Ex 1.1 $p_i \quad (i \geq 0)$

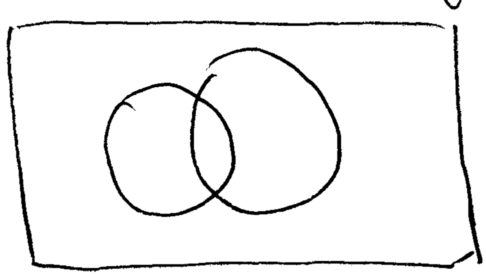
(a)
$$\sum_{i=1}^{\infty} p_i = 1$$
$$\parallel$$
$$p_0 + p_1 + p_2 + p_3 + \sum_{i=4}^{\infty} p_i = 1$$

$$\sum_{i=4}^{\infty} p_i = 1 - .2 - .35 - .25 - .15 = \ldots$$

(b) $P = p_0 + p_1 + p_2 = \ldots$

1.2

U



$$P(\neg C \cap \neg R) = P(\neg(C \cup R)) = \cancel{P}$$

↑ not cloudy ↑ not rainy

$$= 1 - P(C \cup R)$$

$= \cancel{P(\neg C)}$ Now ~~RCR~~ $P(C \cup R) = P(C) + P(R) - P(C \cdot R)$

$$= .4 + .3 - .2 = .5$$

(1.3) ~~####~~ ~~####~~

(a) $P = \left(\frac{8}{14}\right)\cdot\left(\frac{7}{13}\right) = .3077$

$\xrightarrow{+}$ $.4725$

(b) $P = \left(\frac{6}{14}\right)\left(\frac{5}{13}\right) = .1648$

(c) $\left(\frac{8}{14}\right)\left(\frac{6}{13}\right) + \left(\frac{6}{14}\right)\left(\frac{7}{13}\right) = 1 - P(\text{Both men}) - P(\text{Both women})$

draw women 1st

draw men 2nd

$= .4945 \neq 1 - P(BM) - P(BW)$

why not?

$1 = P(\text{Both men}) + P(\text{Both women}) + P(\text{one man, one women})$

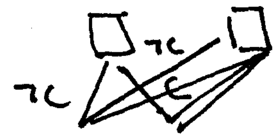
$\Rightarrow P(\text{one men, one woman}) = 1 - ✓ - ✓$

(1.4) ~~35 + 58 + 27~~ ~~P(C) =~~ $P(C) = \frac{35}{120}$; $P(B) = \frac{58}{120}$; $P(C \cap B) = \frac{27}{120}$

(a) $P(C|B) = \dfrac{P(C \cap B)}{P(B)} = \dfrac{27/120}{58/120} = \dfrac{27}{58}$

(b) $P(B|C) = \dfrac{P(B \cap C)}{P(C)} = \dfrac{27/120}{35/120} = \dfrac{27}{35}$

1.5



(a) $p = \left(\frac{1}{4}\right)$

(b) $\cancel{P_{None} + 2P_{one} +}$

(F,F)	(F,N)	(N,F)	(N,N)
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$$\frac{2/4}{3/4} = \frac{2}{3}$$

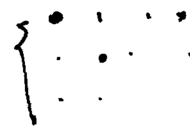
1.6 $P(A) = \frac{4}{52} = \frac{1}{13}$

$$P(A_1 \cap A_2 \mid \text{Different suites}) = \frac{P(A_1 \cap A_2 \cap (S(P_1) \neq S(P_2)))}{P(S(P_1) \neq S(P_2))}$$

$$P(S(P_1) \neq S(P_2)) = \frac{3}{4}$$

x x x x

$P(A_1 \cap A_2 \cap (S(D_1) \neq S(D_2)))$



$= \dfrac{4 \cdot 3 \cdot \binom{4}{1}}{52^2}$?

(1,7) $P(A \cap B) = P(A) \cdot P(B)$

(a) $P(A \cap B^c) \stackrel{?}{=}$

$= P(A) \cdot P(B^c \mid A)$

$= P(A) \cdot (1 - P(B \mid A)) = P(A) \cdot (1 - P(B)) = P(A)P(B^c)$

(b) $P(A^c \cap B^c) = P(\neg(A \cup B))$

$= 1 - P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A) \cdot P(B)$

$\Rightarrow P(A^c \cap B^c) = 1 - P(A) - P(B) + P(A)P(B)$

$= 1 - P(A) + (-1 + P(A))P(B)$

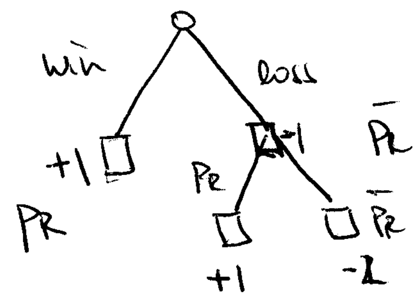
$= (1 - P(A))(1 - P(B)) = P(A^c) \cdot P(B^c)$

19 17 Ross 9-31-03 (

(1.8) $P_R = \frac{18}{38} = \frac{9}{19}$

(a) $P\{X > 0\} = P_R +$

(b) E



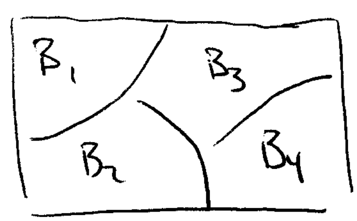
$\text{outcomes} = \{P_R, \bar{P}_R P_R, \bar{P}_R \bar{P}_R\}$ 3.

$$P\{X > 0\} = P_R + \bar{P}_R \cdot P_R = P_R(1 + \bar{P}_R) = P_R(1 + 1 - P_R)$$

$$= P_R(2 - P_R) = \left(\frac{9}{19}\right)\left(2 - \frac{9}{19}\right) = \frac{9}{19}(\quad = .7230$$

(b) $E[X] = 1 P_R + +1 \bar{P}_R P_R - 3 \bar{P}_R^2$

$= .049$!



(1.9) $39 + 33 + 46 + 34$

X = # of students on bus of selected student

Y = one of the 4 ~~buss~~ bus driver chooses

(a) $E[x]$

$p_1 = \frac{39}{152}$; $p_2 = \frac{33}{152}$; $p_3 = \frac{46}{152}$; $p_4 = \frac{34}{152}$

$= .2566$ $= .217$ $= .30$ $= .22$

$E[x] = 39\left(\frac{39}{152}\right) + 33\left(\frac{33}{152}\right) + 46\left(\frac{46}{152}\right) + 34\left(\frac{34}{152}\right)$

$= \frac{1}{152}($ $= 38.69$

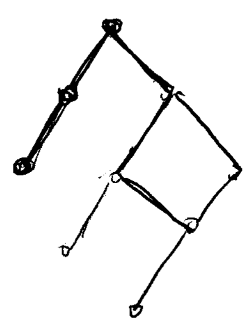
$E[y] = ?$

$p_i = \frac{1}{4}$

$E[y] = \frac{1}{4}(39) + \frac{1}{4}(33) + \frac{1}{4}(46) + \frac{1}{4}(34) = 38$

(b)

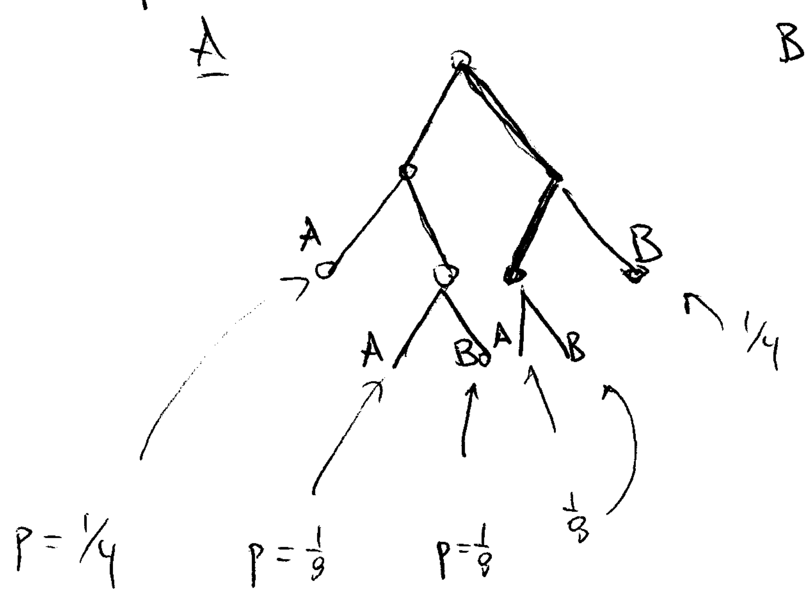
(1.10) $p_i = \frac{1}{2}$



(a) $\underline{X}$ = # of sets played

A ; p^2 ; p^3 ; p^4 ;

4 WW, LWW, LW



$\frac{4}{8} + \frac{1}{4} + \frac{1}{4} =$

(a) $E[X] = 2(\frac{1}{4}) + 3(\frac{1}{8})\cdot 4 + 2(\frac{1}{4})$

$= 1 + \frac{3}{2} = 2.5$

(b) $Var(X) = E[x^2] - E[x]^2$

$= 4(\frac{1}{4}) + 4\cdot 9\cdot \frac{1}{8} + 4(\frac{1}{4}) = 2 + \frac{9}{2} = 6.5$

1.11 ...

1.12 (a) $\mu = 5000 \quad \sigma = 0$

(b) $E[x] = 0 + (25{,}000)(.3) = 2500(3) = 7500$

$Var[x] = E[x^2] - E[x]^2 = 0 + (25\cdot 10^3)^2(.3) - 7500^2$

$= 1.3125\cdot 10^8 \Rightarrow \sigma = 1.1\cdot 10^4$

1.13

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(a) ~~kat~~ ✓

(b) $Var(\bar{X}) = E(\bar{X}^2) - E(\bar{X})^2$

$$= E\left(\frac{1}{n^2}\left(\sum_{i=1}^{n} X_i\right)^2\right) - E(\bar{X})^2$$

$$= \frac{1}{n^2}E\left(\sum_{\substack{i=1\\j=1}}^{n} X_i X_j\right) - \mu^2 \quad \ldots$$

$$Var(\bar{X}) = E((\bar{X} - \mu)^2)$$

$$= E\left[\left(\frac{1}{n}\sum_{i=1}^{n} X_i - \mu\right)^2\right] = E\left[\frac{1}{n^2}\left(\sum_{i=1}^{n} X_i - \sum_{i=1}^{n}\mu\right)^2\right]$$

$$= \frac{1}{n^2} E\left[\left(\sum_{i=1}^{n}(X_i - \mu)\right)^2\right]$$

$= \cancel{\frac{1}{n^2}}$

From proposition 1.3.2. X_i are independent $\Rightarrow$

$$Var\left(\sum_{j=1}^{k} X_j\right) = \sum_{j=1}^{k} Var(X_j)$$

Also $Var(\alpha X) = \alpha^2 Var(X)$

Thus $Var\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right) = \frac{1}{n^2}Var\left(\sum_{i=1}^{n}X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n}Var(X_i)$

due to the independence

$$= \frac{1}{n^2}\cdot n\sigma^2 = \frac{\sigma^2}{n} \checkmark$$

(c) $S^2 = \dfrac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$ sample variance

$$\sum_{i=1}^{n}(X_i - \bar{X})^2 = \sum_{i=1}^{n}(X_i^2 - 2X_i\bar{X} + \bar{X}^2)$$

$$= \sum_{i=1}^{n}X_i^2 - 2\bar{X}\sum_{i=1}^{n}X_i + n\bar{X}^2$$

$$= \sum_{i=1}^{n}X_i^2 \underbrace{- 2n\bar{X}^2 + n\bar{X}^2}$$

$$= \sum_{i=1}^{n}X_i^2 - n\bar{X}^2 \checkmark$$

(d) $E[S^2] = \left(\frac{1}{n-1}\right) E\left[\sum_{i=1}^{n} (X_i - \bar{X})^2\right]$

$= \left(\frac{1}{n-1}\right) E\left[\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right]$

$= \left(\frac{1}{n-1}\right)\left[\cancel{\sum_{i=1}^{n} E[X_i^2]} - n \cancel{\bar{X}^2} \cdot n\right]$

$\cancel{\sum_{i=1}^{n} (\mu^2 + \sigma^2)} -$

$E[X_i^2] - E[X_i]^2 = \sigma^2$ & Also

$\therefore E[X_i^2] = \mu^2 + \sigma^2$ $\qquad \frac{\sigma^2}{n} = E[\bar{X}^2] - E[\bar{X}]^2 = E[\bar{X}^2] - \mu^2$

$\therefore E[\bar{X}^2] = \mu^2 + \frac{\sigma^2}{n}$

$\left(\frac{1}{n-1}\right)\left[\sum_{i=1}^{n} E[X_i^2] - nE[\bar{X}^2]\right]$

$= \left(\frac{1}{n-1}\right)\left[n(\cancel{\mu^2} + \sigma^2) - n\left(\cancel{\mu^2} + \frac{\sigma^2}{n}\right)\right] = \left(\frac{1}{n-1}\right)(n\sigma^2 - \sigma^2) = \sigma^2$ ✓

(1.14) $\mathrm{Cov}(\bar{X},\bar{Y}) = E[\bar{X}\bar{Y}] - E[\bar{X}]\cdot E[\bar{Y}]$

$\equiv$

$E[(\bar{X}-\mu_x)(\bar{Y}-\mu_y)]$

$= E[\bar{X}\bar{Y} - \mu_y\bar{X} - \mu_x\bar{Y} + \mu_x\mu_y]$

$= E[\bar{X}\bar{Y}] - \mu_y\mu_x - \mu_x\mu_y + \mu_x\mu_y$ ✓

(1.15)

(a) ✓

(b) ✓

(c) ✓

(d) ✓

(1.16) $\bar{X} = a\bar{U} + b\bar{V}$ $\quad \bar{Y} = c\bar{U} + d\bar{V}$. $\quad \bar{U}, \bar{V}$ independent

e

$\mathrm{Cov}((a\bar{U}+b\bar{V} - a\mu_u - b\mu_v)($

$= E[\bar{X}\bar{Y}] - E[\bar{X}]E[\bar{Y}]$

J

$$= E[(aU+bV)(cU+dV)] - (aE[U]+bE[V])(cE[U]+dE[V])$$

$$E[XY] = E[X]\cdot E[Y]$$

$$|||$$

$$\sum_{xy} (p_{xy})(xy)$$

$$||$$

$$\sum_x \sum_y p_x \cdot p_y (xy) = \sum x p_x \cdot \sum y p_y =$$

$$= E[\, acU^2 + adUV + bcVU + bdV^2 \,] - \cdots$$

$$= ac\,E[U^2] + ad\,E[U]\cdot E[V] + bc\,E[V]E[U] + bd\,E[V^2] - \cdots$$

$$Var(U) = E[U^2] - E[U]^2 \ldots$$

$$|||$$

$$1$$

(1.17)

(a) $\mathrm{Cov}(X_1 + X_2, X_3 + X_4)$

$$= E[(X_1 + X_2 - \mu_1 - \mu_2)(X_3 + X_4 - \mu_3 - \mu_4)]$$

$$= E[(X_1 - \mu_1 + X_2 - \mu_2)(X_3 - \mu_3 + X_4 - \mu_4)]$$

$$= E[(X_1 - \mu_1)(X_3 - \mu_3) + (X_1 - \mu_1)(X_4 - \mu_4)$$

$$+ \cdots + \cdot \quad] = \cdots$$

(b) ✓

(1.18) X amount stock goes up in 1st period

$\mu_X = 0$

$\sigma_X^2 = \cancel{1^2(1/2)} + E[X^2] - E(X)^2 = (1/2)1 + 1/2(1) = 1$

Y = cumulative amount it goes up in 1st 3 periods

$\mu_Y = np \quad = \cancel{3} \; 3/2$

$\sigma_Y^2 = np(1-p)$ does this work for up & downs?

$n=3$

1.19 $Var(X) = Var(Y) = 1$

$Cov(X,Y) = 2$

$E[X^2] - E[X]^2$

$(1/2)(1)$

$X = \pm 1$ w/ prob 1/2

$E[X] = 0$

$E[X^2] = 1 \Rightarrow Var(X) = 1$

$$E[(X-0)(Y-0)] = E[X \cdot Y]$$

$$= \frac{1}{4}\{(+1,+1), (+1,-1), (-1,+1), (-1,-1)\}$$

$$= \frac{1}{4} \cdot [1 - 1 - 1 + 1] = 0$$

X Dunno

21

(a) $P\{Z < -.66\} = \text{normcdf}(-.66) = .2546$

(b) $P\{-1.64 < Z < +1.64\} = \Phi(1.64) - \Phi(-1.64)$

$= .8990$

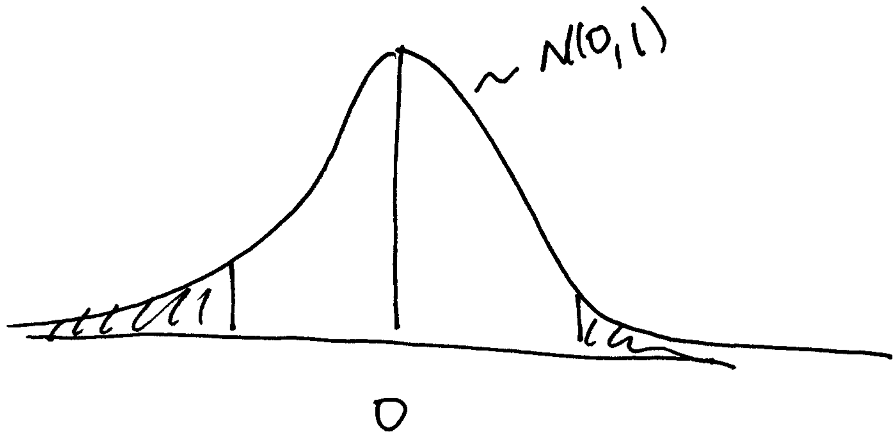
(c) Φ $\text{normcdf}(2.20) - \text{normcdf}(-2.2) = .9722$

22 $P\{-2 < Z < -1\} = \text{normcdf}(-1) - \text{normcdf}(-2) = .1359$

$P\{1 < Z < x\} = \text{normcdf}(x) - \text{normcdf}(1) = .1359$

$\Rightarrow \quad x \approx 2.$

23



Ex 2.4 $Y = a + bX$

$\mu_Y = a + b\mu_X$ $\sigma_Y^2 = b^2\sigma_X^2$

Desire $\mu = \mu_X = \mu_Y$ $\sigma_X^2 = \sigma_Y^2 = \sigma^2$

$\Rightarrow$ $\mu = a + b\mu$ $\sigma^2 = b^2\sigma^2 \Rightarrow b^2 = 1 \Rightarrow b = \pm 1$

$a = \mu(1-b)$

$$= \begin{cases} \mu(1-(1)) = 0 \\ \mu(1-(-1)) = 2\mu \end{cases}$$

$\therefore Y = X$ or $Y = 2\mu - X$

$\mathrm{cov}(X, Y) = \mathrm{cov}(X, 2\mu - X) = \underbrace{\mathrm{cov}(X, 2\mu)}_{=0} - \mathrm{cov}(X, X)$

$= -\sigma^2.$

Ex 2.5 $P\{|X-\mu| < 1\cdot\sigma\} = .667$

$P\{|X-\mu| < 2\cdot\sigma\} = .97$

$P\{|X-\mu| < 3\cdot\sigma\} = .99$

(a) $127.7 - 19.2 < X < 127.7 + 19.2$

(b) $127.7 - 2 \cdot 19.2 < x < 127.7 + 2 \cdot 19.2$

(c) $127.7 - 3(19.2) < x < 127.7 + 3 \cdot 19.2$

Ex 26

let X = life of given Battery $= \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left\{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right\}$

$Y = \sum_{i=1}^{2} X_i$ $\quad$ Y = life of two Batteries.

$P\{Y > 400 \text{ Hrs}\}$

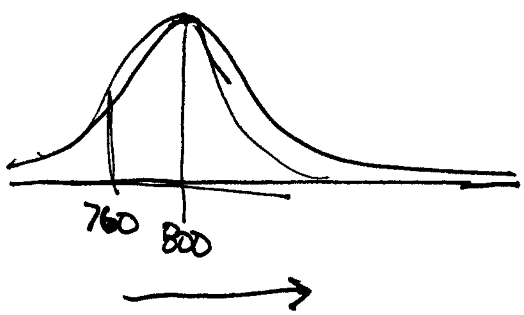
sum of independent normal random variable is also normally distributed. w/ $\mu = \mu_1 + \mu_2$

$\quad$ & $\sigma^2 = \sigma_1^2 + \sigma_2^2$

$\Rightarrow \mu_{\text{2 batteries}} = 800 \text{ hrs}$

$\sigma^2 = 2(50)^2 \Rightarrow \sigma = \sqrt{2} \cdot 50 = 70.71$

$$P\{\bar{X}_1+\bar{X}_2 > 760\} = Pr\{Z > \frac{760-800}{\sqrt{2}\,50}\}$$

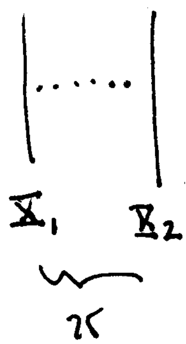


$$= \text{normcdf}(\ \text{"}\ ) = \cancel{.2858}$$

$$= 1 - Pr\{Z < \frac{760-800}{\sqrt{2}\cdot 50}\}$$

$$= .714$$

(b) $Pr\{\bar{X}_2 > \bar{X}_1 + 25\} \Rightarrow Pr\{\bar{X}_1+\bar{X}_2 > 2\bar{X}_1 + 25\}$



$$Pr\{\underbrace{\bar{X}_2 - \bar{X}_1} > 25\}$$

normal w/ $\mu = \mu - \mu = 0$
distributed w/ $\sigma^2 =$
$\sigma^2 + \sigma^2 = 25^2 =$

$$= Pr\{Z > \frac{25-0}{2(50)}\} = 1 - Pr\{Z < \frac{25-0}{2(50)}\} \cong .4$$

(c) $Pr\{\text{Max}(\bar{X}_1,\bar{X}_2) - \text{Min}(\bar{X}_1,\bar{X}_2) > 25\}$?

27 $\Pr\left\{\sum_{i=1}^{100} X_i > 1710\right\}$ σ^2

(a) $\parallel$

$$N\left(\sum_{i=1}^{100} 18, \sum_{i=1}^{100} 1^2\right) = N(1800, 100)$$

$$\Pr\left\{Z > \frac{1710 - 1800}{10}\right\} = \Pr\{Z > -9\}$$

$$\Rightarrow \quad = \Pr\{Z < 9\} = 1$$

$$\Pr\left\{1690 < \sum_{i=1}^{100} X_i < 1710\right\} = \Pr\left\{\frac{1690-1800}{10} < Z < \frac{1710-1800}{10}\right\}$$

$$= \Pr\{-11 < Z < -9\} = \Pr\{9 < Z < 11\}$$

$$= \Phi(11) - \Phi(9) = 0.$$

Ex 2.8 $\mu = 25 \cdot 10^3$

$\sigma = 12 \cdot 10^3$

$\cancel{\mu_{30}} = \cancel{30 \times 25}$ $\quad \bar{\mu} = 25 \cdot 10^3$ $\quad \cancel{\sigma} = \text{Var}(\bar{X}) = \frac{1}{n}\sum_{i=1}^{3} \sigma^2$

$$\Rightarrow \bar{\sigma} = \frac{\sigma}{\sqrt{n}}$$

(a)

$$\Pr\left\{ \bar{X} > 25 \cdot 10^3 \right\} = \Pr\left\{ \frac{\bar{X} - \bar{\mu}}{\bar{\sigma}} > \frac{25 \cdot 10^3 - 25 \cdot 10^3}{\left(\frac{12 \cdot 10^3}{\sqrt{30}}\right)} \right\}$$

$$= \Pr\{Z > 0\} = \Pr\{Z < 0\} = 1/2$$

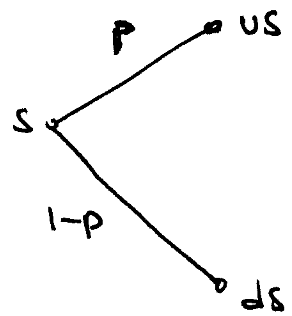
(b) $\Pr\left\{ 23 \cdot 10^3 < \bar{X} < 27 \cdot 10^3 \right\}$

$$= \Pr\left\{ \frac{23 \cdot 10^3 - 25 \cdot 10^3}{2.2 \cdot 10^3} < Z < \frac{27 \cdot 10^3 - 25 \cdot 10^3}{2.2 \cdot 10^3} \right\}$$

$$= \Pr\{-.909 < Z < .909\} = \text{normcdf}(.9) - \text{normcdf}(-.9)$$

$$\cong .63$$

29



$S(t) \geq S(0)\cdot(1.3)$

$\frac{S(t)}{S(0)} \geq 1.3$

a $\quad \frac{S(1)}{S(0)} = \begin{cases} u & \text{w/ prob } p \\ d & \text{w/ prob } 1-p \end{cases}$

$$\frac{S(t)}{S(t-1)} \cdot \frac{S(t-1)}{S(t-2)} \cdots \frac{S(2)}{S(1)} \cdot \frac{S(1)}{S(0)} =$$

Y_i = indicator random variable $\quad Y_i = \begin{cases} 0 & \text{if stock price goes down} \\ 1 & \text{if stock price goes up} \end{cases}$

$$\cancel{S(t)} = S(0)\, u^{\sum_{i=1}^{t} Y_i}$$

$$S(1000) = S(0)\, u^{\sum_{i=1}^{1000} Y_i}\, d^{1000 - \sum_{i=1}^{1000} Y_i}$$

$$= S(0)\, d^{1000} \left(\frac{u}{d}\right)^{\sum_{i=1}^{1000} Y_i}$$

desire

$$\Pr\left\{ \frac{S(1000)}{S(0)} = d^{1000} \left(\frac{u}{d}\right)^{\sum_{i=1}^{1000} Y_i} \geq 1.3 \right\}$$

$$= \Pr\left\{ \left(\frac{u}{d}\right)^{\sum_{i=1}^{1000} x_i} > \frac{1.3}{(.99)^{1000}} \right\}$$

$$\frac{u}{d} = \frac{1.012}{.99} > 1$$

$$= \Pr\left\{ \sum_{i=1}^{1000} x_i > \frac{\ln\left(1.3/(.99)^{1000}\right)}{\ln\left(\frac{1.012}{.99}\right)} \right\}$$

$$\frac{\ln\left(1.3/(.99)^{1000}\right)}{\ln\left(\frac{1.012}{.99}\right)} = 4.69 \cdot 10^2 = 469$$

$$= \Pr\left\{ \sum_{i=1}^{1000} x_i > 469 \right\}$$

Assume Normal distribution w/

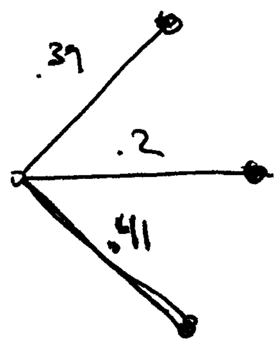
$$\mu = np = 1000(.52) = 520$$

$$+ \ \sigma^2 = np(1-p) = 1000(.52)(.48) \cong 250$$

$$= \Pr\left\{ Z > \frac{469-520}{15.7} \right\}$$

$$\frac{469-520}{15.7} = -3.2 \qquad \cong 1$$

2.10



let $Y_i = \begin{cases} +1 & \text{stock goes up} \\ 0 & \text{stock } \cancel{\text{goes}} \text{ D.N.C. or goes down} \\ \cancel{-1} & \cancel{\text{stock goes down}} \end{cases}$

$Z_i = \begin{cases} 0 & \text{stock DNC or goes up} \\ -1 & \text{stock goes down} \end{cases}$

$$S(700) = S(0)\, u^{n_u} \cdot d^{n_d}$$

$$Y = \sum_{i=1}^{700} Y_i = \#\text{ of times stock goes up}$$

$$Z = \sum_{i=1}^{700} Z_i = \#\text{ of times stock goes down}$$

$$S(700) = S(0)\, U^{\sum Y_i}\, d^{\sum Z_i}$$

$$\Pr\left\{ \frac{S(700)}{S(0)} = U^{\sum Y_i} d^{\sum Z_i} > 10 \right\} \quad ?$$

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(3.1)

(a) $\cancel{E[S(10)]}$

$$\log\left(\frac{S(t+y)}{S(y)}\right) \sim N(\mu t, t\sigma^2)$$

$$\log\left(\frac{S(10)}{S(0)}\right) \sim N(\mu\cdot 10, 10\cdot\sigma^2)$$

$$S(10) = S(0)\cdot N(10\mu, 10\sigma^2)$$

$$E[S(t)] = S(0)e^{t(\mu+\sigma^2/2)}$$

$$\Rightarrow E[S(10)] = 100\cdot\exp\left[10\left(.01+\frac{(.2)^2}{2}\right)\right] = \cdots$$

(b) $\Pr\{S(10) > 100 = S(0)\}$

$$\Leftrightarrow \Pr\left\{\frac{S(10)}{S(0)} > 1\right\} \Leftrightarrow \Pr\left\{\ln\left(\frac{S(10)}{S(0)}\right) > \ln(1) = 0\right\}$$

$$\ln\left(\frac{S(10)}{S(0)}\right) = N(t\mu, t\sigma^2)$$

$$\Rightarrow \Pr\{N(t\mu, t\sigma^2) > 0\} \quad \cdots$$

(c) $\Pr\left\{ \ln\left(\frac{S(10)}{S(0)}\right) < \ln(1.1) \right\}$

$\quad\quad\quad\quad \shortparallel$

$N(10\cdot\mu, t\cdot\sigma^2) < \ln(1.1)$ . . .

(3.2) . . .

(3.3) . . .

(3.4) $X \sim N(m, v^2)$

$E[X] = e^{m + v^2/2}$

$E\left[\frac{S(t)}{S(0)}\right] = e^{t\mu + t\sigma^2/2}$ . . .

(3.5) $Var(S(t)) = E[S(t)^2] - (E(S))^2$

$\quad\quad\quad\quad\quad\quad\quad \shortparallel$

$S_0^2 N(t^2\mu^2, t^2\sigma^4)$

$= \cancel{S_0^2 e^{t^2\mu}} \quad S_0^2 e^{t^2\mu^2 + t^2\sigma^4/2} - S_0^2\left(e^{t\mu + t\sigma^2/2}\right)^2$

$= S_0^2 e^{t^2\mu^2 + t^2\sigma^4/2} - S_0^2 e^{2t\mu + t\sigma^2}$

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$\sum A = 80$

$\sum B = 77$

$\sum C = 72$

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4.1

(a) $r_{eff} = \frac{\text{Amt Repaid} - P}{P}$

$= \frac{P(1+r/2)^2 - P}{P} = (1+r/2)^2 - 1$

$= .1025.$

(b) $r_{eff} = \frac{P(1+r/4)^4 - P}{P} = (1+r/4)^4 - 1 = .103129$

$\approx 10.31\%$

(c) $r_{eff} = \frac{Pe^{r\cdot 1} - P}{P} = e^{r\cdot 1} - 1 = .105 \approx 10.5\%$

4.2 $\cancel{P} \; P(t) = P_0 e^{rt}$

$\parallel$

$2P_0$

$\Rightarrow 2 = e^{rt} \Rightarrow rt = \ln(2)$

$t = \frac{\ln(2)}{r} = \frac{\ln(2)}{.1} \cong 6.9$ yrs

4.3 $P(t) = P_0(\cancel{1}\cancel{+}r/4$

$4P_0 = P_0(1+r)^t$

$4 = (1+r)^t$

$t = \dfrac{\ln(4)}{\ln(1+r)} = 28.4$ yrs

$= 35.34$ yrs !!

4.4 $P(t) = P_0(1+r)^t$

$\|$

$3P_0 = P_0(1+r)^t$

$3 = (1+r)^t$

$$\boxed{\dfrac{\ln 3}{\ln(1+r)} = t}$$

4.5 $P(t) = P_0(1+r/12)^{12 \cdot t}$ ← 1st days investment

$[t]$ = years

$+ P_0(1+r/12)$

— — — —

$P(t) = P_0(1+r/12)^t$

$[t]$ = months

$+ P_0(1+r/12)^{t-1} + P_0(1+r/12)^{t-2} + \cdots + P_0$

$$P(t) = P_0 \sum_{k=0}^{t} (1+r/12)^k$$

$$= P_0 \left[\frac{1-(1+r/12)^{t+1}}{1-(1+r/12)} \right] \Bigg|_{\substack{r=.06 \\ t=60}} \overset{\text{set}}{=} 10^5$$

$$\to \quad P_0 (71.1\ldots) = 10^5$$

$$P_0 = 1400 \, .$$

invest 1400 /month at 6% interest have \$100,000 in 5 years.

4.6 $PV(a) = \frac{a_1}{1+r} + \frac{a_2}{(1+r)^2} + \cdots + \frac{a_n}{(1+r)^n}$

$$PV = \frac{-1000}{(1+.06)} - \frac{1200}{(1+.06)^2} + \frac{800}{(1+.06)^3} + \frac{900}{(1+.06)^4} + \frac{800}{(1+.06)^5}$$

$= -29.0$ Think No

4.7 $r = \{.03, .05, .1\}$

$$PV = \frac{20}{(1+r)} + \frac{20}{(1+r)^2} + \frac{20}{(1+r)^3} + \frac{15}{(1+r)^4} + \frac{10}{(1+r)^5} + \frac{5}{(1+r)^6}$$

V.S.

$$PV = \frac{10}{(1+r)} + \frac{10}{(1+r)^2} + \frac{15}{(1+r)^3} + \frac{20}{(1+r)^4} + \frac{20}{(1+r)^5} + \frac{20}{(1+r)^6}$$

Think B will win for small interest rates where A will win for larger interest rates

(4.8) 60 payments 10% capon ⇒ true payment = \$9,000.

[illegible] =

$$PV = -9000 + \frac{800}{(1+r)} + \frac{800}{(1+r)^2} + \cdots + \frac{800}{(1+r)^{60}} - \frac{10000}{(1+r)^{61}} = \cdots$$

(4.9)

$$PV = -1000 + -\frac{160}{(1+r)} - \frac{160}{(1+r)^2} - \cdots -\frac{160}{(1+r)^{24}} \underset{\text{set}}{=} -4200$$

$$\Rightarrow \sum_{i=1}^{24} \frac{1}{(1+r)^i} = \frac{3200}{160} = 20$$

$$\frac{\frac{1}{(1+r)}^{25} - \frac{1}{(1+r)}}{\left(\frac{1}{1+r} - 1\right)} = 20 \Rightarrow r = \cdots$$

(4.10) ...

(4.11) ...

(4.12) A = 120,000 r = .5

[illegible] (1+.05)(120,000)

.005 · 120,000 = \$600 month

$$PV = -600 + \frac{600}{(1+r)} + \frac{600}{(1+r)^2} + \cdots + \frac{600}{(1+r)^{35}} = 600 \sum_{i=0}^{35} \beta^i = 600 \frac{(1-\beta^{36})}{1-\beta}$$

= 19,821

$120{,}000 + 19{,}821 = \cdots$

(4,13)

-16000 v.s,

High rate better for paying either keep the money for your self + invest.

$\cancel{-10000 \;/\; 10000 \;/\; 1+r}$

$-10000 - 10000 \cdot e^{-r(10)}$

$$= -10{,}000\left(1+e^{-r(10)}\right) = \begin{cases} -18187 & r = .02 \\ -16065 & r = .05 \\ -13678 & r = .1 \end{cases}$$

(4,14)

$$PV = -1000 + \underbrace{30e^{-(.03)(1)} + 30e^{-(.03)(2)} \cdots}_{30\sum_{n=1}^{9} e^{-.05(n)}} + 1030$$

$$= 30 \cancel{\left\{\frac{1-e^{-.05(10)}}{1}\right.}$$

$= \cdots$

9.15 $\left(1+\frac{r}{n}\right)^n$

$$\left(1+\frac{r}{n+1}\right)^{n+1} = \left(1+\frac{r}{n+1}\right)^1\left(1+\frac{r}{n+1}\right)^n > \left(1+\frac{r}{n+1}\right)^n$$

$$= \left(1+\frac{r}{n}\left(\frac{n}{n+1}\right)\right)^n$$

$$\frac{n}{n+1} = \left(1-\frac{1}{n+1}\right)$$

$$= \left(1+\frac{r}{n}-\frac{r}{n(n+1)}\right)^n \overset{?}{>} \left(1+\frac{r}{n}\right)^n$$

iff $\left(1+\frac{r}{n}-\frac{r}{n(n+1)}\right) > 1+\frac{r}{n}$ $\rightarrow\leftarrow$

$F_n = \left(1+\frac{r}{n}\right)^n$ ~~$F_n' = n(1+\frac{r}{}$~~

$H_n = n\ln\left(1+\frac{r}{n}\right) = \ln F_n$

$$\frac{dH_n}{dn} = \frac{1}{F_n}\cdot\frac{dF_n}{dn} = \ln\left(1+\frac{r}{n}\right) + \frac{n}{\left(1+\frac{r}{n}\right)}\left(\frac{-r}{n^2}\right)$$

$$= \ln\left(1+\frac{r}{n}\right) - \frac{\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)}$$

Now:

5

$$\ln(1+x) = \sum$$

$$\frac{1}{1+x} = \sum_{k\geq 0} (-1)^k x^k$$

$$\ln(1+x) = \sum_{k\geq 0} (-1)^k \frac{x^{k+1}}{k+1}$$

$$\ln(1+x) = \sum_{k=0}^{m} (-1)^k \frac{x^{k+1}}{k+1} + \frac{(-1)^{m+1}}{m+2}\frac{d^{m+1}}{dx^{m}}\ln(1+x)\Big|_{x=\xi(x)}$$

$$\ln(1+x) = x + \frac{(-1)^1}{2}\frac{d}{dx}\ln(1+x)\Big|_{\xi} = x - \frac{1}{2(1+\xi)} < x$$

$$=$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{1}{3}\frac{d}{dx} \quad X$$

Work on analysis

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4.16

(a) $100e^{(.06)(\frac{30}{365})} = 100.49$

(b) $100e^{(.06)(\frac{60}{365})} = 100.99$

(c) $100e^{(.06)(\frac{120}{365})} = 101.9$

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4,17

$\cancel{1000\, e^{rt} \neq 2}$

$$1000 \cdot e^{r(3)} + 2000 \cdot e^{r(2)} + 3000 e^{r(1)}$$

$\doteq$

4,18

$$PV(a) = \sum_{i=1}^{n} (1+r)^{-i} x_i$$

$$= (1+r)^{-1}(3) + (1+r)^{-2}(5) + (1+r)^{-3}(-6) + (1+r)^{-4}(5) \Big|_{r=.05}$$

~~4.18~~

4.19 $PV(a) = \dfrac{20}{(1+r)} + \dfrac{10}{(1+r)^2}$

$$PV(b) = \frac{0}{(1+r)} + \frac{34}{(1+r)^2}$$

$PV(a) > PV(b)$

$$\Rightarrow \frac{20}{(1+r)} + \frac{10}{(1+r)^2} > \frac{34}{(1+r)^2} \Rightarrow \frac{24}{(1+r)^2} - \frac{20}{(1+r)} < 0$$

$$24 - 20(1+r) < 0$$

$4 - 20r < 0 \Rightarrow r > \frac{4}{20} = \frac{1}{5} = .2$

4.20 $1500 = 1000e^{(.06)t}$ ...

4.21 $PV(\vec{x}) = \sum_{i=1}^{\infty} x_i e^{-rt_i} =$

$= se^{-r} + (s+t)e^{-2r} + (s+2t)e^{-3r} + \cdots$

$= \sum_{i=1}^{\infty} se^{-ir} + \sum_{i=1}^{\infty} ite^{-r(i+1)}$

$= s\sum_{i=1}^{\infty} e^{-ir} +$ ...

4.22

(a) $\approx D(\cancel{t)(1+r})$

$D(t+h) = D(t)(1+r)^n$

$n =$ # of compounding in time $(t, t+h)$.

$n \cong h$

$D(t+h) = D(t)(1+r)^h \cong D(t)(1+rh)$ small h.

(b) $\dfrac{D(t+h)-D(t)}{h} = D(t)\cdot r$

(c) ✓

(4.23) $\dfrac{100}{1+r} + \dfrac{140}{(1+r)}$

$\{b_i\} = \{100, 140, 131\}$

$\{c_i\} = \{90, 160, 120\}$

$100+140+131 = 371$

$90+160+120 = 370$

$\{B_i\} = \{\cancel{}, 100, 240, 371\}$

$\{C_i\} = \{\cancel{}, 90, 250, 370\}$

$B_n = 371 \geq C_n = 370$ yes

$\{\sum_{i=1}^{k} B_i\} = \{100, 340, 711\}$

$\{\sum_{i=1}^{k} C_i\} = \{90, 340, 710\}$

Then

$$\sum_{i=1}^{n} b_i(1+r)^{-i} \geq \sum_{i=1}^{n} c_i(1+r)^{-i}$$

4.24

$$E = .3\left(\frac{10}{1+r}\right) + .7\left(\frac{40}{1+r}\right)$$

$$= \left(\frac{1}{1+r}\right)$$

$$P(r) = -a + \sum_{i=1}^{n} b_i (1+r)^{-i}$$

$$= -20 + \frac{.3(10)}{(1+r)} + \frac{.7(40)}{1+r} = -20 + \frac{3 + 2.8}{1+r}$$

$$P(r^*) = 0$$

$$\Rightarrow \quad \frac{5.8}{1+r} = 20$$

$$1+r = \frac{5.8}{20} \quad \ldots$$

4.25

$$PV = 1000 \cdot e^{-10(.08)} = \ldots$$

4.26

$$PV(r) = -1000 + \frac{800}{1+r} + \frac{\text{¢}}{(1+r)^2} = 0$$

$$\rightarrow \quad \frac{500}{(1+r)} + \frac{C}{(1+r)^2} = 1000 \quad ..$$

(4.27) $PV(r) = -1000 + \frac{C}{(1+r)} + \frac{500}{(1+r)^2}$

(4.28) Inflation - adjusted rate of return r_a

$$\frac{(1+r)x}{(1+r_i)} \qquad r_i \cong .04$$

$$x \longrightarrow \frac{(1+r)x}{(1+r_i)}$$

$$r_a = \frac{1+r}{1+r_i} - 1 = \frac{1+r-(1+r_i)}{1+r_i} = \frac{r-r_i}{1+r_i} \cong r - r_i$$

$$r_a = \cancel{\frac{.05 - .03}{1+.03}} \; \cancel{\frac{.08}{1.03} - 1} =$$

$$r_a = \frac{1+.05}{1+.03} - 1 = 1.94 \cdot 10^{-2}$$

Ex 4.29

$$P(r) = \sum_{i=0}^{n} c_i (1+r)^{-i}$$

$c_i < 0$

$c_n > 0$

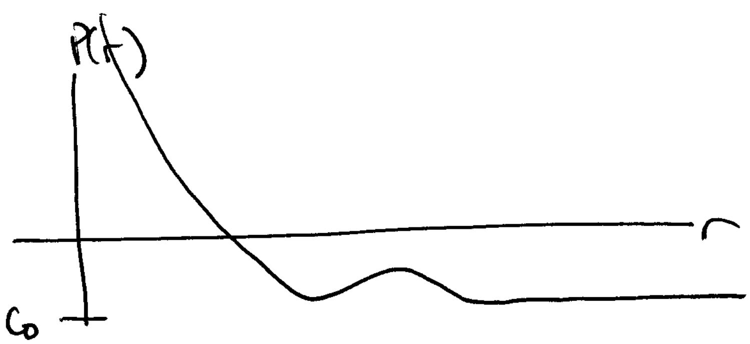
How show unique?

(a) $\exists$ sol $P(r) = 0$ $\quad r > -1$

$P(r=-\infty) = +\infty$ Because $c_n > 0$ $\Big\}$ $\therefore$ $P(r) = 0$.

$P(r=+\infty) = c_0 < 0$

(b) $c_0 + \dfrac{c_1}{1+r}$



How do?

4.30 $-\cancel{1000}$

$$PV(a) = \frac{-1000}{(1+.08)^1} + \frac{900}{(1+.05)^2} + \frac{800}{(1+.05)^3}$$

$$\frac{-1200}{(1+.08)^4} + \frac{700}{(1+.05)^5} = \dots$$

4.31 $r(t)$ non decreasing fn of t

$$\bar{r}(t) = \frac{1}{t}\int_0^t r(s)\,ds = $$ Max os

$$\cancel{\frac{d\bar{r}}{dt} = \frac{r(t)}{t} - \frac{1}{t^2}\int_0^t r(s)\,ds}$$

$$\bar{r}(t_1) < \bar{r}(t_2)$$

$$\frac{1}{t_1}\int_0^{t_1} r(s)\,ds < \frac{1}{t_2}\int_0^{t_2} r(s)\,ds$$

How ?

4.32 ?

4.33 ? ...

4.34 ...

4.35 ?

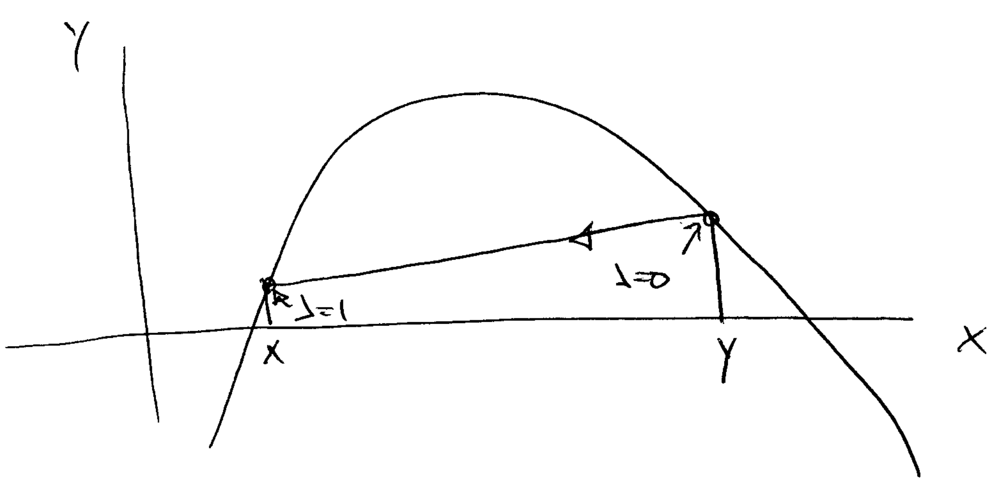
5.1 ?

5.2 ?

5.3 ?

5.4

5.25 $f(\lambda x + (1-\lambda)y) \geq \lambda f(x) + (1-\lambda) f(y) \qquad 0 \leq \lambda \leq 1$



If $g(x) \equiv -f(x)$ is convex then

$-f(\lambda x + (1-\lambda)y) \leq \lambda f(\ldots$

(6.1)

Outcome	Odds
1	1
2	2
3	5

$r_1 = 1$ v.s. -1

$r_2 = 2$ v.s. -1

$r_3 = 5$ v.s. -1

By Arbitrage Theorm

$$\sum_{j=1}^{m} P_j r_j(j) = 0$$

$$r_i(j) = \begin{cases} O_i & j = i \\ -1 & j \neq i \end{cases}$$

$$0 = E_p[r_i(X)] = O_i P_i + (-1)(1-p_i) \Rightarrow p_i = \frac{1}{1+O_i}$$

Not be arbitrage requires

$$\sum p_i = 1 = \sum \frac{1}{1+O_i}$$

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{4}$$

$$p_1 = \frac{1}{1+1} = \frac{1}{2}\ ;\ p_2 = \frac{1}{3}\ ;\ p_3 = \frac{1}{6}$$

$$\sum_i p_i = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1 \quad \checkmark$$

No betting results in a sure win

(6.2) $p_1 = \frac{1}{1+2} = \frac{1}{3}\ ;\ p_2 = \frac{1}{4}\ ;\ p_3 = \frac{1}{5}$ $\{60\}$

$$\sum_i p_i = 1 \Rightarrow p_4 = 1 - \sum_{i=1}^{3} p_i = 1 - \frac{1}{3} - \frac{1}{4} - \frac{1}{5}$$

$$\Rightarrow p_4 = 1 - \frac{20}{60} - \frac{15}{60} - \frac{12}{60}$$

$$= \frac{60-20-15-12}{60} = 40 - \quad \ldots$$

$$p_4 = \frac{1}{1+o_4} \Rightarrow o_4 = \ldots$$

6.3 $p_1 = \frac{1}{1+2} = \frac{1}{3}$; $p_2 = \frac{1}{3}$; $p_3 = \frac{1}{3}$

$\sum p_i = 1$ ✓ no arbitrage is possible

6.4

outcome	odds	p
1	1	1/2
2	2	1/3
3	3	1/6
(1,2)	?	
(1,3)	?	
(2,3)	?	

~~$p = \frac{1}{4}$~~

$\sum_{i=1}^{3} p_i = \cancel{\frac{3}{4} + \frac{1}{3}}$

$= \cancel{\frac{9+4}{12}} = \frac{13}{12}$ →←

$\frac{3}{6} + \frac{2}{6} + \frac{1}{6} = 1$ ✓

?

6.5 $E[x] = \sum_{i=1}^{n} E[x_i] \cdot r_i(j)$

6.6