$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}(\lambda\Delta + 2\mu\epsilon_{xx}) + \frac{\partial}{\partial y}(\mu\epsilon_{xy}) + \frac{\partial}{\partial z}(\mu\epsilon_{xz})$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} \quad ; \quad \epsilon_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad ; \quad \epsilon_{xz} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\rho \frac{\partial^2 u}{\partial t^2} = \lambda \frac{\partial \Delta}{\partial x} + 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y \partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 w}{\partial z \partial x} + \mu \frac{\partial^2 u}{\partial z^2}$$

Now $\Delta \equiv \epsilon_{xx} + \epsilon_{yy} + \epsilon_{zz} \equiv \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$ so the above becomes

$$\rho \frac{\partial^2 u}{\partial t^2} = \lambda \frac{\partial \Delta}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x} + \mu \frac{\partial v}{\partial y} + \mu \frac{\partial w}{\partial z}\right) + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 u}{\partial z^2}$$

$$= \lambda \frac{\partial \Delta}{\partial x} + \frac{\partial}{\partial x}(\mu\Delta) + \mu \nabla^2 u$$

$$= (\lambda + \mu)\frac{\partial}{\partial x}\Delta + \mu \nabla^2 u \qquad \text{eq 2.8 } \checkmark.$$

$$r^2 = x^2 + y^2 + z^2 \qquad 2r\frac{\partial r}{\partial x} = 2x \rightarrow \frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\frac{\partial}{\partial x} = \frac{\partial r}{\partial x}\frac{\partial}{\partial r} = \frac{x}{r}\frac{\partial}{\partial r}$$

$$\frac{\partial^2}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{x}{r}\frac{\partial}{\partial r}\right) = \frac{x}{r}\frac{\partial}{\partial r}\left(\frac{x}{r}\frac{\partial}{\partial r}\right)$$

$x = x(r)$ + not a constant!!

$$= \frac{x^2}{r}\left[\cancel{-\frac{1}{r^2}\frac{\partial}{\partial r}} + \frac{1}{r}\frac{\partial^2}{\partial r^2}\right]$$

$$= \cancel{\frac{x^2}{r^2}\frac{\partial^2}{\partial r^2} - \frac{x^2}{r^3}\frac{\partial}{\partial r}}$$

$$= \frac{x}{r}\frac{\partial}{\partial r}\left(\frac{\sqrt{r^2-y^2-z^2}}{r}\frac{\partial}{\partial r}\right) \qquad x = \sqrt{r^2-y^2-z^2}$$

$$= \frac{x}{r}\left[-\frac{1}{r^2}\cdot x\frac{\partial}{\partial r} + \frac{x}{r}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(\sqrt{r^2-y^2-z^2}\right)\frac{\partial}{\partial r}\right]$$

$$= \frac{x}{r}\left[-\frac{x}{r^2}\frac{\partial}{\partial r} + \frac{x}{r}\frac{\partial^2}{\partial r^2} + \frac{1}{\cancel{r}}\frac{1}{\cancel{2}}\frac{(\cancel{2}r)}{\sqrt{r^2-y^2-z^2}}\frac{\partial}{\partial r}\right]$$

$$= \frac{x^2}{r^2}\frac{\partial^2}{\partial r^2} - \frac{x^2}{r^3}\frac{\partial}{\partial r} + \frac{x}{r\cdot x}\frac{\partial}{\partial r}$$

$$= \frac{x^2}{r^2}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\left[1 - \frac{x^2}{r^2}\right]\frac{\partial}{\partial r}$$

$$= \frac{x^2}{r^2}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\left[\frac{r^2-x^2}{r^2}\right]\frac{\partial}{\partial r} \quad ✓$$

$$\frac{\partial^2 \alpha}{\partial t^2} = c^2 \nabla^2 \alpha$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\cancel{= \frac{x^2}{r^2}\frac{\partial^2}{\partial x^2} + \frac{1}{r}\left(\frac{r^2 - x^2}{r^2}\right)\frac{\partial}{\partial x} + \frac{y^2}{r^2}\frac{\partial^2}{\partial y^2} + \frac{1}{r}\left(\frac{r^2-y^2}{r^2}\right)\frac{\partial}{\partial y} + \frac{z^2}{r^2}\frac{\partial^2}{\partial z^2} + \frac{1}{r}\left(\frac{r^2-z^2}{r^2}\right)\frac{\partial}{\partial z}}$$

$$\cancel{= \frac{1}{r^2}\left[x^2\frac{\partial^2}{\partial x^2} + y^2\frac{\partial^2}{\partial y^2} + z^2\frac{\partial^2}{\partial z^2}\right.}$$

"r" derivatives not x & y & z !!

$$\nabla^2 = \frac{x^2}{r^2}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\left(\frac{r^2 - x^2}{r^2}\right)\frac{\partial}{\partial r} + \frac{y^2}{r^2}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\left(\frac{r^2-y^2}{r^2}\right)\frac{\partial}{\partial r} + \frac{z^2}{r^2}\frac{\partial^2}{\partial r^2} + \frac{1}{r}\left(\frac{r^2-z^2}{r^2}\right)\frac{\partial}{\partial r}$$

$$= \frac{1}{r^2}(x^2+y^2+z^2)\frac{\partial^2}{\partial r^2} + \frac{1}{r^3}\left(3r^2 - (x^2+y^2+z^2)\right)\frac{\partial}{\partial r}$$

$$= \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} = \frac{1}{r^2}\left(r^2\frac{\partial^2}{\partial r^2} + 2r\frac{\partial}{\partial r} \right)$$

$$= \frac{1}{r^2}\left[r^2\frac{\partial^2}{\partial r^2} + \frac{\partial(r^2)}{\partial r}\cdot\frac{\partial}{\partial r}\right]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\right]$$

$$\frac{\partial^2 y}{\partial r^2} + \frac{2}{r}\frac{\partial y}{\partial r}$$

$$\left\{ \frac{\partial^2}{\partial r^2}(ry) = 0 + 2 \cdot 1 \cdot \frac{\partial y}{\partial r} + r\frac{\partial^2 y}{\partial r^2} \right\}$$

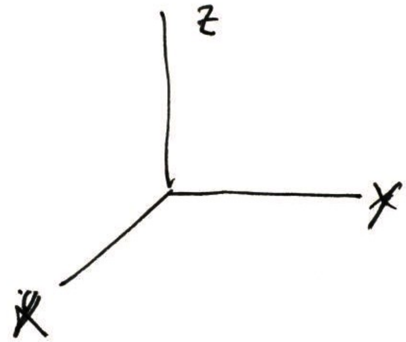
$$\therefore \quad \frac{\partial^2 y}{\partial r^2} + \frac{2}{r}\frac{\partial y}{\partial r} = \frac{1}{r}\left[r\frac{\partial^2 y}{\partial r^2} + 2\frac{\partial y}{\partial r} \right]$$

$$= \frac{1}{r}\left[\frac{\partial^2}{\partial r^2}(ry) - 2\frac{\partial y}{\partial r} + 2\frac{\partial y}{\partial r} \right] = \frac{1}{r}\frac{\partial^2}{\partial r^2}(ry) .$$

$$u = \frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial z} \qquad w = \frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}$$

$$\Delta = \frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}^{\,0} + \frac{\partial w}{\partial z} = \frac{\partial^2\phi}{\partial x^2} + \cancel{\frac{\partial^2\psi}{\partial x\partial z}} + \frac{\partial^2\phi}{\partial z^2} - \cancel{\frac{\partial^2\psi}{\partial x\partial z}}$$

$$= \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2}$$



2.8, 2.9, 2.10

$$\rho\frac{\partial^2}{\partial t^2}(u,v,w) = (\lambda+\mu)\left(\frac{\partial\Delta}{\partial x}, \frac{\partial\Delta}{\partial y}, \frac{\partial\Delta}{\partial z}\right) + \mu\nabla^2(u,v,w)$$

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left(\frac{\partial^2\phi}{\partial t^2}\right) + \frac{\partial}{\partial z}\left(\frac{\partial^2\psi}{\partial t^2}\right) \qquad \text{1st eq}$$

$$\to \quad \rho\frac{\partial}{\partial x}\left(\frac{\partial^2\phi}{\partial t^2}\right) + \rho\frac{\partial}{\partial z}\left(\frac{\partial^2\psi}{\partial t^2}\right) = (\lambda+\mu)\frac{\partial}{\partial x}(\nabla^2\phi) + \mu\nabla^2\left(\frac{\partial\phi}{\partial x} + \frac{\partial\psi}{\partial z}\right)$$

$$= (\lambda+2\mu)\frac{\partial}{\partial x}(\nabla^2\phi) + \mu\frac{\partial}{\partial z}(\nabla^2\psi)$$

$$\& \quad \rho\frac{\partial^2 w}{\partial t^2} = \rho\frac{\partial}{\partial z}\left(\frac{\partial^2\phi}{\partial t^2}\right) - \rho\frac{\partial}{\partial x}\left(\frac{\partial^2\psi}{\partial t^2}\right)$$

$$\underset{\text{set}}{=} (\lambda+\mu)\frac{\partial}{\partial z}(\nabla^2\phi) + \mu\nabla^2 w$$

$$\|$$

$$\mu\frac{\partial}{\partial z}(\nabla^2\phi) - \mu\frac{\partial}{\partial x}(\nabla^2\psi)$$

$$= \cancel{\text{f}} \quad (\lambda+2\mu)\frac{\partial}{\partial z}(\nabla^2\phi) - \mu\frac{\partial}{\partial x}(\nabla^2\psi)$$

~~Add eq~~

$$\cancel{2\rho\frac{\partial}{\partial z}\left(\frac{\partial^2\phi}{\partial t^2}\right)} = \cancel{2(\lambda+2\mu)\frac{\partial}{\partial x}(\nabla^2\phi)}$$

will be sat. if

$$\frac{\partial^2\phi}{\partial t^2} = \frac{(\lambda+2\mu)}{\rho}\nabla^2\phi \quad ✓$$

$$+ \quad \frac{\partial^2\psi}{\partial t^2} = \frac{\mu}{\rho}\nabla^2\psi \quad ✓$$

$c = \cdot f(k^{-1})$

$p = 2\pi f = \omega$ ✓ $\quad \omega t - kx$

$c = \underbrace{\left(\frac{k}{\omega}\right)^{-1}}\!=\left(\frac{f}{p}\right)^{-1}$ ✓ $\quad [f] = 1/L \quad [p] = 1/T \qquad c = \frac{p}{f}$

$$\phi = F(z)\exp(i(pt - fx)$$

$$\psi = G(z)\exp(i(pt - fx))$$

$$\left\{ \begin{array}{l} p = 2\pi\omega \\ f = \frac{2\pi}{\lambda} \\ \underbrace{\lambda = \frac{2\pi}{f}}\ ✓ \quad \omega = \frac{p}{2\pi}\ ✓ \end{array} \right.$$

— — —

$$F(z)(-p^2)e^{i(pt-fx)} = \left(\frac{\lambda+2\mu}{\rho}\right)\nabla^2\left(F(z)e^{i(pt-fx)}\right)$$

$$= \left(\frac{\lambda+2\mu}{\rho}\right)\left(\frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial x^2}\right)\left(F(z)e^{i(pt-fx)}\right)$$

$$F(z)\frac{(-p^2)}{q^2} = F'' + F(z)(i(-7))^2$$

$$= F'' - 7^2 F$$

$$F'' - \left(7^2 - \frac{p^2}{q^2}\right)F = 0$$

$$F'' - (7^2 - h^2)F = 0$$

$$F(z) = A\exp(-qz) + A'\exp(qz)$$

$$q = 7^2 - h^2$$

$$\sigma_{zz} = \lambda\Delta + 2\mu\frac{\partial w}{\partial z}$$

$$= \lambda\nabla^2\phi + 2\mu\frac{\partial}{\partial z}\left[\frac{\partial\phi}{\partial z} - \frac{\partial\psi}{\partial x}\right]$$

$$= \lambda\nabla^2\phi + 2\mu\frac{\partial^2\phi}{\partial z^2} - 2\mu\frac{\partial^2\psi}{\partial z\partial x}$$

$$= \lambda\frac{\partial^2\phi}{\partial x^2} + (\lambda+2\mu)\frac{\partial^2\phi}{\partial z^2} - 2\mu\frac{\partial^2\psi}{\partial z\partial x}$$

$$\sigma_{zz}(z=0) = 0$$

$$= \cancel{\lambda\left[(-if)^2\right] + (\lambda+2\mu)\left[(-q)^2\right] - 2\mu\left[(-q)(-if)\right] = 0}$$

$$\Rightarrow \cancel{-\lambda f^2 + q^2(\lambda+2\mu) - 2i\mu qf = 0}$$

$$= \lambda A(-if)^2 + (\lambda+2\mu)A(-q)^2 - 2\mu B(-s)(-fi) = 0$$

at $z = 0$

$$A\left[(\lambda+2\mu)q^2 - \lambda f^2\right] - 2B\mu isf = 0 \qquad \text{eq } 2.32$$

$$\sigma_{zx} = \mu\epsilon_{xz} = \mu\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]$$

$$= \mu\left[\frac{\partial^2\phi}{\partial x\partial z} + \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\phi}{\partial x\partial z} - \frac{\partial^2\psi}{\partial x^2}\right]$$

$$\sigma_{xz} = \mu\left[2\frac{\partial^2\phi}{\partial x\partial z} + -\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial z^2}\right] \quad \checkmark$$

$$= \mu\left[2A(-i\xi)(-q) - B(-\xi i)^2 + B(-q)^2\right]\Big|_{z=0} = 0$$

$$\Rightarrow 2A(iq\xi) + B\xi^2 + Bq^2 = 0$$

$$2iq\xi A + (\xi^2+q^2)B = 0 \qquad \text{eq } 2.33$$

eq 2.32 $\Rightarrow \dfrac{A}{B} = \dfrac{2\mu is\xi}{((\lambda+2\mu)q^2 - \lambda\xi^2)}$ + into eq 2.33

$$\cancel{(\xi^2+q^2)}$$

$$\frac{A}{B} = \frac{-(\xi^2+q^2)}{2iq\xi} = \frac{2\mu is\xi}{((\lambda+2\mu)q^2 - \lambda\xi^2)}$$

$$\Rightarrow -4q\mu s\xi^2 + (\xi^2+q^2)((\lambda+2\mu)q^2 - \lambda\xi^2) = 0$$

$$4\mu qs\xi^2 = ((\lambda+2\mu)q^2 - \lambda\xi^2)(s^2+\xi^2) \qquad \text{eq } 2.34 \checkmark$$

Squer both sides :

$$16\mu^2q^2s^2\xi^4 = ((\lambda+2\mu)q^2 - \lambda\xi^2)^2(s^2+\xi^2)^2$$

w/ $q^2 = \xi^2 - h^2$ + $s^2 = \xi^2 - k^2$

$$16\mu^2(\gamma^2-h^2)(\gamma^2-k^2)\gamma^4 = \left((\lambda+2\mu)(\gamma^2-h^2)-\lambda\gamma^2\right)^2(\gamma^2-k^2+\gamma^2)^2$$

$$\Rightarrow 16\mu^2(\gamma^2-h^2)(\gamma^2-k^2)\gamma^4 = \left(-(\lambda+2\mu)h^2+2\mu\gamma^2\right)^2(2\gamma^2-k^2)^2$$

$\div\ \mu^2\gamma^8$

$$16\left(1-\frac{h^2}{\gamma^2}\right)\left(1-\frac{k^2}{\gamma^2}\right) = \left(2-\frac{k^2}{\gamma^2}\right)^2\left(2-\frac{(\lambda+2\mu)}{\mu}\frac{h^2}{\gamma^2}\right)^2 \quad \text{eq } 2.35$$

$$h = \frac{p}{c_1} \qquad k = \frac{p}{c_2}$$

$$= \frac{p}{\left[\left(\frac{\lambda+2\mu}{\rho}\right)^{1/2}\right]} \ ; \quad k = \frac{p}{\left[\frac{\mu}{\rho}\right]^{1/2}}$$

$$\frac{h^2}{k^2} = \frac{\frac{p^2}{c_1^2}}{\frac{p^2}{c_2^2}} = \frac{c_2^2}{c_1^2} = \frac{\left(\frac{\mu}{\rho}\right)}{\left(\frac{\lambda+2\mu}{\rho}\right)} = \frac{\mu}{\lambda+2\mu}$$

$$\nu = \frac{\lambda}{2(\lambda+\mu)} = \frac{1}{2(1+\frac{\mu}{\lambda})} \Rightarrow 2\nu\left(1+\frac{\mu}{\lambda}\right) = 1$$

$$1+\frac{\mu}{\lambda} = \frac{1}{2\nu}$$

$$\frac{\mu}{\lambda} = \frac{1}{2\nu} - 1 = \frac{1-2\nu}{2\nu}$$

Sim

$$\frac{h^2}{k^2} = \frac{1}{\frac{\lambda}{\mu}+2} = \frac{1}{\frac{2\nu}{1-2\nu}+2}$$

$$\frac{h^2}{k^2} = \left(\frac{1}{2}\right) \frac{1}{\frac{\nu}{1-2\nu} + \frac{1-2\nu}{1-2\nu}} = \left(\frac{1}{2}\right)\left(\frac{1-2\nu}{1-\nu}\right) \checkmark$$

$\Rightarrow \quad h = \alpha_1 k$

eq 2.35 $\Rightarrow$

$$16\left(1 - \frac{\alpha_1^2 k^2}{f^2}\right)\left(1 - \frac{k^2}{f^2}\right) = \left(2 - \alpha_1^{-2}\frac{\alpha_1^2 k^2}{f^2}\right)^2 \left(2 - k^2 f^{-2}\right)^2$$

$$= \left(2 - \frac{k^2}{f^2}\right)^4 \quad \text{eq 2.36}$$

$\lambda_1 \equiv \frac{k}{f}$ gives

$$16\left(1 - \alpha_1^2 \lambda_1^2\right)\left(1 - \lambda_1^2\right) = \left(2 - \lambda_1^2\right)^4$$

$$\Rightarrow 16\left(1 - \alpha_1^2 \lambda_1^2 - \lambda_1^2 + \alpha_1^2 \lambda_1^4\right) = \Big(16 + \binom{4}{1} 2^3 (-\lambda_1^2) + \binom{4}{2} 2^2 (-\lambda_1^2)^2$$
$$+ \binom{4}{3} 2 (-\lambda_1^2)^3 + \lambda_1^8\Big)$$

$$= 16 - 4\cdot 8 \lambda_1^2 + 6 \cdot 4 \lambda_1^4 - 4 \cdot 2 \lambda_1^6 + \lambda_1^8$$

$$\left\{ \binom{4}{2} = \frac{4\cdot 3}{2} = 6 \; ; \; \binom{4}{3} = \frac{4}{1} = 4 \right\}$$

$\Rightarrow \cancel{16-\alpha_1^2} \quad 16 \cancel{< 16(1+\lambda_1}$

$16 < 16\alpha_1^2$

$$\cancel{16} - 16(\alpha_1^2+1)\lambda_1^2 + 16\alpha_1^2\lambda_1^4 = \cancel{16} - 32\lambda_1^2 + 24\lambda_1^4 - 8\lambda_1^6 + \lambda_1^8$$

$\div\ \lambda_1^2$

$$-16(1+\alpha_1^2) + 16\alpha_1^2\lambda_1^2 = -32 + 24\lambda_1^2 - 8\lambda_1^4 + \lambda_1^6$$

$$\Rightarrow \lambda_1^6 - 8\lambda_1^4 + (24 - 16\alpha_1^2)\lambda_1^2 + -32 + 16(1+\alpha_1^2) = 0$$

$\cancel{16?}$

$-32 + 16 + 16\alpha_1^2$

$16\alpha_1^2 - 16$ ✓