eq 14 Ansorge 07-08-031

$$\lim_{h\to 0} \frac{1}{h} \left\{ \int_{W(t+h)} \phi(\tilde{y}, t+h)\, d(y_1,y_2,y_3) - \int_{W(t)} \phi(x,y,z,t)\, d(x,y,z) \right\}$$

$$= \lim_{h\to 0} \frac{1}{h} \left\{ \int_{W(t)} \phi(x + h u(x,t) + o(h), t+h)(1 + h \nabla\circ u + o(h))\, d(x,y,z) \right.$$

$$\left. - \int_{W(t)} \phi(x,y,z,t)\, d(x,y,z) \right\}$$

taylor expand $\phi(\dots)$ about h

$$= \lim_{h\to 0} \frac{1}{h} \left\{ \int_{\overline{W}(t)} \left[\phi(\vec{x},t) + \phi_t h + \nabla\phi\circ\vec{u}\, h + o(h)\right](1 + h\nabla\circ u + o(h))\, d(x,y,z) \right.$$

$$\left. - \int_{\overline{W}(t)} \phi(x,y,z,t)\, d(x,y,z) \right\}$$

$$= \lim_{h\to 0} \frac{1}{h} \left\{ \int_{\overline{W}(t)} (\cancel{\phi} + \phi_t h + \nabla\phi\circ\vec{u}\, h + \phi\nabla\circ u\, h - \cancel{\phi})\, d(x,y,z) \right\}$$

$$= \int_{\overline{W}(t)} (\phi_t + \nabla\phi\circ\vec{u} + \phi\nabla\circ u)\, d(x,y,z)$$

eq 1.2 ✓

bet eq 1.8

$$\frac{\partial}{\partial t}\vec{q} + \frac{1}{\rho}(\vec{q}\circ\nabla)\vec{q} + \nabla\circ\left(\frac{1}{\rho}\vec{q}\right)\vec{q} + \nabla p = \vec{F}$$

x component:

$$\frac{\partial}{\partial t}q_1 + \frac{1}{\rho}\left(q_1\underline{\frac{\partial}{\partial x}}q_1 + q_2\underline{\underline{\frac{\partial}{\partial y}}}q_1 + q_3\underline{\underline{\underline{\frac{\partial}{\partial z}q_1}}}\right) + \left(\underline{\frac{\partial}{\partial x}}\left(\frac{1}{\rho}q_1\right) + \underline{\underline{\frac{\partial}{\partial y}}}\left(\frac{1}{\rho}q_2\right) + \underline{\underline{\underline{\frac{\partial}{\partial z}}}}\left(\frac{1}{\rho}q_3\right)\right)q_1$$

$$+ \frac{\partial p}{\partial x} = 0$$

$$= \frac{\partial q_1}{\partial t} + \frac{1}{\rho}q_1\frac{\partial q_1}{\partial x} + \frac{\partial}{\partial x}\left(\frac{1}{\rho}q_1\right)q_1 + \frac{1}{\rho}q_2\frac{\partial q_1}{\partial y} + q_1\frac{\partial}{\partial y}\left(\frac{1}{\rho}q_2\right)$$

$$+ \frac{1}{\rho}q_3\frac{\partial q_1}{\partial z} + q_1\frac{\partial}{\partial z}\left(\frac{1}{\rho}q_3\right) + \frac{\partial p}{\partial x} = 0$$

$$\Rightarrow \frac{\partial q_1}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{\rho}q_1\cdot q_1\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}q_1q_2\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}q_1q_3\right)$$

$$+ \frac{\partial p}{\partial x} = 0$$

$$\Rightarrow \frac{\partial q_1}{\partial t} + \frac{\partial}{\partial x}\left(\frac{1}{\rho}q_1q_1 + p\right) + \frac{\partial}{\partial y}\left(\frac{1}{\rho}q_1q_2\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho}q_1q_3\right) = 0 \quad ✓$$

$$\left(1-\left(\frac{u}{c}\right)^2\right)\frac{\partial u}{\partial x}+\left(1-\left(\frac{v}{c}\right)^2\right)\frac{\partial v}{\partial y}-\frac{uv}{c^2}\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=0 \quad \text{eq } 1.55$$

Assum $u=u(x,y)$

$v=v(x,y)$ can be inverted then

$$D=\begin{vmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{vmatrix}\neq 0$$

$$du=\frac{\partial u}{\partial x}\cdot dx+\frac{\partial u}{\partial y}\cdot dy$$

$$dv=\frac{\partial v}{\partial x}\,dx+\frac{\partial v}{\partial y}\cdot dy$$

$$\rightarrow \begin{pmatrix}du\\ dv\end{pmatrix}=\begin{pmatrix}\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y}\\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y}\end{pmatrix}\begin{pmatrix}dx\\ dy\end{pmatrix}$$

$$\Rightarrow \begin{pmatrix}dx\\ dy\end{pmatrix}=\frac{1}{D}\begin{pmatrix}\frac{\partial v}{\partial y} & -\frac{\partial u}{\partial y}\\ -\frac{\partial v}{\partial x} & \frac{\partial u}{\partial x}\end{pmatrix}\begin{pmatrix}du\\ dv\end{pmatrix}$$

so $$dx=\frac{1}{D}\left(\frac{\partial v}{\partial y}\cdot du-\frac{\partial u}{\partial y}\cdot dv\right)$$

$$dy=\frac{1}{D}\left(-\frac{\partial v}{\partial x}\cdot du+\frac{\partial u}{\partial x}\cdot dv\right)$$

$$\Rightarrow \quad \frac{\partial x}{\partial u}=\frac{1}{D}\frac{\partial v}{\partial y}\ ;\ \frac{\partial x}{\partial v}=-\frac{1}{D}\frac{\partial u}{\partial y}\ ;\ \frac{\partial y}{\partial u}=-\frac{1}{D}\frac{\partial v}{\partial x}\ ;\ \frac{\partial y}{\partial v}=\frac{1}{D}\frac{\partial u}{\partial x}$$

or $\frac{\partial u}{\partial x} = D\frac{\partial y}{\partial v}$; $\frac{\partial u}{\partial y} = -D\frac{\partial x}{\partial v}$; $\frac{\partial v}{\partial x} = -D\frac{\partial y}{\partial u}$; $\frac{\partial v}{\partial y} = D\frac{\partial x}{\partial u}$

Thus: Eq 1.55 Becomes:

$$\left(1-\left(\frac{u}{c}\right)^2\right)\left[D\frac{\partial y}{\partial v}\right] + \left(1-\left(\frac{v}{c}\right)^2\right)\left[D\frac{\partial x}{\partial u}\right] - \frac{uv}{c^2}\left[-D\frac{\partial x}{\partial v} + -D\frac{\partial y}{\partial u}\right] = 0$$

$\div D \neq 0$

$$\left(1-\left(\frac{u}{c}\right)^2\right)\frac{\partial y}{\partial v} + \left(1-\left(\frac{v}{c}\right)^2\right)\frac{\partial x}{\partial u} + \frac{uv}{c^2}\left(\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\right) = 0 \quad ✓$$

+ eq of $\nabla \times \vec{u} = 0$ is $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0$ or transformed

$$-D\frac{\partial x}{\partial y} - \left(-D\frac{\partial y}{\partial u}\right) = 0 \qquad \div D$$

$$-\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u} = 0 \qquad ✓$$

$$\nabla \times \vec{x}(u,v,w) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \equiv 0 \\ x & y & z \equiv 0 \end{vmatrix} = \hat{k}\left(\frac{\partial y}{\partial u} - \frac{\partial x}{\partial v}\right) \quad ✓ = 0$$

1.56 w/ $x = \theta_u$ $y = \theta_v$ we get

$$\left(1-\left(\frac{v}{c}\right)^2\right)\theta_{uu} + \left(1-\left(\frac{u}{c}\right)^2\right)\theta_{vv} + \frac{uv}{c^2}\left(\theta_{vu} + \theta_{uv}\right) = 0$$

$$\left(1-\left(\frac{v}{c}\right)^2\right)\theta_{uu} + \left(1-\left(\frac{u}{c}\right)^2\right)\theta_{vv} + \frac{2uv}{c^2}\theta_{uv} = 0 \quad ✓$$

(w, α) used in the hodograph plane

$U = w\cos\alpha \qquad V = w\sin\alpha$

eq 1.57 is

$$\left(1-\left(\frac{v}{c}\right)^2\right)\partial_{uu}\theta + \left(1-\left(\frac{u}{c}\right)^2\right)\partial_{vv}\theta + \frac{2uv}{c^2}\partial_{uv}\theta = 0 \qquad \text{eq } 1.57$$

let $u = w\cos\alpha \qquad v = w\sin\alpha$

$$\begin{cases} w = \sqrt{u^2+v^2} \\ \alpha = \tan^{-1}\left(\frac{v}{u}\right) \end{cases}$$

$$\partial_u = \partial_u w\, \partial_w + \partial_u \alpha\, \partial_\alpha$$

$$\partial_v = \partial_v w\, \partial_w + \partial_v \alpha\, \partial_\alpha$$

$$\frac{\partial w}{\partial u} = ? \qquad \frac{\partial \alpha}{\partial u} = ? \qquad \frac{\partial w}{\partial v} = ? \qquad \frac{\partial \alpha}{\partial v} = ?$$

$$\frac{\partial w}{\partial u} = \frac{u}{\sqrt{u^2+v^2}} \; ; \quad \frac{\partial \alpha}{\partial u} = \frac{\left(-\frac{v}{u^2}\right)}{1+\left(\frac{v}{u}\right)^2}$$

$$\frac{\partial w}{\partial v} = \frac{v}{\sqrt{u^2+v^2}} \; ; \quad \frac{\partial \alpha}{\partial v} = \frac{\left(\frac{1}{u}\right)}{1+\left(\frac{v}{u}\right)^2}$$

$$\Rightarrow \frac{\partial w}{\partial u} = \cos\alpha \; ; \quad \frac{\partial \alpha}{\partial u} = \frac{-v}{u^2+v^2} = -\frac{\sin\alpha}{w}$$

$$\frac{\partial w}{\partial v} = \sin\alpha \; ; \quad \frac{\partial \alpha}{\partial v} = \frac{u}{u^2+v^2} = +\frac{\cos\alpha}{w}$$

$$\partial_v = \cos\alpha \frac{\partial}{\partial w} + -\frac{\sin\alpha}{w}\frac{\partial}{\partial\alpha}$$

$$\frac{\partial}{\partial v} = \sin\alpha \frac{\partial}{\partial v} + \frac{\cos\alpha}{w}\frac{\partial}{\partial\alpha}$$

Then eq 1.57 becomes:

$$\left(1-\left(\frac{v}{c}\right)^2\right)\left(\cos\alpha\,\partial_w - \frac{1}{w}\sin\alpha\,\partial_\alpha\right)^2\Theta$$

$$+\left(1-\left(\frac{u}{c}\right)^2\right)\left(\sin\alpha\,\partial_w + \frac{1}{w}\cos\alpha\,\partial_\alpha\right)^2\Theta$$

$$+\frac{2uv}{c^2}\left(\cos\alpha\,\partial_w - \frac{1}{w}\sin\alpha\,\partial_\alpha\right)\left(\sin\alpha\,\partial_w + \frac{1}{w}\cos\alpha\,\partial_\alpha\right)\Theta = 0$$

$$\Rightarrow \left(1-\left(\frac{v}{c}\right)^2\right)\left(\cos\alpha\,\partial_w - \frac{1}{w}\sin\alpha\,\partial_\alpha\right)\left(\cos\alpha\,\partial_w - \frac{1}{w}\sin\alpha\,\partial_\alpha\right)\Theta$$

$$+\left(1-\left(\frac{u}{c}\right)^2\right)\left(\sin\alpha\,\partial_w + \frac{1}{w}\cos\alpha\,\partial_\alpha\right)\left(\sin\alpha\,\partial_w + \frac{1}{w}\cos\alpha\,\partial_\alpha\right)\Theta$$

$$+\frac{2uv}{c^2}\Big(\sin\alpha\cos\alpha\,\partial_w^2 + \frac{1}{w}\cos^2\alpha\,\partial^2_{w\alpha} - \frac{1}{w^2}\cos^2\alpha\,\partial_\alpha$$

$$-\frac{1}{w}\sin\alpha(\cos\alpha)\partial_w - \frac{1}{w}\sin^2\alpha\,\partial^2_{\alpha w} - \frac{1}{w^2}\sin\alpha(-\sin\alpha)\partial_\alpha$$

$$-\frac{1}{w^2}\sin\alpha\cos\alpha\,\partial^2_{\alpha\alpha}\Big)\Theta = 0$$

$$\Rightarrow \left(1-\left(\frac{V}{c}\right)^2\right)\Big[\cos^2\alpha\,\partial^2_\omega + \frac{1}{\omega^2}\cos\alpha\sin\alpha\,\partial_\alpha - \frac{1}{\omega}\cos\alpha\sin\alpha\,\partial_{\alpha\omega}$$

$$-\frac{1}{\omega}\sin\alpha\cos\alpha\,\partial_{\alpha\omega} + \frac{1}{\omega}\sin^2\alpha\,\partial_\omega + \frac{1}{\omega^2}\sin^2\alpha\,\partial^2_\alpha$$

$$+\frac{1}{\omega^2}\sin\alpha\cos\alpha\,\partial_\alpha\Big]\Theta$$

$$+ \left(1-\left(\frac{U}{c}\right)^2\right)\Big[\sin^2\alpha\,\partial^2_\omega - \frac{1}{\omega^2}\sin\alpha\cos\alpha\,\partial_\alpha + \frac{1}{\omega}\cos\alpha\sin\alpha\,\partial^2_{\omega\alpha}$$

$$+\frac{1}{\omega}\cos^2\alpha\,\partial_\omega + \frac{1}{\omega}\cos\alpha\sin\alpha\,\partial_{\alpha\omega} + \frac{1}{\omega^2}(-1)\sin\alpha\cos\alpha\,\partial_\alpha$$

$$+\frac{1}{\omega^2}\cos^2\alpha\,\partial^2_\alpha\Big]\Theta$$

$$+\frac{2UV}{c^2}\Big[\sin\alpha\cos\alpha\,\partial^2_\omega + \left(\cos^2\alpha - \sin^2\alpha\right)\left(\frac{1}{\omega}\right)\partial^2_{\omega\alpha}$$

$$+\left(\sin^2\alpha - \cos^2\alpha\right)\left(\frac{1}{\omega^2}\right)\partial_\alpha - \frac{1}{\omega}\sin\alpha\cos\alpha\,\partial_\omega$$

$$-\frac{1}{\omega^2}\sin\alpha\cos\alpha\,\partial^2_{\alpha\alpha}\Big]\Theta = 0$$

$$\Rightarrow \left(1-\left(\frac{V}{c}\right)^2\right)\Big[\cos^2\alpha\,\partial^2_\omega - \frac{2}{\omega}$$

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Assume eq 1.58 Results

$$\partial_{\omega\omega}\theta + \frac{1}{\omega^2}\left(1-\frac{\omega^2}{c^2}\right)\partial_{\alpha\alpha}\theta + \frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)\partial_\omega\theta = 0$$

let $\theta(\omega,\alpha) = g(\omega)\sin(m\alpha)$ or $\theta(\omega,\alpha) = g(\omega)\cos(m\alpha)$

$$g''(\omega)\begin{Bmatrix}\sin(m\alpha)\\ \cos(m\alpha)\end{Bmatrix} + \frac{1}{\omega^2}\left(1-\frac{\omega^2}{c^2}\right)m^2(-1)\begin{Bmatrix}\sin(m\alpha)\\ \cos(m\alpha)\end{Bmatrix}g + \cancel{\frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)mg'\begin{Bmatrix}\cos(m\alpha)\\ -\sin(m\alpha)\end{Bmatrix}}$$

$\leftarrow 0$

$$+\frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)g'(\omega)\begin{Bmatrix}\sin(m\alpha)\\ \cos(m\alpha)\end{Bmatrix} = 0$$

$$\Rightarrow \quad g''(\omega) - \frac{m^2}{\omega^2}\left(1-\frac{\omega^2}{c^2}\right)g + \frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)g' = 0$$

$$\Rightarrow \quad g''(\omega) + \frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)g' - \frac{m^2}{\omega^2}\left(1-\frac{\omega^2}{c^2}\right)g = 0 \quad \checkmark$$

$m=0$

$$\frac{g_0''}{g_0'} = -\frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right) = -\frac{1}{\omega} + \frac{\omega}{c^2} \quad \checkmark$$

$m=1$

$$g'' + \frac{1}{\omega}\left(1-\frac{\omega^2}{c^2}\right)g' - \frac{1}{\omega^2}\left(1-\frac{\omega^2}{c^2}\right)g = 0$$

2.9 is $f(\rho) = v_f \cdot \rho \cdot \left(1 - \frac{\rho}{\rho^*}\right)$

1.17 is $x(t) = x_0 + t f'(\rho_0(x_0))$

$$f'(\rho) = v_f \left[\left(1 - \frac{\rho}{\rho^*}\right) + \frac{\rho}{\rho^*}(-1) \right]$$
$$= v_f \left[1 - \frac{2\rho}{\rho^*} \right]$$

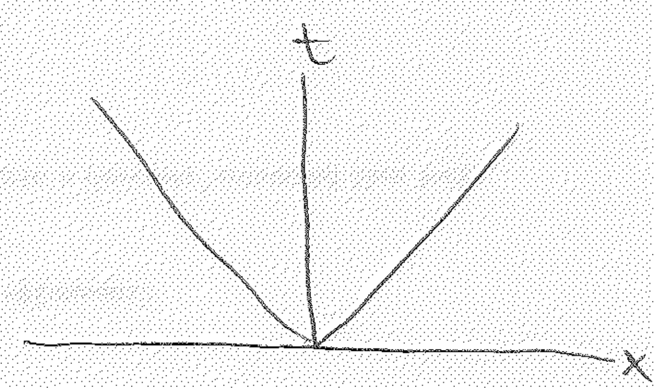
w/ $\rho_0(x) = \begin{cases} \rho^* & x < 0 \\ 0 & x \geq 0 \end{cases}$

$$f'(\rho_0(x_0)) = v_f \cdot \begin{cases} 1-2 & x_0 < 0 \\ 1 & x_0 > 0 \end{cases} = \begin{cases} -v_f & x_0 < 0 \\ v_f & x_0 > 0 \end{cases}$$

$\therefore$

$$x(t) = x_0 + t \begin{cases} -v_f & x_0 < 0 \\ v_f & x_0 > 0 \end{cases}$$

$$\frac{x - x_0}{v_f} = \begin{cases} -t & x_0 < 0 \\ +t & x_0 > 0 \end{cases}$$



eq 2,3 is $\partial_t \rho + \partial_x f = 0$

$$f(\rho) = \rho v(\rho) = \cancel{\rho \cdot v_f \cdot \left(1 - \frac{\rho}{\rho^*}\right)} = v \cdot \rho^* \left(1 - \frac{v}{v_f}\right)$$

$$= \rho v_f \left(1 - \frac{\rho}{\rho^*}\right)$$

$$\frac{\rho}{\rho^*} = 1 - \frac{v}{v_f} \rightarrow \left(1 - \frac{\rho}{\rho^*}\right) v_f = v(\rho)$$

eq 2,3 $\rightarrow \partial_t \rho + f_v \cdot \partial_x v(\rho) = 0$

$$\rightarrow \partial_t \rho + f_v \frac{\partial}{\partial x}\left[v_f \left(1 - \frac{\rho}{\rho^*}\right)\right] = 0$$ ~✓

eq 2,2 is

$$\int_\Omega \left\{ \vec{v} \partial_t \Phi + \vec{f}(\vec{v}) \partial_x \Phi \right\} d(x,t) + \int_{-\infty}^{+\infty} \vec{v}_0(x) \Phi(x,0)\, dx = 0 \quad \forall \Phi \in C_0^1(\Omega)$$

$$\Rightarrow \int_0^\infty \int_{-\infty}^{+\infty} \left[\rho_1 \cdot \partial_t \Phi + \partial_x \Phi \cdot \rho_1 v_f \left(1 - \frac{\rho_1}{\rho^*}\right)\right] d(x,t) + \int_{-\infty}^{\infty} \Phi(x,0)\, \rho_0(x)\, dx$$

$$= \int_0^\infty \int_{-\infty}^{0} \left[\rho^* \partial_t \Phi + 0\right] d(x,t) + \int_{-\infty}^{0} \Phi(x,0)\, \rho^*\, dx =$$

$$= \rho^* \int_{-\infty}^{0} \Phi(x,t')\Big|_0^\infty dx + \rho^* \int_{-\infty}^{0} \Phi(x,0)\, dx =$$

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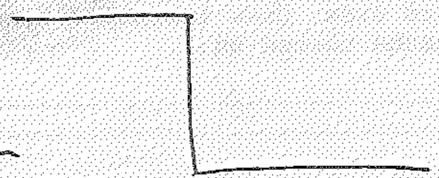
$$= \rho^* \int_{-\infty}^{0} \left(\cancel{\Phi(x,\infty)}^{\,0} - \Phi(x,0) \right) dx + \rho^* \int_{-\infty}^{0} \Phi(x,0)\, dx = 0 \quad ✓$$

$\hat{v} > u_r > 0$



$\to \hat{v} - u_r > 0$

since $\hat{v} - u_\ell$ is the same

sign by

$\rho_\ell(\hat{v} - u_\ell) = \rho_r(\hat{v} - u_r)$

$\hat{v} - u_\ell > 0$

$\to \hat{v} > u_\ell$

ϵ = energy per unit mass

$$+ \frac{\partial \rho}{\partial t} \cdot \epsilon + \rho \frac{\partial \epsilon}{\partial t} + \frac{1}{2}\frac{\partial \rho}{\partial t} u^2$$

$$+ \rho u \frac{\partial u}{\partial t} + u \left\{ \frac{\partial p}{\partial x} + \frac{\partial \rho}{\partial x}\epsilon + \rho \frac{\partial \epsilon}{\partial x} + \frac{1}{2}\frac{\partial \rho}{\partial x} u^2 + \rho u \frac{\partial u}{\partial x} \right\}$$

$$+ \left(\rho\epsilon + \frac{\rho}{2}u^2 + p \right)\frac{\partial u}{\partial x} = 0$$

✓

$$\Rightarrow \quad \frac{\partial \rho}{\partial t}\epsilon + \epsilon u \frac{\partial \rho}{\partial x} + \epsilon \rho \frac{\partial u}{\partial x}$$

$$+ \frac{1}{2}\frac{\partial \rho}{\partial x} u^3 + \frac{\rho}{2}u^2 \frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial \rho}{\partial t} u^2$$

$$+ \rho \frac{\partial \epsilon}{\partial t} + u\rho \frac{\partial \epsilon}{\partial x} + p \frac{\partial u}{\partial x}$$

$$+ \rho u \frac{\partial u}{\partial t} + u \frac{\partial p}{\partial x} + \rho u^2 \frac{\partial u}{\partial x} = 0$$

$$\Rightarrow \quad \epsilon\left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) \right) + \frac{u^2}{2}\left(u\frac{\partial \rho}{\partial x} + \rho \frac{\partial u}{\partial x} + \frac{\partial \rho}{\partial t} \right)$$

$$+ \rho \left(\frac{\partial \epsilon}{\partial t} + u \frac{\partial \epsilon}{\partial x} + \frac{p}{\rho}\frac{\partial u}{\partial x} \right)$$

$$+ u\rho \left(\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + u \frac{\partial u}{\partial x} \right) = 0$$

$\Rightarrow$

$$\left(E + \frac{u^2}{2}\right)\left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u)\right) + \rho\left(\frac{\partial E}{\partial t} + u\frac{\partial E}{\partial x} + \frac{p}{\rho}\frac{\partial u}{\partial x}\right)$$

$$+ u\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial x}\right) = 0 \qquad \text{eq } 3.5 \checkmark$$

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$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{q^2}{\rho} + P\right) = 0$$

$q = \rho u$

$$u\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(\rho u^2 + P) = 0$$

$$u\frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial t} + u^2\frac{\partial \rho}{\partial x} + 2u\rho\frac{\partial u}{\partial x} + \frac{\partial P}{\partial x} = 0$$

$$\Rightarrow \quad \rho\left(\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} + u\frac{\partial u}{\partial x}\right) + u\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + \rho\frac{\partial u}{\partial x}\right) = 0$$

$$\Rightarrow \quad \rho\left(\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} + u\frac{\partial u}{\partial x}\right) + u\left(\cancelto{0}{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u)}\right) = 0$$

$$\Rightarrow \quad \frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial P}{\partial x} + u\frac{\partial u}{\partial x} = 0$$ so eq 3.5 Becomes

$$\frac{\partial \varepsilon}{\partial t} + u\frac{\partial \varepsilon}{\partial x} + \frac{P}{\rho}\frac{\partial u}{\partial x} = 0$$ eq 3.6 ✓

Since eq 3.4 is formulated in the following way

$$\frac{\partial s}{\partial t} + \frac{\partial(us)}{\partial x} = \frac{\rho}{T}\left\{ \underbrace{\frac{\partial \varepsilon}{\partial t} + u\frac{\partial \varepsilon}{\partial x}}_{-\frac{P}{\rho}\frac{\partial u}{\partial x}} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} - \frac{uP}{\rho^2}\frac{\partial \rho}{\partial x}\right\}$$

$$= \frac{\rho}{T}\left\{ -\frac{P}{\rho}\frac{\partial u}{\partial x} - \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} - \frac{uP}{\rho^2}\frac{\partial \rho}{\partial x}\right\}$$ eq 3.7 ✓

$$= \frac{p \cdot P}{T}\left\{ -\frac{p}{\rho^2}\frac{\partial u}{\partial x} - \frac{1}{\rho^2}\frac{\partial \rho}{\partial t} - \frac{u}{\rho^2}\frac{\partial \rho}{\partial x} \right\}$$

$$= -\frac{p \cdot P}{T\rho^2}\left\{ \rho\frac{\partial u}{\partial x} + \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} \right\} = -\frac{P}{T\rho}\left\{ \cancelto{0}{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u)} \right\}$$

$$\rightarrow \quad \frac{\partial s}{\partial t} + \frac{\partial}{\partial x}(us) = 0$$

eq 3.9 $\quad \dfrac{\partial s}{\partial t} + \dfrac{\partial (us)}{\partial x} \geq 0$

$$\int_\Omega \left(\Phi \frac{\partial s}{\partial t} + \Phi \frac{\partial (us)}{\partial x} \right) d(x,t) \geq 0 \qquad \forall \, \Phi \in C_0^1(\Omega) \quad \Phi \geq 0$$

$$= \int_\Omega \left(\frac{\partial (\Phi s)}{\partial t} - s\frac{\partial \Phi}{\partial t} + \frac{\partial}{\partial x}(\Phi us) - us\frac{\partial \Phi}{\partial x} \right) d(x,t) \geq 0$$

$$= \int_\Omega \left(\frac{\partial (\Phi s)}{\partial t} + \frac{\partial}{\partial x}(\Phi us) \right) d(x,t) - \int_\Omega \left(s\frac{\partial \Phi}{\partial t} + us\frac{\partial \Phi}{\partial x} \right) d(x,t) \geq 0$$

$$= \int_\Omega \nabla \cdot (\Phi s, \Phi us) \, d(x,t) - \int_\Omega \left(s\frac{\partial \Phi}{\partial t} + us\frac{\partial \Phi}{\partial x} \right) d(x,t) \geq 0$$

$$\nabla = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\right)$$

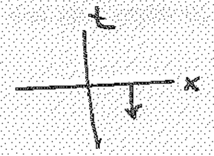
$$= \int_{\partial\Omega} (\Phi s, \Phi u s) \cdot \hat{n} \, d\sigma - \int_{\Omega} \geq 0$$

let $\partial\Omega \to +\infty$ $\{\hat{n} = \text{outward normal}\}$

$\Rightarrow$ only integral that matters is the one over the x-axis

$$= -\int_{-\infty}^{+\infty} \Phi s \Big|_{t=0} dx - \int_{\Omega} \geq 0$$

$\hat{n} = (-1, 0)$



$d\sigma = dx$

$$\Rightarrow \int_{\Omega} \left(s \frac{\partial \Phi}{\partial t} + u s \frac{\partial \Phi}{\partial x} \right) d(x,t) + \int_{-\infty}^{+\infty} \Phi(x,0)\, s(x,0)\, dx \leq 0$$

eq 3.10 ✓

pg 24 Analogue

$$S'(v)\cdot\frac{\partial v}{\partial t} + F'(v)\frac{\partial v}{\partial x} = 0 \qquad \text{eq } 2.1 \qquad \frac{\partial v}{\partial t} + \frac{\partial f(v)}{\partial x} = 0$$

$$\Rightarrow \quad S'(v)\left[-\frac{\partial f(v)}{\partial x}\right] + F'(v)\frac{\partial v}{\partial x} = 0$$

$$\Rightarrow \quad -S'(v)\,f'(v)\frac{\partial v}{\partial x} + F'(v)\frac{\partial v}{\partial x} = 0$$

$$\Rightarrow \quad \left[-S'(v)f'(v) + F'(v)\right]\frac{\partial v}{\partial x} = 0$$

$$\Rightarrow \quad F'(v) = S'(v)f'(v)$$

$$\Rightarrow \quad F(v) = \int_0^v S'(\alpha)f'(\alpha)\,d\alpha + C \qquad \text{eq } 3.13 \ \checkmark$$

If ~~[illegible]~~ $m > 1$ + $\vec{v}$ is

$$\left\langle \nabla_{\vec{v}} S, \frac{\partial \vec{v}}{\partial t} \right\rangle + \left\langle \nabla_{\vec{v}} F, \frac{\partial \vec{v}}{\partial x} \right\rangle = 0$$

$$\frac{\partial \vec{v}}{\partial t} = -\left\langle \nabla_{\vec{v}} f, \frac{\partial \vec{v}}{\partial x} \right\rangle$$

$$\Rightarrow -\left\langle \nabla_{\vec{v}} S, Jf(v)\frac{\partial v}{\partial x} \right\rangle + \left\langle \nabla_{\vec{v}} F, \frac{\partial v}{\partial x} \right\rangle = 0$$

$$\left(\frac{\partial v}{\partial x}\right)^T\left(\nabla_v F - (Jf(v)^T)(\nabla_v S)\right) = 0$$

$$\Rightarrow \quad \nabla_v F - (Jf(\vec{v}))^T \nabla_v S = 0 \qquad \text{eq } 3.14$$

$$\bar{f}(\bar{v}) = \nabla_{\bar{v}} k(\bar{v}) \qquad 3.14. \qquad = \left(\frac{\partial k}{\partial v_i}\right)_{i=1}^{n}$$

$$\nabla_v \tilde{F} - (J f(v))^T \nabla_v \tilde{\delta} = 0 \qquad w/ \quad \tilde{\delta} = \frac{1}{2}\langle v, v\rangle = \frac{1}{2}\left(\sum_{i=1}^{n} v_i^2\right)$$

$$\frac{\partial}{\partial v_i}\tilde{F} - \left(J \begin{pmatrix} \frac{\partial k}{\partial v_1} \\ \vdots \\ \frac{\partial k}{\partial v_n} \end{pmatrix}\right)^T \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix} = 0$$

$$\rightarrow \left(\frac{\partial F}{\partial v_i}\right) - \begin{pmatrix} \frac{\partial^2 k}{\partial v_1^2} & \frac{\partial^2 k}{\partial v_1 \partial v_2} & \cdots & \frac{\partial^2 k}{\partial v_1 \partial v_n} \\ \frac{\partial^2 k}{\partial v_2 \partial v_1} & \cdots & & \\ \vdots & & & \\ \frac{\partial^2 k}{\partial v_n \partial v_1} & \cdots & & \frac{\partial^2 k}{\partial v_n \partial v_n} \end{pmatrix}^T \begin{pmatrix} v_1 \\ \vdots \\ \vdots \\ v_n \end{pmatrix} = 0$$

$$\rightarrow \begin{pmatrix} \frac{\partial F}{\partial v_1} \\ \vdots \\ \frac{\partial F}{\partial v_n} \end{pmatrix} - \begin{pmatrix} \frac{\partial^2 k}{\partial v_1^2} v_1 + \frac{\partial^2 k}{\partial v_1 \partial v_2} v_2 \cdots + \frac{\partial^2 k}{\partial v_1 \partial v_n} v_n \\ \vdots \\ \frac{\partial^2 k}{\partial v_n \partial v_1} v_1 + \frac{\partial^2 k}{\partial v_n \partial v_2} v_2 + \cdots \frac{\partial^2 k}{\partial v_n \partial v_n} \cdot v_n \end{pmatrix} = \bar{0}$$

$$\Rightarrow \quad \frac{\partial F}{\partial v_i} - \sum_{j=1}^{n} \frac{\partial^2 k}{\partial v_i \partial v_j} \cdot v_j = 0 \qquad i=1,\dots,n$$

Book uses m.

Changing to m

$$\frac{\partial F}{\partial v_i} = \sum_{j=1}^{m} \frac{\partial}{\partial v_i}\left(\frac{\partial k}{\partial v_j}\right) v_j$$

$$= \frac{\partial}{\partial v_i}\left(\sum_{j=1}^{m}\left(\frac{\partial k}{\partial v_j}\right) v_j\right) - \frac{\partial k}{\partial v_i}$$ ✓

Integrate wrt v_i

$$\tilde{F}(\vec{v}) = \langle \vec{f}(\vec{v}), \vec{v} \rangle - k(\vec{v}) + \text{const} \qquad \text{eq } 3.15$$ ✓

$m=1 \Rightarrow \quad k(v) = \frac{1}{2}v^2 \qquad f(v) \cancel{=} \frac{v^2}{2}$ Specify a general flux

$\cancel{F(v) =}$ $\quad f(v) \Rightarrow \frac{\partial k}{\partial v} = f(v)$

From eq

$$\frac{\partial F}{\partial v} = \left(\frac{\partial^2 k}{\partial v^2}\right) v = \frac{\partial}{\partial v}\left(\frac{\partial k}{\partial v} \cdot v\right) - \frac{\partial k}{\partial v} \cdot 1$$

How get 3.16?

$F(v) = -k(v) +$

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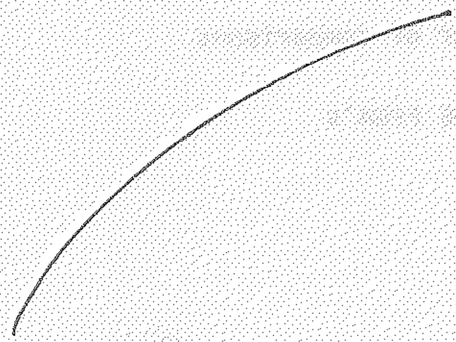
$[\hat{S}]\,\hat{v} \leq [\hat{F}]$ $\qquad$ $\hat{v} \cancel{=} \dfrac{[f]}{[v]}$

$[\hat{S}]\dfrac{[f]}{[v]} \leq [\hat{\tilde{F}}]$ $\qquad$ $\hat{v} = \dfrac{[f]}{[v]}$ for scalar case only



$$F(v) = \begin{cases} \left(\dfrac{S(v) - S(v_r)}{v - v_r}\right)\left(f(v) - f(v_r)\right) - \displaystyle\int_{v_r}^{v} S'(x) f'(x)\, dx \\ 0 \end{cases}$$

$$F'(v) = \begin{cases} \dfrac{S'(v)}{(v - v_r)}\left(f(v) - f(v_r)\right) + (-1)\dfrac{(S(v) - S(v_r))}{(v-v_r)^2}\left(f(v) - f(v_r)\right) \\ \quad + \left(\dfrac{S(v) - S(v_r)}{v - v_r}\right) f'(v) - S'(v) f'(v) & v \neq v_r \\ 0 \end{cases}$$

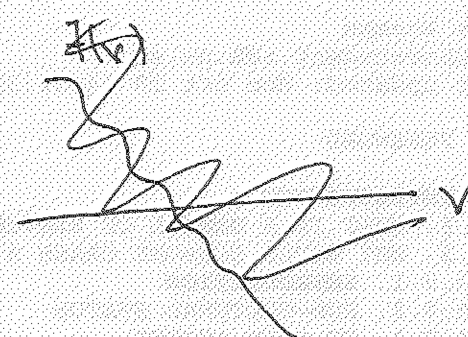
$$\Rightarrow F'(v) = \begin{cases} -S'(v)\left[f'(v) - \dfrac{f(v) - f(v_r)}{v - v_r}\right] \\ \quad + \dfrac{(S(v) - S(v_r))}{v - v_r}\left[f'(v) - \dfrac{f(v) - f(v_r)}{v - v_r}\right] & v \neq v_r \\ 0 & v = v_r \end{cases}$$

$$\Rightarrow \quad f'(v) = \begin{cases} -\left\{ s'(v) - \dfrac{s(v)-s(v_r)}{v-v_r} \right\}^{-1} \left\{ f'(v) - \dfrac{f(v)-f(v_r)}{v-v_r} \right\} & v \neq v_r \\ 0 & v = v_r \end{cases}$$

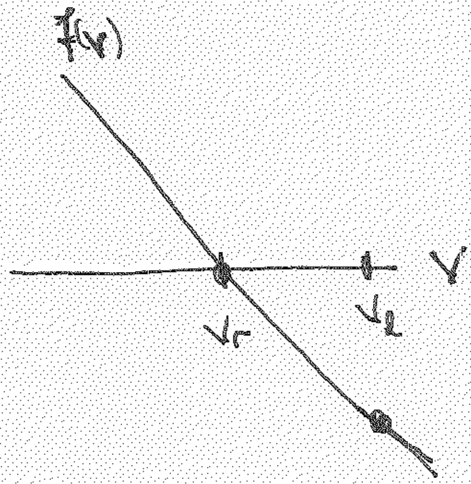
Since $s''(v) > 0 \quad \forall \; v \in \mathbb{R}$

$s'(v) - \dfrac{s(v)-s(v_r)}{v-v_r} > 0$ $\quad v > v_r$ & neg for $v < v_r$

& $f'(v) - \dfrac{f(v)-f(v_r)}{v-v_r} > 0$



$f'(v) < 0 \quad$ for $\; v \neq v_r$



$\Rightarrow \quad v_r < v_\ell$

entropy

$$\frac{d}{dt}\vec{v}(x_i(t),t) = \frac{\partial \vec{v}}{\partial t}(x_i(t),t) + \cancel{\frac{\partial \vec{v}}{\partial x}(x_i(t),t)\, x_i(t)}$$

$$+ \quad \frac{\partial \vec{v}}{\partial x}\cdot \dot{x}_i(t)$$

Since $\frac{\partial \vec{v}}{\partial t} + \frac{\partial}{\partial x} F(\vec{v}(x,t)) = 0$

$$\frac{\partial v}{\partial t} = - J_v \cdot \frac{\partial v}{\partial x}$$

$$\frac{d}{dt}\vec{v}(x_i(t),t) = - J_v \cdot \frac{\partial v}{\partial x}(x_i,t) + \lambda(v)\frac{\partial \vec{v}}{\partial x}$$

$$= - \left[JF(\vec{v}(x_i(t),t)) - \lambda_i(\vec{v}(x_i(t),t))\cdot I \right] \frac{\partial \vec{v}}{\partial x}(x_i(t),t)$$

eq 3.31 ✓

$$V(x,t) = V^*\left(\frac{x-x_0}{t-t_0}\right)$$

$$\partial_t V + \partial_x f(V) = Jf(V^*)\frac{\partial V}{\partial x} + \frac{\partial V}{\partial t} = Jf(V^*)\frac{\partial V^*}{\partial \xi}\cdot \xi_x + \frac{\partial V^*}{\partial \xi}\cdot \xi_t$$

$$= \left(\xi_x Jf(V^*) + \xi_t\right)\frac{\partial V^*}{\partial \xi} = 0$$

$$\Rightarrow \left(Jf(V^*) + \frac{\xi_t}{\xi_x} I\right)\frac{\partial V^*}{\partial \xi} = 0$$

$$\frac{\xi_t}{\xi_x} = \frac{-\frac{(x-x_0)}{(t-t_0)^2}}{\frac{1}{(t-t_0)}} = -\xi$$

$$\Rightarrow \left(Jf(V^*) - \xi I\right)\frac{\partial V^*}{\partial \xi} = 0 \qquad \text{eq } 3.33 \quad \checkmark$$

$$\Rightarrow \quad V^{*\prime}(\xi) = s_i(V^*(\xi)) \qquad \text{eq } 3.34$$
$$\xi = \lambda_i(V^*(\xi))$$

$$\frac{\partial}{\partial \xi} \Rightarrow \qquad 1 = \left\langle \nabla_V \lambda_i, \frac{\partial V^*}{\partial \xi}\right\rangle = \left\langle \nabla_V \lambda_i(\vec{V}), s_i(\vec{V})\right\rangle$$

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$$\partial_t \rho \bar{u} + \rho \partial_t \vec{u} + \langle \bar{u}, \nabla \rho \rangle \bar{u} + \rho \langle \bar{u}, \nabla \rangle \bar{u} + (\rho \nabla \circ u)\bar{u} - \rho \hat{f} + \nabla p$$

$$- (\lambda + \eta) \nabla(\nabla \circ \bar{u}) - \eta \nabla^2 \bar{u} = 0$$

$$\rho \nabla \circ u + \langle u, \nabla \rho \rangle = \nabla \circ (\rho u)$$

$$\Rightarrow \partial_t \rho \bar{u} + \rho \partial_t \bar{u} + (\nabla \circ (\rho \vec{u}) - \rho \nabla \circ \bar{u})\bar{u} + \rho \langle \bar{u}, \nabla \rangle \bar{u}$$

$$+ (\rho \nabla \circ \bar{u})\bar{u} - \rho \hat{f} + \nabla p - (\lambda + \eta) \nabla (\nabla \circ \bar{u}) - \eta \nabla^2 \bar{u} = 0$$

$$\Rightarrow \left(\frac{\partial \rho}{dt} + \nabla \circ (\rho \bar{u}) \right) \bar{u} + \rho \frac{\partial \bar{u}}{dt} - \rho \cancel{(\nabla \circ \bar{u})} \bar{u} + \rho \langle \bar{u}, \nabla \rangle \bar{u}$$

$$+ \rho \cancel{(\nabla \circ \bar{u})} \bar{u} - \rho \hat{f} + \nabla p - (\lambda + \eta) \nabla (\nabla \circ \bar{u}) - \eta \nabla^2 \bar{u} = 0 \quad ✓$$

$$\Rightarrow \frac{\partial \bar{u}}{dt} + \langle \bar{u}, \nabla \rangle \bar{u} - \frac{(\lambda + \eta)}{\rho} \nabla (\nabla \circ \bar{u}) - \frac{1}{\rho} \nabla^2 \bar{u} = -\frac{1}{\rho} \nabla p + \hat{f}$$

eq 5.9 ✓

$$[\lambda] = [\eta] = \frac{g}{cm \cdot s} = \text{Poise}$$