~ ~~13~~ ~~14~~ ~~15~~ ~~16~~ ~~17~~ ~~18~~ ~~19~~ ~~20~~ ~~21~~ ~~22~~ ~~23~~ ~~24~~ ~~25~~ ~~26~~ ~~27~~ ~~28~~ ~~29~~ ~~30~~ ~~31~~ ~~32~~ ~~33~~ ~~34~~ ~~35~~ ~~36~~ ~~37~~ ~~38~~ ~~39~~ ~~40~~ ~~41~~ ~~42~~ ~~43~~ ~~44~~ ~~45~~ ~~46~~ ~~47~~ ~~48~~ ~~49~~ ~~50~~ ~~51~~ ~~52~~ ~~53~~ ~~54~~ ~~55~~ ~~56~~ ~~57~~ ~~58~~ ~~59~~ ~~60~~ ~~61~~ ~~62~~ ~~63~~ ~~64~~ ~~65~~ ~~66~~ ~~67~~ ~~68~~ ~~69~~ ~~70~~ ~~71~~ ~~72~~ ~~73~~ ~~74~~ ~~75~~ ~~76~~ ~~77~~ ~~78~~ ~~79~~ ~~80~~ ~~81~~ ~~82~~ ~~83~~ ~~84~~ ~~85~~ ~~86~~ ~~87~~ ~~88~~ ~~89~~ ~~90~~ ~~91~~ ~~92~~ ~~93~~ ~~94~~ ~~95~~ ~~96~~ ~~97~~ ~~98~~ ~~99~~ ~~100~~ ~~101~~ ~~102~~ ~~103~~ ~~104~~ ~~105~~ ~~106~~ ~~107~~ ~~108~~ ~~109~~ ~~110~~ ~~111~~ ~~112~~ ~~113~~ ~~114~~ ~~115~~ ~~116~~ ~~117~~ ~~118~~ ~~119~~ ~~120~~ ~~121~~ ~~122~~ ~~123~~ ~~124~~ ~~125~~ ~~126~~ ~~127~~ ~~128~~ ~~129~~ ~~130~~ ~~131~~ ~~132~~ ~~133~~ ~~134~~ ~~135~~ ~~136~~ 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~~1343~~ ~~1344~~ ~~1345~~ ~~1346~~

Add row #3 to row #1.

Mult row #3 by -1 + add to row #1

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & x' & -1 & -x' \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & x' & -1 & -x' \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -\frac{1}{2}x' & 0 & -\frac{1}{2}x' \end{bmatrix} = \begin{bmatrix} \frac{1}{5}x' \\ \frac{1}{5}x' \\ 0 \\ \frac{1}{5} \\ \frac{1}{5}(\frac{1}{2}-x') \end{bmatrix} \checkmark$$

Mult row #4 by -1

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & x' & -1 & -x' \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & x' & -1 & -x' \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\frac{1}{2}x' & 0 & -\frac{1}{2}x' \end{bmatrix} = \begin{bmatrix} \frac{1}{5}x' \\ \frac{1}{5}x' \\ 0 \\ \frac{1}{5} \\ \frac{1}{5}(\frac{1}{2}-x') \end{bmatrix} \checkmark$$

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -\frac{1}{2}x' + (-1) & (\frac{1}{2} + x') \end{bmatrix} = \begin{bmatrix} \frac{1}{5}x' \\ \frac{1}{5}x' \\ 0 \\ \frac{1}{5} \\ \frac{1}{5}(\frac{1}{2}-x') \end{bmatrix} \checkmark$$

=

$$\begin{bmatrix} -\frac{9}{5}x' + \frac{9}{5}(x') \\ 0 \\ -\frac{9}{5}x' \\ \frac{9}{5} \\ \frac{9}{5}(\frac{1}{2} + x') - \frac{9}{5}(\frac{1}{2} - x') \end{bmatrix}$$

=

$$\begin{bmatrix} -\frac{29}{5}x' \\ 0 \\ -\frac{9}{5}x' \\ \frac{9}{5} \\ \frac{29}{5}x' \end{bmatrix} \quad \checkmark \checkmark$$

↓ elt at position 5 x 6 is

$$-\frac{9}{5}x' + \frac{9}{5}x' - \frac{9}{5}x' = -\frac{9}{5}x' \quad \text{negative!!} \quad \checkmark$$

∴

~~$$\begin{bmatrix} \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{bmatrix}$$~~

∴ get

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} A_I \\ \cancel{B_I} \\ \cancel{A_{II}} \\ B_{II} \\ A_{III} \\ B_{III} \end{matrix} = \begin{bmatrix} -\frac{2q}{6}x' \\ 0 \\ -\frac{q}{6}x' \\ q \\ -\frac{2q}{6L}x' \end{bmatrix} \quad \checkmark$$

$$\begin{aligned} [A] &= [B] \cdot [X] \\ &= [8] \cdot [x] \\ &= \begin{bmatrix} q \\ \epsilon_0 k \end{bmatrix} \end{aligned}$$

Add row #5 to rows #2 & #4

$$\Leftrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} A_I \\ B_I \\ A_{II} \\ B_{II} \\ A_{III} \\ B_{III} \end{matrix} = \begin{bmatrix} -\frac{2q}{6}x' \\ -\frac{2q}{6L}x' \\ -\frac{q}{6}x' \\ \frac{q}{6} - \frac{2q}{6L}x' \\ -\frac{2q}{6L}x' \end{bmatrix} \quad \checkmark$$

$$\frac{q}{6} - \frac{2q}{6L}x' = \frac{q}{6L}(L - 2x') \quad \checkmark$$

$$\therefore B_{III} = -\frac{2q}{6L}x' ; \quad \cancel{A_{II}} = \frac{q}{6L}(L - 2x') ; \quad B_I = -\frac{2q}{6L}x' \quad \checkmark$$

$$\dagger A_{II} - A_{III} = -\frac{2q}{60} x' \quad \checkmark$$

$$A_{II} - A_{III} = -\frac{q}{60} x' \quad \checkmark$$

$$\Rightarrow A_{II} = -\frac{2q}{60} x' + A_{III} \quad + \quad A_{II} = -\frac{q}{60} x' + A_{III} \quad \checkmark$$

w/ A_{III} undetermined. of this step. thus

$$\phi_I = \left(-\frac{2q}{60} x' + A_{III} \right) + \frac{-2q}{60L} x' x \quad -\frac{L}{2} \leq x \leq -x' \quad //$$

$$\phi_{II} = \left(-\frac{q}{60} x' + A_{III} \right) + \frac{q}{60L} (L - 2x') x \quad -x' \leq x \leq x' \quad //$$

$$\phi_{III} = A_{III} + \frac{-2q}{60} x' x \quad +x' \leq x \leq \frac{L}{2} \quad //$$

As specified in H-E to make the avg. potential zero

$$\frac{L}{L} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \phi dx = 0$$

$$\Rightarrow \int_{-\frac{L}{2}}^{-x'} \phi_I dx + \int_{-x'}^{+x'} \phi_{II} dx + \int_{x'}^{\frac{L}{2}} \phi_{III} dx = 0$$

$$\Rightarrow \left(-\frac{2q}{6} x' + A_{III} \right) \left(-x' + \frac{L}{2} \right) - \frac{2q}{6L} \frac{x'}{2} \left[(-x')^2 - \left(\frac{L}{2} \right)^2 \right]$$

$$+ \left(-\frac{q}{6} x' + A_{III} \right) (2x') + \frac{q}{6L} \frac{(L-2x')}{2} \cdot 0$$

$$+ A_{III} \left(\frac{L}{2} - x' \right) - \frac{2q}{6L} \frac{x'}{2} \left[\frac{L^2}{4} - x'^2 \right] = 0$$

$$\Rightarrow A_{III} \left[\cancel{7} + \frac{L}{2} + \cancel{2x'} + \frac{L}{2} - \cancel{x'} \right] =$$

$$- \left(-\frac{2q}{6} x' \right) \left(-x' + \frac{L}{2} \right) + \frac{2q x'}{6L} \left[x'^2 - \frac{L^2}{4} \right] + \frac{q x'}{6} 2x' +$$

$$+ \frac{q x'}{6L} \left[\frac{L^2}{4} - x'^2 \right]$$

$$\Rightarrow LA_{III} = \frac{2q x'^2}{6} + \frac{2q x' L}{26} + \frac{q x'^3}{6L} - \frac{q x'^2}{46L} + \frac{2q x'^2}{6}$$

$$+ \frac{q x' L^2}{460L} - \frac{q x'^3}{60L}$$

$$\Rightarrow A_{III} = \frac{2q x'}{26} = \frac{q x'}{6} \quad \checkmark$$

Thus

$$\phi_I = \frac{-qx'}{6} + \frac{-2qx'}{6L}x \quad -\frac{L}{2} \leq x \leq -x'$$

$$\phi_{II} = 0 + \frac{q(L-2x')}{6L}x \quad -x' \leq x \leq +x'$$

$$\phi_{III} = \frac{qx'}{6} - \frac{2qx'}{6L}x \quad +x' \leq x \leq \frac{L}{2}$$

or

$$\phi_I = \frac{q}{6L}(-x'L - 2x'x) \quad -\frac{L}{2} \leq x \leq -x'$$

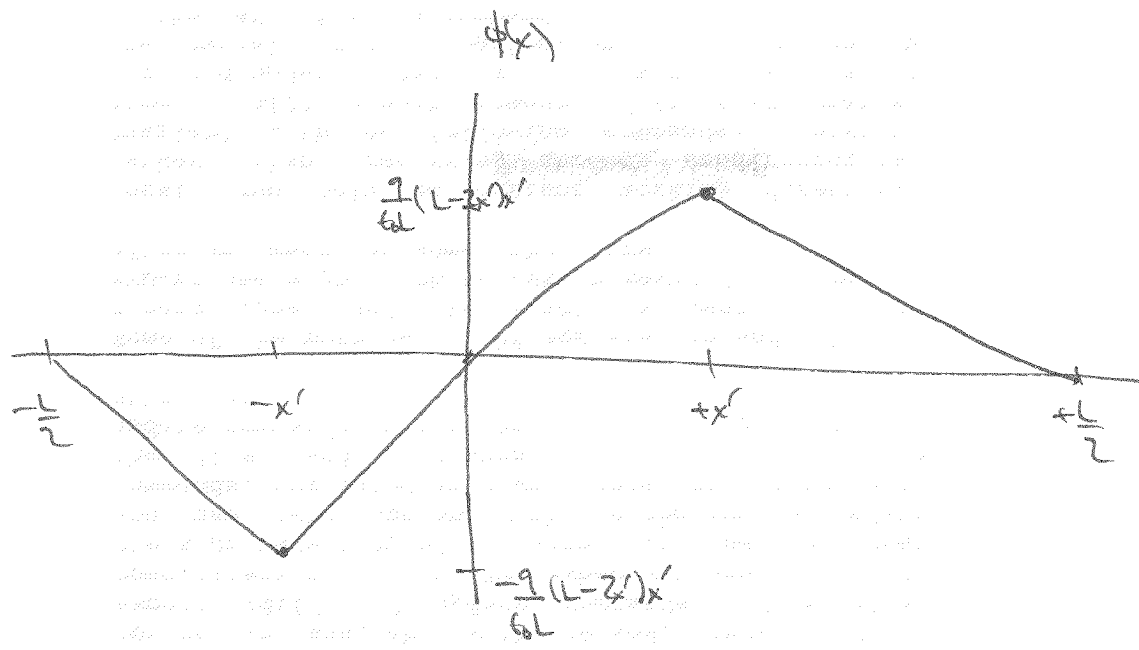
$$\phi_{II} = \frac{q}{6L}(L-2x')x \quad "$$

$$\phi_{III} = \frac{q}{6L}(x'L - 2x'x) \quad "$$

$$\Rightarrow \phi_I = \frac{-q}{6L}(L+2x)x' \quad -\frac{L}{2} \leq x \leq -x'$$

$$\phi_{II} = \frac{q}{6L}(L-2x')x \quad -x' \leq x \leq \frac{L}{2} + x' \quad \text{eq 5-3 } \checkmark$$

$$\phi_{III} = \frac{q}{6L}(L-2x)x' \quad +x' \leq x \leq \frac{L}{2}$$



Then $E = -\frac{d\phi}{dx}$ so

$$E_I = +\frac{q}{6L} 2x' \quad -\frac{L}{2} \leq x \leq -x'$$

$$E_{II} = -\frac{q}{6L} (L-2x') \quad -x' \leq x \leq +x'$$

$$E_{III} = +\frac{q}{6L} 2x' \quad +x' \leq x \leq +\frac{L}{2}$$

The Average electric field at $-x'$ is

$$\begin{aligned} \bar{E}(-x') &= \frac{1}{L} \left[\frac{1}{2} E_I(-x') + \frac{1}{2} E_{II}(-x') \right] = \frac{1}{L} \left(\frac{2x'q}{6L} - \frac{q(L-2x')}{6L} \right) \\ &= \frac{1}{L} \left[-\frac{qL}{6L} + \frac{4x'q}{6L} \right] \\ &= -\frac{q}{26L} (L-4x') \quad \text{eg 5-5} \end{aligned}$$

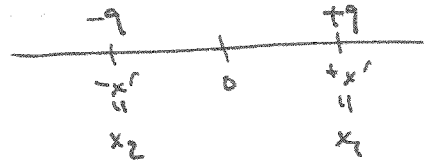
Sim.

$$\vec{E}(+x') = \frac{1}{2} E_{II}(+x') + \frac{1}{2} E_{III}(x')$$

$$= \frac{1}{2} \left[\frac{-q}{6L} (L - 2x') + \frac{q}{6L} 2x' \right]$$

$$= \frac{-q}{26L} (L - 4x') \quad \text{eg } \sigma - 5 \quad \checkmark$$

Then $F_e(x_1) = +q E_e(x_1) = \frac{-q^2}{26L} (L - 4x_1)$



$$\downarrow F_e(x_2) = -q E_e(x_2) = \frac{+q^2}{26L} (L - 4x_2)$$

eg $\sigma - 6$

If $x' = 0$ then since $q - q = 0$ $F = 0$

\downarrow Sim by periodicity at $x' = \frac{L}{2}$ $F = 0$

$$G = F_e - F$$

$$= \frac{-q^2}{2\epsilon_0 L} (L - 2x) - \left[\alpha \left(\frac{-q^2}{2\epsilon_0 L} \right) (L - 2x_p) + (1 - \alpha) \left(\frac{-q^2}{2\epsilon_0 L} \right) (L - 2x_{p+1}) \right]$$

$$= \frac{-q^2}{2\epsilon_0 L} \left[L - 2x - \alpha (L - 2x_p) - (1 - \alpha) (L - 2x_{p+1}) \right]$$

$$= \frac{-q^2}{2\epsilon_0 L} \left[-2x + 2\alpha x_p + (1 - \alpha) 2x_{p+1} \right]$$

$$= \frac{+q^2}{\epsilon_0 L} \left[x - \alpha x_p - (1 - \alpha) x_{p+1} \right]$$

$$= \frac{q^2}{\epsilon_0 L} \left[x - x_{p+1} - \alpha x_p + \alpha x_{p+1} \right]$$

$$= \frac{q^2}{\epsilon_0 L} \left[x - x_{p+1} - \alpha (x_p - x_{p+1}) \right]$$

eq 5-18 ✓

pick $\alpha = \frac{x - x_{p+1}}{x_p - x_{p+1}}$



$$= \frac{x_{p+1} - x}{x_{p+1} - x_p}$$

$$\left\{ \frac{x - x_p}{x_{p+1} - x_p} \right.$$

$$= \frac{x_{p+1} - x_p + x_p - x}{x_{p+1} - x_p}$$

$$= 1 + \frac{x_p - x}{x_{p+1} - x_p} = 1 + \frac{x_p - x}{H} = 1 - \frac{(x - x_p)}{H}$$

then $E_1 = 0$

eq 5-19 ✓

Then take for $F(x) = \alpha F(x_p) + (1-\alpha)F(x_{p+1})$

$$F(x) = \left[1 - \frac{x-x_p}{H} \right] F(x_p) + \left(\frac{x-x_p}{H} \right) F(x_{p+1})$$

$$= \left[\frac{x_{p+1} - x}{H} \right] F(x_p) + \left(\frac{x-x_p}{H} \right) F(x_{p+1})$$

$$= \frac{H - x_{p+1} + x}{H} F(x_p) + \left(\frac{x-x_p}{H} \right) F(x_{p+1})$$

$$x_p = x_{p+1} - H$$

$$= \frac{x_{p+1} - x_p - x_{p+1} + x}{H} F(x_p) + \left(\frac{x-x_p}{H} \right) F(x_{p+1})$$

||

$$\frac{x - (x_{p+1} - H)}{H} = \frac{x - x_{p+1}}{H} + 1$$

$$= 1 + \frac{x - x_{p+1}}{H}$$

∴

$$F(x) = \left[1 - \frac{x-x_p}{H} \right] F(x_p) + \left[1 + \frac{x-x_{p+1}}{H} \right] F(x_{p+1})$$

9 5-20 ✓

What would happen if we had a charge with 1000 nC distributed in space...

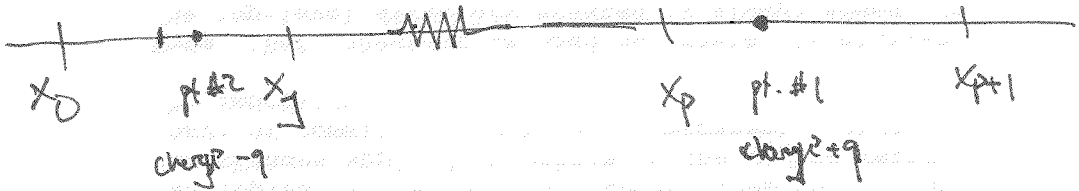
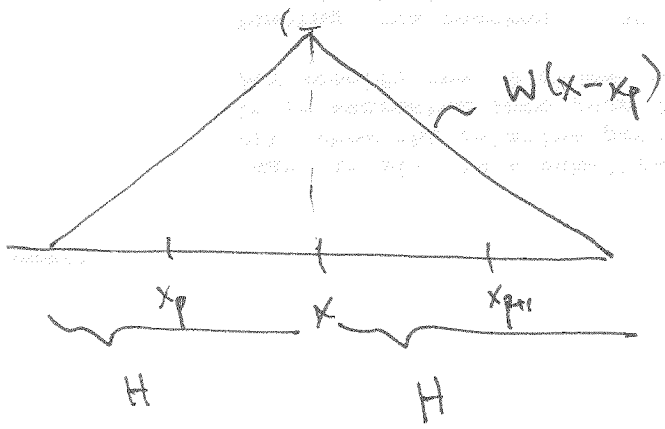


define $W(x-x_p) = \begin{cases} 1 - \frac{|x-x_p|}{H} & |x-x_p| \leq H \\ 0 & \text{else} \end{cases}$

Then

$$\sum_p W(x-x_p) F(x_p) = W(x-x_p) F(x_p) + W(x-x_{p+1}) F(x_{p+1})$$

$$= \left(1 - \frac{(x-x_p)}{H}\right) F(x_p) + \left(1 + \frac{(x-x_{p+1})}{H}\right) F(x_{p+1})$$



$$f(x_0) = \frac{-q}{H} \beta$$

for $\beta = 1 - \frac{x}{H}$

$$f(x) = \frac{-q}{H} (1-\beta)$$



$$P^+(x) = \Pi\left(\frac{x}{H}\right) P'(x)$$

$$\Rightarrow \hat{P}^+(k) = H \Pi\left(\frac{kH}{2\pi}\right) * \hat{P}'(k)$$

explicitly evaluating the convolution eq 5-177 gives, from table A-3

if $f(x)g(x) \xleftrightarrow{F.T.} \int_{-\infty}^{+\infty} \frac{dk'}{2\pi} \hat{f}(k') \hat{g}(k-k')$ then the above becomes:

$$\hat{P}^+(k) = \frac{H}{2\pi} \int_{-\infty}^{+\infty} dk' \hat{P}'(k-k') \Pi\left(\frac{k'H}{2\pi}\right)$$

$$= \frac{H}{2\pi} \int_{-\infty}^{+\infty} dk' \hat{P}'(k-k') \sum_n \delta\left(\frac{k'H}{2\pi} - n\right) \quad \text{define } k_y = \frac{2\pi}{H}$$

$$\sum_n \delta\left(\frac{k'}{k_y} - n\right)$$

$$\sum_n \delta\left(\frac{k' - nk_y}{k_y}\right)$$

From a property of delta fns $\delta\left(\frac{x}{a}\right) = |a| \delta(x)$

So the choice becomes:

$$\hat{p}^+(k) = \frac{H}{2\pi} \cdot |g| \sum_n \hat{p}'(k - n|g) = \sum_n \hat{p}'(k - n|g) = \hat{p}(k)$$

via. eq 5-172
eq 5-179 ✓

Equality of $\hat{p} + \hat{p}^+$

$$\begin{aligned} \hat{p}^+(k) &= \int_{-\infty}^{+\infty} dx p^+(x) e^{-ikx} = \int_{-\infty}^{+\infty} dx \pi \left(\frac{x}{H}\right) p'(x) e^{-ikx} = \int_{-\infty}^{+\infty} dx \sum_n \delta\left(\frac{x}{H} - n\right) p'(x) e^{-ikx} \\ &= \int_{-\infty}^{+\infty} dx \sum_n \delta(x - nH) p'(x) e^{-ikx} \quad \text{w/ } \delta\left(\frac{x}{2}\right) = |a| \delta(x) \\ &= \int_{-\infty}^{+\infty} dx \sum_n \delta(x - nH) p'(x) e^{-ikx} = H \sum_n p'(nH) e^{-iknH} \end{aligned}$$

let $x_p = pH$

$$\hat{p}^+(k) = H \sum_p p'(x_p) e^{-ikx_p} = \hat{p}(k)$$

via. eq 5-173

$$f_{GS} = p^2 \quad p = \cos\left(\frac{\pi}{n}\right) \approx 1 - \frac{1}{2} \frac{\pi^2}{n^2}$$

$$f_{GS} \approx 1 - \frac{\pi^2}{n^2}$$

From 6-24b $t_{GS}^* = \frac{-p \ln(10)}{\ln(f_{GS})}$

$$\forall \ln(1+x) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{x^k}{k} = x - \frac{x^2}{2} \dots$$

$$\text{So } \ln(f_{GS}) \approx -\frac{\pi^2}{n^2} + O\left(\frac{1}{n^4}\right)$$

$$t_{GS}^* = \frac{-p \ln(10)}{-\frac{\pi^2}{n^2}} = \frac{\ln(10)}{\pi^2} (pn^2) \approx .233 (pn^2) \approx \frac{pn^2}{4}$$

$$\forall n = N_g^{1/2}$$

$$t_{GS}^* = \frac{p(N_g)^{1/2}}{4} \quad \text{eg } 6-30 \text{ L } \checkmark$$

qs in 9 6-26 = 9 \therefore on a two dimensional grid ($d=2$)

$$T_{GS} = 9 \cdot n^2 \left(\frac{pn}{4}\right) \approx 2pn^4 = 2p(N_g^{1/2})^4 = 2pN_g^2$$

eg 6-31 \checkmark

$$\sin p = 1 - \frac{\pi^2}{2n^2}$$

$$+ \sqrt{1+x} = 1 + \frac{x}{2} + \frac{x^2}{8} + \dots$$

we have

$$\sqrt{1-p^2} = \sqrt{1 - \left(1 - \frac{\pi^2}{2n^2}\right)^2} = \sqrt{1 - \left(1 - \frac{\pi^2}{n^2} + \frac{\pi^4}{4n^4}\right)}$$

$$= \sqrt{1 - \left(1 - \frac{\pi^2}{n^2} + \frac{\pi^4}{4n^4}\right)} = \sqrt{\frac{\pi^2}{n^2} - \frac{\pi^4}{4n^4}} = \frac{\pi}{n} \sqrt{1 - \frac{\pi^2}{4n^2}}$$

$$= \frac{\pi}{n} \left(1 - \frac{\pi^2}{8n^2}\right)$$

$$\text{So } 1 + \sqrt{1-p^2} = 1 + \frac{\pi}{n} - \frac{\pi^3}{8n^3}$$

$$\text{So } w_2 = \frac{2}{1 + \sqrt{1-p^2}} = \frac{2}{1 + \frac{\pi}{n} - \frac{\pi^3}{8n^3}} \approx \frac{2}{1 + \frac{\pi}{n}} \approx 2\left(1 - \frac{\pi}{n}\right) \quad \text{eg 6-34b } \checkmark$$

$$\therefore \ln s_2 = w_2 - 1 = 1 - \frac{2\pi}{n} \quad \text{eg 6-34c } \checkmark$$

Since $t_{sor}^k = (1 - \frac{2\pi}{n})^t = 10^{-p}$

$t_{sor}^k = \frac{-p \ln(10)}{\ln(1 - \frac{2\pi}{n})}$

$\ln(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1} x^k}{k}$

$\ln(1 - \frac{2\pi}{n}) \approx -\frac{2\pi}{n}$	So	
$t_{sor}^k = \frac{-p \ln(10)}{(-\frac{2\pi}{n})} = (pn) \frac{\ln(10)}{2\pi}$		
Now $\frac{\ln(10)}{2.303} = .36 \dots \hat{=} \frac{1}{3}$		

$t_{sor}^k \approx \frac{pn}{3}$

$n \hat{=} 100$ eq 6-26a is 7 operations
 eq 6-26b is 4 ops as is or 3 operations

~~7~~ ~~4~~ w/ eq 6-27b

for a total of 13 or 12 operations

So $T_{sor} = 12 N_g \frac{p}{3} (N_g)^{1/2} \approx 24 N_g^{1+1/2}$ eq 6-36

Assuming

$$\rho = \cos\left(\frac{\pi}{n}\right) \left(2 - \cos\left(\frac{\pi}{n}\right)\right) \approx \left(1 - \frac{\pi}{n}\right) \left(2 - \left(1 - \frac{\pi}{n}\right)\right)$$

$$= \left(1 - \frac{\pi}{n}\right) \left(1 + \frac{\pi}{n}\right) = 1 - \frac{\pi^2}{n^2} \quad \text{eq 6-39a } \checkmark$$

$$\omega_b = \frac{2}{1 + \sqrt{1 - \rho^2}} = ?$$

since $\rho^2 \approx 1 - \frac{2\pi^2}{n^2}$

$$1 - \rho^2 \approx \frac{2\pi^2}{n^2}$$

$$\sqrt{1 - \rho^2} \approx \frac{\sqrt{2}\pi}{n}$$

so $\omega_b = \frac{2}{1 + \frac{\sqrt{2}\pi}{n}} \approx 2 \left(1 - \frac{\sqrt{2}\pi}{n}\right) = 2 - \frac{2\sqrt{2}\pi}{n} \quad \text{eq 6-39b } \checkmark$

$$T_{\text{slow}} = \omega_b - 1 = 1 - \frac{2\sqrt{2}\pi}{n} \quad \text{eq 6-39c}$$

$$t_{\text{slow}}^* = \frac{-\rho \ln(\omega)}{\ln(\Delta)} = \frac{-\rho \ln(\omega)}{-\frac{2\sqrt{2}\pi}{n}} = \frac{\rho \ln(\omega)}{2\sqrt{2}\pi} \quad (\text{np})$$

Now $\frac{\rho \ln(\omega)}{2\sqrt{2}\pi} = 0.2591 \approx \frac{1}{4} \therefore t_{\text{slow}}^* = \frac{np}{4} \quad \text{eq 6.39d } \checkmark$

$$T_{\text{slow}} \approx 11 \cdot N_g \frac{pn}{4} \approx 3pn^3 \quad \text{if } N_g = n^2 \text{ in a 2d problem.}$$

$$\frac{\phi^{n+1/2} - \phi^n}{\Delta t/2} = -L_x \phi^{n+1/2} - L_y \phi^n + q$$

$$\frac{\phi^{n+1} - \phi^{n+1/2}}{\Delta t/2} = -L_x \phi^{n+1/2} - L_y \phi^{n+1} + q$$

$$\Rightarrow \phi^{n+1/2} - \phi^n = -r L_x \phi^{n+1/2} - r L_y \phi^n + r q$$

$$+ \phi^{n+1} - \phi^{n+1/2} = -r L_x \phi^{n+1/2} - r L_y \phi^{n+1} + r q$$

$$\Rightarrow (I + r L_x) \phi^{n+1/2} = [I - r L_y] \phi^n + r q \quad \text{eq 6-43a } \checkmark$$

$$+ [I + r L_y] \phi^{n+1} = [I + r L_x] \phi^{n+1/2} + r q \quad \text{eq 6-43b } \checkmark$$

$$\Rightarrow \phi^{n+1/2} = [I + r L_x]^{-1} [I - r L_y] \phi^n + r [I + r L_x]^{-1} q$$

then

$$\phi^{n+1} = [I + r L_y]^{-1} [I - r L_x] [I + r L_x]^{-1} [I - r L_y] \phi^n + r [I + r L_y]^{-1} [I - r L_x] [I + r L_x]^{-1} q$$

$$+ r q [I + r L_y]^{-1}$$

$$\equiv M \phi^n + N$$

$$\text{w/ } M = [I + rL_y]^T [I - rL_x] [I + rL_x]^T [I - rL_y]$$

$$N = [I + rL_y]^T [I - rL_x] [I + rL_x]^T q + [I + rL_y]^T r q$$

Now consider

$$[I - rL_x] [I + rL_x]^T \stackrel{?}{=} [I + rL_x]^T [I - rL_x] \quad \text{ie. } \rightarrow \text{ these two}$$

matrices commute?

$$\Leftrightarrow [I + rL_x] [I - rL_x] \stackrel{?}{=} [I - rL_x] [I + rL_x]$$

Yes since each side equals $I - rL_x + rL_x - r^2 L_x^2$ thus M can be

written as

$$M = [I + rL_y]^T [I + rL_x]^T [I - rL_x] [I - rL_y] \quad \text{eq 6-49 b } \checkmark$$

Sim w/ N , N can be written as

$$N = \cancel{[I + rL_y]^T [I + rL_x]^T} [I - rL_x] [I + rL_x]^T q$$

$$= [I + rL_y]^T \left\{ [I + rL_x]^T [I - rL_x] + I \right\} q$$

$$[I + rL_x]^T [I + rL_x]$$

Given ~~xxxx~~

$$(A+N)\phi^{t+1} = N\phi^t + P$$

$$\Leftrightarrow LU\phi^{t+1} = N\phi^t + P$$

$$LU\phi^{t+1} - LU\phi^t = \underbrace{(N-LU)}_{-A}\phi^t + P$$

$$= -(A\phi^t - P)$$

$$= -r$$

$$\Rightarrow LU\Delta\phi^{(t+1)} = -r$$

$$\text{let } y \Rightarrow Ly = -r$$

$$+ U\Delta\phi^{(t+1)} = y$$

$$+ \phi^{(t+1)} = \phi^{(t)} + \Delta\phi^{(t+1)} \quad \text{q 6-74 e.}$$