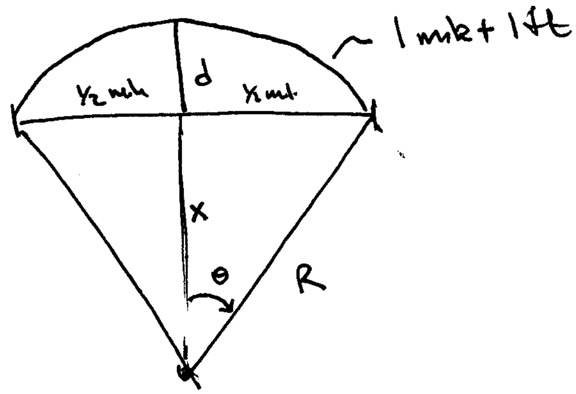


$d = ?$

$x + d = R$ get an equation for x & R

$\Rightarrow d = R - x$ to be able to solve for

$\left(\frac{2\theta}{2\pi}\right)(2\pi R) = 1\text{ mile} + 1\text{ ft.}$ $\Rightarrow$ eq for R in terms of θ.

fraction of circle subtended ↑ circumference of a circle

$\tan\theta = \frac{\frac{1}{2}\text{ mile}}{x}$

$R = \sqrt{x^2 + (\frac{1}{2}\text{ mile})^2}$

Thus given the 4 equations & 4 unknowns

d, R, x, θ

$d = R - x$

$(2\theta)(R) = 1\text{ mile} + 1\text{ ft}$

$\tan\theta = \frac{\frac{1}{2}\text{ mile}}{x}$

$R^2 = (\frac{1}{2}\text{mile})^2 + x^2$

We can solve for everything.

Looks like $R = R(x)$ easily expressed as fn of x

$\theta = \theta(x)$ easily

$$R = \sqrt{(1\,\text{mile})^2 + x^2} \qquad \theta = \tan^{-1}\left(\frac{\tfrac{1}{2}\,\text{mile}}{x}\right)$$

Then x is the solution to the non linear eq.

$$2\tan^{-1}\left(\frac{1/2}{x}\right)\sqrt{1+x^2} = 1 + \frac{1}{5280}$$ all units in miles.

$1 \text{ mile} = 5280 \text{ ft}$

$1 \text{ ft} = \frac{1}{5280} \text{ mi}$

This non linear eq would have to be solved for x, but to very high accuracy since

$d = R(x) - x$ & both R & x will be the same order of magnitude leading to high degree of cancellation.

$$\tan^{-1}(x) = \sum_{n\geq 0} \frac{(-1)^n x^{2n+1}}{(2n+1)}$$ is an alternating series so

$$\left| \tan^{-1}(x) - \sum_{n=0}^{N} \frac{(-1)^n x^{2n+1}}{(2n+1)} \right| \leq \frac{x^{2N+3}}{2N+3}$$

↑

1st neglected term

So for $x = [.4 \quad .8 \quad .95]$ we desire

$$\frac{x^{2N+3}}{2N+3} \leq 10^{-6}$$

So using ~~x2~~ $x = .4$ gives $\frac{(.4)^{2N+3}}{2N+3} \cong 10^{-6}$ ~~gives~~ gives $N=$

plotting $\frac{x^{2N+3}}{2N+3}$ ~~for~~ v.s. N for fixed x gives the ~~following~~. the values of N requested.

HJ 11 Acton

$$\cos((k-1)\theta) = \cos(k\theta)\cos\theta + \sin(k\theta)\sin\theta$$

$$+ \quad \cos((k+1)\theta) = \cos(k\theta)\cos\theta - \sin(k\theta)\sin\theta$$

$$\Rightarrow \quad \cos((k+1)\theta) - 2\cos(k\theta)\cos\theta + \cos((k-1)\theta) = 0 \qquad \text{eq } 1.4$$

Thus $\cos(k\theta) = 2\cos\theta\cos((k-1)\theta) - \cos((k-2)\theta)$ so

eq 1.3 can be written as

$$g(\theta) = \sum_{k=0}^{N} b_k\cos(k\theta) = b_N\left[2\cos\theta\cos((N-1)\theta) - \cos((N-2)\theta)\right] + b_{N-1}\cos((N-1)\theta)$$

$$+ b_{N-2}\cos((N-2)\theta) + \sum_{k=0}^{N-3} b_k\cos(k\theta)$$

$$= (2b_N\cos\theta + b_{N-1})\cos((N-1)\theta) + (-b_N + b_{N-2})\cos((N-2)\theta)$$

$$+ \sum_{k=0}^{N-3} b_k\cos(k\theta)$$

$$= \sum_{k=0}^{N-1} \tilde{b}_k\cos(k\theta)$$

w/ $\tilde{b}_{N-1} = 2b_N\cos(\theta) + b_{N-1}$ & $\tilde{b}_{N-2} = -b_N + b_{N-2}$ &

other $\tilde{b}$'s the same.

Thus the following loop will collapse the original array of b_k's into two #'s $\tilde{b}_0$ + $\hat{b}_1$ which then combine as

$$g(\theta) = \tilde{b}_0 + \hat{b}_1 \cos\theta \quad \text{to give} \quad g(\theta)$$

for $i=N:-1:2$

$b_{i-1} = 2b_i\cos\theta + b_{i-1}$

$b_{i-2} = -b_i + b_{i-2}$

end

$g(\theta) = b_0 + b_1\cos\theta$

Pg 12 Action

$$c_k = (2\cos\theta)\,c_{k+1} - c_{k+2} + b_k \qquad k = 8, 7, \dots, 0$$

$$\cancel{g(\theta)} \Rightarrow b_k = c_k - (2\cos\theta)\,c_{k+1} + c_{k+2}$$

$$g(\theta) = \left(c_8 - (2\cos\theta)\cancelto{0}{c_9} + \cancelto{0}{c_{10}}\right)\cos(8\theta) + \left(c_7 - (2\cos\theta)\,c_8 + \cancelto{0}{c_9}\right)\cos(7\theta)$$

$$+ \left(c_6 - (2\cos\theta)\,c_7 + c_8\right)\cos(6\theta) + \left(c_5 - (2\cos\theta)\,c_6 + c_7\right)\cos(5\theta)$$

$$+ \cdots + \left(c_0 - (2\cos\theta)\,c_1 + c_2\right)\cancel{\cos(}$$

$$= c_8\left[\cancelto{0}{\cos(8\theta) - 2\cos\theta\cos(7\theta) + \cos(6\theta)}\right]$$

$$+ c_7\left[\cancelto{0}{\cos(7\theta) - 2\cos\theta\cos(6\theta) + \cos(5\theta)}\right] + \cdots$$

$$+ c_2\left[\cancelto{0}{\cos(2\theta) - 2\cos\theta\cos\theta + 1}\right] + c_1\left[\cos\theta - 2\cos\theta\right] + c_0$$

$$\Rightarrow g(\theta) = c_0 - c_1\cos\theta$$

For a finite sin series we use the identitys

$$\sin((k-1)\theta) = \sin(k)\cos\theta - \sin\theta\cos(k\theta)$$
$$+ \quad \sin((k+1)\theta) = \sin(k)\cos\theta + \sin\theta\cos(k\theta)$$

$$\Rightarrow \quad \sin((k+1)\theta) - 2\sin(k)\cos\theta + \sin((k-1)\theta) = 0$$

so we must compute $\cos\theta$ for a given θ then $\sin(k\theta)$ can be determined from this scaler.

$$\sin(k\theta) = 2\sin((k-1)\theta)\cos\theta - \sin((k-2)\theta)$$

So

$$g(\theta) \equiv \sum_{k=0}^{N} b_k \sin(k\theta) = b_N\left[2\sin((N-1)\theta)\cos\theta - \sin((N-2)\theta)\right] + \sum_{k=0}^{N-1} b_k \sin(k\theta)$$

$$= (2b_N\cos\theta + b_{N-1})\sin((N-1)\theta) + (-b_N + b_{N-2})\sin((N-2)\theta) + \sum_{k=0}^{N-2} b_k \sin(k\theta)$$

So define $\tilde{b}_{N-1} = 2b_N\cos\theta + b_{N-1}$

$+ \quad \tilde{b}_{N-2} = -b_N + b_{N-2}$

other b's are the same.

Thus using my method $g(\theta)$ can be computed with

For $i = N:-1:2$

$b_{i-1} = 2b_i \cos\theta + b_{i-1}$

$b_{i-2} = -b_i + b_{i-2}$

end

$g(\theta) = b_0 + b_1 \sin\theta$

& this is the same algo as for $\cos(k\theta)$ series.

Using the method in the book consider

$$c_k = (2\cos\theta)c_{k+1} - c_{k+2} + b_k \qquad k = N, N-1, \dots, 0 \quad w/ \quad c_{N+1} \equiv 0 \ \& \ c_{N+2} \equiv 0.$$

$\{ b_k = c_k - (2\cos\theta)c_{k+1} + c_{k+2} \}$

Then

$$g(\theta) = \sum_{k=0}^{N} (c_k - 2\cos\theta\, c_{k+1} + c_{k+2}) \sin(k\theta)$$

$$= \sum_{k=0}^{N} c_k \sin(k\theta) - 2\cos\theta \sum_{k=1}^{N+1} c_k \sin((k-1)\theta) + \sum_{k=2}^{N+2} c_k \sin((k-2)\theta)$$

$$= \text{''} \qquad - 2\cos\theta \sum_{k=1}^{N} c_k \sin((k-1)\theta) + \sum_{k=2}^{N} c_k \sin((k-2)\theta)$$

since $c_{N+1} = c_{N+2} = 0$

$$= c_0 \sin(0\cdot\theta) + c_1 \sin(\theta) - 2\cos\theta \, c_1 \sin(0\cdot\theta)$$

$$+ \sum_{k=2}^{N} c_k \Big[\underbrace{\sin(k\theta) - 2\cos\theta \sin((k-1)\theta) + \sin((k-2)\theta)}_{\equiv 0} \Big]$$

$$= \cancel{c_0} \; c_1 \sin\theta$$

Note: The sum should go from $k=1$ Not $k=0$ since $\sin(0\cdot\theta) \equiv 0$

Thus there is No b_0 term.

For Chebyshev polynomials. $T_n(x)$ satisfy:

$$T_{n+1}(x) - 2x T_n(x) + T_{n-1}(x) = 0 .$$

So to evaluate $\cancel{g(x)}$ $g(\theta) = \sum_{k=0}^{N} b_k T_k(\theta)$

$$= b_N\big(2\theta T_{N-1}(\theta) - T_{N-2}(\theta)\big) + b_{N-1} T_{N-1}(\theta) + b_{N-2} T_{N-2}(\theta) + \sum_{k=0}^{N-3} \cdots$$

$$= (2b_N\theta + b_{N-1}) T_{N-1}(\theta) + (-b_N + b_{N-2}) T_{N-2}(\theta) + \sum \cdots$$

The same techniques will work

by defining $c_k = -c_{k+2} + 2\theta c_{k+1} + b_k \quad k = N, N-1, \dots, 0 \quad c_{N+2} = 0 = c_{N+1}$.

pg 13 Acton

$t = s-1$

$dt = ds$

$$F(b) = \int_1^\infty \frac{e^{-b(s-1)}}{s}\, ds = e^{+b} \int_1^\infty \frac{e^{-bs}}{s}\, ds \qquad \text{let} \quad t = bs$$

$$dt = b\,ds$$

$$= e^{b} \int_b^\infty \frac{e^{-t}}{(t/b)} \left(\frac{dt}{b}\right) = e^{b} \int_b^\infty \frac{e^{-t}}{t}\, dt = e^{b} E_i(b)$$

Pg 11 Acton

$$F(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt = \mathrm{erf}(x)$$

$$F(x) = e^{-x^2} G(x)$$

$$F'(x) = -2x e^{-x^2} G(x) + e^{-x^2} G'(x)$$

$$F''(x) = -2e^{-x^2} G(x) + -2x(-2x) e^{-x^2} G(x)$$

$$-2x e^{-x^2} G'(x) + (-2x) e^{-x^2} G'(x)$$

$$+ e^{-x^2} G''(x)$$

$$= e^{-x^2}\left[(-2+4x^2)G(x) - 4x G'(x) + G''\right]$$

D.E for $F(x)$ is given by

$$F''(x) = C(-2x)e^{-x^2}$$

$$F'(x) = Ce^{-x^2}$$

so

$$F''(x) = -2x(F'(x))$$

$$\Rightarrow F'' + 2xF' = 0$$

w/ $F(0) = 0$

$$F'(0) = \frac{2}{\sqrt{\pi}}$$

put into D.E for $F(x)$ gives

$$(-2+\cancel{4x^2})G(x) - 4xG'(x) + G''(x) + 2x(-\cancel{2x}G(x) + G'(x)) = 0$$

$$-2G(x) - 2xG'(x) + G''(x) = 0 \quad \checkmark \quad \text{eq } 1.10$$

w/ I.C. on $G(\cdot)$ $\Rightarrow$ $G(0) = 0$

$$\& \quad \underset{F'(0)}{\overset{\|}{\frac{2}{\sqrt{\pi}}}} = 0 + G'(0) \quad \checkmark$$

Now $x=0$ is a regular pt for this DE & $\therefore$ a taylor series exists for $f(x)$ about this point. ... rest is algebra

Pg 19 Acton

$$\frac{\sin z}{z} = \prod_{k=1}^{\infty}\left(1-\frac{z^2}{k^2\pi^2}\right)$$

Solve $\sin x = x^n\left(1-\dfrac{x^2}{\pi^2}\right)$

$$\|$$

$$x\cancel{\left(1-\frac{x^2}{\pi^2}\right)}\prod_{k=2}^{\infty}\left(1-\frac{x^2}{k^2\pi^2}\right) = x^n\cancel{\left(1-\frac{x^2}{\pi^2}\right)}$$

$$\Rightarrow \quad x\left(1-\frac{x^2}{4\pi^2}\right)\prod_{k=3}^{\infty}\left(1-\frac{x^2}{k^2\pi^2}\right) = x^n$$ eq on H 20 ✓

π

$$F(b) = \int_0^\infty \frac{\tan^{-1}(bx)}{1+x^2}\,dx \qquad 0 \le b < \infty$$

$$F(1) = \int_0^\infty \frac{\tan^{-1}(x)}{1+x^2}\,dx = \int_0^\infty \tan^{-1}(x)\cdot \frac{d}{dx}(\tan^{-1}(x))\,dx$$

$$= \frac{1}{2}(\tan^{-1}(x))^2 \Big|_0^\infty = \frac{1}{2}\left(\frac{\pi^2}{4} - 0\right) = \frac{\pi^2}{8}$$

$$F(0) = \int_0^\infty \frac{dx}{1+x^2} = \tan^{-1}(x)\Big|_0^\infty = \frac{\pi}{2}$$

$$F(+\infty) = \int_0^\infty \frac{(\pi/2)}{1+x^2}\,dx = \frac{\pi^2}{4}$$

$$F(b) = \int_0^\infty \tan^{-1}(bx)\cdot\frac{1}{1+x^2}\,dx = \cancel{\tan \tan(x)}\,)\ \tan^{-1}(bx)\tan^{-1}(x)\Big|_0^\infty$$

$$- \int_0^\infty \frac{d}{dx}(\tan^{-1}(bx))\tan^{-1}(x)\,dx$$

$$= \tan^{-1}(bx)\tan^{-1}(x)\Big|_0^\infty - \int_0^\infty \frac{\tan^{-1}(x)}{1+b^2x^2}\cdot b\,dx$$

let $v = bx \qquad dv = b\,dx$

so

$$F(b) = \frac{\pi^2}{4} - \underbrace{\int_0^\infty \frac{\tan^{-1}(v/b)}{1+v^2}\,dv}_{\equiv F(1/b)}$$

eq 1.22 ✓

$$F(b) = \int_0^a \frac{\tan^{-1}(bx)}{1+x^2}\,dx + \int_a^\infty \frac{\tan^{-1}(bx)}{1+x^2}\,dx$$

From the continued fraction expansion on pg 7.

$$\frac{\tan^{-1}(x)}{x} \cong \cfrac{1}{1 + \cfrac{x^2}{3 + \cfrac{4x^2}{5}}} = \cfrac{1}{1 + \cfrac{x^2}{\cfrac{15+4x^2}{5}}}$$

$$= \cfrac{1}{1 + \cfrac{5x^2}{15+4x^2}} = \cfrac{1}{\cfrac{15+4x^2+5x^2}{15+4x^2}}$$

$$= \frac{15+4x^2}{15+9x^2} = \frac{3+\left(\frac{4}{5}x^2\right)}{3+\frac{9}{5}x^2}$$

$$\tan^{-1}(bx) = \frac{(bx)\left(3+\frac{4}{5}(bx)^2\right)}{3+\frac{9}{5}(bx)^2}$$

]

<u>MI 31 Acton</u>

$$F(b) = \int_0^1 \frac{\tan^{-1}(bx)}{1+x^2}\, dx + \int_1^{\infty} \frac{\tan^{-1}(bx)}{1+x^2}\, dx$$

using $\tan^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(1/x)$ then $F(b)$ becomes

$$F(b) = \int_0^1 \frac{\tan^{-1}(bx)}{1+x^2}\, dx + \int_1^{\infty} \frac{\frac{\pi}{2} - \tan^{-1}(1/bx)}{1+x^2}\, dx$$

let $y = 1/x$

$dy = -1/x^2\, dx = -y^2 dx \Rightarrow dx = -\frac{dy}{y^2}$

So the 2nd integral becomes

$$\int_1^0 \frac{\frac{\pi}{2} - \tan^{-1}(y/b)}{1+(1/y^2)} \cdot \left(-\frac{dy}{y^2}\right) = \int_0^1 \frac{\frac{\pi}{2} - \tan^{-1}(y/b)}{1+y^2}\, dy$$

① $x = 0, .1, 1.0$

<u>taylor series:</u>

$$\arctan(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots$$

<u>continued-fraction:</u>

$$\tan^{-1}(x) = x \cfrac{1}{1 + \cfrac{(1x)^2}{3 + \cfrac{(2x)^2}{5 + \cfrac{(3x)^2}{7 + \cfrac{(4x)^2}{9 + \cfrac{(5x)^2}{11 + \cdots}}}}}}$$

or <u>Rational Function Approx:</u>

$P_0 = \alpha_0 = 1 \qquad Q_0 = \beta_0 = 1$

$P_1 = \alpha_0 \beta_1 = 3 \qquad Q_1 = \beta_0\beta_1 + \alpha_1 x^2 = 3 + x^2$

Then iterate $n = 2:N$.

$< w/\ \beta_n = 2n-1$
$+\ \alpha_n = 2^n$

$$P_n = \beta_n P_{n-1} + \alpha_n x^2 P_{n-2}$$
$$Q_n = \beta_n Q_{n-1} + \alpha_n x^2 Q_{n-2}$$

continued ...

$$\tan^{-1}(x) = x \cdot 1 / 1 + \cancel{(1}x)^2 / 3 + (2x)^2 / 5 + (3x)^2$$

do the following. Create the following string

$$\frac{\tan^{-1}(x)}{x} = 1/(1+(1x)^2)$$

$$= 1/(1+(1x)^2/(3+(2x)^2/$$

$$= \underbrace{1/(}_{\text{prefix}}\ \underbrace{1+(1x)^2}_{\text{1st iteration}}\ /(\ \underbrace{3+(2x)^2}_{\text{2nd iteration}}\ /(\ \underbrace{5+(3x)^2}_{\text{3rd iteration}}\ )$$

~~1st~~ prefix

conjunction operators...

Add enough parenthesis to complete the postfix as ness.

MA 40 Acton

③ $f = x \ln x$ $\quad x \in (10^{-5}, 10^{-4})$

let $b = 10^{-5}$ then $x = by$ w/ $y \in (1, 10)$

So $\ln x = \ln(by) = \ln(b) + \ln(y) = \ln(10^{-5}) + \ln(y)$

$$= \frac{\log_{10}(10^{-5})}{\log_{10}(e)} + \ln(y)$$

$$\ln(\xi) = \frac{\log_{10}(\xi)}{\log_{10}(e)}$$

$$= \frac{-5}{\log_{10}(e)} + \ln(y)$$

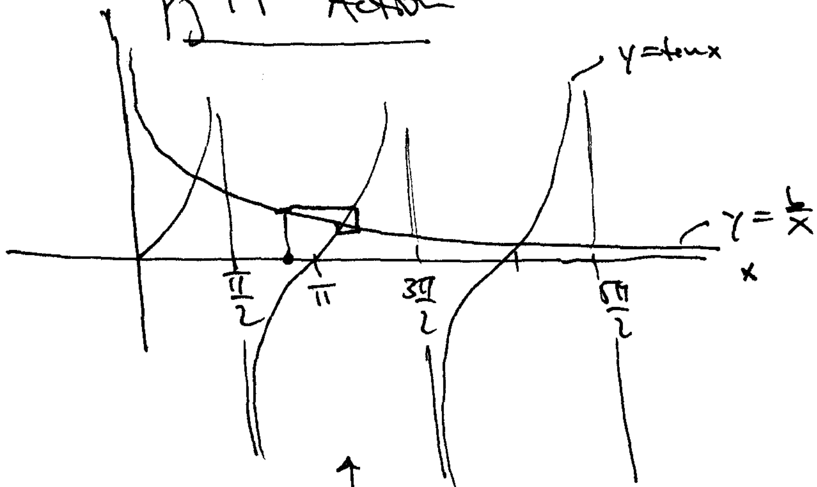
Thus we have to evaluate accurately $\log_{10}(e)$ & $\ln(y)$ for $y \in (1, 10)$

pg 44 Action

$y = \tan x$

$y = \frac{b}{x}$

$b \approx .4$



$y_1 = \frac{.4}{3.0}$

$y_1 = \tan x_2$

$\frac{.4}{3.} = \tan x_2$

$\cancel{y = \tan x}$

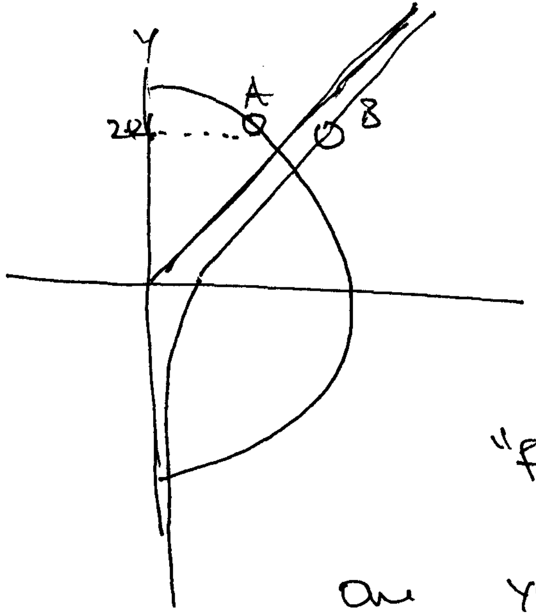
$x = \cancel{\frac{y}{b}}$

$$\underbrace{\frac{\cos x}{\sin x}}_{y} - \underbrace{\frac{1}{bx}}_{y} = 0$$

**

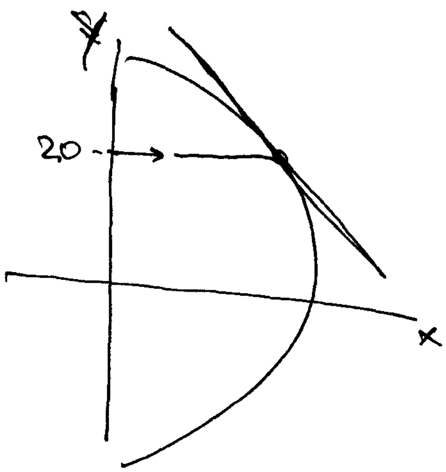
pg 46 Action

~~The~~ ~~[illegible]~~ a decision needs to be made given x or y ~~B~~ (does not matter which) what curve do we go to ~~B~~ stop i.e. Assuming we want to start at $y = 2.0$ (poss. motivated by the fact that $e^x - e^{+y} = 1$ is "flat" in the x as we run in y.

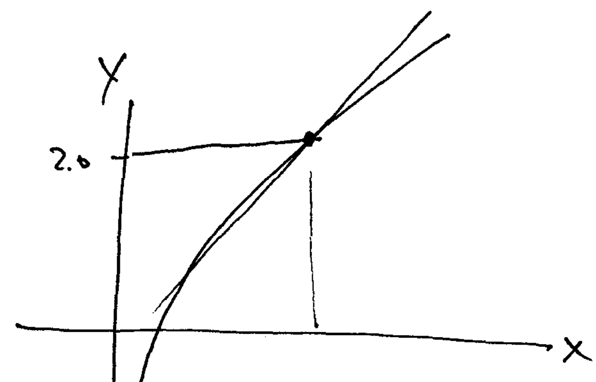


i.e do we stop at A or B to project into x.

The heuristic argument is Stop on the curve that is "flatter" in the direction ⊥ to the one you are moving. i.e



V.S



As the exponential curve is visually flatter we stop on it. i.e use the

eq

$$x = \ln(e^{Y} - 1)$$

Something is ~~not~~ not correct as doing this will miss

$-1+e^x = e^y$

$y = \ln(1+e^x)$

$y = \underbrace{\ln(e^x(e^{-x}+1))}_{\text{Now as product}} = x + \ln(e^{-x}+1) = x+\ln(1-e^{-x})$

expand w/ law logs work on products

Now have asymptotic form of eqs.

$\Rightarrow \underbrace{y-x}_{} = \ln(1-e^{-x})$

$u = \ln(1$

$u = y-x$

$\cancel{y = y+x}$

$v = y+x$ if put into algebraic

Circle eq get a mess.

$y = x+u$

$x =$

$u = \ln(1-e^{-x})$

$y = u+x$

$x = (256-y^6)^{1/10}$

given x, get u,

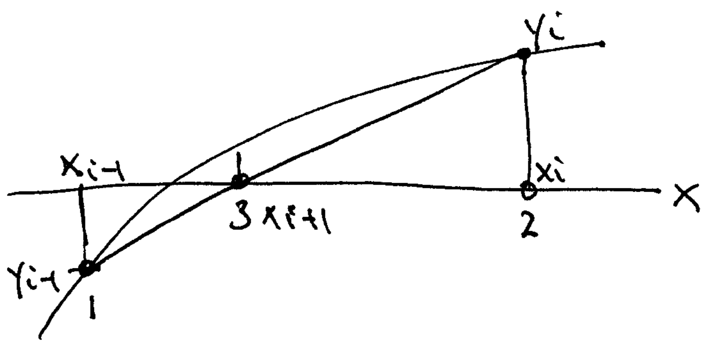
given u & x get y

given y get x again.

Or changing the order of the equations gives

given	get	
Y	X	
X	U	
U, X	Y	reject.

pg 53 Acton



$$y - y_i = \frac{(y_{i-1} - y_i)(x - x_i)}{(x_{i-1} - x_i)}$$

Zero of this line gives point x_{i+1} setting $y = 0$

$$(-y_i)(x_{i-1} - x_i) = (y_{i-1} - y_i)(x - x_i)$$

$$x = x_i - \frac{y_i}{y_{i-1} - y_i}(x_{i-1} - x_i)$$

$$\therefore\ x_{i+1} = \frac{x_i(y_{i-1} - \cancel{y_i})}{y_{i-1} - y_i} - \frac{y_i(x_{i-1} - \cancel{x_i})}{y_{i-1} - y_i}$$

$$= \frac{x_i y_{i-1}}{y_{i-1} - y_i} - x_{i-1}\frac{y_i}{y_i - y_{i-1}}$$

$$x_{i+1} = \alpha(y_{i-1}, y_i)\,x_i + \beta(y_{i-1}, y_i)\,x_{i-1}$$

eq 2.9:

pg 54 Acton

$f(x) = x^2 - N$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f'(x) = 2x$

$$x_{n+1} = x_n - \frac{x_n^2 - N}{2x_n} = x_n - \frac{x_n^2}{2x_n} + \frac{N}{2x_n}$$

$$= \frac{x_n}{2} + \frac{N}{2x_n} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

Eq A:

$$\epsilon_{i+1} = -b + \frac{1}{2}\left[\cancel{\epsilon_i} + b + b - \cancel{\epsilon_i} + \frac{\epsilon_i^2}{b} - \frac{\epsilon_i^3}{b^2} + O(\epsilon_i^4)\right]$$

$$= \cancel{-b} \cancel{+b} + \frac{\epsilon_i^2}{2b} - \frac{\epsilon_i^3}{2b^2} + O(\epsilon_i^4)$$

$$\epsilon_{i+1} = \frac{\epsilon_i^2}{2b} - \;..$$

Eq B:

$$\epsilon_{i+1} = \frac{\cancel{\epsilon_i f'} + \epsilon_i^2 f'' + \cdots - \cancel{\epsilon_i f'} - \epsilon_i^2 \frac{f''}{2} - \frac{\epsilon_i^3}{6} f'''}{f' + \epsilon_i f''}$$

$$\sim \frac{\frac{\epsilon_i^2 f''}{2}}{f'} + O(\epsilon_i^3)$$

$\epsilon_3 = $ ~~$\epsilon_2\epsilon_1 \ldots$~~

~~$\frac{R\epsilon_1 f' - R\epsilon_2 f'}{}$~~

$\epsilon_1\epsilon_2 f'$

Eq 2.15: if ϵ_1 is fixed $\Rightarrow \epsilon_i$ fixed up to i

$$\epsilon_3 = C_0 \epsilon_2$$

or

$$\epsilon_{i+1} = C_0 \epsilon_i$$

Eq A:

$$r_i \sim \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{i+1} \qquad i \to \infty$$

$\therefore \quad C\mathcal{E}_{i+1} \sim M^{r_{\boxed{?}}}$

$C\mathcal{E}_1 \sim M^1 \qquad r_{i=0} = 1$

$C\mathcal{E}_2 \sim M^1 \qquad r_{i=1} \qquad \frac{1+\sqrt{5}}{2}$

$$\therefore \quad C\mathcal{E}_{i+1} \approx M^{r_{i+1}} \sim M^{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{i+1}} \cancel{\times} \left(M^{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{i}}\right)$$

$\sim M$

$C\mathcal{E}_i$

$\therefore \quad C\mathcal{E}_{i+1} \sim C\mathcal{E}_i$

$$c\epsilon_{i+1} = \eta^{r_{i+1}} \sim \eta^{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{i+1}} \qquad i \to \infty$$

$$= \underbrace{\left(\eta^{\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{i}}\right)}_{}^{\left(\frac{1+\sqrt{5}}{2}\right)} \qquad (a^b)^c = a^{bc}.$$

$$(c\epsilon_i)^{\frac{1+\sqrt{5}}{2}}$$

$$\epsilon_{i+1} \sim k\,|\epsilon_i|^{\frac{1+\sqrt{5}}{2}} \qquad i \to \infty$$

Note 1 $10 \sim \sqrt{96}$ & the subtraction results in cancellation, loosing a significant # of digits of accuracy.

Rule: Never subtract two equal quantities.

Note 2: Then small root checks must be consistent of ~~[illegible]~~ worce. i.e.

$$x^3 - 2x - 5 = 0$$

dropping x^3 ~ $x \simeq -\frac{5}{2} \approx -2.5$ ⟸ Ask is this small?

i.e. can we drop the x^3 ? $(2.5)^3 << -2(2.5)$

~~Not~~: No!!

~~Good~~

BE

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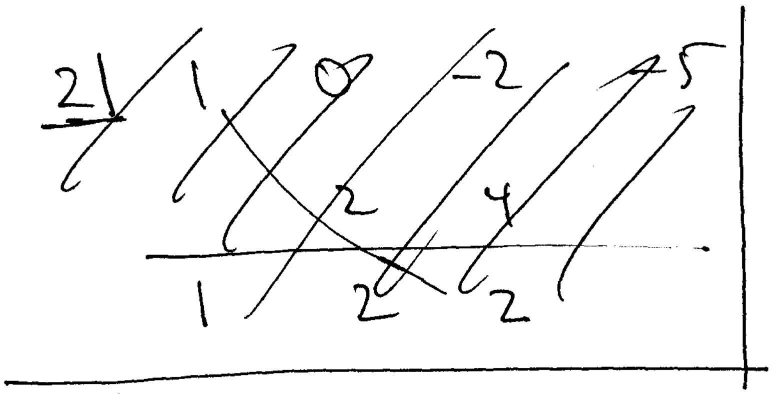
$x^3 - 2x - 5 = 0$ Shift down by 2



$\Rightarrow a(x-2)^3 + b(x-2)^2 + c(x-2) + d = 0$

i.e. taylor about $x = 2$.

Eq A:

$f(2) = 8 - 4 - 5 = -1$

$f'(x) = 3x^2 - 2$ $\qquad f'(2) = 12 - 2 = 10$

$f''(x) = 6x$ $\qquad f''(2) = 12.$

$f'''(x) = 6.$

$$\therefore \; x^3 - 2x - 5 = \frac{6}{3!}(x-2)^3 + \frac{12}{2!}(x-2)^2 + 10(x-2) + -1 = 0$$

$$\Rightarrow \; x^3 - 2x - 5 = (x-2)^3 + 6(x-2)^2 + 10(x-2) - 1 = 0.$$

$z = x - 2$

$z^3 + 6z^2 + 10z - 1 = 0$

Now Assuming $z \ll 1$ largest term (besides 1)

$\sim \; z = \frac{1}{10}(1 - 6z^2 - z^3)$

??

This iteration works because of the subharmonic behavior of x^p for $x < 1$

i.e. $\underline{x^{p+1} \ll x^p} \quad x \to 1$

Thus $e^{-(n+1)x} \ll e^{-nx} \quad x \to +\infty$.

Might be another iteration method that would work?

Eq B:

~~$x^4 + 3.14x^3$~~ +

$$x^4 + .2x^3 + 3.14x^2 + .1x - 4.10 = 0$$

if $x \sim 1$ then $.2x^3 \ll x^4$ & in fact $x = 1$ works

$.1x \ll +4.10$ etc

∴ One method would be to write this eq in terms of $\sum_{p=0} (x-1)^p$ ...

$$y^2 + 3.14y - 4.10 = 0$$

$$1 + 3.14 - 4.10 \simeq 0 \checkmark.$$

Then w/ $y \simeq 1$ $\quad$ $\text{root}_1 \cdot \text{root}_2 \simeq -4.10$

$$\begin{aligned} &\| \\ &1 \end{aligned}$$

$$\rightarrow \text{root}_2 \simeq -4.1 \simeq -4.$$

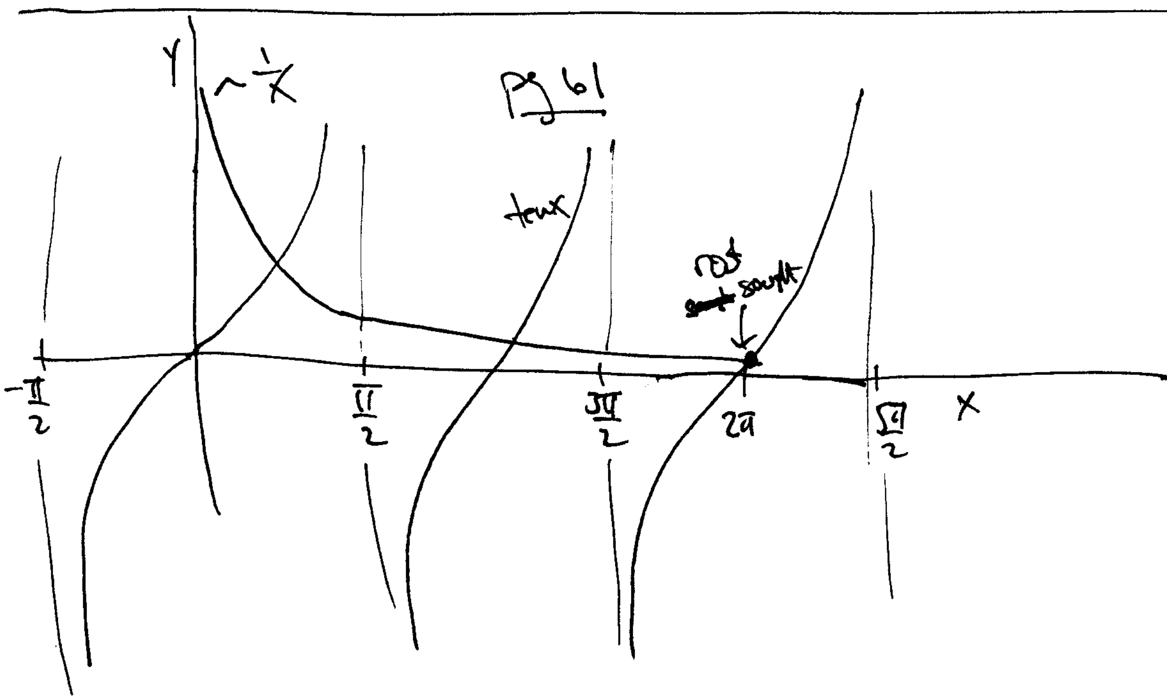
$x = \pm 1$

$\pm 2i$

could one do?

$$y^2 = -3.14y + 4.10 - .2x^3 - .1x \qquad y = x^2$$

& put $y \simeq -4$ (for this value of y y^2 is the largest term) & iterate? ~~Try~~ Try.



$\cot x = x$.

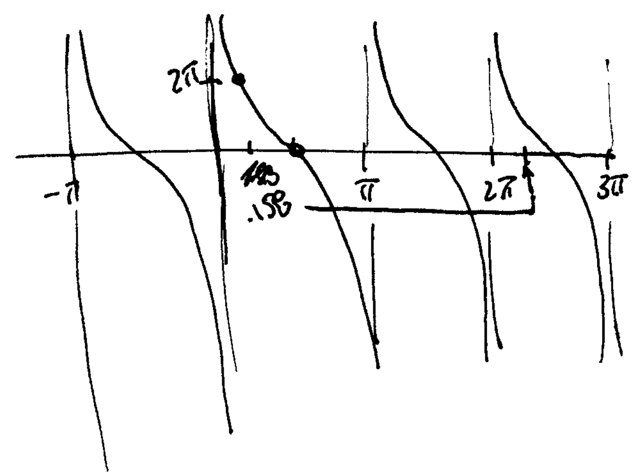
want root such that

$\Rightarrow x \sim 2\pi$.

$\Rightarrow \cot x \sim 2\pi$.

~~$x_1 = .158$~~ $\Rightarrow x_1' = .158$

cot x graph looks like



Add 2π to value to get $x_1 = x_1' + 2\pi$.

Then find. $\underline{x_2} = \cot x_1$

How iterate this procedure?

Get x_2 add 2π. take $\cot x_2$.

$$x_n = \cot x_{n-1} + 2\pi \quad ?$$

~~How does~~ Why do computers Not use tables to evaluate trig. fns?

How would one hard wire a computer to use a table?

i.e. w/ interpolation to produce the values?

$$x = 2\pi + y \qquad y \ll 1$$

$$\tan x = \frac{1}{x}$$

$$\frac{\sin(2\pi + y)}{\cos(2\pi + y)} = \frac{1}{x}$$

Eq A:

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~~$1+x^2 = r^2$~~

~~$1+x^2 = x^2 + 2xd + d^2$~~

~~$x = \frac{1-d^2}{2d}$~~

~~$\Rightarrow (2d)^2 + (1-d^2)^2 = r^2$~~

~~$4d^2 + 1 - 2d^2 + d^4 = r^2$~~

~~$2d^2 + d^4 + 1 = r^2$~~

~~$(d^2+1)^2 = r^2$.~~

Eq A:

$$\boxed{r = d + x} \quad \text{from geometry}$$

$x = r - d.$

$1 + x^2 = r^2$

$1 + (\cancel{r^2} - 2rd + d^2) = \cancel{r^2}$

$\underline{1 - 2rd + d^2 = 0}$

$d = \frac{1+d^2}{2r}.$

$d = \frac{1}{2r} + \frac{d^2}{2r}$

---

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$$\frac{(10^2/4)^n}{(n!)^2} \sim 1$$

$25^n \sim (n!)^2$

$5^n \sim (n!) \qquad n = ?$

n	
1	$5 \sim 1$
2	$25 \sim 4$
⋮	

$n = 10$

$5^{10} \sim 10!$ check?

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$\cos(\pi) = -1$
$\sin(\pi) = 0.$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

~~$1-\cos V =$~~

~~$2\int_0^X f(v)(1-\cos 2\theta)$~~

$$2\int_0^X \frac{f(v)(1-\cos v)}{1-\cos 2v}\, dV.$$

$$\frac{1-\cos v}{\sin^2 v} \stackrel{?}{=}$$

Non singular at $v = \pi$

How remove sing. at π to ~~form~~ a more tractable problem?

Eg A:

$$\int_0^X \frac{e^{-v}}{\sqrt{v}}\, dv = \int_0^X e^{-v} v^{-1/2}\, dv = e^{-v}\frac{v^{1/2}}{1/2}\Big|_0^X + 2\int_0^X \sqrt{v}\, e^{-v}\, dv$$

Eg B:

$$\int_0^X \frac{e^{-t}}{\sqrt{t}}\, dt$$

let $dv = \frac{dt}{2\sqrt{t}}$

$v = \sqrt{t} \Rightarrow t = v^2$

$$= \int_0^{\sqrt{X}} \frac{e^{-v^2}}{\cancel{}}\, dv$$

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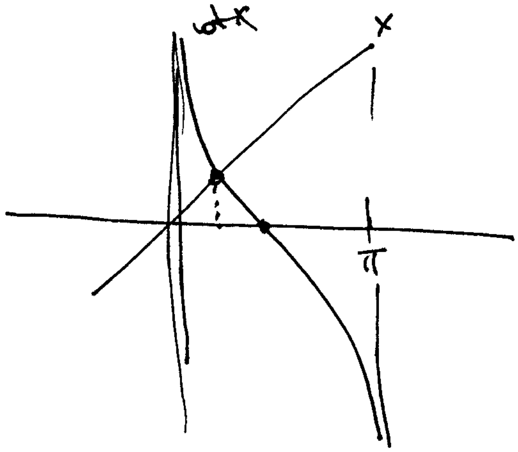


1st guess from table ~

$\cot x = x$ is $.860$

pg 87 Hetan

① small root $x \ll 1$

$x \sim 10^{-4}$. consistent w/

$\Rightarrow \quad x = \dfrac{1+x^2}{10^4}$

iterate $x_{n+1} = \dfrac{1+x_n^2}{10^4}$ w/ $x_0 = 0$ to get as many digits as req.

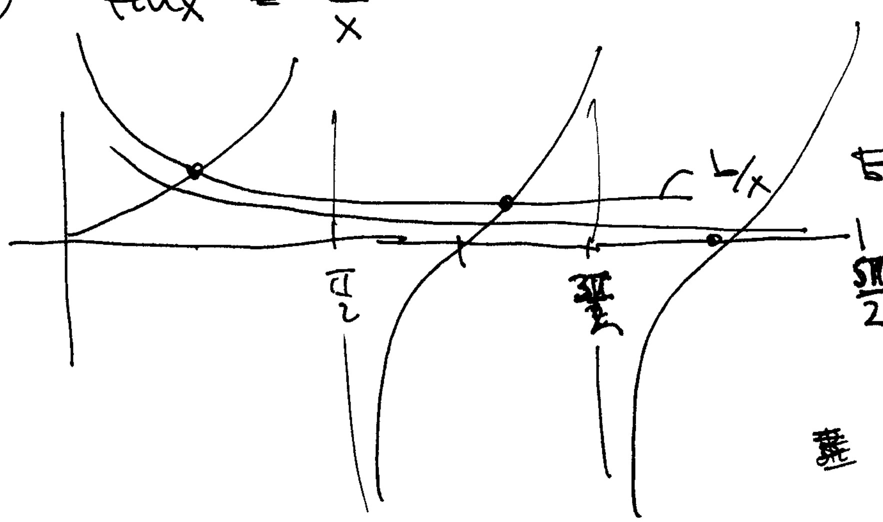
Quadratic formula gives:

$$x = \frac{10^4 \pm \sqrt{10^8 - 4}}{2}$$

Note $\sqrt{10^8 - 4} \simeq 10^4$.

cancellation.

② $\tan x = \dfrac{b}{x}$



$b > 1$.

start at $\frac{\pi}{4}, \pi, 2\pi$.

$f(x) = \sin x - \dfrac{b}{x}\cos x \qquad f'(x) = \cos x + \dfrac{b}{x}\sin x + \dfrac{b}{x^2}\cos x$

$x \sim 1$ All terms are of equal order.

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$$\cos(n+1)\theta = \cos n\theta \cos\theta - \sin(n\theta)\sin\theta$$

$$=$$

$+$

$$\cos(n-1)\theta = \cos n\theta \cos\theta + \sin(n\theta)\sin\theta$$

$\Rightarrow$ Adding:

$$2\cos n\theta \cos\theta = \cos(n+1)\theta + \cos(n-1)\theta$$

$$\Rightarrow \cos(n+1)\theta = \cancel{\cos(n+1)\theta} - \cancel{2\cos}$$

$$= 2\cos(n\theta)\cos\theta - \cos(n-1)\theta \quad \text{eq } 16.1$$

$$T_{n+1} = 2\cos\theta\, T_n - T_{n-1}$$

let $\epsilon_n = C_n - T_n$

$$\Rightarrow \epsilon_{n+1} = 2\cos\theta\, \epsilon_n - \epsilon_{n-1} \quad \text{eq } 16.3$$

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(1)

$$I_n = \int_0^1 x^n e^{x-1}\,dx$$

$$= x^n e^{x-1}\Big|_0^1 - n\int_0^1 x^{n-1} e^{x-1}\,dx$$

$$= 1 - n I_{n-1}$$

$\therefore$ $$I_n + n I_{n-1} = 1$$

~~considering stability considering the solution in terms yields.~~

$$\cancel{\rho^n + n\rho^{n-1} = 1}$$

For stability Assume $\tilde{I}_n$ are the rounded (due to ~~...~~ finite word length truncation) numbers we are computing with.

then let $\varepsilon_n \equiv I_n - \tilde{I}_n$ + the equation that governs the error propagation is

$$\varepsilon_n + n\,\varepsilon_{n-1} = 0$$

~~so that~~ ~~considering the coefficient n frozen~~

$$\Rightarrow \quad \varepsilon_n = -n\,\varepsilon_{n-1}$$

~~n = 0, 1, 2~~. $n = 1, 2, 3, \ldots$

w/ ~~$|\varepsilon_n|$~~ ~~$\varepsilon_0 = 0$~~. w/ ε_0 given.

$$\frac{\varepsilon_n}{\varepsilon_{n-1}} = -n$$

The difference between the truncated exact # + the true value.

$$\ln \varepsilon_n - \ln \varepsilon_{n-1} = \ln(-n)$$

$\varepsilon_0 \neq 0$.

$$\Rightarrow \quad \Delta(\ln \varepsilon_{n-1}) = \ln(-n) = \ln(n e^{i\pi})$$

$$= \ln(n) + i\pi$$

$$\left\{ \ln(z) = \ln|z| + i \operatorname{Arg} z \right\}$$

$$\varepsilon_1 = -1\,\varepsilon_0$$

$$\varepsilon_2 = -2\,\varepsilon_1 = -2(-1)\varepsilon_0 = (-1)^2 2!\,\varepsilon_0$$

$$\vdots$$

$$\varepsilon_n = (-1)^n n!\,\varepsilon_0$$

This expression is <u>not</u> stable. + is unstable to a greater degree than exponential.

(2) $I_0 = \int_0^1 e^{x-1}\,dx = \frac{e^{x-1}}{1}\Big|_0^1 = e^0 - e^{-1}$

$= 1 - \frac{1}{e} = 0.632120$ to 6 signif figs.

now iterate

$\tilde{I}_n = 1 - n\tilde{I}_{n-1}$

gives $\tilde{I}_9 = 0.294399$

$\varepsilon_9 = I_9 - \tilde{I}_9 = .0916123 - .294399$

$= -.202$

= % error of -221.35%

(3) using $I_n = 1 - nI_{n-1}$ in reverse

$I_{n-1} = \frac{1}{n}(1 - I_n)$

Then ε_n = error at step n. is given by

$\varepsilon_{n-1} = (-\frac{1}{n})\varepsilon_n$

so that Assuming ε_9 say is known $\neq 0$.

$\varepsilon_8 = (-\frac{1}{9})\varepsilon_9$

$\varepsilon_7 = (-\frac{1}{8})\varepsilon_8 = (-\frac{1}{8})(-\frac{1}{9})\varepsilon_9 = \frac{(-1)^2}{9\cdot 8}\varepsilon_9$

1/07-02 ?

$$\varepsilon_6 = \left(\frac{-1}{7}\right)\varepsilon_7 = \left(\frac{-1}{7}\right)\frac{(-1)^2}{9\cdot 8}\,\varepsilon_9$$

$$= \frac{(-1)^3}{9\cdot 8\cdot 7}\,\varepsilon_9 = \frac{(-1)^3\,(9-3)!}{9!}\,\varepsilon_9$$

$$\vdots$$

$$\cancel{\varepsilon_i = \frac{(-1)^{9-i}\,(9-i)}{9!}\,\varepsilon_9}$$

This should give a stable algo:

$\cancel{\varepsilon_9} =$ Using $\cancel{\varepsilon_9} \doteq \tilde{I}_9 = .0916123$

gives $\tilde{I}_0 = .63212055\ldots$

w/ % error $-3.05\cdot 10^{-12}\,\%$.

unbelievable..

(4) $C_{n+1} + C_{n-1} = \frac{2x}{n} C_n$

iterating forward

$$\varepsilon_{n+1} = \frac{2x}{n}\varepsilon_n - \varepsilon_{n-1}$$

Not sure might have to iterate backwards?

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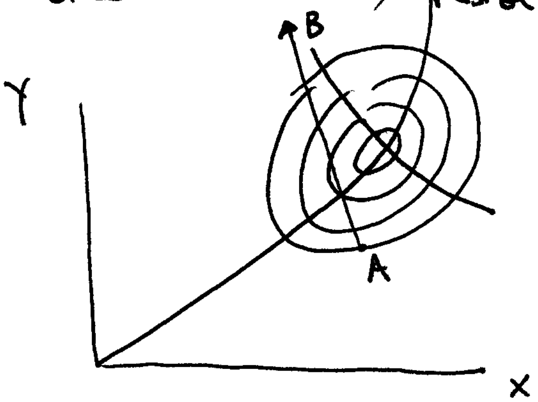
10-23-01 1

Cluster of pts star. n-dim analogue to binary chop.

very stable searching technique.

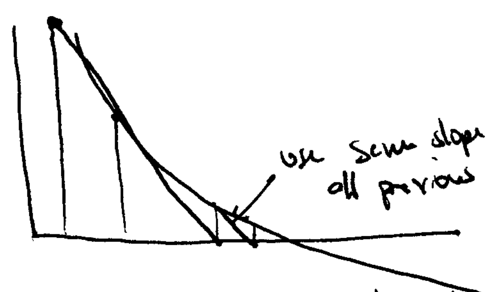
downhill star search

when minimum is at center of star. (i.e. the evaluations produce no movement in any other direction) resize the size of our star & repeat.



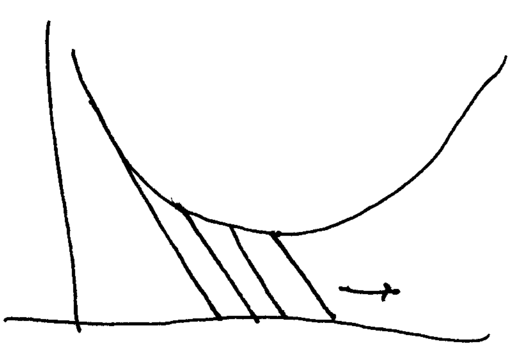
distance to min v.s. length along vector $\overline{AB}$.

practical to evaluate 1st derivatives



use same slope as found on all previous approximations.

to find a minimum one can approximate the slope into the min. Then since an almost parabola has almost a constant second derivative this value of slope will not change much



$h \sim |s|$

$$h \sim \frac{-2(f_0 - f_m)}{g_0}$$

$$\cancel{R} \quad f = f_m + \frac{b}{h^2}(x - x_m)^2 + \frac{c}{h^3}(x - x_m)^3 \qquad h \equiv x_1 - x_0$$

$$\alpha = hr = x_m - x_0 \qquad f_1 = f_m + \frac{b}{h^2}(x_1 - x_m)^2 + \frac{c}{h^3}(x_1 - x_m)^3$$

$$g = \frac{2b}{h^2}(x - x_m) + \frac{3c}{h^3}(x - x_m)^2$$

$$F = f_1 - f_0 = \cancel{f_m} + \frac{b}{h^2}(x_1 - x_m)^2 + \frac{c}{h^3}(x_1 - x_m)^3$$

$$- \cancel{f_m} - \frac{b}{h^2}(x_0 - x_m)^2 - \frac{c}{h^3}(x_0 - x_m)^3$$

$$= \frac{b}{h^2}\left((x_1 - x_m)^2 - (x_0 - x_m)^2\right) + \frac{c}{h^3}\left((x_1 - x_m)^3 - (x_0 - x_m)^3\right)$$

$$= \frac{b}{h^2}\left(((x_1 - \cancel{x_m}) - (x_0 - \cancel{x_m}))(x_1 - x_m + x_0 - x_m)\right)$$

$$+ \frac{c}{h^3}\left((x_1 - \cancel{x_m} - x_0 + \cancel{x_m})((x_1 - x_m)^2 + (x_1 - x_m)(x_0 - x_m) + (x_0 - x_m)^2)\right)$$

$$= \frac{b}{h^2} h(x_0 + x_1 - 2x_m) + \frac{c}{h^3} h($$

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$$f = f_m + \frac{b}{h^2}(x - x_m)^2 + \frac{c}{h^3}(x - x_m)^3$$ why no linear term?

$h = x_1 - x_0$ $\quad\quad$ $\alpha = hr = x_m - x_0$

$$g = \frac{df}{dx} = \frac{2b}{h^2}(x - x_m) + \frac{3c}{h^3}(x - x_m)^2$$ eq 17.2

$$F \equiv f_1 - f_0 = \cancel{f_m} + \frac{b}{h^2}(x_1 - x_m)^2 + \frac{c}{h^3}(x_1 - x_m)^3$$

$$- \cancel{f_m} - \frac{b}{h^2}(x_0 - x_m)^2 - \frac{c}{h^3}(x_0 - x_m)^3$$

$$= \frac{b}{h^2}(x_1 - x_0 + x_0 - x_m)^2 + \frac{c}{h^3}(x_1 - x_0 + x_0 - x_m)^3$$

$$- \frac{b}{h^2}h^2r^2 + \frac{c}{h^3}(h^3r^3)$$

$$= \frac{b}{h^2}(h - hr)^2 + \frac{c}{h^3}(h - hr)^3$$

$$- \frac{b}{h^2}(h^2r^2) + cr^3$$

$$= b(1-r)^2 + c(1-r)^3 - b + cr^3$$

$$= b(\cancel{1} - 2r + r^2) + c(1 - 3r + 3r^2 - \cancel{r^3}) - \cancel{b} + \cancel{cr^3}$$

$$= -2br + br^2 + c - 3rc + 3cr^2$$

$$= b(1-2r) + c(1-3r+3r^2) \qquad \text{eq } 17.3$$

$$b \equiv (g_1 - g_0)h = h\Big[\frac{2b}{h^2}(x_1 - x_0 + x_0 - x_m) + \frac{3c}{h^3}(h-hr)^2$$

$$- \frac{2b}{h^2}(x_0 - x_m) - \frac{3c}{h^3}(x_0 - x_m)^2\Big]$$

$$= \cancel{h}\Big[\frac{2b}{h}(h - rh) + \frac{3c}{\cancel{h^2}}\cancel{h^2}(1-r)^2$$

$$-\frac{2b}{\cancel{h}}(-\cancel{h}r) - \frac{3c}{\cancel{h^2}}(\cancel{h^2}r^2)$$

$$= 2b(1-\cancel{r}) + 3c(1-r)^2 + \cancel{2b}r - 3cr^2$$

$$= 2b + 3c(1-2r+\cancel{r^2}) - \cancel{3cr^2}$$

$$= 2b + 3c(1-2r) \qquad \text{eq } 17.4$$

$$g_0h = \frac{2b}{\cancel{h}}(-\cancel{h}r) + \frac{3c}{\cancel{h^2}}(\cancel{h^2}r^2)$$

$$= -2br + 3cr^2 \qquad \text{eq } 17.5.$$

Thus in summary

$$F \equiv f_1 - f_0 = b(1-2r) + c(1-3r+3r^2) = b + c(1-3r) \underbrace{-2br + 3cr^2}_{g_0 h}$$

$$G \equiv (g_1 - g_0)h = 2b + 3c(1-2r)$$

$$g_0 h = -2br + 3cr^2$$

$$\therefore \quad F = g_0 h + b + c(1-3r) \quad \Rightarrow \quad 2F = 2g_0 h + 2b - 6cr + 2c$$

$$G = 2b + 3c(1-2r) \qquad G = 2b - 6cr + 3c$$

~~then~~ subtracting $\quad 2F - G = 2g_0 h + \underbrace{2c - 3c}_{-c}$

$$c = 2g_0 h + G - 2F = G - 2(F - g_0 h) \qquad \text{eq} \quad 17.6$$

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17.4 is $G = 2b + 3c(1-2r)$

+ 17.5 is $goh = -2br + 3cr^2$

w/ $c = G - 2(F - goh)$ known. Mult ~~top~~ by $+r$ + Adding gives

$$Gr = 3rc(1-2r)$$
$$+goh \qquad + 3cr^2$$

$$Gr + goh = 3rc - 6r^2c + 3cr^2$$

$$0 = -3cr^2 + 3rc - Gr - goh$$

$$\Rightarrow 3cr^2 + (G - 3c)r + goh = 0 \qquad \text{eq } 17.7$$

$c \ll 1$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} \cdot \frac{\left(-b \mp \sqrt{b^2-4ac}\right)}{\left(-b \mp \sqrt{b^2-4ac}\right)}$$

$$= \frac{b^2 - (b^2 - 4ac)}{2a\left(-b \mp \sqrt{b^2-4ac}\right)} = \frac{\cancel{4}^{2}2ac}{2a\left(-b \mp \sqrt{b^2-4ac}\right)}$$

$$= \frac{-c}{b \pm \sqrt{b^2-4ac}}$$

Thus

$$r = \frac{-2g_0h}{(G-3c) + \sqrt{(G-3c)^2 - 4(3c)(g_0h)}}$$

$$= \frac{-2g_0h}{(G-3c) + \sqrt{(G-3c)^2 - 12cg_0h}} \qquad \text{eq 17.8}$$

fit a parabola to $f_0, f_1 + g_1$ Then

$$f = f_m + \frac{b}{h^2}(x-x_m)^2$$

So that

$$F \equiv f_1 - f_0 = \frac{b}{h^2}(x_1 - x_m)^2 - \frac{b}{h^2}(x_0 - x_m)^2$$

$$= \frac{b}{h^2}(x_1 - x_0 + x_0 - x_m)^2 - \frac{b}{\cancel{h^2}}(\cancel{h^2}r^2)$$

$$= \frac{b}{h^2}(h - hr)^2 - r^2 b$$

$$F = b(1-r)^2 - r^2 b$$

$$= b(1-2r+\cancel{r^2}) - \cancel{b}\cancel{r^2} = b(1-2r)$$

$$+ \quad g_1 h = \frac{2b}{h^2}(x_1 - x_m) = \frac{2b}{h^2}(x_1 - x_0 + x_0 - x_m)$$

$$= \frac{2b}{h^2}(h - rh)$$

$$= \frac{2b}{h}(1-r)$$

$$\therefore \quad g_1 h = 2b(1-r)$$

Thus

$$\left.\begin{array}{l} F = b(1-2r) \\ g_1 h = 2b(1-r) \end{array}\right] \text{ 2 eqs 2 unknowns } b + r$$

$$\div \quad \frac{F}{g_1 h} = \frac{1-2r}{2(1-r)} \Rightarrow 2F(1-r) = g_1 h(1-2r)$$

$$2F - g_1 h = 2Fr - 2g_1 h r = 2r(F - g_1 h)$$

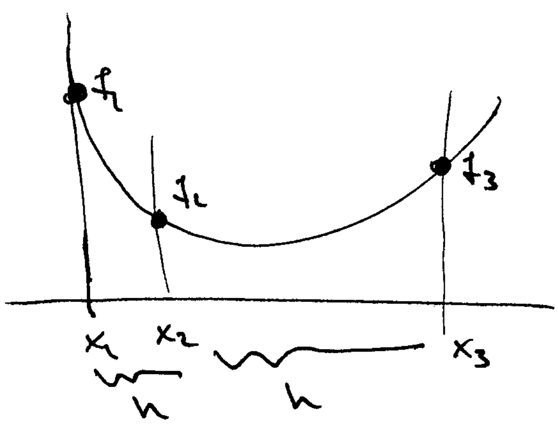
$$\therefore \quad r = \frac{2F - g_1 h}{2(F - g_1 h)} = \frac{F + F - g_1 h}{2(F - g_1 h)}$$

$$= \frac{1 + F/(F - g_1 h)}{2} = \frac{1 - \frac{F}{g_1 h - F}}{2}$$

w/ $b = \dfrac{F}{1-2r}$

$$1-2r = 1-\left(1-\frac{F}{g_i h - F}\right) = \frac{F}{g_i h - F}$$

$$\therefore\ b = \frac{F}{\dfrac{F}{g_i h - F}} = g_i h - F$$



Assume spacing of the points is uniform.

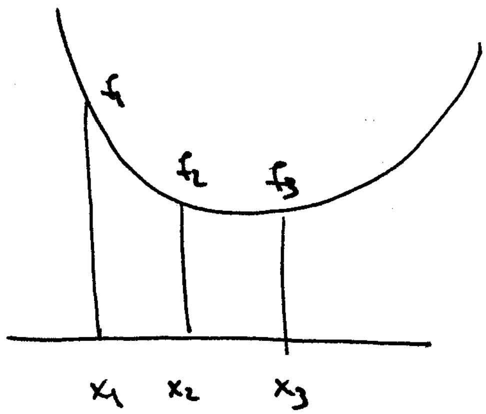
$$\Delta^2 \approx \frac{f_3 - 2f_2 + f_1}{\Delta x^2}$$ Approx of 2nd derivative.

How implement in the case of non equally spaced points?

let $y = a + b(x - x_3) + c(x - x_3)^2$

$f_1 = a + b(x_1 - x_3) +$ $\quad = a + b(-h) + 2^2 c h^2$

$f_2 = a +$



order x_1, x_2, x_3 ∋

$x_1 < x_2 < x_3$.

Let $y = a + b(x-x_3) + c(x-x_3)^2$ pass through these 3 pts

$$f_1 = a + b(-2h) + c(-2h)^2 = a - 2bh + 4c^2h^2$$

$$f_2 = a - hb + ch^2$$

$$f_3 = a$$

$$\Rightarrow \quad f_1 = f_3 - 2hb + 4h^2c$$

$$f_2 = f_3 - hb + h^2c$$

$$\begin{pmatrix} f_1 - f_3 \\ f_2 - f_3 \end{pmatrix} = \begin{pmatrix} -2h & 4h^2 \\ -h & h^2 \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix} = h \begin{pmatrix} -2 & 4h \\ -1 & h \end{pmatrix} \begin{pmatrix} b \\ c \end{pmatrix}$$

$$\begin{pmatrix} b \\ c \end{pmatrix} = \frac{1}{h} \frac{1}{(-2h+4h)} \begin{pmatrix} h & -4h \\ 1 & -2 \end{pmatrix} \begin{pmatrix} f_1 - f_3 \\ f_2 - f_3 \end{pmatrix}$$

$$= \frac{1}{h} \frac{1}{2h} \begin{pmatrix} h & -4h \\ 1 & -2 \end{pmatrix} \begin{pmatrix} f_1 - f_3 \\ f_2 - f_3 \end{pmatrix}$$

$$\Rightarrow \quad b = \frac{1}{2h^2} \left(h(f_1 - f_3) - 4h(f_2 - f_3) \right)$$

$$c = \frac{1}{2h^2} \left((f_1 - f_3) - 2(f_2 - f_3) \right)$$

Thus $f(x) = a + b(x-x_3) + c(x-x_3)^2$

$$\frac{df}{dx} = b + 2c(x-x_3) = 0 \Rightarrow$$

$$x = x_3 - \frac{b}{2c}$$

Thus

$$x_m = x_3 - \frac{1}{2}\frac{\left(h(f_1-f_3) - 4h(f_2-f_3)\right)}{\left(f_1 - f_3 - 2(f_2-f_3)\right)}$$

$$= x_3 - \frac{1}{2}\frac{(hf_1 - hf_3 - 4hf_2 + 4hf_3)}{f_1 - 2f_2 + f_3}$$

$$= x_3 - \frac{h}{2}\frac{(f_1 + 3f_3 - 4f_2)}{f_1 - 2f_2 + f_3}$$

$$= x_3 - \frac{h}{2}\frac{(f_1 - 2f_2 + f_3 + 2f_3 - 2f_2)}{f_1 - 2f_2 + f_3}$$

$$= x_3 - \frac{h}{2} - \frac{h(f_3 - f_2)}{f_1 - 2f_2 + f_3}$$

Almost eq on pg 457 sign mistake.

$$x_2 = x_3 - h$$

$$x_m = x_2 + \frac{h}{2} - \frac{h(f_3 - f_2)}{f_1 - 2f_2 + f_3}$$

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$$x_m = x_2 + \frac{(f_1 - 2f_2 + \cancel{f_3} - \cancel{f_3} + f_2)h}{2(f_1 - 2f_2 + f_3)}$$

$$= x_2 + \frac{f_1 - f_2}{(f_1 - 2f_2 + f_3)} \frac{h}{2}$$

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$$v = b(s-s_m)^2 + v_m \qquad b > 0$$

$$v_1 - v_0 = b\left((s_1-s_m)^2 - (s_0-s_m)^2\right).$$

$$= b(s_1 - \cancel{s_m} - s_0 + \cancel{s_m})(s_1 - s_m + s_0 - s_m)$$

$$= b(s_1 - s_0)(s_1 + s_0 - 2s_m)$$

$$s_m = \frac{-(v_1-v_0)}{2b(s_1-s_0)} + \frac{s_1+s_0}{2}$$

$$\Rightarrow s_m = \frac{s_0+s_1}{2} - \frac{(v_1-v_0)}{2b(s_1-s_0)}$$

let $s_1 - s_0 = h$

$s_m - s_0 = \delta_0$

$$s_m - s_0 = \frac{s_1-s_0}{2} - \frac{v_1-v_0}{2b(s_1-s_0)}$$

$$\delta_0 = \frac{h}{2} - \frac{v_1-v_0}{2bh}$$

$$v'' = 2b$$

$$\therefore \quad \delta_0 = h\left[\frac{1}{2} - \frac{(v_1-v_0)}{v''h^2}\right]$$

let $u'' = u''_{exact} + \delta$

~~$x_2 = \left\{ \frac{1}{2} - \frac{(u_1 - u_0)}{(u''+\delta) h} \right\} h$~~ where ~~$x_2 = x_m = x_0$~~

x_0, x_1 given

x_2 is predicted from $x_2 = x_0 + \delta_0 = x_0 + h\left[\frac{1}{2} - \frac{(u_1 - u_0)}{h^2 u''}\right]$

But an error in the calculation of u'' is made

What effect does this have on x_2?

$$x_2 = x_0 + h\left[\frac{1}{2} - \frac{(u_1 - u_0)}{h^2 \underbrace{(u''+\delta)}_{\text{error of size } \delta}}\right]$$

Also Assume we are very close to the minimum + thus $f(u)$ is approx a parabola.

$$u_1 - u_0 = b(x_1 - x_0)(x_1 + x_0 - 2x_m) \qquad u'' = 2b$$

$$x_2 = x_0 + h\left[\frac{1}{2} - \frac{b(x_1 - x_0)(x_1 + x_0 - 2x_m)}{h^2(u''+\delta)}\right]$$

$$x_2 - x_m = x_0 - x_m + h\left[\frac{1}{2} - \frac{u''(x_1 + x_0 - 2x_m)}{2h(u''+\delta)}\right]$$

$$x_2 - x_m = x_0 - x_m + \frac{h}{2(\upsilon'' + \delta)}\left[\upsilon'' + \delta - \frac{\upsilon''}{h}(x_1 - x_m + x_0 - x_m)\right]$$

$$= \frac{1}{(\upsilon'' + \delta)}\left[(\upsilon'' + \delta)(x_0 - x_m) + \frac{h}{2}(\upsilon'' + \delta) - \frac{\upsilon''}{2}(x_1 - x_m + x_0 - x_m)\right]$$

$$= \frac{1}{(\upsilon'' + \delta)}\left[\left(\upsilon'' + \delta - \frac{\upsilon''}{2}\right)(x_0 - x_m) - \frac{\upsilon''}{2}(x_1 - x_m) + \frac{h}{2}(\upsilon'' + \delta)\right]$$