Some Notes on the Book:
Are You Smart Enough to Work at Google?
by William Poundstone

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Introduction

As I was reading this book I worked a few of the problems. The answers presented are sometimes somewhat different than the ones presented in the book. In many of my answers I phrase the statement in algebra and proceed two work the problem from there. This is often results in a different solution (from the one presented in the book) to the given problem.

More, Less, or the Same Time to Fly with a Wind

Consider the case where a plane is flying against the wind one way and is flying with the wind the other way. Then let L be the distance we want to travel (one way), let v_p be the velocity of the plane and v_w be the velocity of the wind. Then the travel time when flying into the wind is

$$T_1 = \frac{L}{v_p - v_w} \,.$$

Here we must have $v_w < v_p$ or otherwise the plane would not make progress against the wind. The travel time when flying with the wind at our back is given by

$$T_2 = \frac{L}{v_p + v_w} \,.$$

The total travel time T is then

$$T = T_1 + T_2 = \frac{L}{v_p - v_w} + \frac{L}{v_p + v_w} = \frac{2Lv_p}{v_p^2 - v_w^2} \,.$$

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Notice is there is no wind then we have $v_w = 0$ and get $T = \frac{2L}{v_p}$. Since we know that
$$v_p^2 - v_w^2 < v_p^2,$$
we have that
$$T > \frac{2Lv_p}{v_p^2} = \frac{2L}{v_p},$$
showing that it takes *more* time to fly a round trip with a wind that it does without one.

Measuring 9 Minutes with a 4 and a 7 Minute Hourglass

Start with the four and the seven hourglasses going at the same time. When the four runs out flip it again to start timing of four minutes. When the seven minute hourglass ends our official time starts and there will be one more minute left in the four minute hourglass. Once this minute has been used up start the four minute timer, which when it ends we have measured a total of five minutes. When the four minute is up start the four minute timer again. When that final four minute timer is over we have measured $5 + 4 = 9$ minutes the desired amount of time.

The Monte Hall Problem

Pick a box and then decide to not switch. The probability that we win is $\frac{1}{3}$. Pick a box and the decide to switch. The probability that we get the prize in this case is given by
$$P(\text{Win}) = \frac{1}{3}(0) + \frac{2}{3}(1) = \frac{2}{3}.$$
The first term is the case where we initially pick the correct box in the first draw (when we switch away from it there is no way to win). The second term comes when our first pick is incorrect and then given that Monte has to reveal an empty by when we switch we must win.

Anther way to see that switching is the correct option (also mentioned in the book) is to consider a large number of boxes say 100 from which you pick one. Then Monte opens all but your box and one other i.e. he opens 98 other boxes. Do you still think you should keep the box you initially picked?

70% like coffee and 80% like tea

The minimum value of $P(AB)$ is when the two sets A and B have the least overlap of the two sets A and B. If we draw sets these two sets such that they are separated by a maximal distance (so that there is the most separation) we see that $P(AB) \geq 0.5$. The maximum value of $P(AB)$ is when we have the most overlap of the two sets A and B. In this case we have $P(AB) \leq 0.7$.

At 3:15 What is Angle Between the Hands of a Clock?

The minute hand points directly at the three while the hour hand points $\frac{1}{4}$ of the way from 3 to 4 since 15 minutes is one quarter of an hour. The arc length from 3 to 4 is $\frac{1}{12}$ of a circle of $\frac{1}{4}$ and thus this is $\frac{1}{48}$ of a circle the angle (in degrees) is then

$$\frac{360}{48} = \frac{360}{4(12)} = \frac{30}{4} = \frac{15}{2} = 7.5\,,$$

degrees.

How Many Integers Between 1 and 1000 contain a 3?

This problem seems to really be about considering various ranges of numbers and keeping track of the number of natural numbers in each range that contain the digit 3. To get the total number we then add up the numbers in each range. As an example of this procedure we first count the number of natural numbers that contain the digit 3 between 1 and 100. In great detail we find

- Between 1 and 10 there is one number i.e. the number 3.

- Between 11 and 20 we have one number i.e. the number 13.

- Between 21 and 30 we have two numbers i.e. the numbers 23 and 30.

- Between 31 and 40 we have nine numbers i.e. the numbers $31, 32, \cdots, 38, 39$.

- Between 41 and 50 we have one number i.e. the number 43.

- Between 51 and 60 we have one number i.e. the number 53.

- Between 61 and 70 we have one number i.e. the number 63.

- Between 71 and 80 we have one number i.e. the number 73.

- Between 81 and 90 we have one number i.e. the number 83.

- Between 91 and 100 we have one number i.e. the number 93.

This gives a total of $8 + 2 + 9 = 19$ numbers with a three in them in this range. We now count the number of natural numbers with a three in them for the ranges of numbers 200 - 300, 300 - 400 etc. We have

- The range $201 - 299$ has 19 numbers with a three.

- The single number 300 is one number with a three.

- The range $301 - 399$ has $399 - 301 + 1 = 99$ numbers with a three.

- The range $401 - 499$ has 19 numbers with a three.

- The range $501 - 599$ has 19 numbers with a three.

- The range $601 - 699$ has 19 numbers with a three.

- The range $701 - 799$ has 19 numbers with a three.

- The range $801 - 899$ has 19 numbers with a three.

- The range $901 - 999$ has 19 numbers with a three.

This gives a total of $9(19) + 1 + 99 = 271$ numbers with a three.

The Number of Pages in a Book with 1095 Digits

If a book only has the pages $1, 2, \cdots, 9$ then it has 9 digits. If a book has pages $10, 11, \cdots, 98, 99$ then it has
$$2(99 - 10 + 1) = 2(90) = 180 \,,$$
additional digits. If a book has pages $100, 101, \cdots, 998, 999$ then we have
$$3(999 - 100 + 1) = 3(900) = 1800 \,,$$
additional digits. As our book has only 1095 digits its must be a book where the last page has only three digits. Let N be the number of pages in the book. Then the number of digits is given by
$$9 + 180 + 3(N - 100 + 1) = 1095 \,.$$
If we solve the above for N we find $N = 401$.

Fermi Problems

Here are a few Fermi problems I attempted.

How Many Garbage Collectors in California?

We start with the population of California which is about 40 million people in 2013. I'll assume that the typical family size is around 2.5 people and thus we have $\frac{40}{2.5} = 16$ million households. We assume that each of these households must have its trash emptied at least once a week. Assume that a given garbage collector can work 5 days a week (Monday - Friday) for eight hours each day. Assume it takes around three minutes to collect and empty

4

the average household's trash. This estimate would need to include the travel time to get from one household to another and the time to actually put the trash on the truck. Thus one garbage collector can empty $\frac{8(60)}{3} = 160$ (eight hours time 60 minutes in an hour) households each day or (multiply this by 5) 800 households per week. To have all of the households emptied in a week we need to have $\frac{16\,10^6}{800} = 20000$ garbage collectors in California.

Estimate the Number of Taxis in New York City

There are about 2 million people in Manhattan and if we assume that about one-half of them use a taxi about once a week the number of taxis needs to be able to provide one million taxi rides per week. A single taxi can work 24 hours a day (by rotating different drivers) and lets assume that the typical single trip length is about 10 minutes. This time estimate must include not only the time transporting the customer but the time to and from one pickup point to another passenger pickup point. Then one taxi can provide

$$\frac{24(60)}{10} = 24(6) \approx 25(6) = 150\,,$$

trips per day or $7(150) = 1050 \approx 1000$ trips per week. This gives $\frac{10^6}{1000} = 1000$ taxis needed to service the specified demand. A google search indicated that the actual number of taxis in New York is about 10 times the above number or 10 thousand taxis. From the above specification, our estimate of the number of taxis would increase if the length of time a taxi can work were less than 24 hours (say only 8 hours a day) or the average trip time were less than 10 minutes or if the number of people who use taxis were to include the outer boroughs of New York City i.e. The Bronx, Brooklyn, etc.

Estimate the Number of Golf Balls that Would Fit in a Stadium

To do this problem we will estimate the volume of a golf ball and the volume of typical stadium and then divide the two to get the number of golf balls that would fit inside. A golf ball has a diameter of two inches (the actual number is specified to not be smaller than 1.68 inches) and thus assuming a radius of one inch its volume is $\frac{4}{3}\pi R^3$ or assuming the worst possible approximation for π of $\pi \approx 3$ we have a volume of *four* cubic inches.

One way to do this problem is to know that the fans sit facing a football field of known size (something like 100 yards by 50 yards). This gives a area of 5000 yards squared. We next need to calculate how "high" the typical stadium is to get a volume. I found this number harder to come by. One would need to estimate it. It certainly would be larger than the 100 yards of linear length down the field. If we assume a height of 100 yards we get a volume of $5000(100) = 5\,10^7$ cubic yards. Since one yard is three feet is 36 inches one cubic yard is $36^3 = 46656$ cubic inches and we have a volume of about $5\,10^7(50000) = 2.5\,10^{12}$ cubic inches. Dividing this by the volume of a typical golf ball gives $2\,10^{11}$ or 200 billion. This estimate does not include an estimate of the volume of the space where the fans sit (which is typically a triangular region) and thus would *underestimate* the total number of golf balls that could fit in a stadium.

How Many Vacuum Cleaners are Made a Year?

Start with the population of the United States of about 300 million (other geographical units could be treated in a similar manner) . Assume that a household consists of 2.5 people and we have $\frac{300}{2.5} = 120$ million households. If we assume that a household replaces its vacuum cleaner every 5 years then each year we need $\frac{120}{5} = 24$ million vacuums produced every year to meet this demand.

Racing Twenty-Five Horses

One can get the max of a set of numbers by doing sequential maximizations which is really the procedure we are allowed to do in this problem. Here in addition we want to find the three horses that can run the fastest (not just the single fastest horse). We can do this by first running five races of five. The single winner of each of these races (five horses total) can then be run against each other a second time to produce the fastest horse over all. Call this horse M for maximum. If it had happened that in M's first race the actual second and third fastest horses had been run against him they would have lost and not made it to the second race. Thus the second and third place horses from the first race with M need to be considered as contenders for the second and third fastest horses overall. Let N and O be the second and third place finisher respectively in our second race. As in the previous logic the second place finisher in the first race where N was the winner is a contender for the third fastest horse. This gives a total of three horses that did not run in the second race that needed to be considered in a third race. The other two horses to race are the second and third place finishers in the second race. This gives a total of five horses to race in a third race to determine the second and third fastest horses overall.