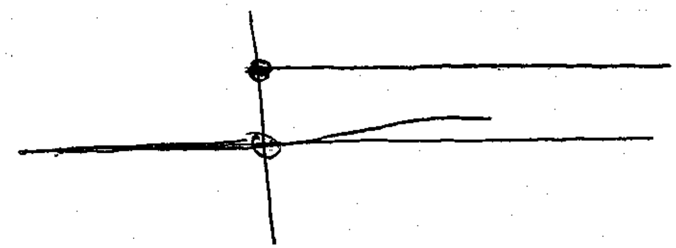
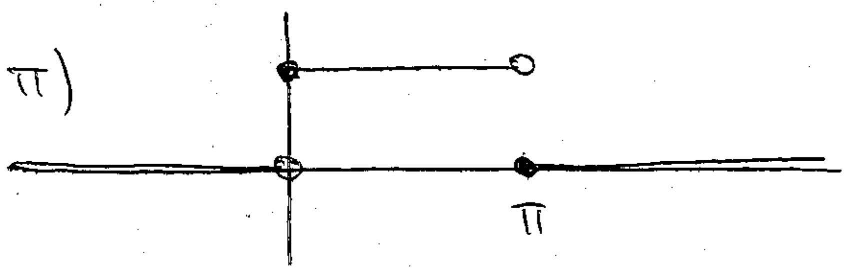
pg 40 Boyce + Diprima:

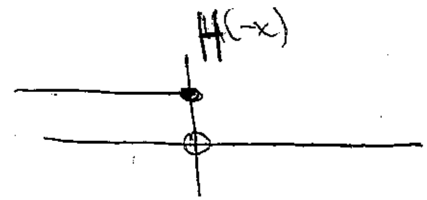
22

$H(t) = \longrightarrow$

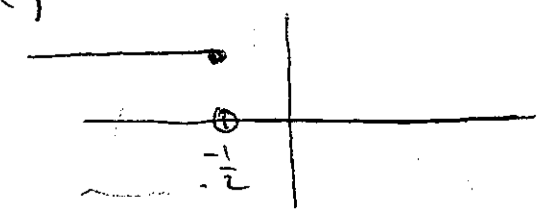


$f(t) = H(t) - H(t-\pi)$

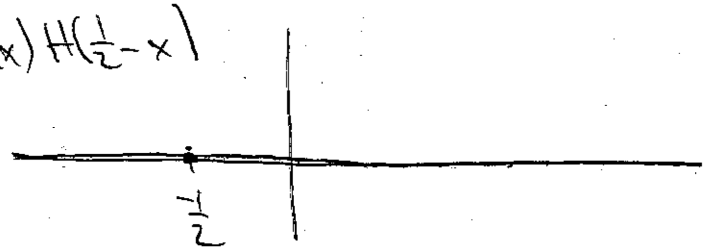




23 $H(x)H(2t-x)$ $t = 1/4$ $H(2t-x) = H(\tfrac{1}{2}-x)$



$H(x)H(\tfrac{1}{2}-x)$



$I_{t=\frac{1}{4}} = 0$

RICE UNIVERSITY

6100 Main Street
Houston, Texas 77005-1892

RECOMMENDATION TO SUPPLEMENT APPLICATION FOR ADMISSION

PART I - *TO BE COMPLETED BY THE APPLICANT. (TYPE OR PRINT)*

NAME OF APPLICANT Weatherwax, John Lloyd
Last First Middle

FIELD OF STUDY CAAM IN THE DEPARTMENT OF Mathematics

I authorize the preparation of a confidential evaluation and understand that the material will be kept confidential both from me and from the public.

DATE 11-27-95 SIGNATURE John Weatherwax

PART II - *TO BE COMPLETED BY THE RESPONDENT AND MAILED DIRECTLY TO THE DEPARTMENT CHAIR.*

Please rate the applicant in the section below. Indicate your basis for comparison:

Graduating Seniors____ First Year Graduate Students____ All Students I Have Known____ Other____________

	Upper 1 or 2%	Upper 10% but not upper 1 or 2%	Upper 25% but not upper 10%	Upper half but not upper 25%	Lower Half	No Basis for Judgment
Apparent Intellectual Ability						
Potential in Field						
Oral Expression						
Written Expression						
Working with Others						
Emotional Maturity						
Imagination and Probable Creativity						
Promise as a Teacher						

(Type or Print)

Evaluators Name:______________________ Date:__________

Position: ______________________ Address: ______________________

Institution:______________________ ______________________

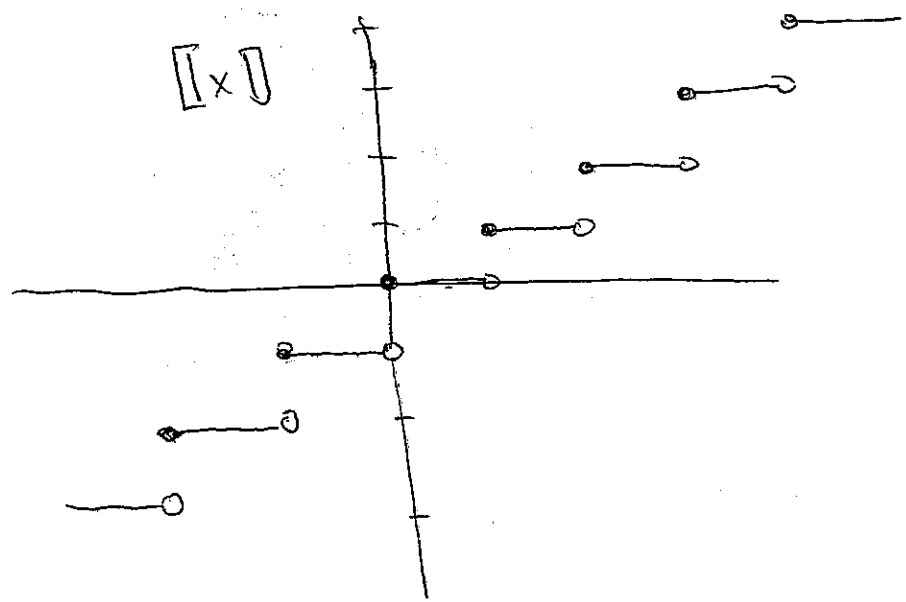
Signature: ______________________ ______________________

The Selection Committee needs your opinion of the candidate's ability to carry on advanced studies in his/her field and to achieve a successful professional career in it. Compare this applicant with others who recently attended or are applying to this or other comparable graduate programs. Please use the back of this form or attach another sheet to this document.

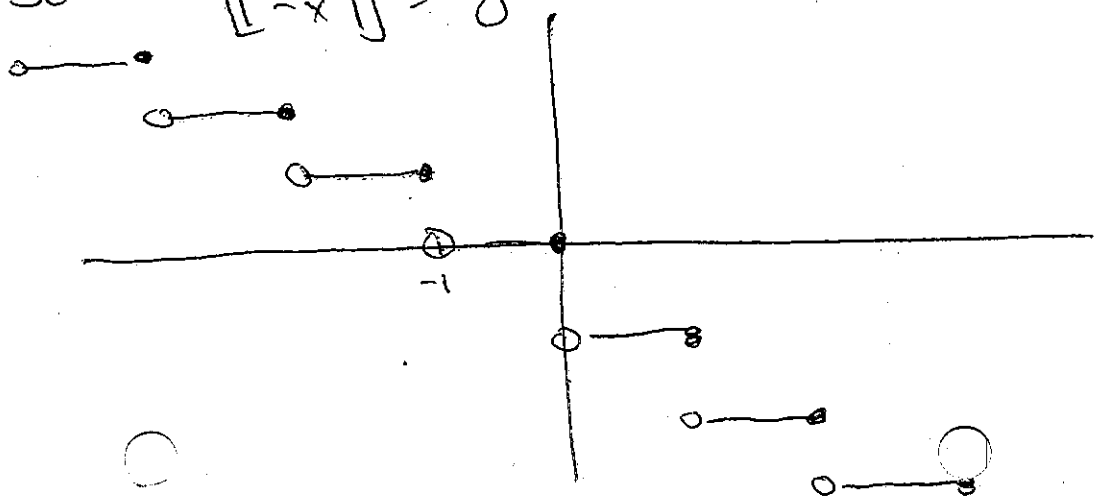
Pg 40 Boyce + Diprima

greatest integer fn

$g(x) = [\![-x]\!]$ ~ $[\![x]\!]$ except when x's are negative put positive + vice versa



So $[\![-x]\!] = g(x)$.



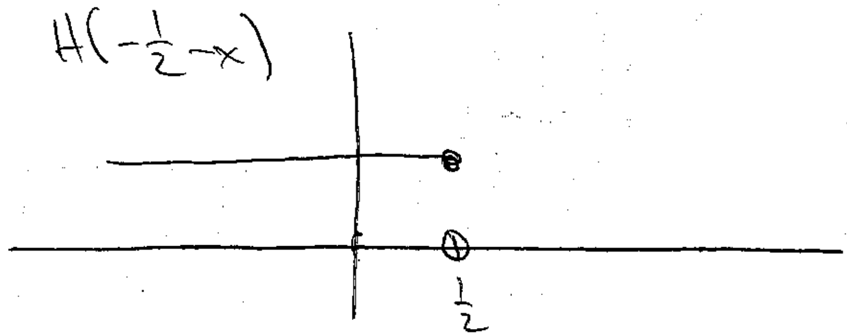
Pg 40 Boyce + Diprima

$t = \frac{1}{2}$

$H(2t - x) = H(1-x)$

All will be 0 $\quad I_t = 0 \quad \forall\, t > 0$

$\therefore$ Must mean $H(-2t-x)$. Then 1st one becomes.



So $\quad I_{\frac{1}{4}} = \frac{1}{2}. \qquad I_2 = 4$

$I_{\frac{1}{2}} = 1 \qquad$ by 2's.

$I_1 = 2$

pg 40 Boyce & Diprima

33 $$f(x) = \underbrace{\frac{f(x)+f(-x)}{2}}_{=\ g \text{ even}} + \underbrace{\frac{f(x)-f(-x)}{2}}_{=\ h \text{ odd}}.$$

Pg 48 Boyce / Diprema

4 $\cos\left(\frac{7\pi}{6}+\frac{7\pi}{4}\right) = \cos\frac{7\pi}{6}\cos\frac{7\pi}{4} - \sin\frac{7\pi}{6}\sin\frac{7\pi}{4}$

$=$

$\frac{(\cancel{44+21})\pi}{12} = \frac{38\pi}{12}$

8 $\sin\frac{\left(\frac{\pi}{6}\right)}{2} = + \sqrt{\frac{1-\cos\pi/6}{2}}$

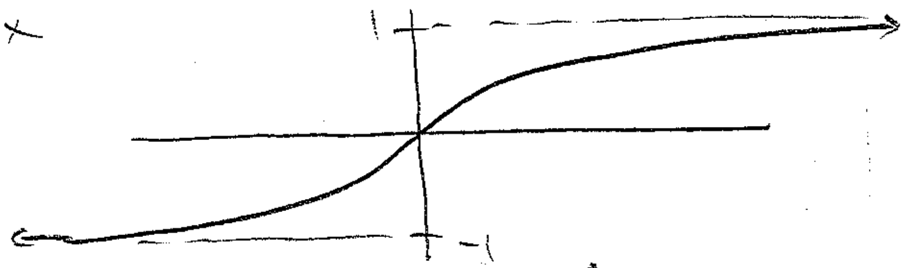
$\sin\frac{x}{2} =$

$\sin^2 x = \frac{1-\cos 2x}{2}$

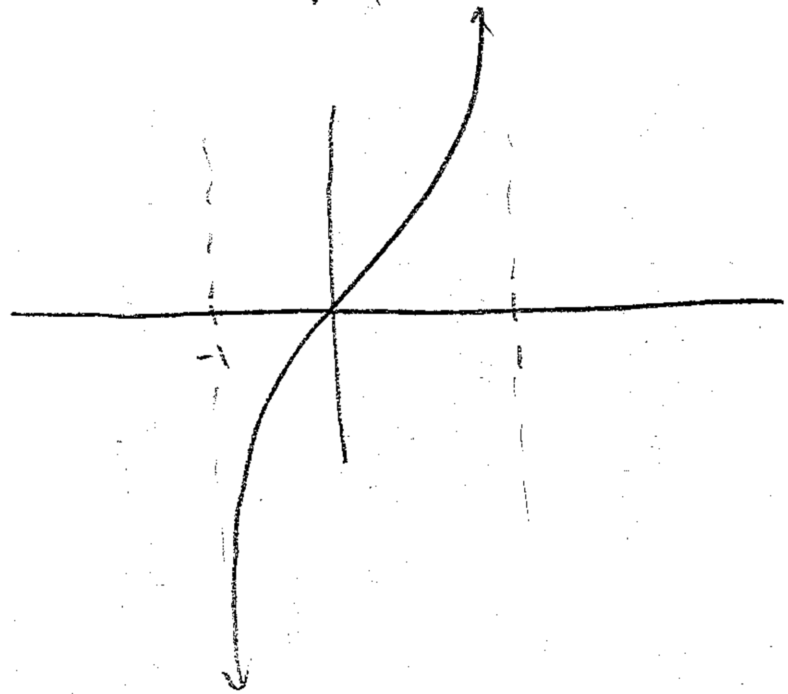
$\sin x = \pm\sqrt{\frac{1-\cos 2x}{2}}$

pg 458 Boyce / Diprima

37) $y = \tanh x$



$\therefore y = \operatorname{arctanh} x$



D: $-1 < x < 1$

R: $-\infty < y < \infty$

$$\frac{d}{dx}\left(\tanh^{-1}(x)\right) = ?$$

$$y = \tanh^{-1}(x)$$

$$x = \tanh(y)$$

$$1 = \operatorname{sech}^2(y)\,\frac{dy}{dx}$$

But $\cosh^2(x) - \sinh^2(x) = 1$

$\therefore$ $\operatorname{sech}^2(x) = 1 - \tanh^2(x)$

$\therefore$ $$\frac{dy}{dx} = \frac{1}{1-\tanh^2(y)} = \frac{1}{1-x^2} \qquad |x| < 1$$

Domain of y.

(b) $$\frac{d}{dx}\left(\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right|\right) = \frac{1}{1-x^2} \qquad |x|<1$$

Thus $$\frac{d}{dx}\left(\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| - \operatorname{arctanh}(x)\right) = 0$$

$\therefore$ $$\frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| = \operatorname{arctanh}(x) + C$$

$$x = 0$$

$$\tfrac{1}{2}\cancelto{0}{\ln(1)} = \cancelto{0}{\operatorname{arctanh}(0)} + C \quad \therefore \quad C = 0$$

$$\therefore \quad \operatorname{arctanh}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

Pg 460 Boyce / Diprima

(33) $\displaystyle\int \frac{dx}{\sqrt{x}(1+x)}$

$u = \sqrt{x} \qquad x = u^2 \qquad du = \frac{dx}{2\sqrt{x}} = \frac{dx}{2u}$

$\therefore \displaystyle\int \frac{2\,du}{1+u^2} = 2\tan^{-1}(u) + C$

$= 2\tan^{-1}(\sqrt{x}) + C$

Pg 634 Boyce / Diprima

(42) False $a_n = \frac{1}{n}$ $b_n = \frac{1}{n}$ both dv.

But $a_n b_n = \frac{1}{n^2}$ conv.

Pg 676 Boyce + DiPrima

(1) $f(x) = (1+x)^{1/2} = \sum_{k=0}^{\infty} \binom{1/2}{k} x^k$

$= \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\cdots\left(\frac{1}{2}-k+1\right)}{k!} x^k$

$\frac{1}{2} - \frac{2(k-1)}{2}$

$\frac{1-2(k-1)}{2} = \frac{-2k+3}{2}$

$= \sum_{k=0}^{\infty} \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\cdots\left(\frac{-2k+3}{2}\right)}{k!} x^k$

$= + \sum_{k=0}^{\infty} \frac{(-1)^k \cdot \overset{-1}{(2\cdot 1-3)}\overset{1}{(2\cdot 2-3)}(2\cdot 3-3)\cdots(2k-3)}{2^k \; k!} x^k$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k (-1)(1)(3)(5)\cdots(2k-3)\,x^k}{2^k\, k!}$$

$$= 1 + \frac{x}{2} + \sum_{k=2}^{\infty} \frac{(-1)^{k-1}\, 1\cdot 3\cdot 5 \cdots (2k-3)\, x^k}{2^k\, k!}$$

Ratio test $\overline{\lim_{k\to\infty}} \dfrac{|a_{k+1}|}{|a_k|} = \overline{\lim_{k\to\infty}} \dfrac{\dfrac{1\cdot 3\cdot 5\cdots(2k-3)(2k-1)|x|^{k+1}}{2^{k+1}(k+1)!}}{\dfrac{1\cdot 3\cdot 5\cdots(2k-3)|x|^k}{2^k\, k!}}$

Pg 676 Boyce/Diprima:

(3) $f(x) = (4+x)^{-1/2} = (4)^{-1/2}(1+x/4)^{-1/2}$

$= \frac{1}{2}(1+x/4)^{-1/2}$

$= \frac{1}{2}\sum_{k=0}^{\infty}\binom{-1/2}{k}\left(\frac{x}{4}\right)^k = \frac{1}{2}\left[\cancel{1} + -\frac{1}{2}\left(\frac{x}{4}\right) + \sum_{k=2}^{\infty}\binom{-1/2}{k}\left(\frac{x}{4}\right)^k\right]$

$= \frac{1}{2}\sum_{k=0}^{\infty}\frac{\overset{k=1}{(-\frac{1}{2})}\overset{k=2}{(-\frac{1}{2}-1)}(-\frac{1}{2}-2)\cdots\overset{k=k}{(-\frac{1}{2}-k+1)}}{k!}\left(\frac{x}{4}\right)^k$ $\quad -1-2k+2$, $-2k+1$, $-(2k-1)$

$= \frac{1}{2}\left[1 + \sum_{k=1}^{\infty}\frac{\overset{k=1}{(-1)^k}(1)(3)(5)\cdots(2k-1)}{2^k k!}\left(\frac{x}{4}\right)^k\right]$ ✓

pg 676 Boyce/DiPrima

$$= \overline{\lim_{k\to\infty}} \; |x| \frac{(2k-1)}{2(k+1)} \longrightarrow |x| < 1 \Rightarrow \rho = 1.$$

Pg 676 Boyce / Diprima

$= \frac{1}{2}\left[1 +\right.$

$\overline{\lim_{k}} \frac{|a_{k+1}|}{|a_k|} = \frac{(2k+1)|x|}{2\cdot(k+1)4} \longrightarrow \frac{|x|}{4} < 1 \Rightarrow \underline{\rho = 4}.$

PJ 676 Boyce / Diprima:

(5) $f = \ln\left(\sqrt{1+x^2} + x\right)$ $\qquad x_0 = 0$

$$f' = \frac{\left(\frac{1}{2}(1+x^2)^{-1/2}(2x) + 1\right)}{\sqrt{1+x^2} + x} = \frac{\frac{x}{\sqrt{1+x^2}} + 1}{\sqrt{1+x^2} + x}$$

$$= 1\left(\frac{1}{\sqrt{1+x^2}}\right) = \sum_{k=0}^{\infty} \binom{-1/2}{k} x^{2k}$$

$$\binom{-1/2}{k} = \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right) \cdots \left(-\frac{1}{2}-k+1\right)}{k!} = \frac{\overset{k=1}{(-1)}\overset{k=2}{(-1-2)} \cdots \overset{-(2k-1)}{(-1-2k+2)}}{2^k k!}$$

$$= \frac{(-1)^k(1 \cdot 3 \cdot 5 \cdots (2k-1))}{2^k k!}$$

$$\therefore \quad f' = \sum_{k=0}^{\infty} \frac{(-1)^k (1 \cdot 3 \cdot 5 \cdots (2k-1))}{2^k k!} x^{2k}$$

$$f = \sum_{k=0}^{\infty} \frac{(-1)^k (1 \cdot 3 \cdots (2k-1))}{2^k k! (2k+1)} x^{2k+1} + C$$

$$f(0) = C$$

$$\|$$

$$\ln 1 = C \Rightarrow C = 0$$

pg 676 Boyce/DiPrima

(14) $f(x) = (8-x)^{1/3}$ $\approx 2\left(1 - \frac{x}{24} + \cdots\right)$

$$= 2\left(1 - x/8\right)^{1/3} = 2\sum_{k=0}^{\infty}\binom{1/3}{k}\left(-\frac{x}{8}\right)^k$$

$$= 2\left[\sum_{k=1}^{\infty}\frac{(1/3)(1/3-1)(\cdots\cdots(1/3-k+1)}{k!}\left(\frac{-x}{8}\right)^k + 1\right]$$

$$= 2\left[1 + -\frac{x}{8}\left(\frac{1}{3}\right) + \frac{1}{2}\frac{x^2}{64}\frac{1}{3}\left(-\frac{4}{3}\right) + \cdots\right]$$

Pg 677 Boyce/ Diprima

1

16

(a) $$f'(x) = \sum_{k=1}^{\infty} \binom{\alpha}{k} k\, x^{k-1}$$

$$= \sum_{k=0}^{\infty} \binom{\alpha}{k+1} (k+1)\, x^{k} \qquad |x| < 1$$

(b) $$(1+x) f'(x) = \sum_{k=0}^{\infty} \left[\binom{\alpha}{k+1}(k+1)\, x^{k} + \binom{\alpha}{k+1}(k+1)\, x^{k+1} \right]$$

$$= \sum_{k=0}^{\infty} \left[\binom{\alpha}{k+1}(k+1) + \binom{\alpha}{k} k \right] x^{k}.$$

As $\sum_{k=0}^{\infty} \binom{\alpha}{k+1}(k+1)\, x^{k+1} = \sum_{k=1}^{\infty} \binom{\alpha}{k} k\, x^{k}$ but putting $k=0$ term Adds nothing

(c) $$\binom{\alpha}{k+1}(k+1) + \binom{\alpha}{k}k = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-(k+1)+1)}{(k+1)!}(k+1)$$

$$+ \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-k+1)}{k!}k$$

$$= \frac{\alpha(\alpha-1)(\alpha-2)\cdots\overset{(\alpha-k+1)}{\downarrow}(\alpha-k)}{k!} + \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}k$$

$$= \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-k+1)}{k!}\left[(\alpha-k) + k\right]$$

$$= \alpha\binom{\alpha}{k}.$$

(d) $\therefore$ $$(1+x)f'(x) = \alpha\sum_{k=0}^{\infty}\binom{\alpha}{k}x^k = \alpha f(x).$$

$$\frac{f'}{f} = \frac{\alpha}{1+x}.$$

$$\ln f = \alpha \ln(1+x) + C_1 \qquad f(0) = 1$$

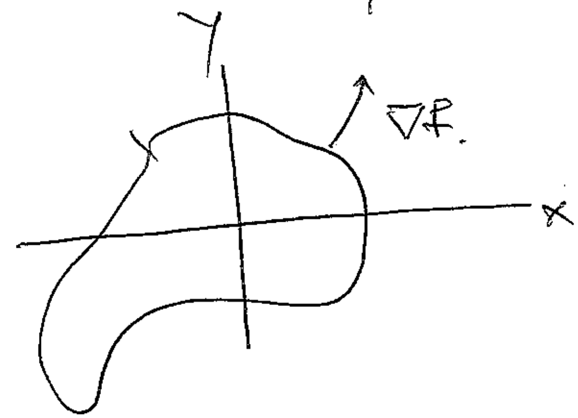
$$\Rightarrow \quad 0 = \alpha \ln 1 + C_1 \quad \Rightarrow \quad C_1 = 0$$

$$\therefore \quad f = (1+x)^{\alpha}$$

Pg 841 Boyce/DiPr

(13) $\nabla f = (x+y)\hat{i} + (2x+2y)\hat{j}$

$f(x,y) = \text{const}$



$\|\nabla f\| = \sqrt{(x+y)^2 + 4(x+y)^2}$

$= \sqrt{5}\,|x+y| \Leftarrow$ rate of change of f Maximal.

(14) $\nabla f = (y^2 + 6zx)\hat{i} + (2xy + 2z^2)\hat{j} + (4yz + 3x^2)\hat{k}.$

$\|\nabla f\| = \sqrt{(\quad)^2 + (\quad)^2 + (\quad)^2}$

$f = \text{const}$ surface