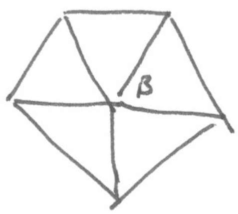
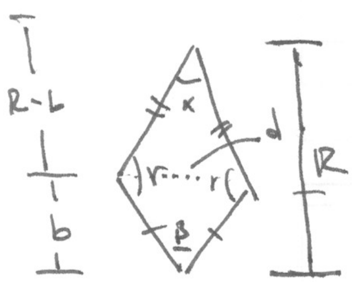
Pg 5 Books



$\therefore \quad 5\beta = 360 \Rightarrow \beta = \cancel{\ } 60 + 10 + 2 = 72$ ✓

$+ \quad 5\alpha = 180 \Rightarrow \alpha = 36$ ✓

I got this from the view that adding each angle up would give 180 just by viewing

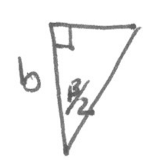


$\alpha + \beta + 2r = 360$

$\Rightarrow 2r = 360 - 36 - 72 = 360 - 108 = 252$

$\therefore r = 100 + 25 + 1 = 126$ ✓

$$A = 5 \cancel{\ } \left[\underbrace{\tfrac{1}{2} d b}_{\text{Area of bottom } \Delta} + \underbrace{\tfrac{1}{2} d (R-b)}_{\text{Area of top } \Delta} \right] = \frac{5}{2} d \left[R \right] = \frac{5R}{2} d$$



$$d = 2b\tan(\beta/2) = 2(R-b)\tan(\alpha/2)$$

$$\Rightarrow \quad b\left[\tan(\beta/2) + \tan(\alpha/2)\right] = R\tan(\alpha/2)$$

$$\therefore \quad b = \frac{R\tan(\alpha/2)}{\tan(\beta/2) + \tan(\alpha/2)}$$

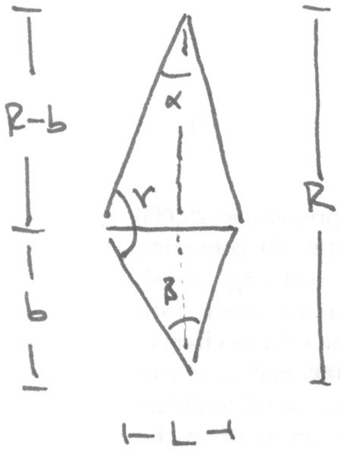
$$\therefore \quad d = \frac{2R\tan(\alpha/2)\tan(\beta/2)}{\tan(\beta/2) + \tan(\alpha/2)} = \frac{2R\sin(\alpha/2)\sin(\beta/2)}{\sin(\beta/2)\cos(\alpha/2) + \sin(\alpha/2)\cos(\beta/2)}$$

$$= \frac{2R\sin(\alpha/2)\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}$$

$$\therefore \quad A = \frac{5R}{\cancel{2}}\left[\frac{\cancel{2}R\sin(\alpha/2)\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}\right] = 5R^2\,\frac{\sin(\alpha/2)\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}$$

If $\alpha = 36$, $\beta =$

pg 5 Banks



$$A = 5 A_{diamond}$$

$$A_{diamond} = \frac{1}{2}L(R-b) + \frac{1}{2}Lb = \frac{1}{2}LR$$

$L = 2b\tan(\beta/2)$ + $L = 2(R-b)\tan(\alpha/2)$

equating these two + solving for b gives

$$b = (R-b)\frac{\tan(\alpha/2)}{\tan(\beta/2)}$$

$$\left(1 + \frac{\tan(\alpha/2)}{\tan(\beta/2)}\right) b = \frac{\tan(\alpha/2)}{\tan(\beta/2)} R \Rightarrow b = \left(\frac{\frac{\tan(\alpha/2)}{\tan(\beta/2)}}{1 + \frac{\tan(\alpha/2)}{\tan(\beta/2)}}\right) R$$

$$\Rightarrow b = \left(\frac{\tan(\alpha/2)}{\tan(\alpha/2) + \tan(\beta/2)}\right) R$$

$\cancel{\sin(\alpha/2)}$

Then $$A_{dia} = \frac{1}{2}\, 2\left(\frac{\tan(\alpha/2)\tan(\beta/2)}{\tan(\alpha/2)+\tan(\beta/2)}\right)R^2$$

$$= \left(\frac{\sin(\alpha/2)\sin(\beta/2)}{\left(\sin(\alpha/2)\cos(\beta/2)+\sin(\beta/2)\cos(\alpha/2)\right)}\right)R^2$$

$$= \left(\frac{\sin(\alpha/2)\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}\right)R^2$$

$$= \left(\frac{\sin(\alpha/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}\right)\sin(\beta/2)\,R^2$$

$\cancel{\sin(\alpha/2) = \frac{AC}{R-b}}$; [illegible]

$\cancel{\sin(\alpha/2) = \frac{R-b}{AC}}$; $\cancel{\cos(\alpha/2) = \frac{L/2}{AC}}$; $\sin(\beta/2) = \frac{L/2}{r}$

$\cancel{\sin(\alpha/2) = \frac{R-b}{AC}}$; $\cos(\beta/2) = b/r$

$\sin(\alpha/2) = \dfrac{L/2}{AC}$; $\cos(\alpha/2) = \dfrac{R-b}{AC}$,

Now $\sin(\alpha/2) = \dfrac{1}{2}\dfrac{r\sqrt{3-t}}{\phi}$

$$\sin(\alpha/2)\cos(\beta/2)+\sin(\beta/2)\cos(\alpha/2) = \left(\frac{L/2}{AC}\right)\left(\frac{b}{r}\right)+\left(\frac{L/2}{r}\right)\frac{(R-b)}{AC}$$

$$= (\text{[illegible]})\left(\frac{L/2}{r}\right)\frac{1}{AC}R$$

So $$\frac{\sin(\alpha/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)} = \frac{\frac{L/2}{AC}}{\left(\frac{L/2}{r}\right)\frac{1}{AC}\cdot R} = \frac{r}{R}$$

$\therefore$

$$A_{\text{diamond}} = r\sin(\beta/2)R$$

So $$A = 5\left(\frac{r}{R}\right)\sin(\beta/2)\,R^2$$

$$= 5\left(1/\phi^2\right)\sin(\beta/2)R^2 \qquad \text{eq } 1.4 \ \checkmark$$

Now Perimiter $= P = 10\cdot(R-b)(\cos(\alpha/2))^{-1}$

$$= 10\cdot\frac{L}{2}(\sin(\alpha/2))^{-1} = \frac{5L}{\sin(\alpha/2)}$$

$$= \frac{5L}{\left(\frac{L/2}{AC}\right)} = 10\,AC$$

$$= \frac{5L}{\sin(\alpha/2)} = \frac{10\,b\tan(\beta/2)}{\sin(\alpha/2)} = 10\,\frac{\tan(\beta/2)}{\sin(\alpha/2)}\cdot\frac{\tan(\alpha/2)}{(\tan(\alpha/2)+\tan(\beta/2))}\,R$$

$$= \frac{10\cancel{\sin(\alpha/2)}\sin(\beta/2)}{\cancel{\sin(\alpha/2)}\left(\sin\left(\frac{\alpha+\beta}{2}\right)\right)}R = \frac{10\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}R \qquad \text{eq } 1.5 \ \checkmark$$

$$\frac{A_{\text{total pentagon}}}{A_{\text{base pentagon}}} = \frac{\left[5 \, \frac{\sin(\alpha/2)\sin(\beta/2)}{\sin\left(\frac{\alpha+\beta}{2}\right)}\right] R^2}{5\left(\frac{1}{2}\right) L b}$$

$$= \frac{5(1/2) L R}{5(1/2) L b} = \frac{R}{b} = 2\phi \quad ?$$

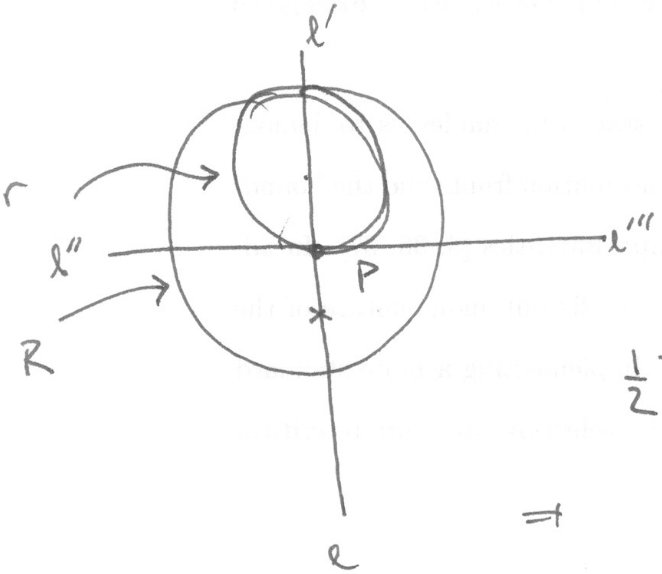
$1:1.9$ to $1:\phi = 1:1.618$

$$A_r = \frac{4}{13}(1.618 - .76) + \frac{3}{13}(1.618) = .6373$$

$$A_w = \frac{3}{13}(1.618) + \frac{3}{13}(1.618 - .76) + 80 \cdot .001065 = .6246$$

$$A_B = .3559 \text{ ft}^2$$

$\%R = 39.4\%$ $\quad \%B = 38.6\%$ $\quad \%W = \cancel{20}\ 21.9\%$



Computing the torques about the line ll' one gets

$$\frac{1}{2}\pi r^2 = \frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2$$

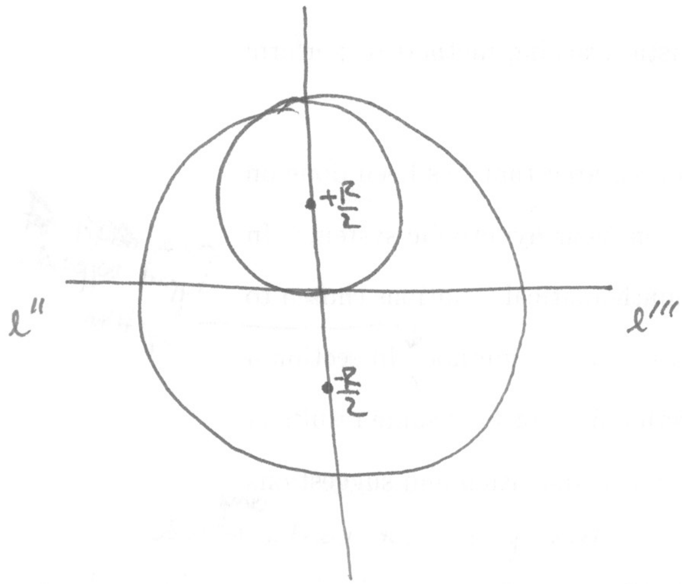
$$\Rightarrow \quad r^2 = R^2 - r^2$$

Compute about $l''\, l'''$ to get

$$\left(\frac{1}{2}R\right)\left(\frac{1}{2}\pi R^2\right) = (r)\left[\frac{1}{2}\pi R^2 - \pi r^2\right] \qquad \div\ \pi r^3$$

$$\Rightarrow \quad \frac{1}{4}\left(\frac{R}{r}\right)^3 = \frac{1}{2}\left(\frac{R}{r}\right)^2 - 1$$

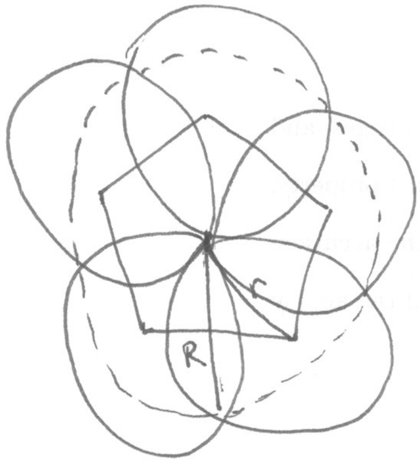
This is the right idea but incorrect #'s trying again



$$\left(\frac{R}{2}\right)\left(\frac{1}{2}\pi R^2\right) = \frac{R}{2}\left[\frac{1}{2}\pi R^2 - \pi r^2\right] \qquad \div \frac{R}{2}\cdot\pi r^2$$

$$\Rightarrow \quad \frac{1}{2}\left(\frac{R}{r}\right)^2 = \frac{1}{2}\left(\frac{R}{r}\right)^2 - 1$$

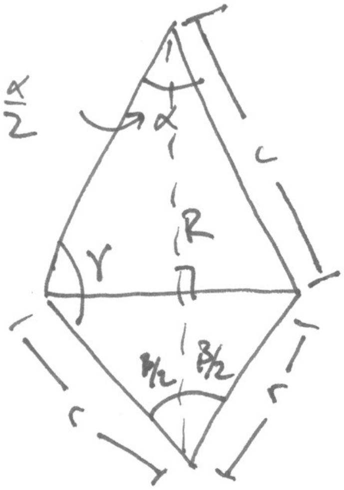
Mult by 2



$$\frac{R}{r} = \phi$$

?

Folded Pentagram?



$$\frac{r}{R} = \frac{\sin(\alpha/2)}{\sin(\gamma)}$$

$$+ \quad \frac{r}{c} = \frac{\sin(\alpha/2)}{\sin(\beta/2)} \qquad \text{eqs } 2.1 \ \checkmark$$

Now

let $\quad p = \frac{r}{R}$ $\qquad\qquad A_\Delta = \frac{1}{2} b \cdot h$

$$A = 5 \cdot A_\Delta = 5\left[\underbrace{\tfrac{1}{2} b \cdot h}_{\text{for top } \Delta} + \underbrace{\tfrac{1}{2} b \cdot h}_{\text{for bottom } \Delta}\right] = 5\left[\right.$$

The common base $b = 2 \cdot r \sin(\beta/2)$

+ the heights for the different "h"'s are

$h_{upper} = c\cos(\alpha/2)$

+ $h_{lower} = r\cos(\beta/2)$

Thus

$$A = \cancel{\tfrac{5}{2}\left[2 r \sin(\beta/2) + 2\right.} = \frac{5}{2} \cdot 2 r \sin(\beta/2) \cdot \underbrace{\left[h_{upper} + h_{lower}\right]}_{R}$$

$$\therefore A = 5 \sin(\beta/2) \cdot r R \Rightarrow \frac{A}{R^2} = 5\left(\frac{r}{R}\right)\sin(\beta/2) \qquad \text{eq } 2.2$$

$$P = 2\cdot 5\cdot c = 10\cdot r\,\frac{\sin(\beta/2)}{\sin(\alpha/2)}$$

so $$\frac{P}{R} = 10\left(\frac{r}{R}\right)\frac{\sin(\beta/2)}{\sin(\alpha/2)} = 10\rho\,\frac{\sin(\beta/2)}{\sin(\alpha/2)}$$ eq 2.3 ✓

+ $\alpha + \beta + 2\gamma = 360$

let $\beta = 72^\circ$ then

$\alpha + 2\gamma = 288$

so if $\alpha = 0 \Rightarrow \gamma = 144.$ (1st line in table 2.1)

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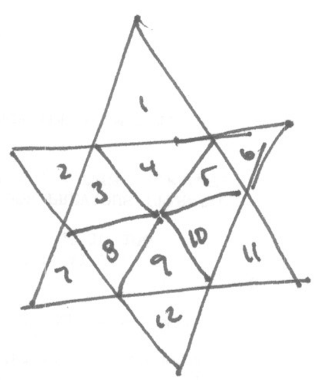
given an equilateral triangle

$h = r\sin(\alpha)$

$\alpha = \frac{180}{3} = 60$

so $h = r\sin(60^\circ) = \frac{\sqrt{3}}{2} r$

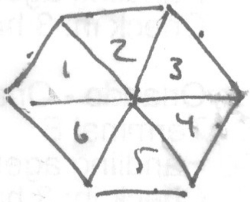
so $A = \frac{1}{2} r \frac{\sqrt{3}}{2} r = \frac{\sqrt{3}}{4} r^2$



$A_{\text{hexagram}} = 12 \cdot \frac{\sqrt{3}}{4} r^2 = 3\sqrt{3}\, r^2$

$P_{\text{hexagram}} = 12r$

$$A_{hexagon} = 6 \cdot \frac{\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{2} r^2$$



$$P_{hexagon} = 6r$$

Note that $R = 2\sin(60°)r = 2\frac{\sqrt{3}}{2}r = \sqrt{3}\,r$

$$\rho = \frac{r}{\sqrt{3}\,r} = \frac{1}{\sqrt{3}} \quad ✓$$

$\cancel{A = 3\sqrt{3}r^2}$; w/ $r = \frac{R}{\sqrt{3}}$

$$A_{hexagram} = 3\sqrt{3}\, r^2 = 3\sqrt{3}\,\frac{R^2}{3} = \sqrt{3}\,R^2$$

$$\& \; P_{hexagram} = 12r = \frac{12R}{\sqrt{3}} = 4\sqrt{3}R$$

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Problem: Assuming characteristics similar to the US Flag

width = 1.0
length = 1.9 radius = ?

$$A_{star} = 3\sqrt{3}\, r^2$$

Then $A_{star}\% = \frac{3\sqrt{3}\,r^2}{1 \cdot 1.9} = \ldots$

$\&$ $A_{backgrnd\,\%} = 1 - A_{star\,\%} = \ldots$

Pg 19 Becks

$$\left.\frac{A}{P}\right|_{\text{hexagon}} = \frac{\frac{3\sqrt{3}}{2} r^2}{6r} = r\left(\frac{\sqrt{3}}{4}\right) \qquad \sqrt{3} \approx$$

$$\left.\frac{A}{P}\right|_{\text{square}} = \frac{r^2}{4r} = \frac{1}{4} r = .25r$$

$$\left.\frac{A}{P}\right|_{\text{eq. triangle}} = \frac{\frac{\sqrt{3}}{4} r^2}{3r} = \frac{\sqrt{3}}{12} r =$$

$$\left.\frac{A}{P}\right|_{\text{circle}} = .5r$$

The $\frac{A}{P}$ is largest for the hexagon.

Pg 21 Becks

Problem

$$A_1 = \frac{\sqrt{3}}{4} r^2 \qquad P_1 = 3r \qquad A_\Delta(s) = \frac{\sqrt{3}}{4} r^2$$

$$A_2 = A_1 + \left(\frac{P_1}{r}\right) A_\Delta(r/2) \qquad P_2 = \left(\frac{P_1}{r}\right)\left(\frac{4}{3} r\right) = 4r$$

$$= \cancel{}$$

$$= \frac{\sqrt{3}}{4} r^2 + 3\left(\frac{\sqrt{3}}{4} \frac{r^2}{4}\right) = \frac{\sqrt{3}}{4} r^2 \left(1 + \frac{3}{4}\right)$$

~~$A_3 =$~~

So $\frac{A_2}{A_1} = 1 + \frac{3}{4} = \frac{7}{4}$ & $\frac{P_2}{P_1} = \frac{4}{3}$

$F_1 = 3$; $F_2 = F_1 \cdot 4$; ... $F_n = 3 \cdot 4^{n-1}$ "Faces" or "edges"

$l_1 = r$; $l_2 = \frac{r}{3}$; $l_3 = \frac{r}{9}$, ... $l_n = \frac{r}{3^{n-1}}$ lengths of each edge

Then

$$A_n = A_{n-1} + F_{n-1} \cdot A_\Delta\left(\frac{l_{n-1}}{3}\right)$$

$$A_n = A_{n-1} + 3 \cdot 4^{n-2} \frac{\sqrt{3}}{4}\left(\frac{l_{n-1}^2}{9}\right)$$

$$= A_{n-1} + 3 \cdot 4^{n-2} \frac{\sqrt{3}}{4} \frac{1}{9}\left(\frac{r^2}{9^{n-1}}\right)$$

$$= \quad A_n - A_{n-1} = \frac{3\sqrt{3}\, 4^{n-3}}{9^n} r^2 = \frac{3\sqrt{3}}{4^3}\left(\frac{4}{9}\right)^n r^2$$

$$\Delta A_n = \frac{3\sqrt{3}}{4^3}\left(\frac{4}{9}\right)^n r^2$$

$$\sum_{n=1}^{N-1} \Rightarrow \quad A_N - A_1 = \sum_{n=1}^{N-1} \frac{3\sqrt{3}}{4^3}\left(\frac{4}{9}\right)^n r^2$$ ... to sum & have result...

In addition

$$P_n = F_n l_n = 3 \cdot 4^{n-1}\left(\frac{r}{3^{n-1}}\right) = 3\left(\frac{4}{3}\right)^{n-1} r \quad \text{So } P_\infty = \infty$$

$a > b$

$4^3 \quad 3^4$

$16 \quad 81$

$\frac{4}{64} <$

$a^b = b^a$

$b\ln(a) = a\ln(b)$

let $a = kb \quad (k > 1)$

$\Rightarrow b\ln(kb) = kb\ln(b) \Rightarrow \ln(k) + \ln(b) = k\ln(b)$

$\Rightarrow$ ~~$b\ln(k) + b\ln$~~ $\quad \ln(k) = (k-1)\ln(b)$

$\Rightarrow \ln(b) = \frac{\ln(k)}{k-1} = \ln\left(k^{\frac{1}{k-1}}\right)$

$\Rightarrow \quad b = k^{\frac{1}{k-1}} \qquad a = k^{1+\frac{1}{k-1}} = k^{\frac{k}{k-1}} \qquad k > 1$ ✓

$k \to 1 \quad a(k) \to ? \quad b(k) \to ?$

let $n = \frac{1}{k-1} \Rightarrow k-1 = \frac{1}{n} \Rightarrow k = \frac{n+1}{n}$

So $b = \left(1+\frac{1}{n}\right)^n$ ✓ + $\quad a = \left(1+\frac{1}{n}\right)^{\left(\frac{n+1}{n}\right)n} = \left(1+\frac{1}{n}\right)^{n+1}$ ✓

$k \to 1 \Rightarrow n \to +\infty \quad \therefore a(k\to 1) = e = b(k \to 1)$

$$\frac{da}{dk} = ?$$

$$\ln(a) = \frac{k}{k-1}\ln(k)$$

$$\frac{d}{dk} \rightarrow \qquad \frac{1}{a}\frac{da}{dk} = \frac{1}{k-1}\ln(k) - \frac{k}{(k-1)^2}\ln(k) + \frac{1}{k-1}$$

$$\Rightarrow$$

$$\frac{da}{dk} = \frac{a}{k-1}\left[\ln(k) + 1 - \frac{k}{k-1}\ln(k)\right] = \cancel{\frac{a}{k-1}\left[1 - \frac{k}{k-1}\right.}$$

$$= \frac{a}{k-1}\left[1 + \left(\frac{k-1-k}{k-1}\right)\ln(k)\right]$$

$$= \frac{a}{k-1}\left[1 - \frac{1}{k-1}\ln(k)\right]$$

$$\ln(b) = \frac{1}{k-1}\ln(k)$$

$$\frac{d}{dk} \rightarrow \qquad \frac{1}{b}\frac{db}{dk} = -\frac{1}{(k-1)^2}\ln(k) + \frac{1}{k(k-1)}$$

$$\frac{db}{dk} = \frac{b}{(k-1)}\left[\frac{1}{k} - \frac{1}{k-1}\ln(k)\right]$$

So
$$\frac{da}{db} = \frac{da/dk}{db/dk} = \frac{a\left(1 - \frac{1}{k-1}\ln(k)\right)}{b\left(\frac{1}{k} - \frac{1}{k-1}\ln(k)\right)} = \frac{a\left(k-1 - \ln(k)\right)}{b\left(\frac{k-1}{k} - \ln(k)\right)} =$$

$$= \frac{ak\left(k-1 - \ln(k)\right)}{b\left(k-1 - k\ln(k)\right)}$$

at $a = e = b$ + $k = 1$

$$\left.\frac{da}{db}\right|_{(e,e)} = \frac{+\frac{1}{(k-1)^2}\ln(k) - \frac{1}{k(k-1)}}{-\frac{1}{k^2} + \frac{1}{(k-1)^2}\ln(k) - \frac{1}{k(k-1)}}$$

$$\frac{\text{"}0\text{"}}{\text{"}0\text{"}}$$

$$\left.\frac{da}{db}\right|_{(e,e)} = \frac{\cancel{a}\left(k-1-\ln(k)\right) + ak\left(1-\frac{1}{k}\right)}{\cancel{b}\left(1-\ln(k)-1\right)} =$$

$$= \frac{k-1-\ln(k)+k-1}{-\ln(k)} = \frac{\ln(k)+2(k-1)}{\ln(k)} = 1 + \frac{2(k-1)}{\ln(k)}$$

$$= 1 + \frac{2}{\frac{1}{k}} = 1 + 2k = 3 \quad ?$$

$$\frac{0}{0}$$

Mg 100 Bricks

$$F = G\frac{mM}{r^2}$$

$$+ F = mg_0 = \text{ w/ } r = R$$

$$\cancel{m}g_0 = \frac{G\cancel{m}M}{R^2} \Rightarrow GM \Rightarrow g_0R^2$$

$$\therefore F = \frac{(GM)m}{r^2} = \frac{mg_0R^2}{r^2}$$

— — —

$$v^2 = 0 = v_0^2 - 2g_0R^2\left(\frac{1}{R} - \frac{1}{r}\right) \Rightarrow r = \dots$$

$$\frac{2gR^2}{R} - v_0^2 = \frac{2g_0R^2}{r} \Rightarrow \frac{1}{r} = \frac{1}{R} - \frac{v_0^2}{2g_0R^2}$$

$$\Rightarrow r = \frac{1}{\left(\frac{1}{R} - \frac{v_0^2}{2g_0R^2}\right)}$$

$R_e \triangleq 6.37 \cdot 10^6$

$R_M \triangleq 1.74 \cdot 10^6$

$$\sim \quad \frac{v_e}{v_M} = \frac{\sqrt{2 g_e R_e}}{\sqrt{2 g_M R_M}} \overset{?}{=} \left\{ \frac{1.74 \cdot 10^6}{6.37 \cdot 10^6} \right. = .27$$

$$\uparrow$$

$$\square^2 = \cancel{.746} \cdot 7.46 \cdot 10^{-2}$$

$$\sqrt{\square} = .5$$

$$F = \cancel{m} g_0 \frac{r}{R} = g \cancel{m}$$

$$\Rightarrow g = g_0 \frac{r}{R}$$

$$= \cancel{m \frac{dv}{dt}} = m v \frac{dv}{dr} = -gm = -g_0 \frac{r}{R} m$$

$$\Rightarrow -v \frac{dv}{dr} = g_0 \frac{r}{R}$$

$$\int_{v_0}^{v} v \, dv = -\frac{g_0}{R} \int_0^{R} r \, dr$$

$$\| $$

$$\frac{v^2}{2} - \frac{v_0^2}{2} = \cancel{-\frac{g_0 (R)^2}{R \, 2}} \rightarrow -\frac{g_0}{R} \frac{r^2}{2} \rightarrow v^2 = v_0^2 - \frac{g_0 r^2}{R} \quad \checkmark$$

$$v_*^2 = g_0 R$$

$$F = \cancel{m}\frac{dv}{dt} = -\cancel{m}g_0\frac{r}{R}$$

$$\Rightarrow \quad \frac{d^2r}{dt^2} + \frac{g_0}{R}r = 0 \qquad r(0) = 0, \; r'(0) = v_*$$

OR

$$v^2 = v_0^2 - \frac{g_0}{R}r^2$$

$$\left(\frac{dr}{dt}\right)^2 = v_0^2 - \frac{g_0}{R}r^2 = g_0 R - g_0\left(\frac{r^2}{R}\right)$$

$$= \frac{g_0}{R}(R^2 - r^2)$$

$$\frac{dr}{dt} = \pm\sqrt{\frac{g_0}{R}}\sqrt{(R^2 - r^2)}$$

$$F = mg(r) = mg_0 \frac{r}{R}$$

$$\rightarrow \quad g(r) = g_0 \frac{r}{R}$$

$$\cancel{\frac{g_0 r}{R}} \neq \quad \sum F = -mg(r) = m\frac{dv}{dt}$$

$$\parallel$$

$$-\frac{g_0 r}{R} = \frac{dv}{dt} = \frac{dv}{dr}\cdot\frac{dr}{dt} = v\frac{dv}{dr} \quad \checkmark \quad \text{eq } 10.16$$

So $$v\frac{dv}{dr} = -\frac{g_0 r}{R}$$

$$\int_{v_*}^{v} v\,dv = -\frac{g_0}{R}\int_0^r r'\,dr'$$

$$v^2 = \cancel{}\; g_0 R - \frac{g_0}{R} r^2 \Rightarrow v = \sqrt{\frac{g_0}{R}}\sqrt{R^2 - r^2} = \frac{dr}{dt}$$

$$\Rightarrow \sqrt{\frac{g_0}{R}}\,dt = \frac{dr}{\sqrt{R^2 - r^2}}$$

$$\sqrt{\frac{g_0}{R}}\int_0^{t_0} dt = \int_0^R \frac{dr}{\sqrt{R^2 - r^2}}$$

$$v^2 = v_{*_0}^2 - \frac{g_0 r^2}{R}$$

& we require that $v(r=R)$ = previously calculated escape velocity $\sqrt{2g_0R}$

So $$2g_0R = v_{*_0}^2 - \frac{g_0(R^2)}{R}$$

→ $$v_{*_0}^2 = 3g_0R$$

Problem:

If $v_0 = \sqrt{2g_0R} = 11.18$ km/s

Then using $v^2 = v_0^2 - \frac{g_0}{R}r^2$ one can calculate the ~~initial~~ velocity at $r=R$ where the force of gravity changes from a ~$O(r)$ to a $O(r^2)$ term

& using ~~eq 10.19~~

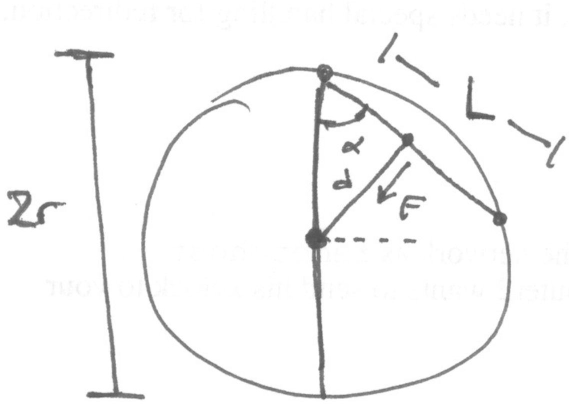
$v^2 = 2g_0R - \frac{g_0}{R}r^2$ we can calculate the time required to reach $r=R$ as is done in eq 10.19

$$\left(\frac{dr}{dt}\right)^2$$

then using the velocity at $r=R$ as the initial velocity in eq 10.13 we can calculate the point $r=R_{peak}$ at which $v=0$ to determine the highest position attained then using 10.13 as chance we can calculate the time taken to get the

$$\left(\frac{dr}{dt}\right)^2 = v_0^2 - 2g_0R^2\left(\frac{1}{R} - \frac{1}{r}\right)$$ via integration

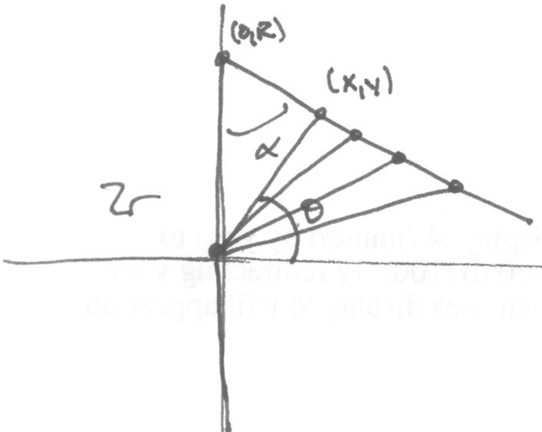
10.3 Bonks



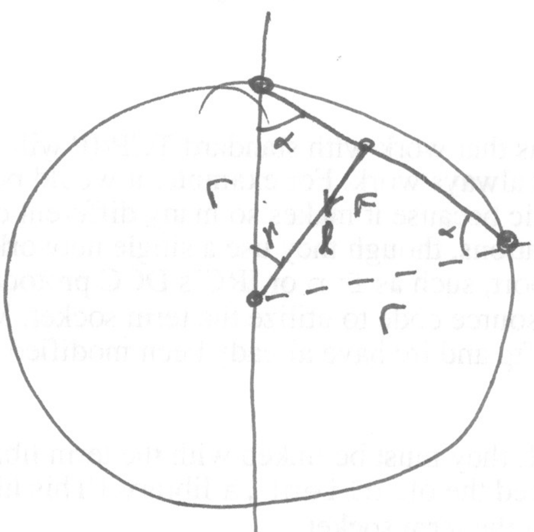
$L = 2r\cos\alpha$

$F = mg_0 \frac{d}{R}$

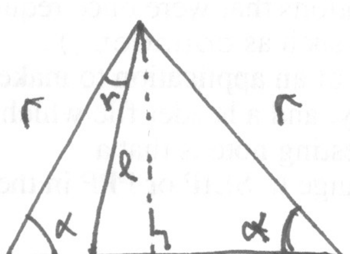
w/ $d = ?$



$d = \sqrt{x^2 + y^2}$



Both angles are α since this is an isosceles triangle. Then given a point on this "Eq." Non equal side what is the distance to the origin



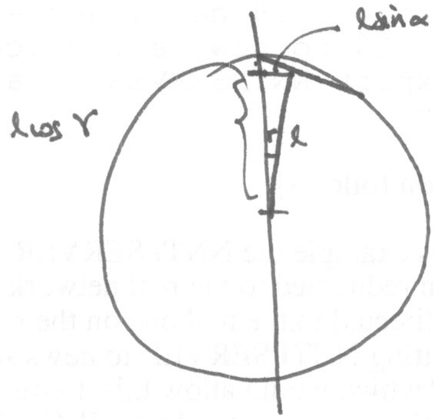
$l = ?$ $\quad \frac{\sin\alpha}{l} = \frac{\sin(180 - \alpha - r)}{r}$

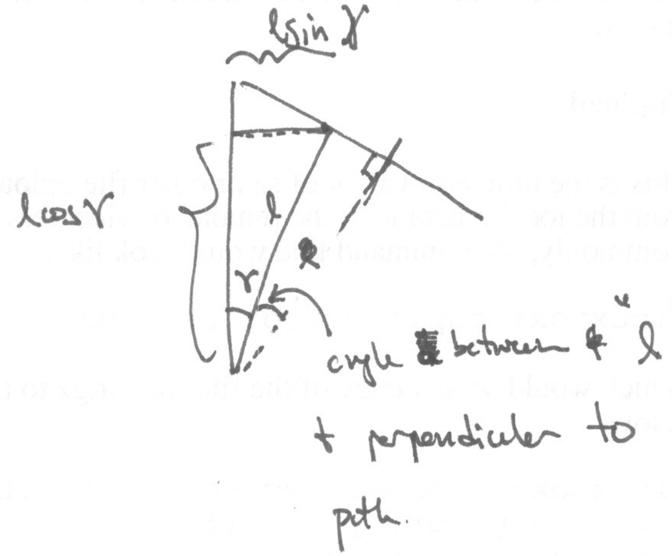
$$\sin(\pi - x) = \sin(\pi)\cos(x) - \cos(\pi)\sin(x)$$
$$= \sin x$$

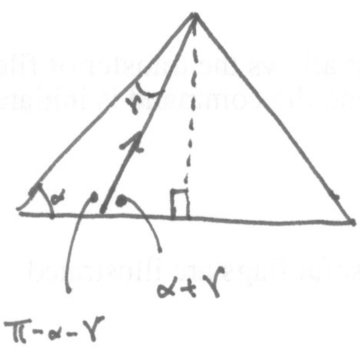
$$\therefore \quad \frac{\sin\alpha}{l} = \frac{\sin(\alpha+\gamma)}{r}$$

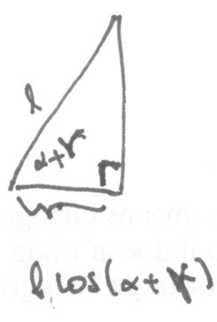
$$\Rightarrow \quad l = r\left[\frac{\sin\alpha}{\sin(\alpha+\gamma)}\right] \qquad 0 \le \gamma \le \pi - 2\alpha$$

Then $\quad F = mg_0 \dfrac{l}{R} \cdot \cos(\alpha+\gamma)$









So in total the Force is given by

$$F(\gamma) = \frac{m g_0 R}{R} \, \frac{\sin\alpha}{\sin(\alpha+\gamma)} \cos(\alpha+\gamma)$$

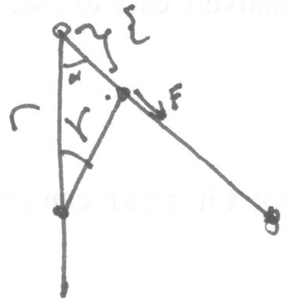
$$= (m g_0 \sin\alpha) \cot(\alpha+\gamma)$$

Then

$$= \quad \frac{dv}{dt} = \frac{F(\gamma)}{m}$$

$$\parallel$$

$$\frac{d^2\xi}{dt^2} = \frac{F(\gamma(t))}{m}$$



$$\frac{\sin(\gamma)}{\xi} = \frac{\sin(\pi-\alpha-\gamma)}{r} = \frac{\sin(\alpha+\gamma)}{r}$$

$$\therefore \quad \xi = r \, \frac{\sin(\gamma)}{\sin(\alpha+\gamma)}$$

Thus D.E. for $\gamma(t)$ becomes

⋮

$$q_3 = f(d, B, D, v_p, \rho, \mu, \sigma, g)$$

$$\frac{q_3}{BDv_p} = f\left(d/D,\ B/D,\ Re,\ We,\ Fr\right)$$

$$q_3 = \frac{C_d}{\nu} BDv_p^2 \qquad \nu = \frac{\mu}{\rho} \ \text{kinematic viscosity}$$

$$q = \frac{rBDv_r}{W} + \frac{rBHv_p}{W} + \frac{C_d}{\nu} BDv_p^2 \qquad \text{w/ } W = \sqrt{v_p^2 + v_r^2}$$

$$V = q\left(\frac{L}{v_p}\right) = \cancel{\frac{rBL}{\sqrt{v_p^2+v_r^2}}}\left[D\right.$$

$$= \frac{L}{v_p}\left[\frac{rBDv_r}{\sqrt{\ \ }} + \frac{rBHv_p}{\sqrt{\ \ }} + \frac{C_d}{\nu} BDv_p^2\right]$$

$$V = \frac{rBL}{\sqrt{v_p^2 + v_r^2}}\left[\frac{Dv_r}{v_p} + H\right] + \frac{C_d}{\nu}(BDL)v_p \qquad \text{eq 12.7 ✓}$$

$$\Rightarrow \frac{V v_r}{BHLr} = \frac{v_r}{\sqrt{v_p^2 + v_r^2}}\left[\frac{D}{H}\cdot\frac{v_r}{v_p} + 1\right] + \frac{C_d}{\nu}\frac{D}{H}\frac{v_p v_r}{r}$$

$$\Rightarrow Z = \frac{1}{\sqrt{1 + \frac{v_p^2}{v_r^2}}}\left[\frac{D}{H}\cdot\frac{v_r}{v_p} + 1\right] + \frac{C_d}{\nu}\frac{D}{H}\frac{v_p^2}{r}\cdot\frac{v_r}{v_p}$$

$$[\nu] = \frac{[\mu]}{[\rho]} = \frac{\quad}{\frac{kg}{m^3}}$$

$$Z = \frac{1}{\sqrt{1+s^2}}\left[ns+1\right] + 7ns$$

$$\frac{dz}{ds} = \frac{n}{\sqrt{1+s^2}} + \frac{(ns+1)(-\frac{1}{2})(2s)}{(1+s^2)^{3/2}} + 7n \quad = 0$$

$\Rightarrow$ . . .

13 124 Becky

$$\frac{dy}{dx} = \frac{n-y}{m-x}$$

$$\frac{d}{dx} \Rightarrow \Rightarrow \frac{d^2y}{dx^2} = \left(\frac{1}{m-x}\right)\left[\frac{dn}{dx} - \frac{dy}{dx}\right] - \frac{(n-y)}{(m-x)^2}\cdot\left[\frac{dm}{dx} - 1\right]$$

$$\Rightarrow (m-x)\frac{d^2y}{dx^2} = \frac{dn}{dx} - \cancel{\frac{dy}{dx}} - \frac{(n-y)}{(m-x)}\frac{dm}{dx} + \underbrace{\frac{(n-y)}{(m-x)}}_{= \cancel{\frac{dy}{dx}}}$$

$$\Rightarrow \frac{dn}{dx} = (m-x)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dm}{dx} \qquad (\text{eq } 13.2) \quad ✓$$

$$v = kV$$

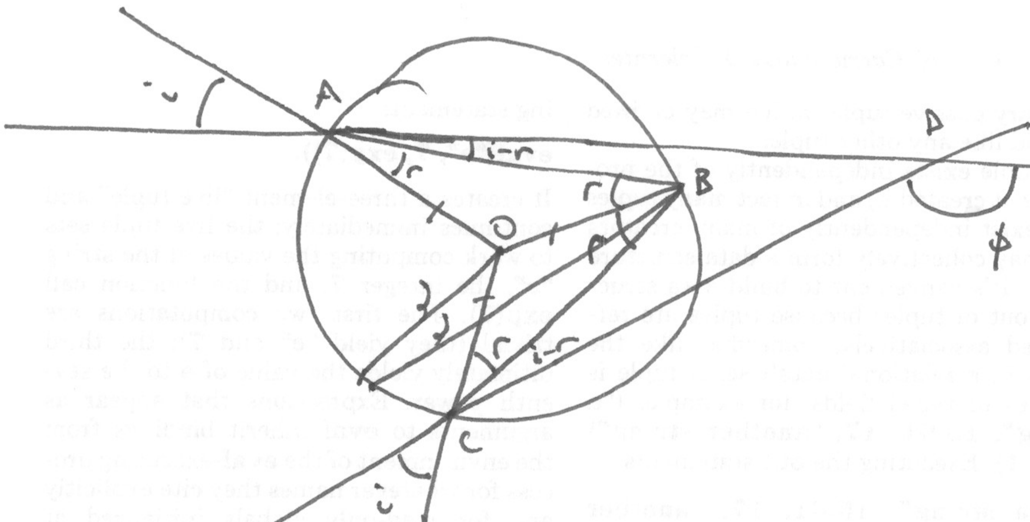
$$v^2 = k^2 V^2$$

$$dx^2 + dy^2 = k^2(dm^2 + dn^2)$$

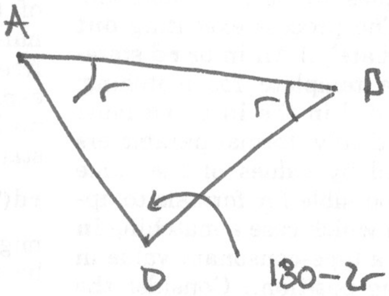
$$1 + \left(\frac{dy}{dx}\right)^2 = k^2\left\{\left(\frac{dm}{dx}\right)^2 + \left(\frac{dn}{dx}\right)^2\right\}$$

$m = g(n)$ known

pg 168 Berks



Then from Δ ABO



sim for ΔOBC

Then for ~~pool~~ quadrilateral ADCO

we get $i + \angle ADC + i + 2(\angle AOB) = 360$

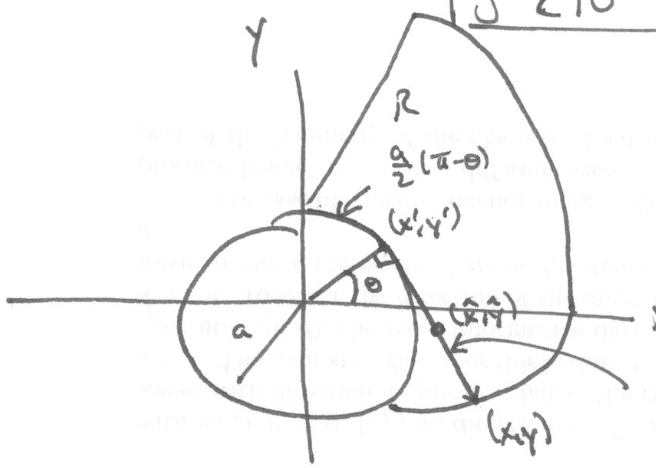
$= 2i + \angle ADC + 2(180 - 2r) = 360$

$= 2i - 4r = -\angle ADC$

Then $\phi = 180 - \angle ADC = 180 + 2i - 4r$

so $\phi = 2(i-r) + 180 - 2r$ eq 18.3 ✓

Pg 270 Banks



$R - \frac{a}{2}(\pi-\theta)$ at $90°$ from x',y'

$\Rightarrow$ direction given by

For $\theta \in [0, \frac{\pi}{2})$ $\quad x' = a\cos\theta$ $\qquad \frac{(-y', +x')}{\sqrt{x'^2+y'^2}} = \frac{(-a\sin\theta, a\cos\theta)}{a}$ *

$\qquad\qquad\qquad\qquad y' = a\sin\theta$

+ The amount of length wrapped along this section is

$$2\pi a \frac{\left(\frac{\pi}{2}-\theta\right)}{2\pi} = \frac{a(\pi-2\theta)}{2} \qquad = (-\sin\theta, \cos\theta) = \hat{t}$$

Thus

$$\hat{x}(\xi) = x' + \xi\hat{t}_x$$
$$\hat{y}(\xi) = y' + \xi\hat{t}_y$$

desire $\xi \ni$ $(\hat{x}(\xi)-x')^2 + (\hat{y}(\xi)-y')^2 = \left(R - \frac{a}{2}(\pi-2\theta)\right)^2$

$$\Rightarrow \quad \xi^2\left[(-\sin\theta)^2 + (\cos\theta)^2\right] = (\qquad)^2$$

$$\Rightarrow \quad \xi = \left|R - \frac{a}{2}(\pi-2\theta)\right|$$

Thus

$$x = a\cos\theta + \left(R - \frac{a}{2}(\pi - 2\theta)\right)(-\sin\theta)$$

& $$y = a\sin\theta + \left(R - \frac{a}{2}(\pi - 2\theta)\right)\cos\theta$$

$\Rightarrow$ $$x = a\cos\theta - R\sin\theta + \frac{a}{2}(\pi - 2\theta)\sin\theta$$

& $$y = a\sin\theta + R\cos\theta - \frac{a}{2}(\pi - 2\theta)\cos\theta$$

$\Rightarrow$ $$(x + R\sin\theta) = a\cos\theta + a\left(\frac{\pi}{2} - \theta\right)\sin\theta \qquad 0 \le \theta \le \frac{\pi}{2}$$

$$(y - R\cos\theta) = a\sin\theta - a\left(\frac{\pi}{2} - \theta\right)\cos\theta$$

Basically eq 26.14 + 26.15.