$$k = \rho \left.\frac{dp}{d\rho}\right|_{\rho=\rho_0} = \rho_0 \frac{dp}{d\rho}(\rho_0)$$

Assum $k \equiv \rho \frac{dp}{d\rho}$ is constant

$$k \equiv \rho_0 \frac{dp}{d\rho}$$

$$\frac{k}{\rho_0} = \frac{dp}{d\rho} \Rightarrow p = p_0 + \frac{k}{\rho_0}(\rho - \rho_0) = p_0 + \frac{k}{\rho_0}(\cancel{\rho_0} + \rho_0 s - \cancel{\rho_0})$$

$$= p_0 + ks \qquad \text{eq } 1.42$$

— — —

$$u = -\frac{\partial \phi}{\partial x}$$

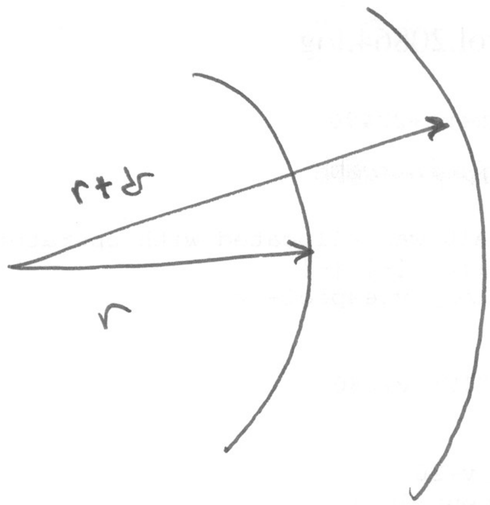
$$u = -c^2 \frac{\partial}{\partial x} \int_0^t s\,dt + u_0$$

$$-\frac{\partial \phi}{\partial x} = -c^2 \frac{\partial}{\partial x} \int_0^t s\,dt + u_0$$

$$\frac{\partial \phi}{\partial x} = c^2 \frac{\partial}{\partial x} \int_0^t s\,dt + u_0$$

$$\phi = c^2 \int_0^t s\,dt + \underbrace{u_0 x}_{\equiv \phi_0} \Rightarrow \phi = c^2 \int_0^t s\,dt + \phi_0$$

$$\frac{\partial \phi}{\partial t} = c^2 s \qquad \text{eq } 1.80$$



Mass gain at rate = ? ~~$4\pi r^2 \rho\, dr$~~

$u_r > 0$ for flow outward

$$4\pi r^2 \rho u_r \Big|_{r=r+dr} - 4\pi r^2 \rho u_r \Big|_{r=r} = \text{Mass flow } \underline{\text{into}} \text{ spherical shell.}$$

due to velocity of fluid per unit time

$$= \frac{\partial}{\partial r}\left(4\pi r^2 \rho \frac{\partial\phi}{\partial r}\right)\cdot dr \qquad \text{By calc}$$

Rate of increase of density $= \frac{\partial\rho}{\partial t} 4\pi r^2 dr$

$$\frac{\partial\rho}{\partial t}\cdot r^2 = \frac{\partial}{\partial r}\left(r^2 \rho \frac{\partial\phi}{\partial r}\right) \qquad \text{eq } 1.81$$

$\rho = \rho_0(1+s)$

$$\frac{\partial\rho}{\partial t} = \rho_0 \frac{\partial s}{\partial t} \Rightarrow \cancel{\rho_0}\frac{\partial s}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \rho \frac{\partial\phi}{\partial r}\right) \cong \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \cancel{\rho_0} \frac{\partial\phi}{\partial r}\right)$$

$$\Rightarrow \frac{\partial s}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial\phi}{\partial r}\right) \qquad \text{eq } 1.82$$

$$\phi r = A(ct-r)$$

$$c^2 s = \frac{\partial\phi}{dt} = \frac{1}{r}$$

$$\int_a^b sr\,dr = \left[-\frac{1}{c}A(ct-r)\right]\Big|_a^b = 0$$ by taking a + b to be outside region of support of ~~p~~ radial pressure wave

$$\Rightarrow \int_a^b sr\,dr = 0 \quad \rightarrow$$ S cannot be of one sign only.

Q: Numerical methods based on Huygens Principle?

$J_- = U - 2\sqrt{gh} = \text{const} \Rightarrow$ line in $(U, +2\sqrt{gh})$ plan

$q_e = U_e h_e$ is specified in hodograph plan

$\Rightarrow U - \cancel{4}\sqrt{gh} = \frac{q_e}{h} \cancel{\times}\, 2\sqrt{gh}$

plot $Uh = \text{constant}$

$\Rightarrow \frac{U(2\sqrt{gh})^2}{4g} = \text{constant}$

$\Rightarrow U(2\sqrt{gh})^2 = 4gC \quad \propto \quad xy^2 = C_2$

End wall $U=0$

$\Rightarrow J_{+e} = \cancel{U}^{0} + 2\sqrt{gh} = 2\sqrt{gh_e}$ h_e determined from J_{+e}

Iteration method: (1) position the (x,t) location of the grid of characteristics
(2) Solve to update the values in the cells maintaining consistency in invariants
(3) Re position the (x,t) characteristic intersections.

Get real reference Crandell 1956 pg 400

* Crandell, S. H. Engineering analysis N.Y. McGraw Hill 1956

$$j = \rho u_1 h_1 = \rho u_2 h_2 \quad \Rightarrow \quad u_2 = \frac{u_1 h_1}{h_2}$$ put in below:

$$+ \quad \frac{1}{2}\rho g h_1^2 + \rho u_1^2 h_1 = \frac{1}{2}\rho g h_2^2 + \rho u_2^2 h_2 \quad *$$

$$\frac{1}{2}\cancel{\rho} g h_1^2 + \cancel{\rho} u_1^2 h_1 = \frac{1}{2}\cancel{\rho} g h_2^2 + \cancel{\rho} h_2\left(\frac{u_1^2 h_1^2}{h_2^2}\right)$$

$$\frac{1}{2} g h_1^2 + u_1^2 h_1 = \frac{1}{2} g h_2^2 + \frac{u_1^2 h_1^2}{h_2}$$

$$h_1\left[1 - \frac{h_1}{h_2}\right] u_1^2 = \frac{1}{2} g (h_2^2 - h_1^2)$$

$$u_1^2 = \frac{\frac{1}{2} g (h_1+h_2)\cancel{(h_2 - h_1)}}{\frac{h_1}{h_2}\cancel{(h_2 - h_1)}} = \frac{g h_2 (h_1 + h_2)}{2h_1} \quad \text{eq } 4.27 \ \checkmark$$

$+$ From eq $*$ $\Rightarrow$

$$\frac{1}{2} g h_1^2 + \frac{g h_2 (h_1+h_2)}{2\cancel{h_1}}\cancel{h_1} = \frac{1}{2} g h_2^2 + u_2^2 h_2$$

$$\frac{1}{2} g (h_1^2 - h_2^2) + \frac{g h_1 h_2}{2} + \frac{g h_2^2}{2} = + u_2^2 h_2$$

$$\frac{g h_1 (h_1 + h_2)}{h_2} = u_2^2 \quad \text{eq } 4.27 \ \checkmark$$

$$h = h_0 + h_1 e^{\alpha x} e^{\beta t}$$
$$u = u_0 + u_1 e^{\alpha x} e^{\beta t}$$

(4.3) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} = 0$$

(4.6) $$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial x} + h\frac{\partial u}{\partial x} = 0$$

$$\Rightarrow \quad u_1 \beta \cancel{e^{\alpha x}} \cancel{e^{\beta t}} + (u_0 + u_1 e^{\alpha x} e^{\beta t}) u_1 \alpha \cancel{e^{\alpha x}} \cancel{e^{\beta t}} + g(h_1 \alpha \cancel{e^{\alpha x}} \cancel{e^{\beta t}}) = 0$$

$$+ \quad h_1 \beta \cancel{e^{\alpha x}} \cancel{e^{\beta t}} + (u_0 + u_1 \cancel{e^{\alpha x}} \cancel{e^{\beta t}}) h_1 \alpha \cancel{e^{\alpha x}} \cancel{e^{\beta t}} + (h_0 + h_1 e^{\alpha x} e^{\beta t}) u_1 \alpha \cancel{e^{\alpha x}} \cancel{e^{\beta t}} = 0$$

$$\Rightarrow \quad u_1 \beta + u_0 u_1 \alpha + g h_1 \alpha = 0$$
$$h_1 \beta + u_0 h_1 \alpha + h_0 u_1 \alpha = 0$$

$$\begin{bmatrix} (u_0\alpha + \beta) & \alpha g \\ \alpha h_0 & u_0\alpha + \beta \end{bmatrix} \begin{bmatrix} u_1 \\ h_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ ✓

$$\Rightarrow \quad (u_0\alpha + \beta)^2 - \alpha^2 h_0 g = 0$$

$$\beta + u_0 \alpha = \pm |\alpha| \sqrt{g h_0} = \pm \alpha \sqrt{g h_0}$$

$$\beta = \alpha(-u_0 \pm \sqrt{g h_0}) = -\alpha(u_0 \pm \sqrt{g h_0})$$

$$\alpha = -\frac{\beta}{\dot{X}_\pm}$$

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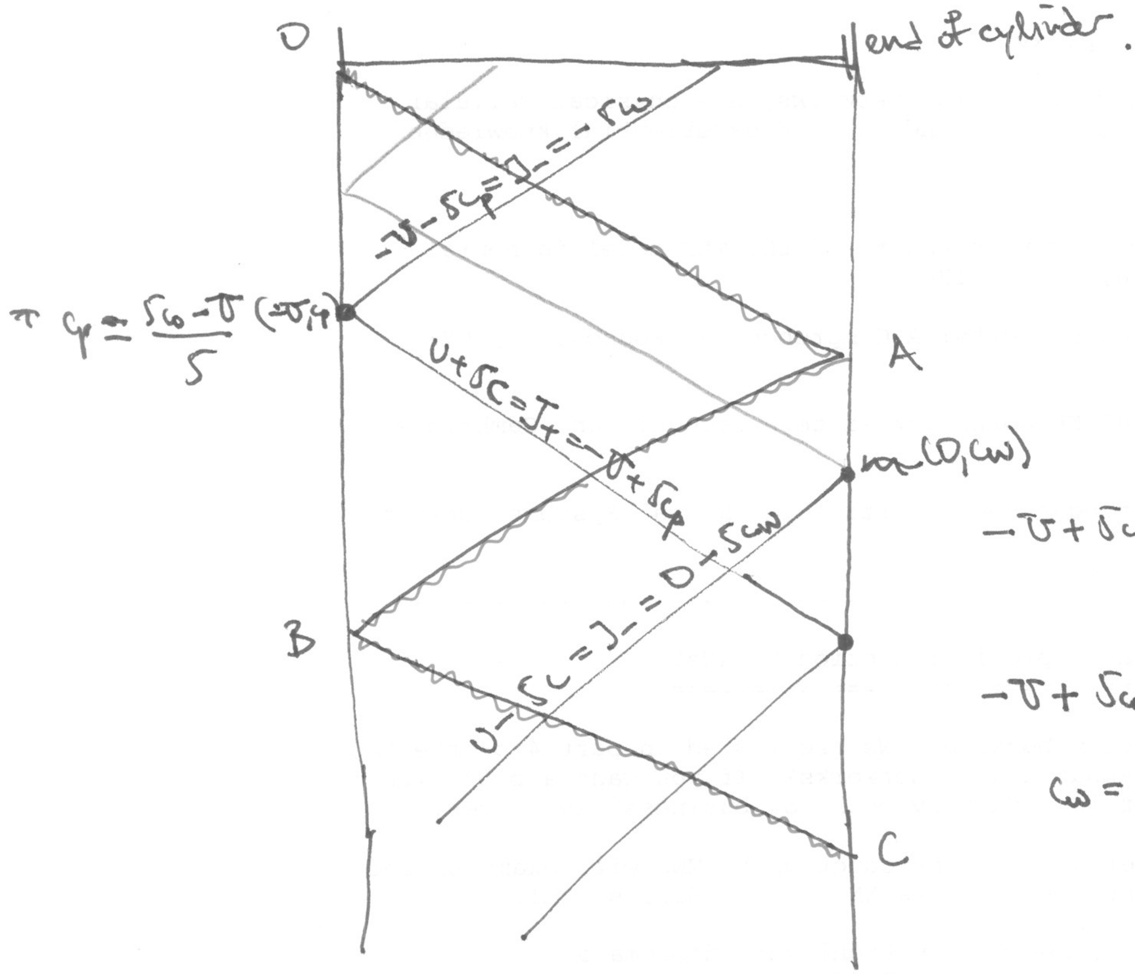
$$u + \delta c = -v + \delta c_p$$
$$u - \delta c = -v + \delta c_p$$

$\Rightarrow$ ~~...~~c_p $\quad 2u = -2v \Rightarrow u = -v$

~~...~~ $\quad$ then $\quad -v + \delta c = -v + \delta c_p$

$\Rightarrow c = c_p$

Think characteristic picture looks like



$$-v + \delta c_p = u_w + \delta c_w$$
$$= \delta c_w$$
$$-v + \delta c_0 - v = \delta c_w$$
$$c_w = c_0 - \frac{2}{\delta} v$$

$$j = \rho_1 u_1 = \rho_2 u_2$$

$$= \frac{u_1}{v_1} = \frac{u_2}{v_2} \Rightarrow u_1 = jv_1 \ \& \ u_2 = jv_2 \quad \text{eq } 5.33$$

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

$$\Rightarrow p_1 + \frac{1}{v_1} j^2 v_1^2 = p_2 + \frac{1}{v_2} j^2 v_2^2$$

$$\Rightarrow p_1 + v_1 j^2 = p_2 + v_2 j^2$$

$$\Rightarrow j^2 = \frac{p_2 - p_1}{v_1 - v_2} \quad \text{eq } 5.34$$

$$(v_1 - v_2) j^2 = p_2 - p_1$$

$$v_1 j^2 - v_2 j^2 = p_2 - p_1$$

$$\Rightarrow j u_1 - j u_2 = p_2 - p_1$$

$$\Rightarrow u_1 - u_2 = \frac{p_2 - p_1}{j} = (p_2 - p_1) \cdot \sqrt{\frac{v_1 - v_2}{p_2 - p_1}} = \sqrt{(p_2 - p_1)(v_1 - v_2)}$$

eq 5.35.

$$\tfrac{1}{2}u_1^2 + w_1 = \tfrac{1}{2}u_2^2 + w_2 \qquad w = e + \frac{P}{\rho}$$

$$w_1 + \tfrac{1}{2}\dot{j}^2 V_1^2 = \cancel{\tfrac{1}{2}}\, w_2 + \tfrac{1}{2}\dot{j}^2 V_2^2 \qquad \text{eq } 5.37$$

$$w_1 - w_2 = \tfrac{1}{2}\dot{j}^2 (V_2^2 - V_1^2) = \tfrac{1}{2}\dot{j}^2 (V_2+V_1)(V_2-V_1)$$

$$= \tfrac{1}{2}\left(\frac{P_2-P_1}{V_1-\cancel{V_2}}\right)(V_2+V_1)(\cancel{V_2-V_1})(-1)$$

$$= \tfrac{1}{2}(V_1+V_2)(P_1-P_2) \qquad \text{eq } 5.39$$

$$e_1 - e_2 + P_1V_1 - P_2V_2 = \cdots$$

$$e_1 - e_2 = -P_1V_1 + P_2V_2 + \tfrac{1}{2}\left[V_1P_1 - V_1P_2 + V_2P_1 - V_2P_2\right]$$

$$= \tfrac{1}{2}\left[-V_1P_1 - V_1P_2 + V_2P_1 + V_2P_2\right]$$

$$= -\tfrac{1}{2}\left[V_1P_1 + V_1P_2 - V_2P_1 - V_2P_2\right]$$

$$= -\tfrac{1}{2}(V_1 - V_2)(P_1 + P_2) = \tfrac{1}{2}(V_2 - V_1)(P_1 + P_2) \qquad \text{eq } 5.40 \checkmark$$

$$P = A\rho^{\gamma} \qquad e = C_v T$$

$$P \cdot V = \frac{P}{\rho} = A\rho^{\gamma-1} \qquad PV = nRT \Rightarrow T = \left(\frac{P}{\rho}\right)\frac{1}{nR}$$

$$\Rightarrow e = \left(\frac{C_v}{nR}\right)\frac{P}{\rho}$$

w/ $n=1$ $\quad e = \left(\frac{C_v}{R}\right)\frac{P}{\rho}$

$$C_p - C_v = R \qquad \text{+} \quad \frac{C_p}{C_v} = \gamma$$

$$\text{+} \quad \frac{C_v}{R} = \frac{1}{\gamma-1} \qquad \frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

$$\Rightarrow \frac{C_v}{R} = \frac{1}{\gamma-1}$$

$\therefore$

$$w = \frac{1}{\gamma-1}\frac{P}{\rho} + \frac{P}{\rho}$$

$$= \left(\frac{\gamma-1+1}{\gamma-1}\right)\frac{P}{\rho} = \left(\frac{\gamma}{\gamma-1}\right)\frac{P}{\rho} \quad \text{eq } 5.41$$

Then

$$\frac{\gamma}{\gamma-1} P_1 V_1 - \frac{\gamma}{\gamma-1} P_2 V_2 = \frac{1}{2}(V_1+V_2)(P_1-P_2)$$

$$\frac{\gamma}{\gamma-1} P_1 - \frac{\gamma}{\gamma-1} P_2\left(\frac{V_2}{V_1}\right) = \frac{1}{2}\left(1+\frac{V_2}{V_1}\right)(P_1-P_2)$$

$$\left(-\frac{1}{2}(P_1-P_2) - \frac{\gamma}{\gamma-1}P_2\right)\left(\frac{V_2}{V_1}\right) = \frac{P_1-P_2}{2} - \frac{\gamma}{\gamma-1}P_1$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{\frac{P_1-P_2}{2} - \frac{\gamma}{\gamma-1}P_1}{-\frac{(P_1-P_2)}{2} - \frac{\gamma}{\gamma-1}P_2} \cdot \frac{2(\gamma-1)}{2(\gamma-1)}$$

$$= \frac{(\gamma-1)(P_1-P_2) - 2\gamma P_1}{-(P_1-P_2)(\gamma-1) - 2\gamma P_2} = \frac{\gamma P_1 - \gamma P_2 - P_1 + P_2 - 2\gamma P_1}{-\gamma P_1 + P_1 + \gamma P_2 - P_2 - 2\gamma P_2}$$

$$= \frac{-\gamma P_1 - \gamma P_2 - P_1 + P_2}{-\gamma P_1 + P_1 - \gamma P_2 - P_2}$$

$$= \frac{-(\gamma+1)P_1 \cancel{+} -(\gamma-1)P_2}{-(\gamma-1)P_1 - \cancel{\gamma}(\gamma+1)P_2} = \frac{(\gamma+1)P_1 + (\gamma-1)P_2}{(\gamma-1)P_1 + (\gamma+1)P_2} \quad \text{eq } 5.42 \checkmark$$

Then $j^2 = ? = \dfrac{P_2 - P_1}{V_1 - V_2} = \dfrac{1}{V_1}\dfrac{P_2-P_1}{\left(1 - \frac{V_2}{V_1}\right)}$

$$= \cancel{\frac{1}{V_1}\frac{(P_2 - P_1)}{}} \quad = \frac{1}{V_1}\frac{(P_2-P_1)}{\left[\frac{(\cancel{\gamma}-1)P_1 + (\cancel{\gamma}+1)P_2 - (\cancel{\gamma}+1)P_1 - (\cancel{\gamma}-1)P_2}{(\gamma-1)P_1 + (\gamma+1)P_2}\right]}$$

$$= \frac{1}{V_1}\frac{(P_2-P_1)}{\left(\frac{(-2P_1 + 2P_2)}{(\gamma-1)P_1 + (\gamma+1)P_2}\right)} = \frac{1}{2V_1}\left((\gamma-1)P_1 + (\gamma+1)P_2\right) \quad \text{eq } 5.43$$

04-07-03 r

$$\dot{j}^2 v_1^2 = \cancel{\frac{v_1}{2}}\cancel{f}(\cdots\cdots \cancel{\frac{v_1}{2}}\cancel{f}(r_1-\cdot$$

$$= \frac{v_1}{2}((r-1)p_1 + (r+1)p_2)$$

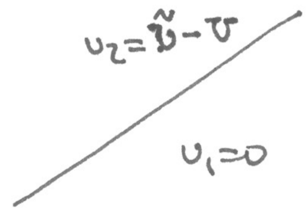
$$u^2 = \quad ''$$

Sim

$$\dot{j}^2 v_2^2 = \frac{v_2^2}{2v_1}((r-1)p_1 + (r+1)p_2)$$

$$= \frac{v_2}{2}((r+1)p_1 + (r-1)p_2) = \frac{v_1}{2}\frac{((r+1)p_1+(r-1)p_2)^2}{((r-1)p_1+(r+1)p_2)} \quad \text{eq } 5.44$$

eq 5.45 is eq 5.35 w/ $u_1 = 0$



$$u_2 = -U$$

$$U = \sqrt{(p_2-p_1)(v_1-v_2)} = \sqrt{(p_2-p_1)\,v_1\left(1-\frac{v_2}{v_1}\right)}$$

$$= \sqrt{(p_2-p_1)\,v_1\left(\frac{+2(p_2-p_1)}{(\gamma-1)p_1+(\gamma+1)p_2}\right)} = (p_2-p_1)\sqrt{\frac{2v_1}{(\gamma-1)p_1+(\gamma+1)p_2}}$$

eq 5.45

$\frac{p_2}{p_1}$ = How get?

$\left(\frac{dx}{dt}\right)_{shock}$ = How   Upert + Cpa

$$\begin{bmatrix} c^2-u^2 & -uv & -uv & c^2-v^2 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ dx & dy & 0 & 0 & du \\ 0 & 0 & dx & dy & dv \end{bmatrix}$$

determinant of 1st 4 columns

$$(c^2-u^2)\begin{vmatrix} 1 & -1 & 0 \\ dy & 0 & 0 \\ 0 & dx & dy \end{vmatrix} + dx \begin{vmatrix} -uv & -uv & c^2-v^2 \\ 1 & -1 & 0 \\ 0 & dx & dy \end{vmatrix} = 0$$

$$= (c^2-u^2)\left[1\cdot 0 - dy(-dy)\right]$$

$$+ dx\left[-uv(-dy) - 1\left(-uv\,dy - dx(c^2-v^2)\right)\right] = 0$$

$$= (c^2-u^2)(dy\,dy) + uv\,dx\,dy + uv\,dx\,dy + (c^2-v^2)\,dx\,dx = 0$$

$$\Rightarrow (c^2-u^2)\left(\frac{dy}{dx}\right)^2 + 2uv\left(\frac{dy}{dx}\right) + (c^2-v^2) = 0 \quad \checkmark$$

$$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{-2uv \pm \sqrt{4u^2v^2 - 4(c^2-u^2)(c^2-v^2)}}{2(c^2-u^2)}$$

$$\frac{dy}{dx} = \frac{-uv \pm \sqrt{u^2v^2 - (c^2-u^2)(c^2-v^2)}}{c^2-u^2}$$

$$= \frac{-uv \pm \sqrt{\cancel{u^2v^2} - c^4 + c^2v^2 + c^2u^2 - \cancel{u^2v^2}}}{c^2-u^2}$$

$$= \frac{-uv \pm \sqrt{c^2(u^2+v^2-c^2)}}{c^2-u^2}$$

$$= \frac{-uv \pm c\sqrt{u^2+v^2-c^2}}{c^2-u^2} \qquad \text{eq } 5.50 \ \checkmark$$

$$\left(\frac{dy}{dx}\right)_+ \cdot \left(\frac{dy}{dx}\right)_- = \left(\frac{-uv + c\sqrt{u^2+v^2-c^2}}{c^2-u^2}\right)\left(\frac{-uv - c\sqrt{u^2+v^2-c^2}}{c^2-u^2}\right)$$

$$= \frac{u^2v^2 - c^2(u^2+v^2-c^2)}{(c^2-u^2)^2}$$

$$= \frac{(uv)^2 - c^2u^2 - c^2v^2 + c^4}{(c^2-u^2)^2}$$

$$= \frac{(u^2-c^2)(v^2-c^2)}{(c^2-u^2)(c^2-u^2)} = +\frac{(v^2-c^2)}{(u^2-c^2)} \ \checkmark$$

For the Riemann differential eq for:

$$\begin{bmatrix} c^2-u^2 & -uv & -uv & c^2-v^2 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ dx & dy & 0 & 0 & du \\ 0 & 0 & dx & dy & dv \end{bmatrix}$$

elliminate a column + take determinate. drop 2 col.

$$\begin{vmatrix} c^2-u^2 & -uv & c^2-v^2 & 0 \\ 0 & -1 & 0 & 0 \\ dx & 0 & 0 & du \\ 0 & dx & dy & dv \end{vmatrix} = 0$$

$$(c^2-u^2)\begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & du \\ dx & dy & dv \end{vmatrix} + dx \begin{vmatrix} -uv & c^2-v^2 & 0 \\ -1 & 0 & 0 \\ dx & dy & dv \end{vmatrix} = 0$$

$$(c^2-u^2)(-1)(-dy\,du) + dx\,1\,(c^2-v^2)\,dv = 0$$

$$(c^2-u^2)\,du\,dy + (c^2-v^2)\,dv\,dx = 0$$

$$(c^2-v^2)\frac{dv}{du} + (c^2-u^2)\frac{dy}{dx} = 0$$

$$\frac{dv}{du} = -\frac{(c^2-u^2)}{(c^2-v^2)}\left(\frac{dy}{dx}\right)$$ eq 5.52 ✓

What if I eliminate the 3rd column

$$\begin{vmatrix} c^2-u^2 & -uv & c^2-v^2 & 0 \\ 0 & 1 & 0 & 0 \\ dx & dy & 0 & du \\ 0 & 0 & dy & dv \end{vmatrix} = 0$$

see pg 6 ...

What if I eliminate the 1st column (seems to be the most complicated ...)

$$\begin{vmatrix} -uv & -uv & c^2-v^2 & 0 \\ 1 & -1 & 0 & 0 \\ dy & 0 & 0 & du \\ 0 & dx & dy & dv \end{vmatrix} = 0$$

$$dy \begin{vmatrix} -uv & c^2-v^2 & 0 \\ -1 & 0 & 0 \\ dx & dy & dv \end{vmatrix} - du \begin{vmatrix} -uv & -uv & c^2-v^2 \\ 1 & -1 & 0 \\ 0 & dx & dy \end{vmatrix} = 0$$

$$\rightarrow dv\,dy(c^2-v^2) - du(-1)(-uv\,dy - dx(c^2-v^2))$$

$$-du(-1)(-uv\,dy) = 0$$

$$\rightarrow dv\,dy(c^2-v^2) - \underline{du\,dy\,uv} - du\,dx(c^2-v^2) - \underline{du\,dy\,uv} = 0$$

$$dv\,dy(c^2-v^2) - 2du\,dy\,uv - du\,dx(c^2-v^2) = 0$$

$$\Rightarrow \quad \frac{dv}{dw}\cdot dy(c^2-v^2) - 2wv\,dy - dx(c^2-v^2) = 0$$

$$\Rightarrow \quad \frac{dv}{dw}(c^2-v^2) - 2wv - \frac{dx}{dy}(c^2-v^2) = 0$$

$$\frac{dv}{dw} = \frac{2wv}{c^2-v^2} + \frac{dx}{dy}$$

$$= \frac{\cancel{2wv}}{c^2-v^2}$$

$$\left(\frac{dv}{dw}\right)_+ = \frac{2wv}{c^2-v^2} + \left(\frac{dx}{dy}\right)_+ = \frac{2wv}{c^2-v^2} + \cancel{\left(c^2-v^2\right)}$$

$$\cancel{\left(\frac{dx}{dy}\right)_+ = \left[\left(\frac{c^2-v^2}{c^2-v^2}\right)\right]} \qquad \left(\frac{dy}{dx}\right)$$

Why is this not giving the same result?

$$(1)\quad \begin{vmatrix} c^2-u^2 & c^2-v^2 & 0 \\ dx & 0 & du \\ 0 & dy & dv \end{vmatrix} = 0$$

$$\Rightarrow -dx\left(dv\,(c^2-v^2)\right) - du\left(dy\,(c^2-u^2)\right) = 0$$

$$= -\frac{dv}{du}\cdot(c^2-v^2) = (c^2-u^2)\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{du} = -\frac{(c^2-u^2)}{(c^2-v^2)}\frac{dy}{dx}$$

same as 5.52 ✓

$$\left(\frac{dv}{du}\right)_+ = -\frac{1}{(dy/dx)_-}$$

$$\left(\frac{dv}{du}\right)_- = -\frac{1}{(dy/dx)_+}$$

$$\left(\frac{dy}{dx}\right)_\pm = \frac{-uv \pm c\sqrt{(u^2+v^2)-c^2}}{(c^2-u^2)}$$

$$q^2 + \frac{2}{\gamma-1}c^2 = q^{*2} = \text{constant}$$

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$$\phi = \frac{A(ct-r)}{r} + \cancel{\frac{B(ct+r)}{r}}^{\,0}$$

$$q = -\frac{\partial\phi}{\partial r} = -\left[-\frac{1}{r^2}A(ct-r) + \frac{1}{r}\frac{\partial A}{\partial \xi}(ct-r)(-1) \right]$$

$$= \frac{1}{r^2}A(ct-r) + \frac{1}{r}\frac{\partial A}{\partial \xi}(ct-r) \qquad \text{eq } 5.18$$

$$p' = -\frac{\rho_0}{r}\frac{\partial}{\partial t}A(ct-r)$$

$$* \quad \frac{\partial R}{\partial t} = \frac{1}{R^2}A(ct-r) - \frac{1}{R}\frac{\partial}{\partial r}A(ct-r) \qquad \text{sign mistake?}$$

$$R = \alpha ct \qquad \omega = r - ct$$

$$\omega = R - ct = \alpha ct - ct = (\alpha-1)ct \qquad \Rightarrow ct = \frac{\omega}{\alpha-1}$$

Then eq * becomes

$$\alpha c = \frac{1}{(\alpha ct)^2}A(-\omega) - \frac{1}{(\alpha ct)}\frac{\partial}{\partial r}(-\omega)$$

$$\Rightarrow \alpha c = \frac{(\alpha-1)^2}{\alpha^2\omega^2}A(-\omega) - \frac{\alpha-1}{\alpha\omega}\frac{\partial A(-\omega)}{\partial r}$$

$$A(-\omega) = \frac{c\alpha^3}{1-\alpha^2}\omega^2 + a(\omega)^{(\alpha-1)/\alpha}$$

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$$\left(\frac{\partial}{\partial t} + \frac{r}{t}\frac{\partial}{\partial r}\right)(q, p, \rho) = 0$$

$$\frac{\partial \rho}{\partial t} + q\frac{\partial \rho}{\partial r} + \rho\left(\frac{\partial q}{\partial r} + \frac{2q}{r}\right) = 0 \qquad \dot{r} \equiv \frac{r}{t}$$

$$\frac{q - \dot{r}}{\rho}\frac{d\rho}{d\dot{r}} + \frac{dq}{d\dot{r}} + \frac{2q}{\dot{r}} = 0$$

$$(q - \dot{r})\frac{dq}{d\dot{r}} = -\frac{1}{\rho}\frac{dp}{d\dot{r}}$$

$$\frac{p}{\rho^{\gamma}} = \text{const} \qquad c = \frac{dp}{d\rho} = \frac{\gamma p}{\rho}$$

$$\frac{1}{\rho}\frac{d\rho}{d\dot{r}} = -\frac{(q-\dot{r})}{c^2}\frac{dq}{d\dot{r}}$$

into eq 5.67 gives

$$-\frac{(q-\dot{r})^2}{c^2}\frac{dq}{d\dot{r}} + \frac{dq}{d\dot{r}} + \frac{2q}{\dot{r}} = 0 \qquad \text{eq } 5.70 \checkmark$$

$$Q = \frac{q}{c} \qquad X = \frac{\dot{r}}{c}$$

$$dq = c\,dQ$$

$$d\dot{r} = c\,dX$$

Then we get

$$\text{Eq} \quad -\frac{(cQ - c\bar{X})^2}{c^2}\frac{dQ}{d\bar{X}} + \frac{dQ}{d\bar{X}} + \frac{2Q}{\bar{X}} = 0$$

$$\Rightarrow \quad -(Q-\bar{X})^2\frac{dQ}{d\bar{X}} + \frac{dQ}{d\bar{X}} + \frac{2Q}{\bar{X}} = 0$$

$$\Rightarrow \quad \frac{dQ}{d\bar{X}}\left[1-(Q-\bar{X})^2\right] = -2Q/\bar{X}$$

$$\frac{dQ}{d\bar{X}} = \frac{2Q}{\bar{X}((Q-\bar{X})^2-1)} \qquad \text{eq } 5.71$$

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$$\begin{vmatrix} \cot\phi & 0 & 2q & 0 \\ 0 & \cot\phi & 0 & -2q \\ ds_+ & ds_- & 0 & 0 \\ 0 & 0 & ds_+ & ds_- \end{vmatrix} = 0$$

$$\Rightarrow \cot\phi \begin{vmatrix} \cot\phi & 0 & -2q \\ ds_- & 0 & 0 \\ 0 & ds_+ & ds_- \end{vmatrix} + ds_+ \begin{vmatrix} 0 & 2q & 0 \\ \cot\phi & 0 & -2q \\ 0 & ds_+ & ds_- \end{vmatrix} = 0$$

$$\Rightarrow \cot\phi(-ds_-) \begin{vmatrix} 0 & -2q \\ ds_+ & ds_- \end{vmatrix} + ds_+(-\cot\phi) \begin{vmatrix} 2q & 0 \\ ds_+ & ds_- \end{vmatrix} = 0$$

$\div$ by $\cot\phi$

$$(-ds_-)(2q\, ds_+) - ds_+(2q\, ds_-) = 0$$

$$-4q\, ds_-\, ds_+ = 0$$

$$\Rightarrow \quad ds_-\, ds_+ = 0 \quad ✓$$

$$\begin{vmatrix} \cot\phi & 0 & 2q & 0 \\ 0 & \cot\phi & 0 & 0 \\ ds_+ & ds_- & 0 & dq \\ 0 & 0 & ds_+ & d\psi \end{vmatrix} = 0$$

This expression ~~was~~ was chosen because it results in an expression indep. of ds_-

$$\Rightarrow (\cot\phi) \begin{vmatrix} \cot\phi & 2q & 0 \\ ds_+ & 0 & dq \\ 0 & ds_+ & d\psi \end{vmatrix} = 0$$

$$\Rightarrow (\cancel{\cot\phi})(\cot\phi)(-dq\, ds_+) - \cancel{\cot\phi}\,(ds_+)(2q\, d\psi) = 0$$

$\div$ by $\cot\phi$

$$-\cot\phi\, dq\, ds_+ - ds_+ 2q\, d\psi = 0 \qquad \div \text{ by } ds_+$$

$$\Rightarrow \quad -\cot\phi\, dq - 2q\, d\psi = 0$$

$$= \quad \cot\phi \frac{dq}{q} + 2 d\psi = 0$$

$$\Rightarrow \quad \cot\phi \ln q + 2\psi = \text{const. along } ds_+$$

Similarly: consider

$$\begin{vmatrix} \cot\phi & 0 & 0 & 0 \\ 0 & \cot\phi & -2q & 0 \\ ds_+ & ds_- & 0 & dq \\ 0 & 0 & ds_- & d\psi \end{vmatrix} = 0$$

expand about 1st row:

$$\Rightarrow \quad \cancel{\cot\phi} \begin{vmatrix} \cot\phi & -2q & 0 \\ ds_- & 0 & dq \\ 0 & ds_- & d\psi \end{vmatrix} = 0$$

$$\Rightarrow \quad \cot\phi\,(-ds_-)\,dq - ds_-(-2q\,d\psi) = 0$$

÷ by ds_-

$$-\cot\phi\,dq + 2q\,d\psi = 0$$

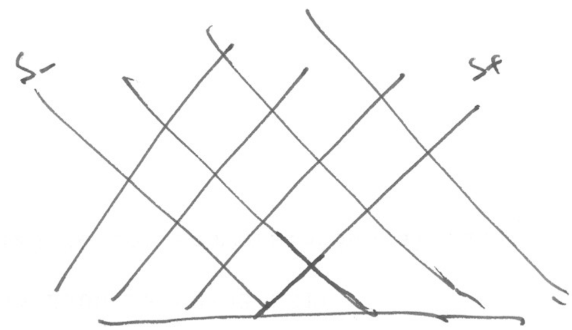
$$-\frac{1}{2}\cot\phi\,\frac{dq}{q} + d\psi = 0$$

$$\frac{1}{2}\cot\phi\,\frac{dq}{q} - d\psi = 0$$

$$\frac{1}{2}\cot\phi\,\ln q - \psi = \cancel{\text{Const}(s_+)}$$

$$\cancel{C(s_+)}$$

$$C(s_-)$$



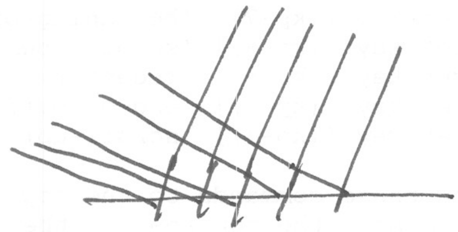
$J_\pm = a \pm 2b$

Assum J_+ constat $= a + 2b$ $\Rightarrow$ $a = A(b)$ + $b = B(a)$

$J_- = a - 2b$

$= a - 2B(a)$

$= A(b) - 2b$



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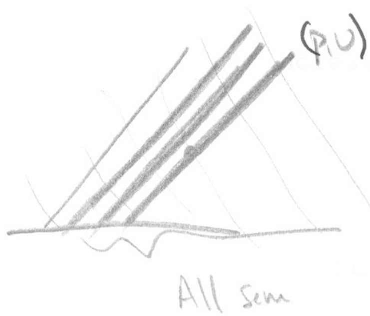
$a - 2b = c$



(p, U) take on constant values on each characteristic

$J_+(i) = a + 2b$ $J_- = \text{const} = a - 2b$

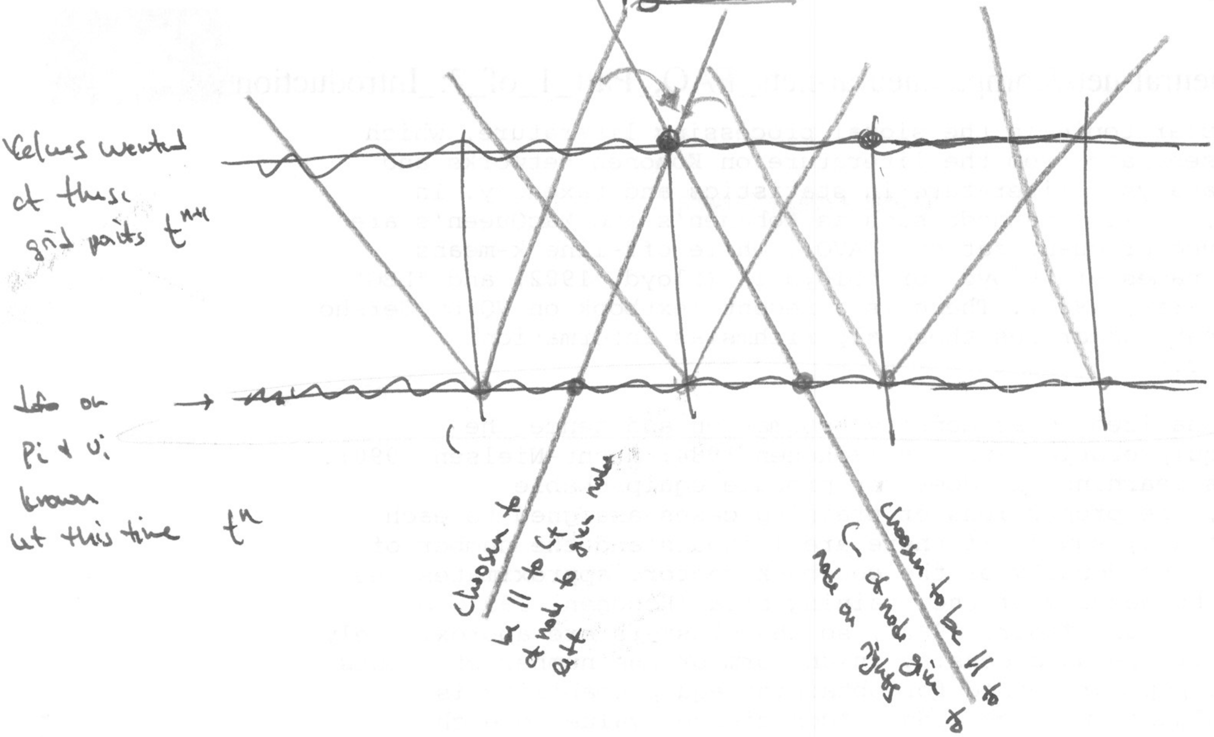
$\Rightarrow a = A(b)$
or $b = B(a)$ } throughout simple wave region

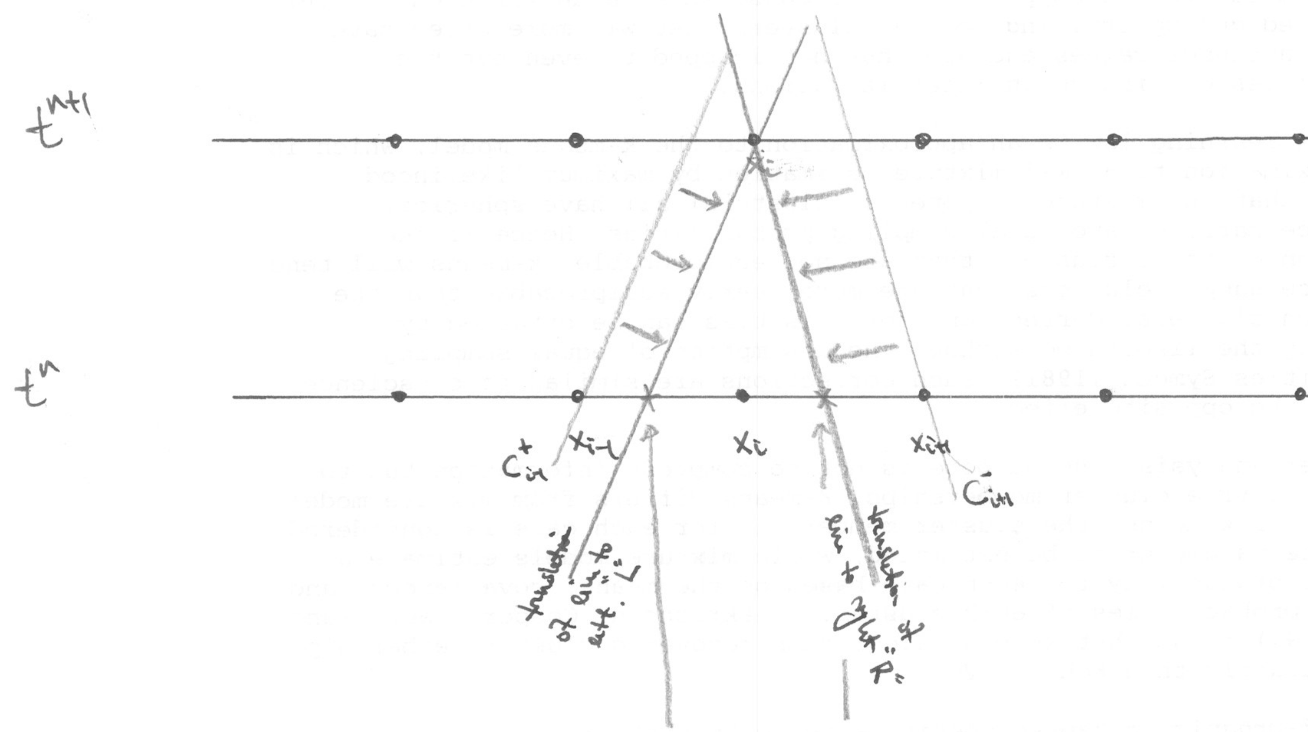
$J_+(i) = A(b) + 2b$

$J_+(i) = a + 2B(a)$

since $\forall$ C_+ characteristic $J_+(i)$ is known or at least a constant $a \therefore b$ or $b \therefore a$ is

constant along each characteristic.





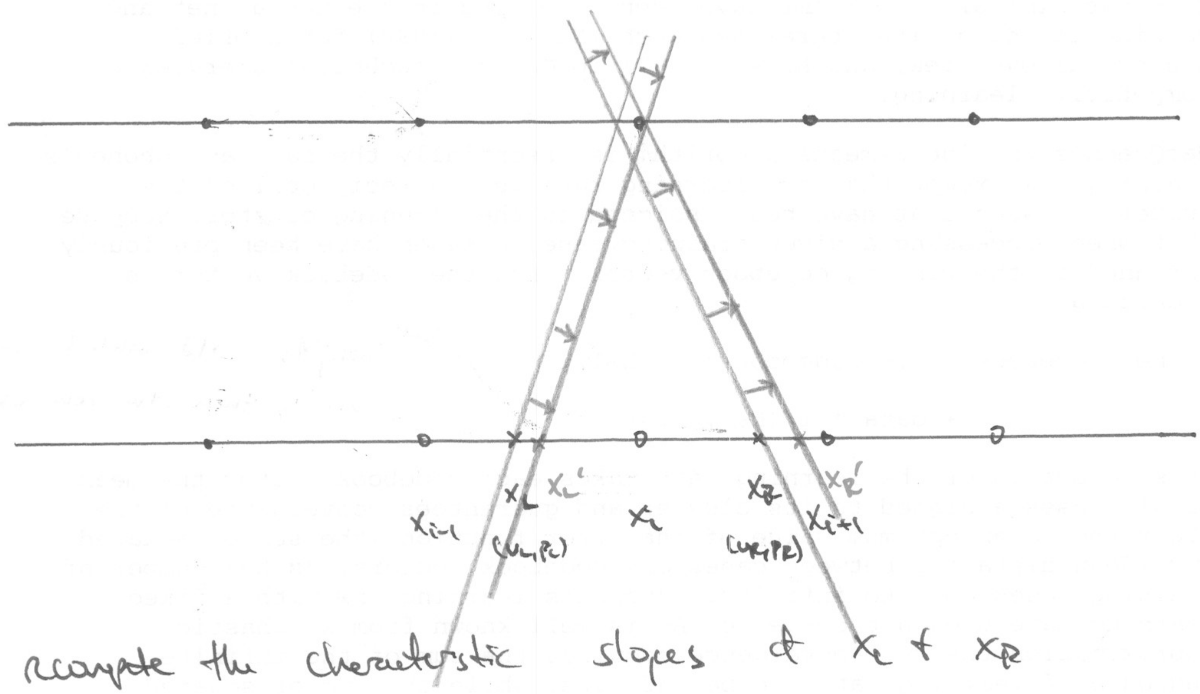
Data computed by interpolation between (x_{i-1}, x_i) + (x_i, x_{i+1}) call them (u_L, p_L) + (u_R, p_R)

Then computing J_+ at left location + J_- at right location (Riem R)

(Riem L)

would give us the value to place at node x_i.

But then I could iterate this procedure: given state (u_L, p_L) + (u_R, p_R) I could



I could recompute the characteristic slopes at x_L + x_R

Again translating the two nearest C_+ + C_- characteristics would allow us to iterate on this method to any allowable degree of accuracy.

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$$G = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j \qquad (a_{ij} = a_{ji})$$

$$\frac{\partial G}{\partial x_k} = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \left[\delta_{ik} x_j + \delta_{jk} x_i \right]$$

$$= \sum_{j=1}^{n} a_{kj} x_j + \sum_{i=1}^{n} a_{ik} x_i \qquad \text{transpose}$$

$$= 2 \sum_{j=1}^{n} a_{ij} x_j$$