(c) $\displaystyle\int_{-\infty}^{\infty} e^{-\xi^2} \cos 2\alpha\xi \, d\xi$

$$\oint_C e^{-\xi^2} d\xi = 0 \qquad \int_{-\infty}^{\infty} e^{-z^2} dz = \sqrt{\pi}$$

$$\int_0^{\alpha} e^{-(L+i\eta)^2} i \, d\eta = e^{-L^2} i \int_0^{\alpha} e^{-2iL\eta + \eta^2} d\eta \quad \alpha \quad e^{-L^2} \xrightarrow[L\to\infty]{} 0$$

2nd term $\displaystyle -e^{\alpha^2} \int_{-L}^{L} e^{-\xi^2 - 2i\xi\alpha} d\xi \xrightarrow{L\to\infty} -e^{-\alpha^2} \int_{-\infty}^{\infty} e^{-\xi^2} \cos 2\xi\alpha \, d\xi$

As $\displaystyle i \int_{-\infty}^{\infty} e^{-\xi^2} \sin 2\xi\alpha \, d\xi = 0$

$$\sqrt{\pi}\, e^{-\alpha^2} = \sqrt{\pi}\, e^{-x^2/4t}$$

Z

1. $U = F(x,t)$ sol to PDE $2xU_t - U_{xx} = \delta(x)\delta(t)$

$G(x,t) = \alpha F(\beta x, \gamma t)$ $\qquad \delta(cx) = \frac{1}{|c|}\delta(x)$

$G_x = \alpha F_{x_1} \beta$

$G_{xx} = \alpha F_{x_1 x_1} \beta^2$ $\qquad x_1 = \beta x$, $x_2 = \gamma t$

$G_t = \alpha \gamma F_{x_2}$

Then $2xG_t - G_{xx} = \delta(x)\delta(t) \Rightarrow 2x\alpha\gamma F_{x_2} - \alpha\beta^2 F_{x_1 x_1} = \beta\gamma\delta(x_1)\delta(x_2)$

$$= 2\frac{(\overbrace{\beta x}^{= x_1})}{\beta}\alpha\gamma F_{x_2} - \alpha\beta^2 F_{x_1 x_1} = \beta\gamma\delta(x_1)\delta(x_2)$$

$$\Rightarrow 2x_1 F_{x_2} - \frac{\beta^3}{\gamma}F_{x_1 x_1} = \frac{\beta^2}{\alpha}\delta(x_1)\delta(x_2)$$

$$\therefore \frac{\beta^3}{\gamma} = 1 \quad \& \quad \frac{\beta^2}{\alpha} = 1$$

Thus $\gamma = B^3$ & $\alpha = B^2$

$\therefore$ a sol in the form of. $\quad B^2 F(Bx, B^3 t) \underbrace{\equiv}_{} F(x,t)$

Anoth

uniqueness ← only unique if $G(x,t)$ Also sat BC & I.C.

B.C $\quad U(0,t) = C_1$

$\Rightarrow \quad F(0,t) = C_1$ (know) $\therefore$

Thus $C_1 = G(0,t) = \alpha F(0, \gamma t) = \alpha C_1 \quad \Rightarrow \quad C_1 = 0$ or $\alpha = 1$

B.C $\quad U(\infty, t) = C_2$

$G(\infty, t) = \alpha F(\infty, \gamma t) = \alpha C_2 = C_2 \quad \Rightarrow \quad C_2 = 0$, or $\alpha = 1$

I.C $\quad U(x,0) = C_3$

$C_3 = G(x,0) = \alpha F(Bx, 0) = \alpha C_3 \quad \Rightarrow \quad C_3 = 0$ or $\alpha = 1$.

Thus $C_1, C_2, C_3 \equiv 0$.

(b) $B^2 F(Bx, B^3t) = F(x,t)$

Solved by $F(x,t) = \frac{1}{x^2} f\left(\frac{x^3}{t}\right)$ or $\frac{1}{t^{2/3}} f\left(\frac{x^3}{t}\right) = F(x,t)$

$$F_x = \frac{1}{t^{2/3}} f' \cdot \frac{3x^2}{t} = \frac{3x^2}{t^{5/3}} f'$$

$$F_{xx} = \frac{6x}{t^{5/3}} f' + \frac{3x^2}{t^{5/3}} f'' \left(\frac{3x^2}{t}\right)$$

$$= \frac{9x^4}{t^{8/3}} f''(\xi)$$

$$F_t = -\frac{2}{3}\frac{1}{t^{5/3}} f + \frac{1}{t^{2/3}} f' \left(-\frac{x^3}{t^2}\right) = -\frac{2}{3}\frac{f}{t^{5/3}} - \frac{x^3 f'}{t^{8/3}}$$

Put in

$$2xU_t - U_{xx} = 0$$

$$-\frac{4}{3}\frac{xF}{t^{5/3}} - \frac{2x^4}{t^{8/3}}F' - \frac{6x}{t^{5/3}}F' - \frac{9x^4}{t^{8/3}}F'' = 0$$

$$-\frac{2x}{3t^{5/3}}\left(2F + 9F'\right) - \frac{x^4}{t^{8/3}}\left(2F' + 9F''\right) = 0$$

$$\xi = \frac{x^3}{t}$$

$\div$ by x & Mult by $t^{5/3}$ to get

$$\frac{x^3}{t^{5/3}t}$$

$$-\frac{2}{3}(2F + 9F') - \xi(2F' + 9F'') = 0$$

Solve for F.

(3) As sol to $u_t - u_{xx} = \delta(t-\tau)\,\delta(x-\xi) \quad -\infty < x < \infty, \; t \geq 0$

is
$$\frac{e^{-(x-\xi)^2/4(t-\tau)}}{\sqrt{4\pi(t-\tau)}}$$

taking diff op inside integration gives

$$\int_{\xi=-\infty}^{\infty} \int_{\tau=0^-}^{t} p(\xi,\tau)\,\delta(x-\xi)\,\delta(t-\tau)\,d\xi\,d\tau = p(x,t)$$

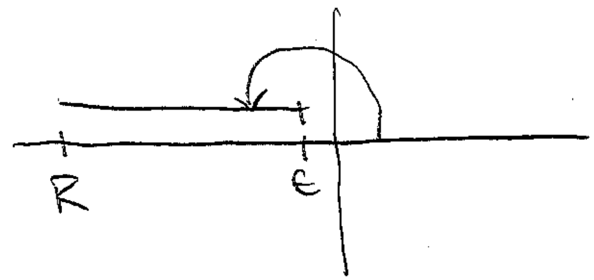
Show
$$(\partial_t - \partial_{xx}) \int_{\xi=-\infty}^{\infty} f(\xi)\,\frac{e^{-(x-\xi)^2/4t}}{\sqrt{4\pi t}} = f(x)\,\delta(t)$$

same eq as asked to solve

$$\frac{e^{-(x-\xi)^2/4t}}{\sqrt{4\pi t}} \;\;\Big\Vert\;\; \delta(t)\,\delta(x-\xi)$$

(a) $$\int_{C_B^+} f\,ds = \int_{\epsilon}^{R} \frac{(-dr)\,e^{-rt-|x|ir^{1/2}}}{2(ir^{1/2})}$$

$$= i\int_{\epsilon}^{R} \frac{e^{-rt-ir^{1/2}|x|}}{2r^{1/2}}\,dr$$



$-\pi < \theta < \pi$

$s = re^{i\theta}$

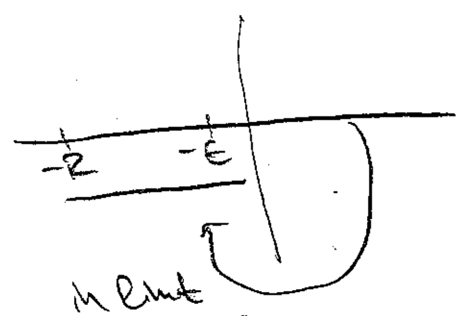
$\theta \to \pi$

$\therefore$ s on C_B^+ is $s = re^{i\pi}$

$s = -r$ $\qquad s^{1/2} = r^{1/2}e^{i\pi/2}$

$ds = -dr$ $\qquad = ir^{1/2}$

(b) $$\int_{C_B^-} f\,ds$$



$s = re^{-i\pi}$ $\qquad s^{1/2} = \ldots e$

$=$

$$\int_0^t dt' \int_0^L dy\, G_{R'}^{N}(\bar{x}, \bar{y}, t-t')$$

$s = r(-1) \qquad s^{1/2} = r^{1/2} e^{-i\pi/2} = -i r^{1/2}$

Thus $ds = -dr$

$$\int_{C_B^-} = \int (-dr) \frac{e^{-rt + ir^{1/2}|x|}}{2(-i) r^{1/2}} = -i \int_{r=R}^{\epsilon} dr \, \frac{e^{-rt + ir^{1/2}|x|}}{2r^{1/2}}$$

$$= i \int_{\epsilon}^{R} \frac{e^{-rt + ir^{1/2}|x|}}{2r^{1/2}}.$$

$C_\epsilon \qquad s = \epsilon e^{i\theta} \qquad -\pi < \theta < \pi$

$ds = \epsilon e^{i\theta} d\theta \, i$

$$\int_{C_\epsilon} = \epsilon i \int_{-\pi}^{\pi} \frac{e^{\epsilon e^{i\theta} - \epsilon^{1/2} e^{i\theta/2}}}{2\epsilon^{1/2} e^{i\theta/2}} d\theta$$

$$\left| \quad \right| \leq M(2\pi)\epsilon^{1/2} \xrightarrow{\epsilon \to 0} 0$$

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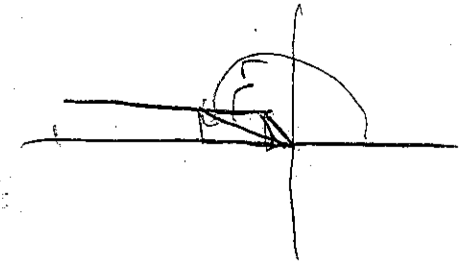
(a) $\int_{C_B^+} F\,ds = \int_{\xi=-\epsilon}^{\xi=-R} \frac{e^{t(\xi+i\epsilon)-\sqrt{\xi+i\epsilon}\,|x|}}{2\sqrt{\xi+i\epsilon}}\,d\xi$ $\Big|_{S=}$

$L\ -\epsilon+i\epsilon$
$-R+i\epsilon$



$\sqrt{S} = r^{1/2} e^{i\theta/2}$ $\quad 0 \le \theta < \pi$

Path C_B^+

$\xi = re^{i\theta}$ $\quad \theta \approx$ take limit $\epsilon \to 0$ $\quad \theta = \pi$

$\therefore\ \xi = ri$ $\qquad d\xi = i\,dr$

$i = e^{i\pi}$
$i^{1/2} = e^{i\pi/2} =$

$$\int_{C_B^+} F\,ds = i\int_{\epsilon}^{R} \frac{e^{itr - r^{1/2}|x|}}{2r^{1/2}}\,dr$$

$$\int_{C_R^+ + C_R^-} = \int_{\pi}^{\pi/2} \frac{Re^{i\theta}\, i\, d\theta}{2R^{1/2}e^{i\theta/2}} \cdot e^{tRe^{i\theta} - R^{1/2}e^{i\theta/2}|x|}$$

on C_R^+ $s = Re^{i\theta}$ θ from π to $\pi/2$

On θ from π to $\pi/2$ $e^{i\theta}$ is from -1 to i & Always has a negative $e^{i\theta} = \cos\theta + i\sin\theta$

But $\theta/2$ has $e^{i\theta/2}$ $\frac{\pi}{2}$ to $\pi/4$ has $\cos\theta > 0$ thus helping convergence

$\therefore$ $e^{tR\cos\theta}\, e^{itR\sin\theta} \xrightarrow{R\to\infty} 0$

$\frac{\pi}{2} < \theta < \pi$

Thus $\int_{C_R^+} \xrightarrow{R\to\infty} 0$

$$\int_{C_R^-} = \int_{-\pi/2}^{-\pi} \frac{e^{tRe^{i\theta} - R^{1/2}e^{i\theta/2}|x|}}{2R^{1/2}e^{i\theta/2}} \cdot iRe^{i\theta} d\theta$$

$s = Re^{i\theta}$ θ from $-\frac{\pi}{2}$ to $-\pi$

$s^{1/2} =$

$$\left| \text{integrand} \right| = \frac{R^{1/2} e^{t R \cos\theta} e^{-R^{1/2} \cos\theta/2}}{2} \xrightarrow{R \to \infty} 0$$

<0 $\quad$ <0

for θ $\quad -\frac{\pi}{2}$ to $-\pi$ $\quad \cos\theta$ is <u>neg</u>

$\cos\theta/2$ is $-\frac{\pi}{4}$ to $-\frac{\pi}{2}$ <u>positive</u>

$$\int_{C_R^-} \xrightarrow{R \to \infty} 0.$$

$k = r^{1/2}$

$dk = +\frac{1}{2} \frac{1}{r^{1/2}} dr.$

(b) $$U(x,t) = \frac{1}{2\pi}\left[\int_{-\infty}^{0} e^{-ikx} e^{-k^2 t}\, dk + \int_{0}^{\infty} e^{-ikx} e^{-k^2 t}\, dk\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{2}\int_{0}^{\infty} e^{ikx} e^{-k^2 t}\, dk + \frac{1}{2}\int_{0}^{\infty} \cdots \right]$$

$$= \frac{1}{\pi}\int_{0}^{\infty} e^{-k^2 t} \cos kx \, dk.$$

(c) $$k\sqrt{t} = \xi$$

$$dk = d\xi/\sqrt{t}$$

$$U(x,t) = \frac{1}{\pi}\frac{1}{\sqrt{t}}\int_{0}^{\infty} e^{-\xi^2} \cos\left(\frac{\xi x}{t^{1/2}}\right) d\xi = \frac{1}{2\pi\sqrt{t}}\int_{-\infty}^{\infty} e^{-\xi^2} \cos\left(\frac{\cancel{\xi x}}{t^{1/2}}\right) d\xi$$

$2\alpha\xi$

Pg 61 Kevorkian

$$U(x_1,\ldots,x_n,t) \equiv w(x_1,\ldots,x_n,t)\exp\left\{-\frac{1}{2}\sum_{i=1}^{n} a_i x_i\right\}$$

$$\frac{\partial U}{\partial x_i} = \frac{\partial w}{\partial x_i}\exp\{\ \} + w\exp\{\ \}\left(-\frac{a_i}{2}\right)$$

$$\frac{\partial^2 U}{\partial x_i^2} = \frac{\partial^2 w}{\partial x_i^2}\exp\{\ \} + -a_i w\exp\{\ \} + w\exp\{\ \}\frac{a_i^2}{4}$$

$$\therefore \sum_{i=1}^{n} \partial_{x_i}^2 U + a_i\partial_{x_i}U = \sum_i \partial_{x_i}^2 w\,\cancel{\exp\{\ \}} + \left(-\frac{a_i^2}{2}+\frac{a_i^2}{4}\right) w\,\cancel{\exp\{\ \}}$$

$$= -\frac{a_i^2}{4}$$

$$+ a w\,\cancel{\exp\{\ \}} = 0$$

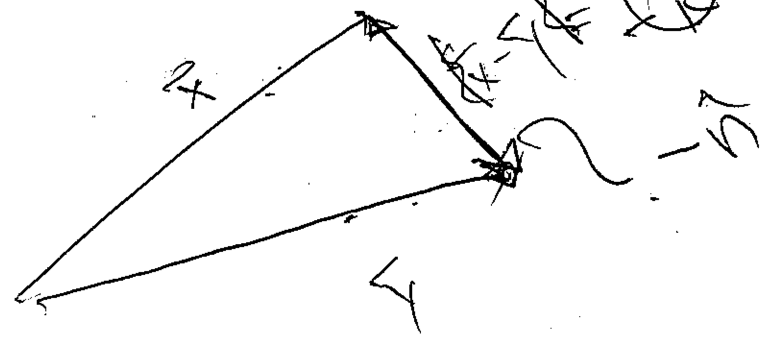
$$\sum_{i=1}^{n}\frac{\partial^2 w}{\partial x_i} + \left(-\frac{1}{4}\sum a_i^2 + a\right)w = 0$$

$$\hat{n} \cdot \nabla_y \frac{1}{|\bar{x}-\bar{y}|} = \quad \frac{1}{|\bar{x}-\bar{y}|^2}$$

$$\nabla_y\left(|\bar{x}-\bar{y}|^{-1}\right) = -|\bar{x}-\bar{y}|^{-2}\,\nabla_y|\bar{x}-\bar{y}|$$

$$= \nabla_y \sqrt{\sum_{i=1}^{3}(x_i-y_i)^2}$$

$$= -\frac{1}{\cancel{|\bar{x}-\bar{y}|}}$$



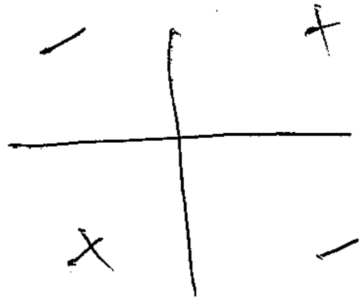
$$\frac{1}{2}\,\frac{2\sum(-1)}{\sqrt{\sum}}$$

$$\frac{\sum_i (x_i-y_i)(-1)\hat{y}_i}{1}$$

$$\frac{(\bar{x}-\bar{y})}{|\bar{x}-\bar{y}|^3} = \frac{-\hat{n}}{|\bar{x}-\bar{y}|}$$

Pg 74 Kevorkian

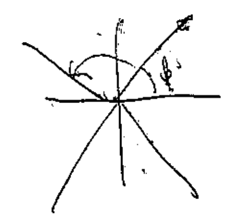
1



$$1-\left(x^2\sin^2\phi - 2xy\cos\phi\sin\phi + y^2\cos^2\phi\right)$$

$$\therefore \int_0^{2\pi} = 2\int_0^{\pi/2} + 2\int_{\pi/2}^{\pi}$$

$$\xi = \phi - \frac{\pi}{2}$$



$$2\int_{\xi=0}^{\pi/2}\left[1-\left(x^2\cos^2\xi + 2xy\sin\xi\cos\xi \; y^2\sin^2\xi\right)\right]^{1/2} d\xi$$

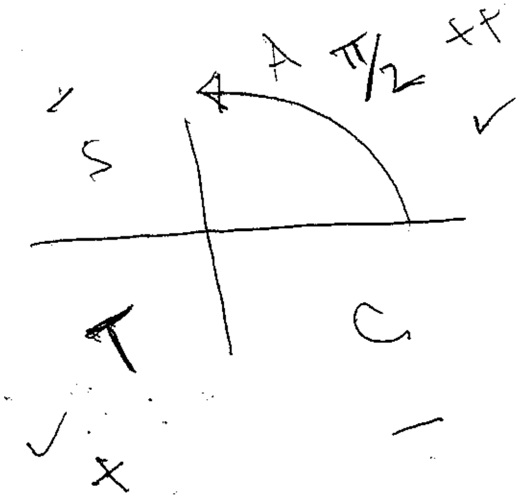
$$\sin\left(\xi+\frac{\pi}{2}\right)$$
$$= \sin\xi\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\cos\xi$$
$$= \cos\xi$$

$$\left[1 - \left(x^2 \sin^2\phi - 2xy \sin\phi\cos\phi + y^2\cos^2\phi\right)\right]$$

$$1 - \sin^2\phi$$

$$\left(x^2 + y^2\right)\sin^2\phi$$

$$x^2\sin^2\phi - y^2\sin^2\phi$$



$$= 2\int_0^{\pi/2} \quad + \quad 2\int_{\pi/2}^{\pi} \left(\quad \right)$$

pg 47 Greenspan

$$\frac{\partial \bar{q}_1}{\partial z} = \frac{1}{2}\frac{\partial}{\partial \tau}\nabla \times \bar{q}_0$$

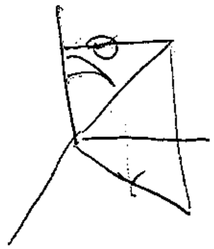
$$\bar{q}_1 = \nabla \times \left(\frac{1}{2} z \frac{\partial}{\partial \tau}\bar{q}_0\right) + \nabla \times B$$

$\bar{q}_0$ independent of z from 26.2

$$\frac{1}{2}\cancel{\nabla(z)} \times \frac{\partial \bar{q}_0}{\partial \tau} + \frac{z}{2}\nabla \times \frac{\partial \bar{q}_0}{\partial \tau}$$

$$\frac{1}{2}\hat{k} \times \frac{\partial \bar{q}_0}{\partial \tau}$$

How??



$$\hat{r}_P = \left(\cancel{\phi}\,\overset{\sin}{\cancel{\cos}}\theta\ \overset{\cos}{\cancel{\sin}}\phi,\ \cancel{\rho}\,\overset{\sin}{\cancel{\cos}}\theta\ \sin\phi,\ \cancel{r}\,\overset{\cos}{\cancel{\sin}}\theta\right)$$

$$\hat{r}_Q = \left(\overset{\sin}{\cancel{\cos}}\theta'\cos\phi',\ \overset{\sin}{\cancel{\cos}}\theta'\sin\phi',\ \overset{\cos}{\cancel{\sin}}\theta'\right)$$

Then $\hat{r}_P \circ \hat{r}_Q = \cos\gamma$

$$= \overset{\sin}{\cancel{\cos}}\theta\ \overset{\sin}{\cancel{\cos}}\theta'\ \cos\underline{\phi\ \cos}\phi' + \overset{\sin}{\cancel{\cos}}\theta\ \overset{\sin}{\cancel{\cos}}\theta'\ \sin\underline{\phi\ \sin}\phi' + \overset{\cos}{\cancel{\sin}}\theta\ \overset{\cos}{\cancel{\sin}}\theta'$$

$$\overset{\sin}{\cancel{\cos}}\theta\ \overset{\sin}{\cancel{\cos}}\theta'\left[\cos(\phi - \phi')\right] + \overset{\cos}{\cancel{\sin}}\theta\ \overset{\cos}{\cancel{\sin}}\theta'$$

Pg 124 bevortion

$$h_t + (uh)_x = 0$$

$$(uh)_t + \left(u^2 h + \frac{h^2}{2}\right)_x = 0$$

$$u h_t + u_t h + \left(u^2 h + \frac{h^2}{2}\right)_x = 0$$

$$\Rightarrow u(-u_x h - \cancel{u h_x}) + u_t h + 2 u u_x h + \cancel{u^2 h_x} + h h_x = 0$$

$$\Rightarrow h u_t + u u_x h + h h_x = 0$$

$$\Rightarrow u_t + u u_x + h_x = 0 \qquad 22b \quad ✓$$

Pg 129 Kevorkian

eq (35b)

$$u_{2t} = -\frac{1}{2}(u_1^2)_x - h_{2x}$$

By $O(\varepsilon)$ term

As $h(x,0) = 1$

$h_x \equiv 0$

$$\therefore \quad u_{2t}(x,0) = -\underbrace{u_1(x,0)}_{=0}\, u_{1x}(x,0) - \underbrace{h_{2x}(x,0)}_{=0}$$

$$\equiv 0$$

Pg 134 Kevorkian $\left(\frac{a^2}{r}\right)^{1/r-1} = \rho$

$$\rho_t + (u\rho)_x = 0$$

$$= \left(\frac{1}{r-1}\right)\left(\frac{a^2}{r}\right)^{\frac{1}{r-1}-1} \frac{2aa_t}{r} + u_x \left(\frac{a^2}{r}\right)^{1/r-1} + u\frac{1}{r-1}\left(\left(\frac{a^2}{r}\right)^{\frac{1}{r-1}-1} \frac{2aa_x}{r}\right) = 0$$

$$= a_t + ua_x + \frac{u_x \left(a^2/r\right)^{\cancel{\frac{1}{r-1}}}}{\frac{1}{r-1}\left(\frac{a^2}{r}\right)^{\cancel{\frac{1}{r-1}}-1} \frac{2a}{r}} = 0$$

$$\frac{(r-1)u_x}{2\left(\frac{a^2}{\cancel{r}}\right)^{-1} \frac{\cancel{a}}{\cancel{r}}} = \frac{(r-1)}{2} a u_x$$

1

<u>pg 137 Kevorkian</u>

$\rho_t + \nabla \circ (\rho \bar{u}) = 0$

$\rho_t = \epsilon \rho_0 \tilde{\rho}_t$

$\rho \bar{u} = \epsilon a_0 \rho_0 + \epsilon^2 \rho_0 a_0 \tilde{\rho} \tilde{\bar{u}}$

$\nabla \circ (\rho \bar{u}) = \epsilon^2 \rho_0 a_0$

$\rho_t + \nabla \rho \circ \bar{u} + \rho \nabla \circ \bar{u} = 0$

$\epsilon \rho_0 \hat{\rho}_t + \epsilon \rho_0 \nabla \tilde{\rho}_t \circ \epsilon a_0 \hat{u} + (\rho_0 + \epsilon \rho_0 \hat{\rho}) \nabla \circ (a_0 \epsilon \bar{u}) = 0$

<u>$O(\epsilon)$</u> (w/o time & length being normalized)

$\cancel{\rho_0} \hat{\rho}_t + a_0 \cancel{\rho_0} \nabla \circ \hat{u} = 0$ $\qquad t = \frac{L_0}{a_0} t' \quad x = L_0 x'$

$$\frac{\partial \hat{\rho}}{\partial t} = \frac{a_0}{L_0} \frac{\partial \hat{\rho}}{\partial t'} \quad \& \quad \partial_x = \frac{1}{L_0} \partial_{x'}$$

$\Rightarrow$ Dropping primes

$$\frac{\alpha_0}{\cancel{L_0}} \partial_t \hat{\rho} + \frac{a_0}{\cancel{L_0}} \nabla \circ \hat{u} = 0$$

$$(\rho \bar{u})_t + \nabla(\rho u^2) + \nabla p = 0$$

$$\Rightarrow \frac{a_0}{\cancel{L_0}} \left(\rho_0 (1 + \epsilon \hat{\rho})(\epsilon a_0 \tilde{u}) \right)_t + \frac{1}{\cancel{L_0}} \nabla \circ \left(\rho_0 (1 + \epsilon \hat{\rho}) \epsilon a_0 \hat{u} \circ \epsilon a_0 \hat{u} \right)$$

$$+ \frac{1}{\cancel{L_0}} \nabla p_0 (1 + \epsilon \hat{p}) = 0$$

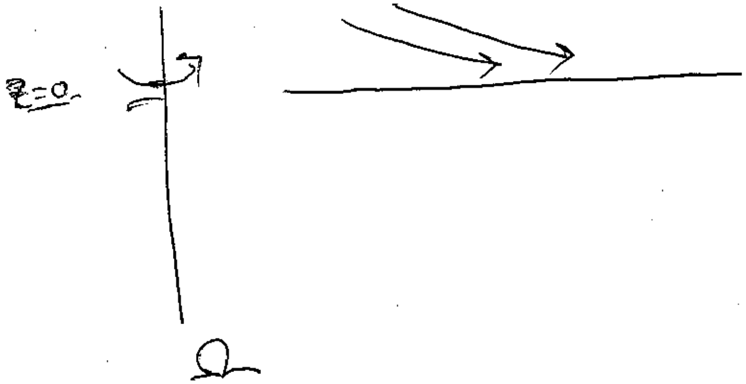
$$\Rightarrow \underline{O(\epsilon)} \quad a_0^2 \rho_0 \hat{u}_t + \nabla \hat{p} = 0$$

$$\rho_t + \nabla \circ (\rho \bar{U}) = 0$$

$$\frac{\bar{U}}{a_0} = \hat{\bar{U}} \qquad \frac{\rho}{\rho_0} = \tilde{\rho}$$

But $a_0^2 = \frac{\gamma p_0}{\rho_0}$

$$\Rightarrow \quad \rho_0 \hat{v}_t + \nabla \hat{p} = 0$$



Solve $\nabla \cdot \bar{q} = 0$

$$\partial_t \bar{q} + 2\hat{k} \times \bar{q} = -\nabla P + E \nabla^2 \bar{q}.$$

B.C. $q \xrightarrow[z \to -\infty]{} 0$ $\quad p(z=0^-) = p(z=0^+) = p_0.$

$$\bar{q}_{air} = \bar{q}_{H_2O} \quad z=0$$

$$\nu_{air} \frac{\partial \bar{q}_{air}}{\partial z} = \nu_{H_2O} \frac{\partial \bar{q}_{H_2O}}{\partial z}. \quad z=0$$

only depends on z, t $\quad \therefore \quad \nabla \cdot \bar{q} = \partial_z W = 0$

$\Rightarrow W = \text{Const}(t)$

Pg 138 k.

$$\int_{t=0^-}^{0^+} u_{tt}\, dt - \cancel{\int_{0^-}^{0^+} u_{xx}}^{\,0} = \delta(x)$$

$$u_t \Big|_{0^-}^{0^+} = \delta(x)$$

$$\Rightarrow \quad u_t(x, 0^+) - 0 = \delta(x)$$

$$\xi = x +$$

$$u_{tt} = \partial_t^2 u(x,t) = \partial$$

$$\partial_t u(x,t) = \partial_\xi U(\xi, \sigma)\, \partial_t \xi + \partial_\sigma U(\xi, \sigma)\, \partial_t \sigma$$

$$\partial_t^2 u(x,t) = \partial_\xi^2 U(\xi, \sigma)\, (\partial_t \xi)^2 + \partial_{\xi\sigma}^2 U(\xi, \sigma)\, \partial_t \xi\, \partial_t \sigma + \ldots$$

$$\partial_t u = \partial_\xi \bar{u}\,(1) + \partial_\eta \bar{u}\,(-1)$$

$$\partial_t^2 u = \partial_\xi^2 \bar{u} - \partial_{\xi\eta} \bar{u} - \partial_{\eta\xi} \bar{u} + \partial_\eta^2 \bar{u}$$

$$\partial_x u = \partial_\xi \bar{u} + \partial_\eta \bar{u}$$

$$\partial_{xx}^2 u = \partial_\xi^2 \bar{u} + 2\partial_{\xi\eta} u + \partial_\eta^2 \bar{u}$$

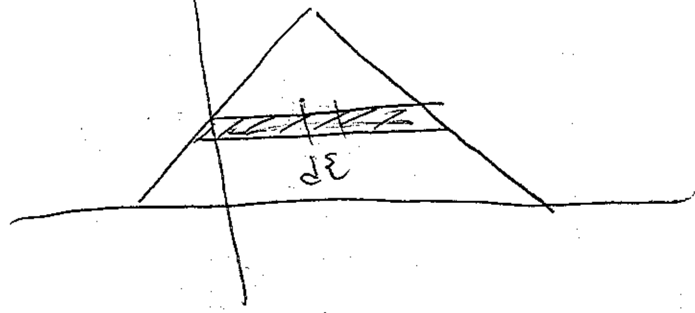
$$\therefore \quad \partial_{tt}^2 u - \partial_{xx}^2 u = 0$$

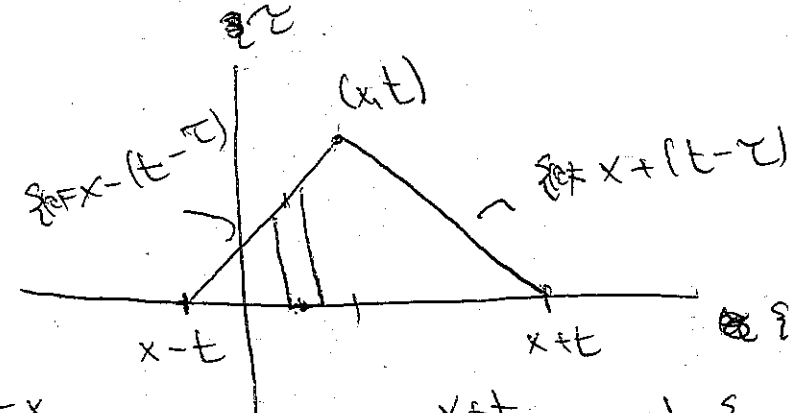
$$\Rightarrow \quad -4\partial_{\xi\eta} \bar{u} = 0$$

$$\bar{u}_\xi = \phi'(\xi)$$

$$\bar{u} = \phi(\xi) + \psi(\eta)$$

Pg 141 Kevorkian





$\Rightarrow \quad \tau = x+t-\xi$

$\xi = x - t + \tau$

$\xi + t - x = \tau$

$$\int_{\xi=x-t}^{x} \int_{\tau=0}^{\xi+t-x} p(\xi,\tau)\,d\tau\,d\xi + \int_{\xi=x}^{x+t} \int_{\tau=0}^{x+t-\xi} p(\xi,\tau)\,d\tau\,d\xi$$

$$= \frac{1}{2} \int_{\xi=x-t}^{x} h(\xi) \int_{\tau=0}^{\xi-x+t} \delta(\tau)\,d\tau\,d\xi + \frac{1}{2} \int_{\xi=x}^{x+t} h(\xi) \int_{\tau=0}^{x+t-\xi} \delta(\tau)\,d\tau$$

$$= \frac{1}{2}\left[\int_{x-t}^{x} h(\xi)\,d\xi + \int_{x}^{x+t} h(\xi)\,d\xi \right]$$

$$= \frac{1}{2} \int_{x-t}^{x+t} h(\xi)\,d\xi$$

pg 144 k

1

$$\xi = x + t \qquad \Rightarrow \qquad \cancel{\xi_t} =$$
$$\sigma = x - t$$

$$x = \tfrac{1}{2}(\xi + \sigma) \qquad t = \tfrac{1}{2}(\xi - \sigma)$$

$$\partial_t = \partial_\xi \cdot \partial_t \xi + \partial_t \sigma \, \partial_\sigma \qquad \partial_x = \partial_x \xi \, \partial_\xi + \partial_x \sigma \, \partial_\sigma$$
$$= \partial_\xi - \partial_\sigma \qquad\qquad = \partial_\xi + \partial_\sigma$$

Add $\quad \partial_\xi = \frac{1}{2}(\partial_t + \partial_x)$

Check

$$\partial_\xi = \partial_\xi t \, \partial_t + \partial_\xi x \, \partial_x$$
$$= \tfrac{1}{2}\partial_t + \tfrac{1}{2}\partial_x .$$

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$H_1(\xi, \sigma) = h_1(x, t)$

$h_1(x, 0) = \hat{h}(x)$

$U_1(\xi, \sigma) = u_1(x, t)$

$u_1(x, 0) = \hat{u}(x)$

$\therefore \quad h_1(x, 0) = \frac{1}{2}\left[F(x) + G(x)\right] = \hat{h}(x)$

$u_1(x, 0) = \frac{1}{2}\left[F(x) - G(x)\right] = \hat{u}(x)$

$F(x) = \hat{h} + \hat{u}$

$G(x) = \hat{h} - \hat{u}$.

$x - t = 0$

$x - t = 1$

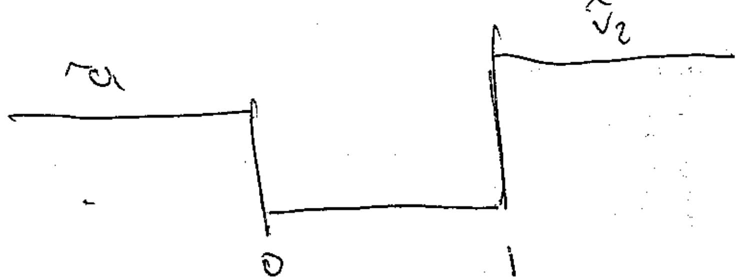
$\hat{h}(\sigma)$: changes values when $\sigma = 0, 1$,

(1) $H_1 = \frac{1}{2}\{\hat{h}_2 + \hat{u}_2$

$h_{1t} = -u_{1x}$

$f_s'(x) = f'(x) + [f(x_0^+) - f(x_0^-)]\delta(x - x_0)$

$\therefore \; u_s'(x) = 0 + \hat{u}_1\delta(x) + \hat{u}_2\delta(x-1)$



$\therefore \; h_{1t} = \hat{u}_1\delta(x) - \hat{u}_2\delta(x-1)$

pg 147 [illegible]

$$\frac{\hat{h}_2 - \hat{v}_2}{2} = \hat{h}_2 \Rightarrow \hat{h}_2 - \hat{v}_2 = 2\hat{h}_2 \Rightarrow \hat{h}_2 = -\hat{v}_2$$

$$\frac{\hat{v}_2 - \hat{h}_2}{2} = \hat{v}_2 \Rightarrow \cancel{\hat{v}_2 - \hat{h}_2 = 0} \Rightarrow$$

$$\hat{v}_2 - \hat{h}_2 = 2\hat{v}_2$$

$$\Rightarrow -\hat{h}_2 = \hat{v}_2$$

4

$$\frac{\hat{h}_1 + \hat{v}_1}{2} = \hat{h}_1 \Rightarrow \hat{v}_1 = \hat{h}_1$$

$$\frac{\hat{h}_1 + \hat{v}_1}{2} = \hat{v}_1 \Rightarrow \hat{h}_1 = \hat{v}_1$$

pg 151 Kevorkian

$$u_{tt} - u_{xx} = \delta(t)h(x)$$

$$\int_{0^-}^{0^+} dt \rightarrow$$

$$u_t(x,0^+) - \cancel{u_t(x,0^-)}^{\,0} - 0 = h(x)$$

$$u_t(x,0^+) = h(x) \quad \checkmark$$

$$\int_{0^-}^{t} \qquad u_t(x,t) - \cancel{u_t(x,0^-)}^{\,0} = h(x)$$

+ integrate again from $\int_{0^-}^{0^+} dt$

$$= u(x,0^+) - u(x,0^-) = 0$$

+ if $u(x,0^-) = 0$ done

$$\therefore u(x,0^+) = u(x,0^-)$$

$\underline{t < x}$

$$u(x,t) = \frac{1}{2}\int_{\tau=0}^{t} \delta(\tau) \int_{\xi=x-t+\tau}^{x+t-\tau} h(\xi)\, d\xi$$

$$= \frac{1}{2}\int_{x-t}^{x+t} h(\xi)\, d\xi$$

$\underline{t > x}$

$$u(x,t) = \frac{1}{2}\int_{-x+t}^{x+t} h(\xi)\, d\xi + 0 \quad \text{As} \quad t - x > 0$$

PJ 152 kevorkian

$$v_t = u(x,t)$$

$$v_{tt} = u_t(x,t)$$

$$v_{xx} = \int_0^t u_{xx}(x,\tau)\,d\tau = \int_0^t u_{tt}(x,\tau)\,d\tau = u_t(x,t) - \underbrace{u_t(x,0)}_{0}$$

$$= v_{tt}$$

$$v(x,0^+) = 0$$

$$v_t(x,0^+) = f(x)$$

Pg 153 Kevorkian

$$w(x,t) = u(x,t) - g(t)$$

$$w_{tt} - w_{xx} = u_{tt} - u_{xx} - \ddot{g} = -\ddot{g}$$

$$w(x,0^+) = u(x,0^+) - g(0^+) = -g(0^+)$$

$$w_t(x,0^+) = -\dot{g}(0^+)$$

$$w(0,t) = 0$$

pg 279 Kevorkian eq 54 ??

Don't think correct.

$$[f_j] = f_j^+ - f_j^-$$

$$= f_j(u_i(\xi^+(t), t), \xi^+(t), t) - f_j($$

$$= \sum_{k=1}^{n} \bar{A}_{jk}(\xi^+(t), t)\, u_k(\xi^+(t), t) - \sum_{k=1}^{n} \bar{A}_{jk}(\xi^-(t), t)\, u_k(\xi^-(t)$$

$$= \sum_{k=1}^{n} \bar{A}_{jk}(\xi^{+,-}, t)\, [u_k]$$

Sim $$[\phi_j] = \sum_{k=1}^{n} \bar{B}_{jk} [u_k]$$

Thus $\dot{\xi}(t)[f_j] = [\phi_j]$

$\Rightarrow$ $\dot{\xi}(t) \sum \bar{A}_{jk}[u_k] = \sum_k \bar{B}_{jk}[u_k]$

$$\sum \left(\dot{\xi}(t)\bar{A}_{jk} - \bar{B}_{jk}\right)[u_k] = 0$$

$\forall j$ for u_k $\left.\begin{array}{l} \frac{dt}{ds} = \bar{A}_{jk} \\ \frac{dx}{ds} = \bar{B}_{jk} \end{array}\right\}$ $\frac{dx}{dt} = \dot{\xi}(t) = \frac{\bar{B}_{jk}}{\bar{A}_{jk}}$

$\therefore$ $\dot{\xi}(t)\bar{A}_{jk} - \bar{B}_{jk} = 0$

$$\iint_D \left(\xi f_{j,t} + \xi \phi_{j,x}\right) dx\,dt - \Lambda_j \xi = 0$$

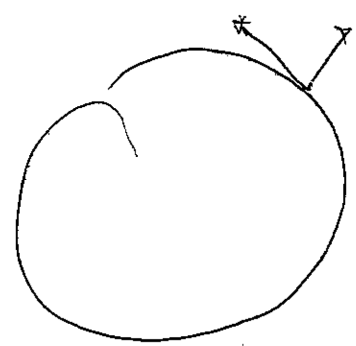
$$+ \iint_D \left(\xi_t f_j + \xi_t \phi_j\right) dx\,dt + \iint_D \xi \Lambda_j \,dx\,dt = 0$$

$$= \iint_D \left((\xi f_j)_t + (\xi \phi_j)_t\right) dx\,dt$$

ξ.

$$\partial_t f_j + \partial_x \phi_j - \Lambda_j = 0$$

$$\int \nabla \cdot \bar{A}\, d^2x = \int \bar{A} \cdot n \, ds$$



$$\int \nabla \cdot \bar{A}\, d^3x = \int \bar{A} \cdot \hat{n}\, dS$$

$dl = \perp$ to normal.

pg 206 kevorkian

$$J(x,y) = \phi_x f_y - f_x \phi_y \qquad \begin{vmatrix} \phi_x & \phi_y \\ f_x & f_y \end{vmatrix}$$

$$\neq 0.$$

$$u \equiv U(\phi(x,y), f(x,y))$$

$$u_x = U_\xi \phi_x + U_\eta f_x \qquad u_y = U_\xi \phi_y + U_\eta f_y$$

$$u_{xx} = U_{\xi\xi} \phi_x^2 + \cancel{U_\xi \phi_{xx} + U_{\eta\eta} f_x^2 + U_\eta f_{xx}}$$

$$+ U_{\xi\eta} \phi_x f_x + U_\xi \phi_{xx} + U_{\eta\xi} \phi_x f_x + U_{\eta\eta} f_x^2 + U_\eta f_{xx}$$

$$= U_{\xi\xi} \phi_x^2 + 2U_{\xi\eta} \phi_x f_x + U_{\eta\eta} f_x^2 + U_\xi \phi_{xx} + U_\eta f_{xx}$$

$$u_{yy} = U_{\xi\xi} \phi_y^2 + 2U_{\xi\eta} \phi_y f_y + U_{\eta\eta} f_y^2 + U_\xi \phi_{yy} + U_\eta f_{yy}$$

$$u_{xy} = U_{\xi\xi} \phi_x \phi_y + U_{\xi\eta} \phi_x f_y + \cancel{U_{\eta\xi} \phi_y f_x} + U_\xi \phi_{xy} + U_{\eta\xi} \phi_y f_x + U_{\eta\eta} f_x f_y$$

$$+ U_\eta f_{xy}.$$

$$= U_{\xi\xi} \phi_x \phi_y + U_{\xi\eta}(\phi_x f_y + \phi_y f_x) + U_{\eta\eta} f_x f_y + U_\xi \phi_{xy} + U_\eta f_{xy}.$$

(1) $a u_{xx} + 2b u_{xy} + c u_{yy} + d u_x + e u_y + f u = g$

$$\Leftrightarrow \quad a\phi_x^2 U_{\xi\xi} + a2\phi_x \psi_x U_{\xi\eta} + a\psi_x^2 U_{\eta\eta} + a\phi_{xx} U_\xi + a\psi_{xx} U_\eta$$

$$+ \; 2b\phi_x\phi_y U_{\xi\xi} + 2b(\phi_x\psi_y + \psi_x\phi_y) U_{\xi\eta} + 2b\psi_x\psi_y U_{\eta\eta} + 2b\phi_{xy} U_\xi + 2b\psi_{xy} U_\eta$$

$$+ \; c\phi_y^2 U_{\xi\xi} + 2c\phi_y\psi_y U_{\xi\eta} + c\psi_y^2 U_{\eta\eta} + c\phi_{yy} U_\xi + c\psi_{yy} U_\eta$$

$$+ \; d\phi_x U_\xi + d\psi_x U_\eta + e\phi_y U_\xi + e\psi_y U_\eta + fU = g$$

$$\Leftrightarrow \quad (a\phi_x^2 + 2b\phi_x\phi_y + c\phi_y^2) U_{\xi\xi} + 2(a\phi_x\psi_x + b(\phi_x\psi_y + \psi_x\phi_y) + c\phi_y\psi_y) U_{\xi\eta}$$

$$+ \; (a\psi_x^2 + 2b\psi_x\psi_y + c\psi_y^2) U_{\eta\eta} + (a\phi_{xx} + 2b\phi_{xy} + c\phi_{yy} + d\phi_x + e\phi_y) U_\xi +$$

$$+ \; (a\psi_{xx} + 2b\psi_{xy} + c\psi_{yy} + d\psi_x + e\psi_y) U_\eta + fU = g.$$

$$\Leftrightarrow \quad M \cdot U + \acute{F} U = \acute{G}$$

$$M = A\partial_\xi^2 + 2B\partial_{\xi\eta}^2 + C\partial_\eta^2 + D\partial_\xi + E\partial_\eta$$

$$\Delta \equiv b^2 - ac \stackrel{?}{=} \frac{B^2 - AC}{J^2} = \frac{1}{J^2}\Big(\big(a\phi_x t_x + b(\phi_x t_y + t_x\phi_y) + c\phi_y t_y\big)^2$$

$$- \big(a\phi_x^2 + 2b\phi_x\phi_y + c\phi_y^2\big)\big(a t_x^2 + 2b t_x t_y + c t_y^2\big)\Big)$$

$$= \frac{1}{J^2}\Big[\cancel{a^2\phi_x^2 t_x^2} + b^2(\phi_x t_y + t_x\phi_y)^2 + c^2\phi_y^2 t_y^2 + 2ab\phi_x t_x(\phi_x t_y + t_x\phi_y)$$

$$+ 2a\phi_x t_x c\phi_y t_y + 2bc\phi_y t_y(\phi_x t_y + t_x\phi_y)$$

$$- \cancel{a^2\phi_x^2 t_x^2} - 2ab t_x t_y \phi_x^2 - ac t_y^2\phi_x^2 - 2ab t_x^2\phi_x\phi_y - 4b^2\phi_x\phi_y t_x t_y$$

$$- 2bc\phi_x\phi_y t_y^2 - ca t_x^2\phi_y^2 - 2bc\phi_y^2 t_x t_y - c^2 t_y^2\phi_y^2\Big]$$

$$= \frac{1}{J^2}\Big[b^2 \underline{\phi_x^2 f_y^2} + 2b^2 \cancel{\phi_x f_y} f_x \phi_y + b^2 f_x^2 \phi_y^2 + c^2 \cancel{\phi_y^2 f_y^2}$$

$$+ 2ab \cancel{\phi_x^2} f_x f_y + 2ab \cancel{\phi_x \phi_y} f_x^2 + 2ac \phi_x \phi_y f_x f_y$$

$$+ 2bc \cancel{\phi_x} \phi_y f_y^2 + 2bc \cancel{\phi_y^2} f_x f_y - 2ab \cancel{\phi_x^2 f_x f_y} - \underline{ac \phi_x^2 f_y^2}$$

$$- 2ab \cancel{\phi_x} \phi_y f_x^2 - \overset{-2}{4b^2} \cancel{\phi_x} \phi_y f_x f_y - 2\cancel{bc} \phi_x \phi_y f_y^2 - ac \phi_y^2 f_x^2 - 2\cancel{bc \phi_y^2} f_x f_y$$

$$- c^2 \cancel{\phi_y^2 f_y^2} \Big]$$

$$= \frac{1}{J^2}\Big[(b^2 - ac) \phi_x^2 f_y^2 + b^2 \phi_y^2 f_x^2 + 2ac \phi_x \phi_y f_x f_y \cancel{+ ac \phi_x^2 f_y^2}$$

$$- 2b^2 \phi_x \phi_y f_x f_y - ac \phi_y^2 f_x^2 \Big]$$

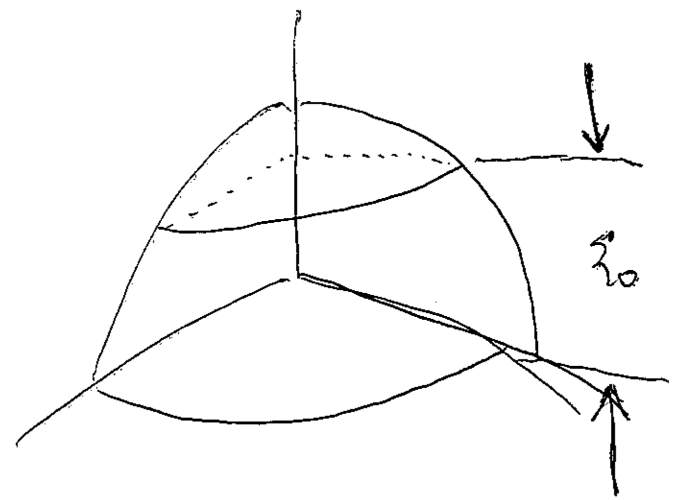
$$= \frac{1}{J^2}\Big[(b^2 - ac)(\phi_x^2 f_y^2 + \phi_y^2 f_x^2) - 2(b^2 - ac) \phi_x \phi_y f_x f_y \Big]$$

$$= \frac{1}{\delta^2}(b^2 - ac)\left(\phi_x^2 f_y^2 - 2\phi_x\phi_y f_x f_y + \phi_y^2 f_x^2\right) = b^2 - ac \ \checkmark$$

$$\parallel$$

$$(\phi_x f_y - \phi_y f_x)^2$$

$$a(x,y)\phi_x^2 + 2b(x,y)\phi_x\phi_y + c(x,y)\phi_y^2 = 0$$



$$y' = \frac{dy}{dx} =$$

$$\phi(x,y) = \xi_0 = \text{const}$$

$$\frac{d}{dx} \rightarrow \quad \phi_x + \phi_y \frac{dy}{dx} = 0. \Rightarrow \frac{dy}{dx} = -\phi_x/\phi_y$$

$$a\frac{\phi_x^2}{\phi_y^2} + 2b\frac{\phi_x}{\phi_y} + c = 0$$

or $\quad a(y')^2 - 2by' + c = 0 \qquad \Delta > 0$

$$y' = \frac{b+\sqrt{\Delta}}{a} \qquad y' = \frac{b-\sqrt{\Delta}}{a}$$

$$\Rightarrow A = 0 \quad + \quad C = 0$$

$$\Rightarrow M(U) + FU = G \iff 2BU_{\xi\eta} + DU_{\xi} + EU_{\eta} + FU = G$$

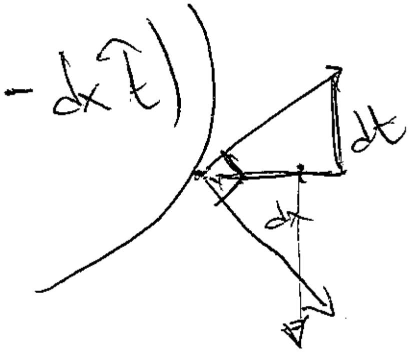
$$U_{\xi\eta} = -\frac{D}{2B}U_{\xi} - \frac{E}{2B}U_{\eta} - \frac{E}{2B}U + \frac{G}{2B}$$

PJ 281 lecture

$\hat{dt} = dx\,\hat{x} + dt\,\hat{t}$

$d\hat{n} = \pm(dt\,\hat{x} - dx\,\hat{t})$

take +.



$$\iint_{D_1} \left(\ell_t f_j + \ell_x \phi_j + \ell \Lambda_j\right) dx\,dt$$

$$\iint_{D_1} (\ell f_j)_t - \ell f_{j,t} + (\ell \phi_j)_x - \ell \phi_{j,x} + \Lambda_j \ell \quad dx\,dt$$

$$0.$$

$$-\iint_{D_1} \ell\left(f_{j,t} + \phi_{j,x} - \Lambda_j\right) dx\,dt + \iint_{D_1} \left((\ell f_j)_t + (\ell \phi_j)_x\right) dx\,dt$$

$$= -\int_{\Gamma_1} \ell\left(f_j \hat{t} + \phi_j \hat{x}\right) \cdot \left(dt\,\hat{x} - dx\,\hat{t}\right)$$

$$= \int_{\Gamma_1} h(f_j\,dx - \phi_j\,dt)$$

$$\iint_{D_2}$$

$$= -$$

pg 285 Kevorkian (typo should

$$\mathring{\xi}[\rho] = [u\rho] \Rightarrow \mathring{\xi}(\rho^+ - \rho^-) = u^+\rho^+ - u^-\rho^- \quad (1)$$

$$\mathring{\xi}[\rho u] = [\rho u^2 + p] \Rightarrow \mathring{\xi}(\rho^+u^+ - \rho^-u^-) = \rho^+u^{+2} + p^+ - \rho^-u^{-2} - p^- \quad (2)$$

$$\mathring{\xi}\left[\frac{\rho u^2}{2} + \frac{p}{\gamma-1}\right] = \left[u\left(\frac{\rho u^2}{2} + \frac{\gamma}{\gamma-1}p\right)\right]$$

$$v = u^+ - \mathring{\xi}(t)$$

eq (1) becomes $\rho^+(u^+ - \mathring{\xi}(t)) - \rho^-(u^- - \mathring{\xi}) = 0$

$$= [\rho v] = 0.$$

eq (2) $0 = \rho^+(u^{+2} - \mathring{\xi}u^+) - \rho^-(u^{-2} - \mathring{\xi}u^-) + p^+ - p^-$

~~But $[\rho v] = \rho^+ v^+$~~

$$\therefore \quad 0 = \rho^+\left(u^{+2} - 2\mathring{\xi}u^+ + \mathring{\xi}^2\right) - \rho^-\left(u^{-2} - 2\mathring{\xi}u^- + \mathring{\xi}^2\right)$$

$$+ \rho^+\mathring{\xi}u^+ - \rho^+\mathring{\xi}^2 - \rho^-\mathring{\xi}u^- + \rho^-\mathring{\xi}^2$$

$$+ p^+ - p^-$$

$$0 = \left[\rho v^2 + p\right] + \underbrace{\left(\rho^+u^+ - \rho^-u^-\right)}_{} \mathring{\xi} \qquad \left(\rho^+u^+ - \rho^-u^- = \mathring{\xi}[\rho]\right)$$

$$- \left(\rho^+ - \rho^-\right)\mathring{\xi}^2$$

$$\therefore \quad 0 = \left[\rho v^2 + p\right]$$

eq 3 becomes

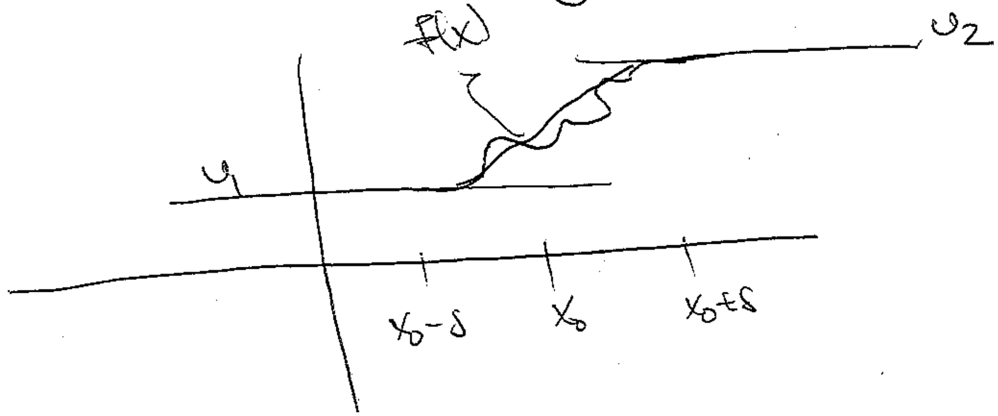
$$\mathring{\xi}\left(\rho^+\frac{u^{+2}}{2} - \rho^-\frac{u^{-2}}{2}\right) + \mathring{\xi}\left(\frac{p^+}{\gamma-1} - \frac{p^-}{\gamma-1}\right)$$

eq 75c becom

Do

Finish

Pg 291 [illegible]



f(x) should be mono increasing.

$$U(x) = \begin{cases} 0 & -\tfrac{1}{2} < x < 0 \\ \tfrac{1}{2} & 0 < x < \tfrac{1}{2} \end{cases} \Bigg\} \text{ U periodic of period } 1.$$

Then

$$\hat{U}(k) = \frac{1}{1} \int_{-1/2}^{1/2} U(x) e^{-i2\pi k}$$

$$\frac{1}{2\pi} \int_{-1}^{1} \frac{1}{2} e^{-ikx}\, dx = \frac{1}{4\pi} \frac{e^{-ikx}}{-ik} \Big|_{-1}^{1}$$

$$= \frac{1}{2\cancel{4}\pi} \frac{(\cancel{i})}{k} \left(e^{-ik} - e^{ik} \right)$$

$$\parallel$$

$$\frac{1}{2\pi} \frac{\sin k}{k} \quad \cancel{2i} \sin k \quad - \quad \cancel{?}$$

pg 293 kenotare

$$v(1-h_1) = 0 - v_1 h_1$$

$$v = \frac{v_1 h_1}{h_1 - 1}$$

$$v\,(-v_1 h_1) = \left(0 + \frac{1}{2}\right) - \left(v_1^2 h_1 + \frac{h_1^2}{2}\right)$$

$$\therefore\ v = \frac{v_1^2 h_1 + \frac{h_1^2}{2} - \frac{1}{2}}{v_1 h_1} \quad \Leftarrow \ *$$

get $v_1 = \dfrac{h_1 - 1}{h_1}\, v$

Then
$$v = \frac{v^2 \frac{(h_1-1)^2}{h_1^2} \cancel{h_1} + \frac{h_1^2}{2} - \frac{1}{2}}{\frac{h_1-1}{h_1}\, v\, \cancel{h_1}}$$

$$\bar{\sigma}^2 = \frac{\sigma^2(h_1-1)^2 + h_1^3/2 - h_1/2}{h_1(h_1-1)}$$

$$= [illegible]$$

$$= [illegible]$$

$$\left(1 - \frac{(h_1-1)^2}{h_1(h_1-1)}\right)\sigma^2 = \frac{1}{2}\frac{(h_1^2-1)}{h_1-1} = \frac{1}{2}(h_1+1)$$

$$\left(\frac{\cancel{h_1^2} - h_1 - (\cancel{h_1^2} - 2h_1 + 1)}{h_1(h_1-1)}\right)\sigma^2 = \frac{1}{2}(h_1+1)$$

$$\frac{\cancel{h_1-1}}{h_1(\cancel{h_1-1})}\,\bar{v}^2 = \frac{1}{2}(h_1+1)$$

$$\bar{v} = \pm\sqrt{\frac{h_1(h_1+1)}{2}}$$

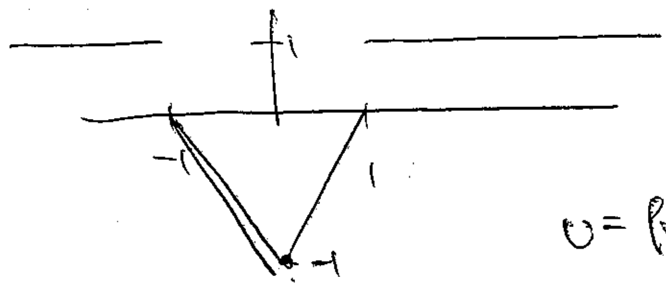
w/ ma get:

$$\bar{v} = \pm\sqrt{\frac{h_1(h_1+1)}{2}}$$

$$v_1 = \pm(h_1-1)\sqrt{\frac{h_1+1}{2h_1}}$$ Same!!

Pg 300 Ice warm

$$U(x,0) = \begin{cases} 1 & |x|>1 \\ -1+|x| & \text{if } |x|<1 \end{cases}$$



$$U = \frac{\rho_{max}/2 - \rho}{\rho_{max}/2}$$

if $= 1 \Rightarrow \rho = 0$

if $= -1 \Rightarrow \rho = \rho_{max}$

$X_0(\tau) = \tau$

$t_0(\tau) = 0$

$$U_0(\tau) = \begin{cases} 1 & \text{if } |\tau|>1 \\ -1+|\tau| & \text{if } |\tau|<1 \end{cases}$$

f eq $\quad U_t + UU_x = 0$

$\frac{dt}{ds} = 1 \qquad \frac{dx}{ds} = U \qquad \frac{dU}{ds} = 0$

$t = s + C_1 \qquad U = C_2$

$t(\tau, s) = s + C_1(\tau) \qquad U(s,\tau) = C_2(\tau)$

$t(\tau, s=0) = C_1(\tau) = 0 \qquad U(0,\tau) = U_0(\tau)$

$\Rightarrow t = s$

$\frac{dx}{ds} = U$

$= U_0(\tau)$

$\Rightarrow X = U_0(\tau)s + x_0(\tau)$

$= U_0(\tau)s + \tau$

pg 298 Kevorkian

$$U = \frac{\gamma+1}{4} u_1 + \left(\left(\frac{r+1}{4} u_1 \right)^2 + 1 \right)^{1/2} \sim \frac{r+1}{2} u_1 \quad \checkmark$$

$$\frac{r+1}{4} u_1 \left[1 + \left(1 + \left(\frac{4}{r+1} \right)^2 \frac{1}{u_1^2} \right)^{1/2} \right] \approx \frac{r+1}{4} u \left[1 + 1 + \right.$$

pg 293 keno-tree

$$v(1-h_1) = 0 - v_1 h_1$$

$$v = \frac{v_1 h_1}{h_1 - 1}$$

$$v(-v_1 h_1) = \left(0 + \frac{1}{2}\right) - \left(v_1^2 h_1 + \frac{h_1^2}{2}\right)$$

$$\therefore\ v = \frac{v_1^2 h_1 + \frac{h_1^2}{2} - \frac{1}{2}}{v_1 h_1} \Leftarrow *$$

get $v_1 = \frac{h_1 - 1}{h_1} v$

Then $v = \dfrac{v^2 \frac{(h_1-1)^2}{h_1^2} \cancel{h_1} + \frac{h_1^2}{2} - \frac{1}{2}}{\frac{h_1-1}{h_1} v \cancel{h_1}}$

$$\bar{\sigma}^2 = \frac{\sigma^2(h_1-1)^2 + h_1^3/2 - h_1/2}{h_1(h_1-1)}$$

$$= \cancel{\tfrac{1}{2}\left(\frac{(h_1^2-2h_1+1)+h_1^3-h_1}{(h_1-1)h_1}\right)}$$

$$= \cancel{\tfrac{1}{2}}\ \cancel{\frac{(h_1-1)(h_1^2+3h_1-2)}{h_1(\ldots)}}$$

$$\left(1 - \frac{(h_1-1)^2}{h_1(h_1-1)}\right)\sigma^2 = \frac{1}{2}\frac{(h_1^2-1)}{h_1-1} = \frac{1}{2}(h_1+1)$$

$$\left(\frac{\cancel{h_1^2}-h_1-(\cancel{h_1^2}-2h_1+1)}{h_1(h_1-1)}\right)\sigma^2 = \frac{1}{2}(h_1+1)$$

$$\frac{\cancel{h_1-1}}{h_1(\cancel{h_1-1})}\,\bar{U}^2 = \frac{1}{2}(h_1+1)$$

$$\bar{U} = \pm\sqrt{\frac{h_1(h_1+1)}{2}}$$

w/ M_a get:

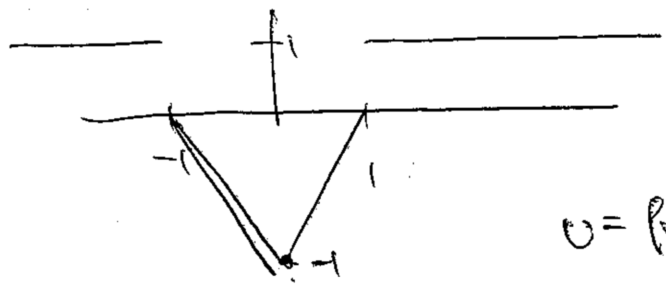
$$\bar{U} = \pm\sqrt{\frac{h_1(h_1+1)}{2}}$$

$$U_1 = \pm(h_1-1)\sqrt{\frac{h_1+1}{2h_1}}$$

Same!!

Pg 300 levortian

$$u(x,0) = \begin{cases} 1 & |x|>1 \\ -1+|x| & \text{if } |x|<1 \end{cases}$$



$$u = \frac{\rho_{max}/2 - \rho}{\rho_{max}/2}$$

$$x_0(\tau) = \tau$$

$$t_0(\tau) = 0$$

$$u_0(\tau) = \begin{cases} 1 & \text{if } |\tau|>1 \\ -1+|\tau| & \text{if } |\tau|<1 \end{cases}$$

if $= 1 \Rightarrow \rho = 0$

if $= -1 \Rightarrow \rho = \rho_{max}$

f eq $\quad u_t + uu_x = 0$

$$\frac{dt}{ds} = 1 \qquad \frac{dx}{ds} = u \qquad \frac{du}{ds} = 0$$

$$t = s + c_1 \qquad u = c_2$$

$$t(\tau,s) = s + c_1(\tau) \qquad u(s,\tau) = c_2(\tau)$$

$$t(\tau,s=0) = c_1(\tau) = 0 \qquad u(0,\tau) = u_0(\tau)$$

$$\Rightarrow t = s$$

$$\frac{dx}{ds} = u = u_0(\tau)$$

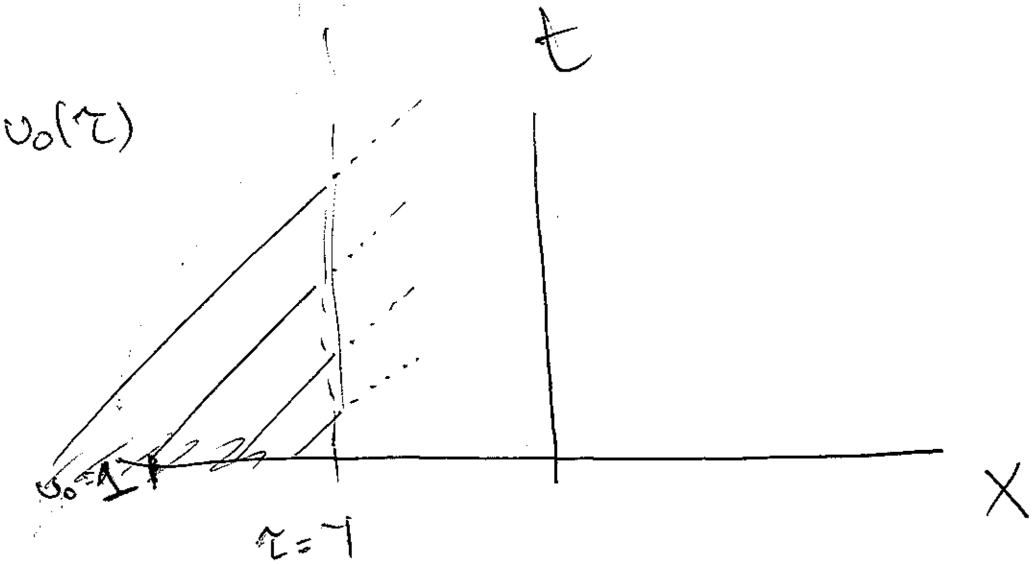
$$\Rightarrow x = u_0(\tau)s + x_0(\tau) = u_0(\tau)s + \tau$$

Pg 298 kevorkian

$$U = \frac{\gamma+1}{4} u_1 + \left(\left(\frac{\gamma+1}{4} u_1 \right)^2 + 1 \right)^{1/2} \sim \frac{\gamma+1}{2} u_1 \quad \checkmark$$

$$\frac{\gamma+1}{4} u_1 \left[1 + \left(1 + \left(\frac{4}{\gamma+1} \right)^2 \frac{1}{u_1^2} \right)^{1/2} \right] \approx \frac{\gamma+1}{4} u \left[1 + 1 + \right.$$

$x = v_0(\tau)\, t + \tau \qquad v = v_0(\tau)$



if $x < -1 \qquad v =$

$\tau < -1$

$v = +1$ on

$x = +t + \tau$

$\Rightarrow \; x - t = \tau$

$-1 < \tau < 0$

$v = -1 - \tau \quad = -(1+\tau)$

$x = (-1-\tau)t + \tau \;= -(1+\tau)t + \tau$

$\Downarrow$

$x + t = \tau(1-t) \;\Rightarrow\; \tau = \frac{x+t}{1-t}$

$\tau = -1 \qquad \infty =$

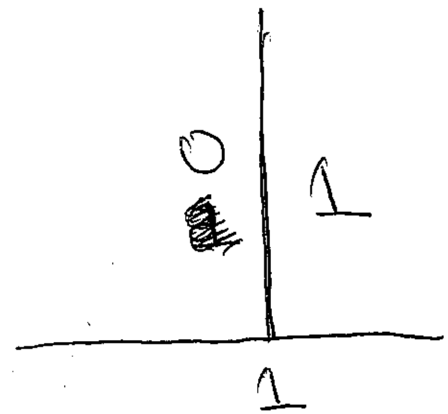
$x = -1 \qquad$ i.e. intersection of Right

Characteristics from $\tau \in (-\infty, -1)$ + Left Char in $(-1, 1)$ occur immediately.

Pg 303 Kevorkian

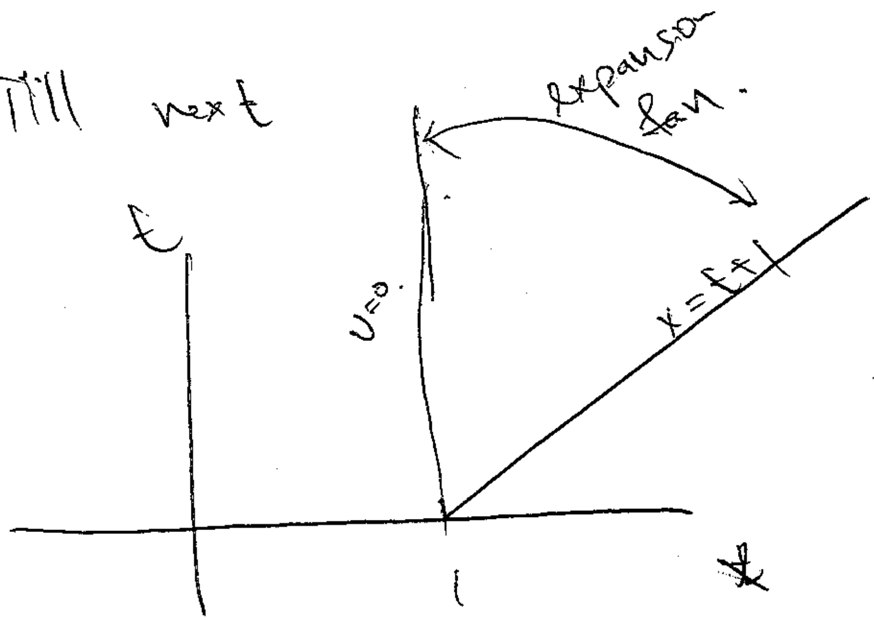
$u(1^-, 0) = 0 = -1+1$

$u(1^+, 0) = 1$



$\therefore \quad u(1^-, 0) < u(1^+, 0)$

Till next



$\underline{\tau = 1}$

$x = u_0(1^-, 0)\,t + 1^- = 1$

$u = u_0(1^-, 0) = 0.$

$\underline{\tau = 1}$

$x = u_0(1^+, 0)\,t + 1^+ = t+1$

$u = u_0(1^+, 0) = 1.$

get eq 113 pg 303 Kevorkian

$$\frac{d\xi}{dt} = \frac{1}{2}\left(U^{+} + U^{-}\right) = \frac{1}{2}\left(1 + \frac{\xi^{2} - 1}{t}\right)$$

get eq 113

$$\dot{\xi} = \frac{1}{2}(u^+ + u^-) = \frac{1}{2}\left[\frac{\xi-1}{t} + \frac{\xi+1}{t-1}\right]$$

$$= \frac{1}{2}\left[\frac{(\xi-1)(t-1) + t(\xi+1)}{t(t-1)}\right]$$

$$= \cancel{\frac{1}{2}\left[\frac{\xi t - \xi - t + 1 - \xi - t}{t(t-1)}\right]}$$

$$= \cancel{\frac{1}{2}}$$

$$= \frac{1}{2}\Big[$$

$$U(s) = u(X(s), Y(s))$$

$$\dot{U} = P(s)\dot{X} + Q(s)\dot{Y}$$

$$A(s)u_{xx} + 2B(s)u_{xy} + C(s)u_{yy} = -D(s) \quad \text{from (1).}$$

$$\dot{P} = u_{xx}\dot{X} + u_{xy}\dot{Y}$$

$$\dot{Q} = u_{yx}\dot{X} + u_{yy}\dot{Y}$$

Char curve. does not det unknowns u_{xx}, u_{xy}, u_{yy}.

$$\begin{vmatrix} A & 2B & C \\ \dot{X} & \dot{Y} & 0 \\ 0 & \dot{X} & \dot{Y} \end{vmatrix} = A(\dot{Y}^2) - \dot{X}(2B\dot{Y} - C\dot{X})$$

$$= A\dot{Y}^2 - 2B\dot{X}\dot{Y} + C\dot{X}^2$$

$$\frac{dy}{dx} = \lambda^{\pm}$$

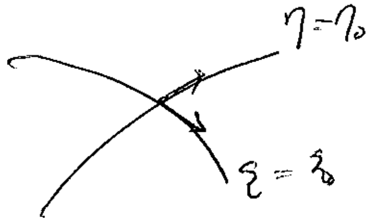
$$Y_\xi(\xi,\eta) - \lambda^- X_\xi = 0$$

$$Y_\eta(\xi,\eta) - \lambda^+ X_\eta = 0.$$

$$u_{xy} = \begin{vmatrix} A & -D & C \\ \mathring{X} & \dot{P} & 0 \\ 0 & \mathring{Q} & \mathring{Y} \end{vmatrix} = A(\mathring{P}\mathring{Y}) - \mathring{X}(-D\mathring{Y} - \mathring{Q}C)$$

$$= A\mathring{P}\mathring{Y} + \mathring{X}D\mathring{Y} + \mathring{X}\mathring{Q}C = 0.$$

$$\mathring{P} + \frac{C}{A}$$



<u>eq 13:</u>

$$P_\eta X_\xi - P_\xi X_\eta + Q_\xi X_\xi\left(-\frac{D}{A}X_\eta - P_\eta\right)$$
$$- X_\eta\left(-P_\xi - \frac{D}{A}X_\xi\right) = 0 \checkmark$$

<u>Pg 395 Kevorkian</u>

why?

$$\sum_{k=1}^{n}\sum_{l=1}^{n} \frac{\partial y_k}{\partial t} V_{lk}\, \bar{\omega}_j \circ \bar{\omega}_l = \sum_k \sum_l \frac{\partial y_k}{\partial t} V_{lk}\, \delta_{jl}\, \bar{\omega}_j \circ \bar{\omega}_l$$

$$= \sum_{k=1}^{n} \frac{\partial y_k}{\partial t} V_{jk}\, \bar{\omega}_j \circ \bar{\omega}_j$$

pg 397 Kevorkian 1

$$s = \log\left(\frac{p}{\rho^r}\right)$$

$$s_t = \frac{1}{(p/\rho^r)}\left(\frac{p}{\rho^r}\right)_t$$

$$s_x = \frac{1}{(\cdot)}\left(\frac{p}{\rho^r}\right)_x$$

$$\Rightarrow \left(\frac{p}{\rho^r}\right)_t + u\left(\frac{p}{\rho^r}\right)_x = E$$

$$\Leftrightarrow \quad s_t + u s_x = \left(\frac{p}{\rho^r}\right)E = e^{+s}E \equiv h$$

$$p e^{-s} = \rho^r \quad \Rightarrow \quad \frac{dp}{d\rho} e^{-s} = r\rho^{r-1}$$

$$\rho = (p e^{-s})^{1/r} \qquad \frac{dp}{d\rho} = c^2 = r\rho^{r-1}e^s$$

$$\left.\frac{\partial p}{\partial \rho}\right|_s = r(pe^{-s})^{r-1/r}e^s = r p^{\frac{r-1}{r}} e^{\left(\frac{r}{r} - \left(\frac{r-1}{r}\right)\right)s}$$

$$= r p^{\frac{r-1}{r}} e^{s/r}$$

$$c(p,s) = \sqrt{r p^{r-1/r} e^{s/r}}$$

$\rho, u, S,$ $\qquad P^r = \rho e^{-S}$

$\cancel{\rho = P^{1/r} e^{-S/r}}$

$\cancel{\rho_t = \frac{1}{r} P_t P^{\frac{1}{r}-1} e^{-S/r} + P^{1/r}\left(-\frac{S_t}{r}\right) e^{-S/r}}$

$= \cancel{e^{-S/r}}$

but w/ $\frac{\partial}{\partial t} \longrightarrow$

$$r \rho^{r-1} \rho_t = P_t e^{-S} + P(-S_t) e^{-S}$$

$$\rho_t = \frac{e^{-S}}{r\rho^{r-1}} (P_t - P S_t)$$

$$= \frac{1}{c^2} (P_t - P S_t)$$

But $\rho^r e^{+S} = P$

$c^2 = \left.\frac{\partial P}{\partial \rho}\right|_S = r\rho^{r-1} e^S$

& $\rho_x = \frac{1}{c^2} (P_x - P S_x)$

$\therefore$ cons mass gives

$$\frac{1}{c^2}\left(P_t - P S_t + u P_x - u P S_x\right) + \rho u_x = Q$$

$$\rho = P^{1/r} e^{-S/r} = Q.$$

But $S_t + uS_x = h(x,t,u,S)$

$\Rightarrow \frac{1}{c^2}\left(p_t - p\,h(x,t,u,s) + u p_x\right) + \rho u_x = Q$

$$p_t + u p_x = c^2 Q + p h - \rho c^2 u_x$$

As $c^2 = \gamma \rho^{\gamma-1} e^S$

$$\rho c^2 = \gamma \rho^{\gamma} e^{S} = \gamma p.$$

$$\Rightarrow \quad p_t + u p_x + \gamma p u_x = c^2 Q + h = q(x,t,u,p,S) \qquad (1)$$

$$\Rightarrow \quad u_t + u u_x + \frac{p_x}{\rho} = 0 \qquad (2)$$

$$S_t + u S_x = h. \qquad (3)$$

$$\Rightarrow \begin{pmatrix} u \\ p \\ s \end{pmatrix}_t + \begin{pmatrix} u & \rho^{-1} & 0 \\ \gamma p & u & 0 \\ 0 & 0 & u \end{pmatrix} \begin{pmatrix} u \\ p \\ s \end{pmatrix}_x = \begin{pmatrix} 0 \\ q \\ h \end{pmatrix}$$

$\Rightarrow f = q\bar{b}_2 + h\bar{b}_3$

$$u - 2\sqrt{h} = -2 \Rightarrow \sqrt{h} = \frac{u+2}{2}$$

$$\frac{dx}{dt} = u + \sqrt{h} =$$

$$\begin{vmatrix} u-\lambda & \rho^{-1} & 0 \\ \gamma p & u-\lambda & 0 \\ 0 & 0 & u-\lambda \end{vmatrix} = (u-\lambda)\Big[(u-\lambda)^2 - \underbrace{\frac{\gamma p}{\rho}}_{c^2}\Big] = 0$$

$\lambda = u$ or

$$\bar{w}_j = \sum_{k=1}^{n} \cancel{w}_{kj} \bar{b}_k \qquad A\bar{w}_j = \lambda_j \bar{w}_j$$

$\mathcal{B}$

$$A\bar{w}_j = \sum_{k=1}^{n} W_{kj} A \underbrace{\bar{b}_k}_{\bar{e}_k} = \lambda_j \bar{w}_j$$

$c^2 = \frac{\gamma p}{\rho}$

$c^2\rho = \gamma p$.

$$W_{1j} A\bar{b}_1 + W_{2j} A \bar{b}_2 + W_{3j} A \bar{b}_3 = \lambda_j \bar{w}_j$$

$$W_{1j}\begin{pmatrix} u \\ \gamma p \\ 0 \end{pmatrix} + W_{2j}\begin{pmatrix} \rho^{-1} \\ u \\ 0 \end{pmatrix} + W_{3j}\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix} = \lambda_j\left[w_{1j}\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + w_{2j}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + w_{3j}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right]$$

is $\bar{W}$. just ~~eigenvectors~~ eigenvectors of $\{A_{ij}\}$?

$$\begin{pmatrix} u-\lambda_j & \rho^{-1} & 0 \\ \gamma p & u-\lambda_j & 0 \\ 0 & 0 & u-\lambda_j \end{pmatrix}\begin{pmatrix} W_{1j} \\ W_{2j} \\ W_{3j} \end{pmatrix} = \bar{0}$$

pg 399 leveraken

$$\Rightarrow \quad (u-\lambda_j)w_{1j} + \frac{w_{2j}}{\rho} = 0$$

$$c^2 \rho w_{1j} + (u-\lambda_j) w_{2j} = 0$$

$$(u-\lambda_j) w_{3j} = 0.$$

Yes W is just evector of A.

$$\begin{pmatrix} u \\ p \\ s \end{pmatrix}$$

$$\frac{w_{21}}{w_{11}} = \rho c \quad \text{thus 1st evector is} \quad \bar{f} = \Big($$

$$w_{21} = \rho c w_{11} \qquad \begin{pmatrix} 1 \\ \rho c \\ 0 \end{pmatrix} = \bar{b}_1 + \rho c \bar{b}_2 .$$

$$\frac{w_{22}}{w_{12}} = -\rho c \qquad V = \begin{pmatrix} 1/2 & 1/2\rho c & 0 \\ 1/2 & -1/2\rho c & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sum_{k=1}^{n} v_{jk}\left(\frac{\partial u_k}{\partial t} + \lambda_j \frac{\partial u_k}{\partial x}\right) = \sum_{k=1}^{n} v_{jk} f_k \qquad j = 1,2,3.$$

for $j=1$:

$$\frac{1}{2}\left(u_t + (u+c)u_x\right) + \frac{1}{2\rho c}\left(p_t + (u+c)p_x\right) = \frac{1}{2\rho c} q$$

$j=2$

$$\frac{1}{2}\left(u_t + (u-c)u_x\right) - \frac{1}{2\rho c}\left(p_t + (u-c)p_x\right) = -\frac{1}{2\rho c} q$$

pg 405 Kevorkian

1

$$\cancel{\bar{w}_1 = -A_{12}\bar{b}_1 + (A_{11}-\lambda)\bar{b}_2}$$

$$\cancel{\bar{w}_2 = -A_{12}\bar{b}_1 +}$$

$$\bar{U}_\xi = U_\xi \bar{b}_1 + V_\xi \bar{b}_2$$

$$\bar{U}_\eta = U_\eta \bar{b}_1 + V_\eta \bar{b}_2$$

$$\bar{U}_\xi = \left(U_\xi V_{11}\bar{w}_1 + U_\xi V_{21}\bar{w}_2\right)$$

$$+ \left(V_\xi V_{12}\bar{w}_1 + V_\xi V_{22}\bar{w}_2\right)$$

$$= \left(U_\xi V_{11} + V_\xi V_{12}\right)\bar{w}_1 + \left(U_\xi V_{21} + V_\xi V_{22}\right)\bar{w}_2.$$

$$\bar{U}_\eta = \left(U_\eta V_{11} + V_\eta V_{12}\right)\bar{w}_1 + \left(U_\eta V_{21} + V_\eta V_{22}\right)\bar{w}_2$$

$$\bar{f} = \left(f_1 V_{11} + f_2 V_{12}\right)\bar{w}_1 + \left(f_1 V_{21} + f_2 V_{22}\right)\bar{w}_2.$$

$$\bar{w}_1 \circ \left(\lambda_1 \bar{U}_x + \bar{U}_t\right) = \bar{w}_1 \circ \bar{f}$$

Now $\bar{U}_x = \phi_x \bar{U}_\xi + f_x \bar{U}_\eta = \cancel{\phi_x(\cdots)\bar{w}_1 + f_x(\cdots)}\bar{w}_2$

$+ \quad \cancel{\bar{U}_t = -\lambda_1 \phi_x \bar{U}_\xi - \lambda_2 f_x U_\eta = -\lambda_1\phi_x(\cdots}$

$$= \left(\phi_x U_\xi V_{11} + \phi_x V_\xi V_{12} + f_x U_\eta V_{11} + f_x V_\eta V_{12}\right)\bar{w}_1$$

$$+ \left(\phi_x U_\xi V_{21} + \phi_x V_\xi V_{22} + f_x U_\eta V_{21} + f_x V_\eta V_{22}\right)\bar{w}_2$$

$$+ \quad \bar{U}_t = \left[-\lambda_1 \phi_x \left(U_\xi V_{11} + V_\xi V_{12} \right) - \lambda_2 f_x \left(U_\eta V_{11} + V_\eta V_{12} \right) \right] \bar{w}_1$$

$-\lambda_1 \phi_x \bar{U}_\xi - \lambda_2 f_x \bar{U}_\eta$

$$+ \left[-\lambda_2 \phi_x \left(U_\xi V_{21} + V_\xi V_{22} \right) - \lambda_2 f_x \left(U_\eta V_{21} + V_\eta V_{22} \right) \right] \bar{w}_2$$

1st eq: 74a $\quad \bar{w}_1 \circ (\lambda_1 \bar{U}_x + \bar{U}_t) = \bar{w}_1 \circ \bar{f}$

$$\Rightarrow \lambda_1 \left[\cancel{\phi_x U_\xi} V_{11} + \cancel{\phi_x V_\xi} V_{12} + f_x U_\eta \underline{V_{11}} + f_x V_\eta \underline{\underline{V_{12}}} \right]$$

$$+ \; -\lambda_1 \cancel{\phi_x U_\xi} V_{11} - \lambda_1 \cancel{\phi_x V_\xi} V_{12} - \lambda_2 f_x U_\eta \underline{V_{11}} - \lambda_2 f_x V_\eta \underline{\underline{V_{12}}}$$

$$= f_1 V_{11} + f_2 V_{12}$$

$$\Rightarrow \cancel{\lambda_1 f_x \left(U_\eta V_{11} + V_\eta V_{12} \right)} =$$

$$f_x \left[(\lambda_1 - \lambda_2) U_\eta V_{11} + (\lambda_1 - \lambda_2) V_\eta V_{12} \right] = \cdots$$

$$f_x (\lambda_1 - \lambda_2) \left(U_\eta V_{11} + V_\eta V_{12} \right) = f_1 V_{11} + f_2 V_{12} \qquad (1)$$

eq 2: 74b $\quad \bar{w}_2 \circ \longrightarrow$

$$\Rightarrow \lambda_2 \left[\phi_x U_\xi V_{21} + \phi_x V_\xi V_{22} + f_x \cancel{U_\eta} V_{21} + \cancel{f_x V_\eta} V_{22} \right]$$

$$- \lambda_1 \phi_x \left(U_\xi V_{21} + V_\xi V_{22} \right) - \lambda_2 f_x \left(\cancel{U_\eta V_{21}} + \cancel{V_\eta V_{22}} \right) =$$

$$f_1 V_{21} + f_2 V_{22}.$$

$$\Rightarrow \quad \phi_x(\lambda_2 - \lambda_1)(U_\xi V_{21} + V_\xi V_{22}) = f_1 V_{21} + f_2 V_{22}. \qquad (2)$$

~~How get 79a, 79b?~~

$$\frac{X_\xi}{T_\xi} = \lambda_1 \qquad \frac{X_\eta}{T_\eta} = \lambda_2 \qquad U_x = J\tilde{T}_v$$

$$\| \qquad\qquad X_\eta = \lambda_1 T_\eta \qquad \begin{pmatrix} X_\eta = \lambda_1 T_\eta \\ X_\xi = \lambda_2 T_\xi \end{pmatrix}$$

$$-\frac{\phi_t}{\phi_x} \qquad\qquad X_\xi = \lambda_2 T_\xi$$

~~$\phi_x = T_\xi$~~

$$\frac{T_\eta}{X_\xi T_\eta - T_\xi X_\eta} = \frac{\cancel{T_\eta}}{(\lambda_2 - \lambda_1)T_\xi \cancel{T_\eta}} = \frac{1}{T_\xi(\lambda_2 - \lambda_1)} \overset{??}{=} \phi_x$$

$\therefore$ eq (1) [If true for ϕ_x then $t_x = \dfrac{-T_\xi}{|J|} = \dfrac{-1}{T_\eta(\lambda_2 - \lambda_1)}$

$$\Rightarrow \quad \frac{-(\lambda_1 - \lambda_2)}{T_\eta(\lambda_2 - \lambda_1)}(U_\eta V_{11} + V_\eta V_{12}) = f_1 V_{11} + f_2 V_{12}$$

$$U_\eta V_{11} + V_\eta V_{12} = T_\eta(f_1 V_{11} + f_2 V_{12}). \quad *$$

eq (2) gives 79b w/ $\phi_x = \dfrac{1}{T_\xi(\lambda_2 - \lambda_1)}$.

pg 422 kevorkian

$$Vh_1 = u_1 h_1 \quad \Rightarrow \quad h_1 = 0 \text{ or } \underline{u_1 = V}.$$

$$Vu_1 h_1 = u_1^2 h_1 + \frac{h_1^2}{2}$$

$$V^2 h_1 =$$

$$h_1\left(V^2 - u_1^2 - \frac{h_1}{2}\right) = 0.$$

$$\Rightarrow h_1 = 0 \text{ or } \quad h_1 = 2(V^2 - u_1^2)$$

$$\Rightarrow \quad h_1 \equiv 0.$$

Pg 423 Kevorkian

$$h = \begin{cases} 1 & x \le -t \\ \frac{1}{9}\left(2 - \frac{x}{t}\right)^2 & -t \le x \le 2t \\ 0 & 2t \le x \end{cases}$$

$$\sqrt{h} = \begin{cases} 1 & '' \\ \frac{1}{3}\left(2 - \frac{x}{t}\right) & '' \\ 0 & '' \end{cases}$$

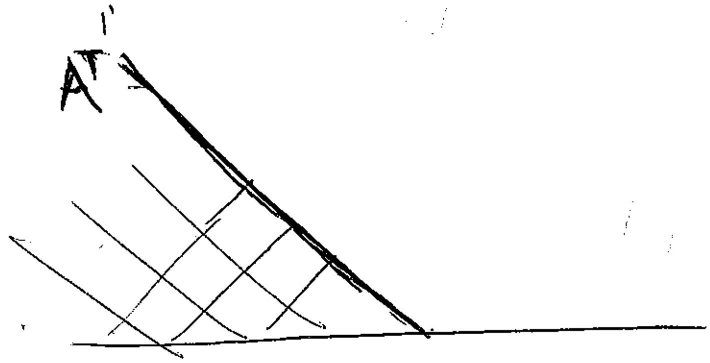
Then

$$u = 2(1 - \sqrt{h}) = \begin{cases} 0 & x \le -t \\ 2\left(1 - \frac{1}{3}\left(2 - \frac{x}{t}\right)\right) & -t \le x \le 2t \\ 2 & 2t \le x \end{cases}$$ Not true

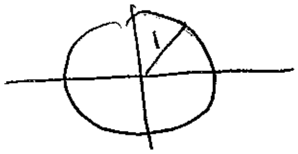
$$= \begin{cases} 0 & x \le -t \\ \cancel{\quad\quad} = \frac{2}{3} + \frac{x}{3t} - \frac{1}{3}\left(2 + \frac{x}{t}\right) \\ \frac{2}{3}\left(3 - 2 + \frac{x}{t}\right) = \frac{2}{3}\left(1 + \frac{x}{t}\right) & -t \le x \le 2t \end{cases}$$

$A =$

$$\begin{pmatrix} h \\ u \end{pmatrix}_t + \begin{pmatrix} u & h \\ 1 & u \end{pmatrix}\begin{pmatrix} h \\ u \end{pmatrix}_x = \vec{0}$$



(1)



$$u = \frac{1}{(1-(x_1^2+x_2^2)+\epsilon)}$$

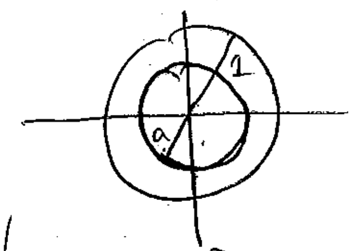
$v = 1$

$u = O(1)$ $\exists\ k(x_1, x_2)$?

$|u| \le k(x_1,x_2)\cdot 1 \quad \forall\ \epsilon\in(0,\underline{\epsilon_0(x_1,x_2)})$

$$\frac{1}{|1-(x_1^2+x_2^2)+\epsilon|} \le \frac{1}{1-(x_1^2+x_2^2)}\cdot 1 \quad \forall\ \epsilon\in(0,1).$$

if $D: x_1^2+x_2^2 \le a^2 < 1$



Then $$\frac{1}{|1-(x_1^2+x_2^2)+\epsilon|} \le \frac{1}{(1-a^2)}\cdot 1 \quad \forall\ x_1,x_2\in D$$
$\forall\ \epsilon\in(0,1)$

(2)

$u = x_1\epsilon^{\alpha}$, $v = x_2\epsilon^{\beta}$ $\underline{\alpha \ge \beta}$

$u \overset{?}{=} O(v)$ $|x_1\epsilon^{\alpha}| \overset{?}{\le} k(x_1,x_2)|x_2\epsilon^{\beta}| \quad \forall\ \epsilon\in(0,\epsilon(x_1,x_2))$

$$\left| \frac{x_1}{x_2} \epsilon^{\alpha - \beta} \right| \le k(x_1, x_2) \qquad \underline{\alpha - \beta} \ge 0$$

pick $k(x_1, x_2) = \frac{x_1}{x_2}$ Then

$$|x_1 \epsilon^{\alpha}| \le \frac{x_1}{x_2} |x_2 \epsilon^{\beta}| \qquad \forall\ \epsilon \in (0,1)$$

$x_2 \ne 0$. x_1 finite

$\hat{D} : 0 < x_1 \le \bar{X}_1 < \infty \qquad 0 < \bar{X}_2 < x_2 < \infty$

Then pick $k(x_1, x_2) = \frac{\bar{X}_1}{\bar{X}_2}$

Then $$\left(\frac{x_1}{\bar{X}_1}\right) \cdot \frac{\bar{X}_2}{x_2} \cdot \epsilon^{\alpha - \beta} \le 1 \qquad \forall\ \epsilon \in (0,1) \checkmark$$

$w = O(r) \Rightarrow v = O(r)$ ✓

1st example $v = O(1)$

is $1 = O(v)$?

$$1 \leq \frac{k(x_1, x_2)}{\left(1-(x_1^2+x_2^2)+\epsilon\right)} \quad \forall\ \epsilon \in (0, \epsilon_0(x_1 x_2))$$

pick $k(x_1,x_2) = \left[1-(x_1^2+x_2^2)+1\right] \quad \forall\ \epsilon \in (0,1)$.

Then $1 \overset{?}{\leq} \dfrac{1-(\quad)+1}{1-(\quad)+\epsilon}$

Note uniform this direction
pick $k = 1-0+1$.

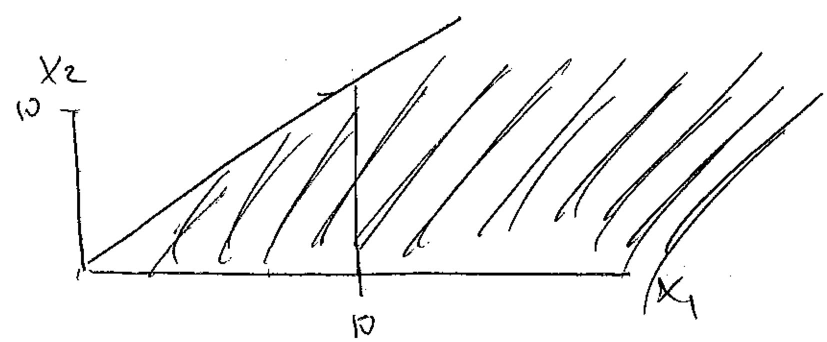
$\Rightarrow \cancel{1}-(\cancel{\ }) + \epsilon \leq \cancel{1}-(\cancel{\ })+1$
$\epsilon \leq 1$ ✓ yes

$\cancel{1}-(\ )+\epsilon \leq \cancel{1}$
$\epsilon - (\ ) \leq 1$

in 2nd Example is $v \overset{?}{=} O(u)$

① $\underline{x_1\epsilon^{\alpha} = o(x_2\epsilon^{\beta})}$ $\quad \alpha -$

$\Rightarrow \forall\, \delta \in \mathbb{R} \quad \left(\dfrac{x_1}{x_2}\right)\epsilon^{\alpha-\beta} \leq \delta \qquad 0 \leq \epsilon \leq \left(\delta \dfrac{x_2}{x_1}\right)^{1/(\alpha-\beta)}$

② 

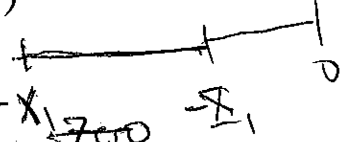
$x_1 - x_2 > 0 \quad \therefore \quad x_2 - x_1 < 0$

$e^{(x_2-x_1)/\epsilon} = o(\epsilon^{\beta})$

$e^{(x_2-x_1)/\epsilon} \overset{?}{\le} \delta\epsilon^{\beta} \quad \forall\, \epsilon \in (0, \epsilon_0(x_i,\delta))$

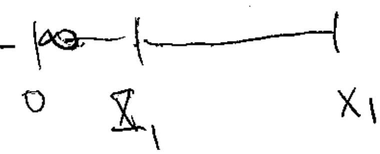
$e^{(x_2-x_1)/\epsilon} = o(\epsilon^{\beta}) \qquad \lim_{\epsilon\to 0} \epsilon^{-\beta} e^{(x_2-x_1)/\epsilon} = 0$

not uniformly valid since $\epsilon = \epsilon_0(\underline{\underline{x_1}}, \underline{\underline{x_2}}, \delta)$



But if ~~x_2 is made as small up to $-x_1$ zero~~

~~& x_1 is made as large up to ∞~~



$\Leftrightarrow$: x_2 is made larger to ∞ or x_1

& x_1 is made smaller to 0

$0 < \bar{X}_1 \le x_1 < \infty$

& $\quad 0 < x_2 \le x_1 - \bar{X}_1 \;\to\; -x_1 < x_2 - x_1 \le -\bar{X}_1$

Then $e^{-(\bar{X}_1)/\epsilon} \le e^{(x_2-x_1)/\epsilon} \le \delta\epsilon^{\beta}$

$\{\epsilon^{n-1}\}$

$\epsilon^n \stackrel{?}{=} o(\epsilon^{n-1})$ $\quad \lim_{\epsilon \to 0} \frac{\epsilon^n}{\epsilon^{n-1}} = \lim_{\epsilon \to 0} \epsilon = 0$ ✓

$\log\epsilon,\ 1,\ \epsilon\log\epsilon,\ \epsilon,\ \epsilon^2\log^2\epsilon,\ \epsilon^2,\ \ldots$ $\quad |1| \le \delta|\log\epsilon|$

$1 = o(\log\epsilon) \quad \epsilon \to 0 \qquad \frac{1}{|\log\epsilon|} \longrightarrow 0$ ✓

$\epsilon\log\epsilon = o(1) \quad \epsilon \to 0 \qquad |\epsilon\log\epsilon| \longrightarrow 0$ ✓

$\epsilon = o(\epsilon\log\epsilon) \quad \epsilon \to 0 \qquad |\cancel{\epsilon}| \le \delta|\cancel{\epsilon}\log\epsilon|$

$\epsilon^2\log^2\epsilon = o(\epsilon) \quad \epsilon \to 0 \qquad 1 \le \delta|\log\epsilon|$

$\epsilon^2 = o(\epsilon^2\log^2\epsilon) \qquad |\epsilon^2\log^2\epsilon| \xrightarrow{\epsilon \to 0} 0$ ✓

$$U(x_i;\epsilon) - \sum_{n=1}^{M} \phi_n(\epsilon)u_n(x_i) = o(\phi_M) \quad \text{as } \epsilon \to 0$$

$o(\phi_M) = \underline{\phi_{M+1}}$ By def of ~~$O(\phi_{M+1})$~~

asymptotic sequence

+ $\phi_{M+1} = O(\phi_{M+1})$ w/ $k = \underline{1}$.

$$U_1(x_i) = \lim_{\epsilon \to 0} \frac{U(x_i;\epsilon)}{\phi_1}$$

$$U(x_i;\epsilon) - \phi_1(\epsilon)\, U_n(x_i) = o(\phi_1)$$

$$\Rightarrow \cancel{\lim_{\epsilon\to 0} \frac{U(x_i;\epsilon) - \phi_1(\epsilon) U_n(x_i)}{\phi_1(\epsilon)}} \Rightarrow \lim_{\epsilon \to 0} \left(\frac{U(x_i;\epsilon) - \phi_1(\epsilon)\, U_n(x_i)}{\phi_1(\epsilon)} \right) = 0$$

$$\Rightarrow \lim_{\epsilon \to 0} \frac{U(x_i;\epsilon)}{\phi_1(\epsilon)} - U_n(x_i) = 0$$

$$\Rightarrow U_1(x_i) = \lim_{\epsilon \to 0} \frac{U(x_i;\epsilon)}{\phi_1(\epsilon)}$$