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$p = f(\rho)$

$p_\alpha = f'(\rho)\rho_\alpha = c^2 \rho_\alpha$

$\therefore$ II_+ : $u_\alpha = -\frac{c}{\rho}\frac{p_\alpha}{c^2} = -\frac{p_\alpha}{\rho c}$

II_- : $u_\alpha = \frac{c}{\rho}\frac{p_\alpha}{c^2} = \frac{p_\alpha}{\rho c}$

Multiply II_+ w/ one in book

$u_\alpha^2 = \frac{c}{\rho} \rho_\alpha \frac{p_\alpha}{\rho c} = \frac{\rho_\alpha p_\alpha}{\rho^2}$ Now $\tau_\alpha = -\frac{\rho_\alpha}{\rho^2}$

$\Rightarrow$ $du^2 + dp\, d\tau = 0$

Normally $p = p(\rho) = p(\rho, S)$

$p_\rho(\rho, S) > 0$ By 2.04

$\Rightarrow$ $p = p(\rho, S)$ is invertible as fn of ρ

$\Rightarrow$ $\rho = \rho(p, S)$

$$dp = c^2 d\rho + f_s\, dS$$

$\Rightarrow \rho_t + u\rho_x + \rho u_x = 0$ mult by c^2

$$\Rightarrow \cancel{p_t} \quad c^2\rho_t + c^2 u\rho_x + c^2\rho u_x = 0$$

$$\underbrace{c^2\rho_t}_{p_t - f_s S_t} + \underbrace{c^2 u\rho_x}_{u(p_x - f_s S_x)} + \text{''} = 0$$

$$\Rightarrow p_t + u p_x - f_s\cancelto{0}{(S_t + u S_x)} + c^2\rho u_x = 0$$

$$\Rightarrow p_t + u p_x + \rho c^2 u_x = 0$$

Following hints in book.

$$p_t + u p_x + \rho c^2 u_x \pm c\rho u_t \pm c\rho u u_x \pm c p_x = 0$$

$$\Rightarrow p_t + (u+c)p_x + \rho c\{u_t + (c+u)u_x\} = 0$$

$$\& \quad p_t + (u-c)p_x - \rho c\{u_t + (-c+u)u_x\} = 0$$

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04-06-01

$$\rho_t + U\rho_x + \rho U_x = 0$$

$$\rho U_t + \rho U U_x + P_x = 0$$

$P = f(\rho)$ known

2 eqs + 2 unknown ρ, U

$P'(\rho) = c^2$

$$\Rightarrow \quad \rho_t + U\rho_x + \rho U_x = 0$$

$$\rho U_t + \rho U U_x + c^2 \rho_x = 0$$

$$\Leftrightarrow \quad \rho_t + U\rho_x + \rho U_x = 0$$

$$U_t + \frac{c^2}{\rho}\rho_x + U U_x = 0$$

$$\begin{pmatrix} \rho \\ U \end{pmatrix}_t + \begin{pmatrix} U & \rho \\ c^2/\rho & U \end{pmatrix} \begin{pmatrix} \rho \\ U \end{pmatrix}_x = 0$$

Find eigenvalues + eigenvectors of $\begin{pmatrix} U & \rho \\ c^2/\rho & U \end{pmatrix}$

$$\begin{vmatrix} U-\lambda & \rho \\ c^2/\rho & U-\lambda \end{vmatrix} = 0$$

$$(U-\lambda)^2 - c^2 = 0$$

$$(U-\lambda)^2 = c^2$$

$$|U-\lambda| = c$$

$$\Rightarrow \quad U-\lambda = -c \qquad U-\lambda = +c$$

$$\lambda = U+c \qquad \lambda = U-c$$

Then eigenvectors are

$\lambda_1 = u - c \quad \lambda_2 = u + c$

For λ_1

$$\begin{pmatrix} u-\lambda_1 & \rho \\ c^2/\rho & u-\lambda_1 \end{pmatrix}\begin{pmatrix} r_{11} \\ r_{12} \end{pmatrix} = 0$$

$$\begin{pmatrix} c & \rho \\ c^2/\rho & c \end{pmatrix}\begin{pmatrix} r_{11} \\ r_{12} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} c & \rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} r_{11} \\ r_{12} \end{pmatrix} = 0$$

$$r_{11} = -\frac{\rho}{c} r_{12} \qquad \vec{r}_1 = \begin{pmatrix} -\frac{\rho}{c} r_{12} \\ r_{12} \end{pmatrix} \propto \begin{pmatrix} -\rho \\ c \end{pmatrix} \propto \begin{pmatrix} \rho \\ -c \end{pmatrix} \checkmark$$

For λ_2

$$\begin{pmatrix} u-\lambda_2 & \rho \\ c^2/\rho & u-\lambda_2 \end{pmatrix}\begin{pmatrix} r_{21} \\ r_{22} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -c & \rho \\ c^2/\rho & -c \end{pmatrix}\begin{pmatrix} r_{21} \\ r_{22} \end{pmatrix} = 0$$

$$\Rightarrow \begin{pmatrix} -c & \rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} r_{21} \\ r_{22} \end{pmatrix} = 0 \qquad r_{21} = \frac{\rho}{c} r_{22}$$

$$\vec{r}_2 = \begin{pmatrix} \rho/c \, r_{22} \\ r_{22} \end{pmatrix} \propto \begin{pmatrix} \rho \\ c \end{pmatrix} \checkmark$$

Multiply quasilinear form by $\vec{r}_1^T$ & $\vec{r}_2^T$

$$\Rightarrow (\rho \quad -c)\begin{pmatrix}\rho\\u\end{pmatrix}_t + (\rho \quad -c)\begin{pmatrix}c & \rho\\ c^2/\rho & c\end{pmatrix}\begin{pmatrix}\rho\\u\end{pmatrix}_x = 0$$

$$\Rightarrow \rho\rho_t - cu_t + (\rho c - c^3/\rho \quad \rho^2 - c^2)\begin{pmatrix}\rho\\u\end{pmatrix}_x = 0 \quad \checkmark$$

$$\Rightarrow \rho\rho_t - cu_t + \underbrace{\rho c\rho_x - \frac{c^3}{\rho}\rho_x} + (\rho^2 - c^2)u_x = 0$$

$$+ \frac{c}{\rho}(\rho^2 - c^2)\rho_x + (\rho^2 - c^2)u_x = 0$$

$$(\rho^2 - c^2)\left[\frac{c}{\rho}\rho_x + u_x\right] = 0$$

$$(X \quad Y)\begin{pmatrix}\rho\\u\end{pmatrix}_t + \underbrace{(X \quad Y)\begin{pmatrix}a & b\\ c & d\end{pmatrix}}\begin{pmatrix}\rho\\u\end{pmatrix}_x = 0$$

$$\lambda_i (X \quad Y)($$

$$v^T A = \lambda v^T$$

$A^T v = \lambda v$. Find eigenvectors of A^T ← note transpose

$$A^T = \begin{pmatrix} c & c^2/\rho \\ \rho & c \end{pmatrix}$$

$$A^T - \lambda I = \begin{pmatrix} c-\lambda & c^2/\rho \\ \rho & c-\lambda \end{pmatrix} \qquad \lambda = \lambda_1 \begin{pmatrix} c & c^2/\rho \\ \rho & c \end{pmatrix}$$

$$\begin{pmatrix} c & c^2/\rho \\ \rho & c \end{pmatrix}\begin{pmatrix} l_{11} \\ l_{12} \end{pmatrix} = \vec{0} \Rightarrow \begin{pmatrix} c & c^2/\rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} l_{11} \\ l_{12} \end{pmatrix} = 0$$

$$c\,l_{11} + \frac{c^2}{\rho} l_{12} = 0$$

$$l_{11} = -\frac{c}{\rho} l_{12}$$

$$\vec{l}_1 = \begin{pmatrix} -c/\rho\, l_{12} \\ l_{12} \end{pmatrix} \propto \begin{pmatrix} -c \\ \rho \end{pmatrix}$$

$\lambda = \lambda_2$

$$\begin{pmatrix} -c & c^2/\rho \\ \rho & -c \end{pmatrix}\begin{pmatrix} l_{21} \\ l_{22} \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} -c & c^2/\rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} l_{21} \\ l_{22} \end{pmatrix} = 0$$

$$l_{21} = c/\rho\, l_{22}$$

$$\vec{l}_2 = \begin{pmatrix} c/\rho\, l_{22} \\ l_{22} \end{pmatrix} \propto \begin{pmatrix} c \\ \rho \end{pmatrix}$$

Then $l_1^T \begin{pmatrix} \rho \\ u \end{pmatrix}_t + l_1^T \underbrace{\begin{pmatrix} u & \rho \\ c^2/\rho & u \end{pmatrix}}_{A \text{ not } A^T} \begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$

$$\Rightarrow (-c \quad \rho)\begin{pmatrix} \rho \\ u \end{pmatrix}_t + (-c \quad \rho)\begin{pmatrix} u & \rho \\ c^2/\rho & u \end{pmatrix}\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$-c\rho_t + \rho u_t + \underbrace{(-cu + c^2 \quad -c\rho + \rho u)\begin{pmatrix} \rho \\ u \end{pmatrix}_x}_{c(-u+c)\rho_x + \rho(-c+u)u_x} = 0$$

$$-c(\rho_t + (u-c)\rho_x) + \rho(u_t + (u-c)u_x) = 0$$

$$\boxed{\Rightarrow (\rho_t + (u-c)\rho_x) - \frac{\rho}{c}(u_t + (u-c)u_x) = 0 \quad **}$$

$$l_2^T \begin{pmatrix} \rho \\ u \end{pmatrix}_t + l_2^T \begin{pmatrix} u & \rho \\ c^2/\rho & u \end{pmatrix}\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$\Rightarrow (c \quad \rho)\begin{pmatrix} \rho \\ u \end{pmatrix}_t + (c \quad \rho)\begin{pmatrix} u & \rho \\ c^2/\rho & u \end{pmatrix}\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$\Rightarrow c\rho_t + \rho u_t + (cu + c^2 \quad c\rho + \rho u)\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$\Rightarrow \quad \underbrace{c\rho_t} + \rho u_t + \underbrace{c(u+c)}\rho_x + \rho(u+c)u_x = 0$$

$$c[\rho_t + (u+c)\rho_x] + \rho[u_t + (u+c)u_x] = 0$$

$$\Rightarrow \boxed{\rho_t + (u+c)\rho_x + \frac{\rho}{c}(u_t + (u+c)u_x) = 0} \quad **$$

$\Rightarrow$ In characteristic form

$$d\rho - \frac{\rho}{c}du = 0 \quad \text{along } \frac{dx}{dt} = u - c$$

$$\& \quad d\rho + \frac{\rho}{c}du = 0 \quad \text{along } \frac{dx}{dt} = u + c$$

$$\Rightarrow \quad I_+: \quad x_\alpha = (u+c)\,t_\alpha$$

$$I_-: \quad x_\beta = (u-c)\,t_\beta$$

$$II_+: \quad \rho_\alpha + \frac{\rho}{c}u_\alpha = 0 \Rightarrow u_\alpha = -\frac{c}{\rho}\rho_\alpha$$

$$II_-: \quad \rho_\beta - \frac{\rho}{c}u_\beta = 0 \Rightarrow u_\beta = \frac{c}{\rho}\rho_\beta$$

To write in terms of the pressure $\quad p = p(\rho)$

$$\frac{dp}{dx} = p'(\rho)\frac{d\rho}{dx} = c^2\frac{d\rho}{dx}$$

$$\rho_x = \frac{p_x}{c^2}$$

$$\Rightarrow \quad I_+ : \quad x_\alpha = (u+c)t_\alpha$$

$$I_- : \quad x_\beta = (u-c)t_\beta$$

$$II_+ : \quad u_\alpha = -\frac{c}{\rho}\cdot\frac{P_\alpha}{c^2} = -\frac{P_\alpha}{\rho c}$$

$$II_- : \quad u_\beta = +\frac{c}{\rho}\frac{P_\beta}{c^2} = \frac{P_\beta}{\rho c}$$

As an alternate means of deriving these equations one can write The Differential eqs in terms of primitive variables u & P.

Then the eqs become:

$$\frac{P_t}{c^2} + u\frac{P_x}{c^2} + \rho u_x = 0$$

$$u_t + u u_x + \frac{P_x}{\rho} = 0 \qquad U = \begin{pmatrix} P \\ u \end{pmatrix}$$

$$\Leftrightarrow \quad P_t + uP_x + c^2\rho u_x = 0$$

$$u_t + \frac{P_x}{\rho} + u u_x = 0$$

$$\begin{pmatrix} P \\ u \end{pmatrix}_t + \underbrace{\begin{pmatrix} u & c^2\rho \\ 1/\rho & u \end{pmatrix}}_{A}\begin{pmatrix} P \\ u \end{pmatrix}_x = 0$$

eigenvalues & vectors of $A^T \Rightarrow \begin{pmatrix} u & 1/\rho \\ c^2\rho & u \end{pmatrix}$

$$\begin{vmatrix} u-\lambda & \rho^{-1} \\ c^2\rho & u-\lambda \end{vmatrix} = (u-\lambda)^2 - c^2 = 0 \quad \Rightarrow \lambda = u \pm c$$

Right eigenvectors of A^T (left eigenvectors of A) are

For $\lambda = u - c$

$$\begin{pmatrix} c & 1/\rho \\ c^2\rho & c \end{pmatrix}\begin{pmatrix} \ell_{11} \\ \ell_{12} \end{pmatrix} = \vec{0} \qquad \begin{pmatrix} c & 1/\rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_{11} \\ \ell_{12} \end{pmatrix} = 0$$

$$\ell_{11} = \frac{-1}{c\rho}\ell_{12} \qquad \vec{\ell}_1 = \begin{pmatrix} -1 \\ c\rho \end{pmatrix}$$

For $\lambda_2 = u + c$

$$\begin{pmatrix} -c & 1/\rho \\ c^2\rho & -c \end{pmatrix}\begin{pmatrix} \ell_{21} \\ \ell_{22} \end{pmatrix} = \vec{0} \rightarrow \begin{pmatrix} -c & 1/\rho \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_{21} \\ \ell_{22} \end{pmatrix} = \vec{0}$$

$$\ell_{21} = 1/c\rho \; \ell_{22} \qquad \vec{\ell}_2 = \begin{pmatrix} 1 \\ c\rho \end{pmatrix}$$

Thus characteristic equations then are

$$\ell_1^T \begin{pmatrix} P \\ u \end{pmatrix}_t + \ell_1^T \begin{pmatrix} u & c^2\rho \\ 1/\rho & u \end{pmatrix}\begin{pmatrix} P \\ u \end{pmatrix}_x = 0$$

$$-P_t + c\rho u_t + \begin{pmatrix} -u + c & -c^2\rho + uc\rho \end{pmatrix}\begin{pmatrix} P \\ u \end{pmatrix}_x = 0$$

$$-P_t - (u-c)P_x + \rho c u_t + c\rho(-c + u)u_x = 0$$

$$P_t + (u-c)P_x - \rho c\left[u_t + (u-c)u_x\right] = 0$$

$$dP - \rho c\, du = 0 \qquad dx = (u-c)dt$$

$$du = \frac{dP}{\rho c}$$

$$l_2^T \begin{pmatrix} p \\ u \end{pmatrix}_t + l_2^T \begin{pmatrix} u & c^2\rho \\ 1/\rho & u \end{pmatrix} \begin{pmatrix} p \\ u \end{pmatrix}_x = 0$$

$$p_t + c\rho u_t + \begin{pmatrix} u+c & \dfrac{c^2\rho + cu\rho}{c\rho(u+c)} \end{pmatrix} \begin{pmatrix} p \\ u \end{pmatrix}_x = 0$$

$$\Rightarrow \quad p_t + (u+c)\, p_x + c\rho \left[u_t + (u+c) u_x \right] = 0$$

$$dp + c\rho \, du = 0 \qquad dx = (u+c)\,dt$$

$$du = -\frac{1}{c\rho} dp$$

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$$\rho_t + u\rho_x + \rho u_x = 0 \qquad c^2 = p'(\rho)$$
$$\rho u_t + \rho u u_x + p_x = 0$$

$$\Rightarrow \quad \rho_t + u\rho_x + \rho u_x = 0$$
$$\rho u_t + \rho u u_x + c^2 \rho_x = 0 \Rightarrow u_t + u u_x + \frac{c^2}{\rho}\rho_x = 0$$

let $\vec{u} = \begin{pmatrix} \rho \\ u \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} \rho \\ u \end{pmatrix}_t + \underbrace{\begin{pmatrix} u & \rho \\ \frac{c^2}{\rho} & u \end{pmatrix}}_{f'(\vec{u})} \begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

eigenvalues / eigenvectors of $f'(u)$ then become

$$\begin{vmatrix} u-\lambda & \rho \\ \frac{c^2}{\rho} & u-\lambda \end{vmatrix} = 0 \Rightarrow (u-\lambda)^2 - c^2 = 0$$
$$\Rightarrow \lambda = u \pm c$$

Then eigenvectors become for $\lambda_1 = u - c$

$$\begin{pmatrix} c & \rho \\ \frac{c^2}{\rho} & c \end{pmatrix} \begin{pmatrix} r_1^1 \\ r_2^1 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} r_1^1 \\ r_2^1 \end{pmatrix} = \begin{pmatrix} -\rho \\ c \end{pmatrix}$$

eigenvector for $\lambda_2 = u+c$

$$\begin{pmatrix} -c & \rho \\ \frac{c^2}{\rho} & -c \end{pmatrix}\begin{pmatrix} r_1^2 \\ r_2^2 \end{pmatrix} = 0 \rightarrow \begin{pmatrix} r_1^2 \\ r_2^2 \end{pmatrix} = \begin{pmatrix} \rho \\ c \end{pmatrix}$$

Then decomposing $f'(u) = \begin{pmatrix} u & \rho \\ \frac{c^2}{\rho} & u \end{pmatrix} = R(u)\Lambda R^{-1}(u)$

w/ $R(u) = \begin{pmatrix} -\rho & \rho \\ c & c \end{pmatrix}$ $\Lambda(u) = \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix}$

+ $R^{-1}(u) = \begin{pmatrix} -\frac{1}{2\rho} & \frac{1}{2c} \\ \frac{1}{2\rho} & \frac{1}{2c} \end{pmatrix}$

Then check $\underline{R(u)\Lambda(u)R^{-1}(u)} \stackrel{?}{=} f'(u)$

$$= \begin{pmatrix} u & \rho \\ \frac{c^2}{\rho} & u \end{pmatrix}$$ yes ! correct.

Then :

$$\begin{pmatrix} \rho \\ u \end{pmatrix}_t + R(u)\Lambda(u)R^{-1}(u)\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$ + mult by $R^{-1}(u)$ from left

$$R^{-1}(u)\begin{pmatrix} \rho \\ u \end{pmatrix}_t + \Lambda(u)R^{-1}(u)\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$\begin{pmatrix} -1/\rho & 1/c \\ 1/\rho & 1/c \end{pmatrix}\begin{pmatrix} \rho \\ u \end{pmatrix}_t + \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix}\begin{pmatrix} -1/\rho & 1/c \\ 1/\rho & 1/c \end{pmatrix}\begin{pmatrix} \rho \\ u \end{pmatrix}_x = 0$$

$$\Rightarrow \begin{pmatrix} -\frac{\rho_t}{\rho} + \frac{u_t}{c} \\ \frac{\rho_t}{\rho} + \frac{u_t}{c} \end{pmatrix} + \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix}\begin{pmatrix} -\frac{\rho_x}{\rho} + \frac{u_x}{c} \\ \frac{\rho_x}{\rho} + \frac{u_x}{c} \end{pmatrix} = 0$$

Mult by ρc in diagonal matrix $\begin{pmatrix} \rho c & 0 \\ 0 & \rho c \end{pmatrix}$ { commutes w/ $\begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix}$ }

$$\Rightarrow \begin{pmatrix} -c\rho_t + \rho u_t \\ c\rho_t + \rho u_t \end{pmatrix} + \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix}\begin{pmatrix} -c\rho_x + \rho u_x \\ c\rho_x + \rho u_x \end{pmatrix} = \vec{0} \quad (**)$$

2 eqs:

$$(-c\rho_t + \rho u_t) + (u-c)(-c\rho_x + \rho u_x) = 0$$

&

$$(c\rho_t + \rho u_t) + (u+c)(c\rho_x + \rho u_x) = 0$$

But to write in terms of pressure remember $P'(\rho) = c^2$

$$P_t = c^2\rho_t \Rightarrow \rho_t = \frac{P_t}{c^2}$$

$$P_x = c^2\rho_x \Rightarrow \rho_x = \frac{P_x}{c^2}$$

matrix eq (**)

$$\Rightarrow \begin{pmatrix} -\frac{P_t}{c} + \rho u_t \\ \frac{P_t}{c} + \rho u_t \end{pmatrix} + \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix} \begin{pmatrix} -\frac{P_x}{c} + \rho u_x \\ \frac{P_x}{c} + \rho u_x \end{pmatrix} = \vec{0}$$

mult by $\begin{pmatrix} -c & 0 \\ 0 & c \end{pmatrix}$

$$\Leftrightarrow \begin{pmatrix} P_t - c\rho u_t \\ P_t + \rho c u_t \end{pmatrix} + \begin{pmatrix} u-c & 0 \\ 0 & u+c \end{pmatrix} \begin{pmatrix} P_x - \rho c u_x \\ P_x + \rho c u_x \end{pmatrix} = 0$$

or

$$P_t - c\rho u_t + (u-c)(P_x - \rho c u_x) = 0$$

$$P_t + \rho c u_t + (u+c)(P_x + \rho c u_x) = 0$$

(How do w/ matrix multiplication ? Do I get these 2 eqs?)

or

$$P_t + (u-c)P_x - \rho c\left(u_t + (u-c)u_x\right) = 0$$

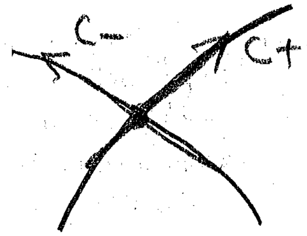
$$\& \quad P_t + (u+c)P_x + \rho c\left(u_t + (u+c)u_x\right) = 0$$

$$\Leftrightarrow \quad dp(x(t),t) - \rho(x(t),t)c(x(t),t)\,du(x(t),t) = 0 \quad \text{Along } \frac{dx(t)}{dt} = u(x(t),t) - c(x(t),t))$$

$$\& \quad dp(x(t),t) + \rho(x(t),t)c(x(t),t)\,du(x(t),t) = 0 \quad \frac{dx(t)}{dt} = u(x(t),t) + c(x(t),t)$$

eq 34.03

Along $\frac{dx}{dt} = u - c$, we have 2 identical statements



$$du - \frac{c}{\rho} d\rho = 0 \iff du - \frac{dp}{\rho c} = 0$$

$$\left\{ d\tau = d\rho^{-1} = -\rho^{-2} d\rho \Rightarrow d\rho = -\rho^{2} d\tau \right\}$$

1st eq is then equivalent to

$$du + c\rho\, d\tau = 0 \qquad \text{eq (2)} \Rightarrow \rho c = \frac{dp}{du} \text{ along } C_-$$

$$\Rightarrow du + \frac{dp}{du} d\tau = 0$$

$$du^2 + dp\, d\tau = 0$$

For C_+ characteristics

$$du + \frac{c}{\rho} d\rho = 0 \iff du + \frac{dp}{\rho c} = 0$$

$$\Updownarrow$$

$$du - c\rho\, d\tau = 0 \Rightarrow \frac{du}{d\tau} = c\rho$$

$$\Rightarrow du + \frac{dp\, d\tau}{du} = 0 \qquad du^2 + dp\, d\tau = 0$$

$$\ell(\rho) = \int_{\rho'}^{\rho} \frac{c}{\rho} d\rho$$

$$\ell(0) = 0 = \int_{\rho'}^{0} \frac{c}{\rho} d\rho$$

Need only Assumption that $\ell(0) = 0$
Just pick $\rho' \ni \int_{\rho'}^{0} \frac{c}{\rho} d\rho = 0$

$\ell(\tilde{\rho}) \quad \hat{\rho} > 0$

$$\ell(\hat{\rho}) = \underbrace{\int_{\rho'}^{0} \frac{c}{\rho} d\rho}_{0} + \int_{0}^{\hat{\rho}} \frac{c}{\rho} d\rho$$

$$0 + \int_{0}^{\hat{\rho}} \frac{c}{\rho} d\rho > 0 \quad \forall\ \tilde{\rho} > 0 \Rightarrow \ell(\rho) > 0 \ \forall \rho$$

$$c = \sqrt{A\gamma}\, \rho^{(\gamma-1)/2} \quad \text{Then} \quad \ell(\rho) = \int_{\rho'=0}^{\rho} \frac{c}{\rho} d\rho = \int_{\rho'=0} \sqrt{A\gamma}\, \rho^{\frac{\gamma-1}{2}-1} d\rho$$

$$= \sqrt{A\gamma}\, \frac{\rho^{\frac{\gamma-1}{2}}}{\left(\frac{\gamma-1}{2}\right)} = \frac{2}{\gamma-1} \sqrt{A\gamma}\, \rho^{\frac{\gamma-1}{2}}$$

$$= \frac{2}{\gamma-1} c$$

$\rho' = 0 \Rightarrow \ell(0) = 0$ in this case.

Riemann invariants then become:

$$2r = u + \ell(\rho) = u + \frac{2}{\gamma-1}c \Rightarrow r = \frac{u}{2} + \frac{c}{\gamma-1}$$

$$-2s = u - \ell(\rho) = u - \frac{2}{\gamma-1}c \Rightarrow -s = \frac{u}{2} - \frac{c}{\gamma-1}$$

$\Rightarrow$ $\frac{dx}{dt} = u+c$ $\quad \frac{u}{2} + \frac{c}{\gamma-1}$ constant along C_+

$\frac{dx}{dt} = u-c$ $\quad \frac{u}{2} - \frac{c}{\gamma-1}$ constant along C_-

If $\gamma = 3 \Rightarrow$ $2r = u+c$ for C_+ : $\frac{dx}{dt} = 2r$ constant

& $-2s = u-c$ for C_- : $\frac{dx}{dt} = -2s$ constant

$u + \frac{2}{\gamma-1}\sqrt{A\gamma}\,\rho^{\frac{\gamma-1}{2}}$ const on Γ^+

$u - \frac{2}{\gamma-1}\sqrt{A\gamma}\,\rho^{\frac{\gamma-1}{2}}$ const on Γ^+

$$\frac{u}{2} + \frac{c}{\gamma-1} = \overset{r_0}{\text{const}} \text{ on } \Gamma^+ \text{ on } \frac{dx}{dt} = u+c$$

$$\frac{u}{2} - \frac{c}{\gamma-1} = \overset{-s_0}{\text{const}} \text{ on } \Gamma^- \text{ on } \frac{dx}{dt} = u-c$$

$$u = 2\left(r_0 - \frac{c}{\gamma-1}\right) \text{ on } \Gamma^+ \Rightarrow \frac{dx}{dt} = 2\left(r_0 - \frac{c}{\gamma-1}\right) + c$$

$$u = 2\left(-s_0 + \frac{c}{\gamma-1}\right)$$

$$\text{II} : \quad u_\beta - \frac{c}{\rho}\rho_\beta = 0$$

$$\frac{d}{d\beta}u - \frac{d}{d\beta}\int^{\rho(\beta)}\frac{c}{\tilde\rho}d\tilde\rho = 0$$

$$\Rightarrow \quad \frac{d}{d\beta}\left(u - \int^{\rho}\frac{c\,d\tilde\rho}{\tilde\rho}\right) = 0$$

$$\Rightarrow \quad u - \int^{\rho}\frac{c\,d\hat\rho}{\hat\rho} = -2s(\alpha)$$

$$\frac{d}{d\beta}\int^{\rho(\beta)}\frac{c(\tilde\rho)}{\tilde\rho}d\tilde\rho$$

$$= \frac{c(\rho)}{\rho}\frac{d\rho}{d\beta}$$

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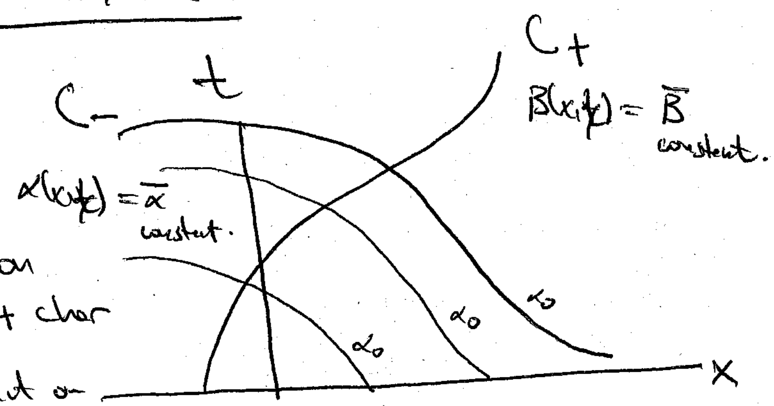


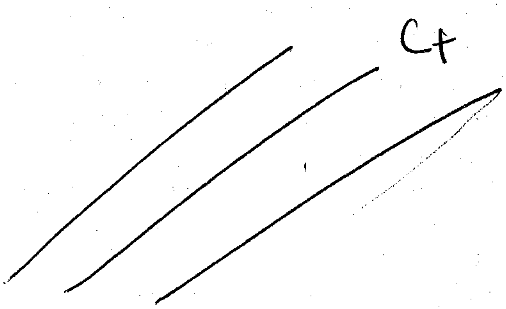
Riemann inv

$U + \ell(\rho) = 2r(\beta)$ constant on each C+ char

$U - \ell(\rho) = -2s(\alpha)$ constant on each C− char.

If $-2s(\alpha) = U - \ell(\rho) =$ constant $\forall\ \alpha$, in flow region

C+ char $\quad X_\beta = (U+c)t_\beta$



$U - \ell = U_0 - \ell_0$

$$\frac{dx}{dt} = U + c$$

$U = ?\quad c = ?$ gas velocity.

$U = \ell + U_0 - \ell_0$ For simple wave by 40.01

$$\therefore\quad \frac{dx}{dt} = c(\rho) + \ell(\rho) + U_0 - \ell_0$$

$$\frac{dc + d\ell}{d\ell} = \frac{dc + \frac{1}{\rho c}\,dp}{\frac{1}{\rho c}\,dp} \quad \text{or} \quad = \frac{dc + \frac{c}{\rho}\,d\rho}{\frac{c}{\rho}\,d\rho}$$

$$\frac{\rho\, dc + c\, d\rho}{c\, d\rho} = \frac{d(\rho c)}{c\, d\rho} = \frac{1}{c}\frac{d(\rho c)}{d\rho}$$

By 205 $\rho^2 c^2 = -g_r$

$\therefore \frac{d}{d\rho} \rightarrow$

$$2(c\rho)\frac{d(\rho c)}{d\rho} = -\frac{d}{d\rho} g_r$$

$$= -\frac{d}{dr} g_r \frac{dr}{d\rho}$$

$$\Rightarrow 2(c\rho)\frac{d(\rho c)}{d\rho} = -g_{rr}(-1\rho^{-2}) = \frac{g_{rr}}{\rho^2}$$

$\therefore$ Above becomes

$$\frac{dc + d\ell}{d\ell} = \frac{1}{c}\frac{g_{rr}}{\rho^2 (2c\rho)} = \frac{r\, g_{rr}}{2c^2\rho^2} = -\frac{r\, g_{rr}}{2 g_r} > 0.$$

For Forward facing simple waves:

$$u - \frac{2}{\gamma-1} c = u_0 - \frac{2}{\gamma-1} c_0$$

$$u - u_0 = -\frac{2}{\gamma-1}(c_0 - c)$$

$$\Rightarrow \quad \underbrace{\frac{(\gamma-1)}{\gamma+1}}(u-u_0) = \frac{2}{\gamma+1}(c - c_0)$$

$$\mu^2 (u - u_0) =$$

If $\quad \mu^2 = \dfrac{\gamma-1}{\gamma+1}$

Then $\quad 1 - \mu^2 = \dfrac{\gamma+1-\gamma+1}{\gamma+1} = \dfrac{2}{\gamma+1}$

& the above becomes

$$\mu^2(u-u_0) = (1-\mu^2)(c-c_0)$$

Now: $\quad p = p_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma} \qquad \rho = \,?$

$$\Longleftarrow$$

$$\frac{2}{\gamma-1} c = u - u_0 + \frac{2}{\gamma-1} c_0$$

$$c = \frac{(\gamma-1)}{2}(u-u_0) + c_0 \qquad \text{Then} \quad c^2 = \gamma A \rho^{\gamma-1}$$

$$\Rightarrow \rho = \left(\frac{1}{\gamma A} c^2\right)^{\frac{1}{\gamma-1}}$$

$$\rho = \left(\frac{1}{\gamma A}\right)^{\frac{1}{\gamma-1}} c^{\frac{2}{\gamma-1}}$$

$$= \left(\frac{1}{\gamma A}\right)^{\frac{1}{\gamma-1}} \left[c_0 + \frac{(\gamma-1)}{2}(u-u_0)\right]^{\frac{2}{\gamma-1}}$$

$$= \underbrace{\left(\frac{1}{\gamma A}\right)^{\frac{1}{\gamma-1}} c_0^{\frac{2}{\gamma-1}}}_{\rho_0} \left[1 + \frac{(\gamma-1)}{2}\frac{(u-u_0)}{c_0}\right]^{\frac{2}{\gamma-1}}$$

$$= \rho_0\left(1 + \frac{(\gamma-1)}{2}\frac{(u-u_0)}{c_0}\right)^{\frac{2}{\gamma-1}}$$

Then $p = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma} = p_0\left(1 + \frac{(\gamma-1)}{2}\left(\frac{u-u_0}{c_0}\right)\right)^{\frac{2\gamma}{\gamma-1}}$

$$c = c_0 + \frac{(\gamma-1)}{2}(u-u_0)$$

$$+ \quad u + c = c_0 + \underbrace{\left(\frac{\gamma-1}{2} + 1\right)}u - \left(\underbrace{\frac{\gamma-1}{2}} + 1 - 1\right)u_0$$

$$= c_0 + u_0 + \frac{\gamma+1}{2}(u-u_0)$$

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$x_\beta = v t_\beta$ particle paths. $(x,t) = (x(\beta), t(\beta))$

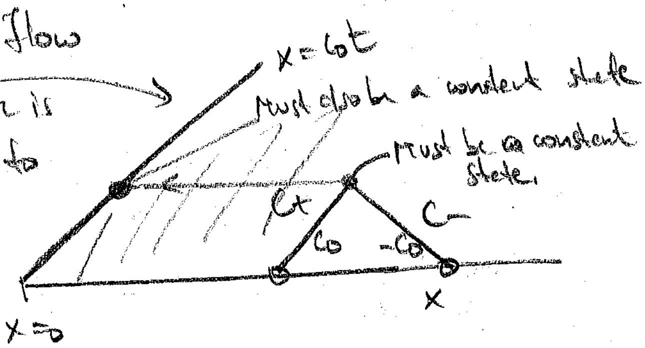
But also $x(\beta) = \xi(\beta) + (v(\beta) + c(\beta)) t(\beta)$

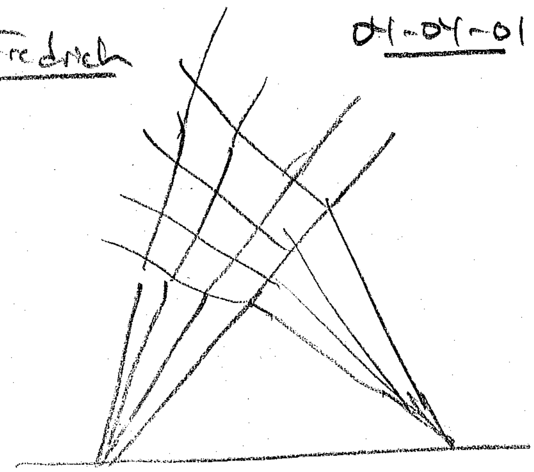
$x_\beta = (v' + c')t + (v+c)t' + \xi'$

$\cancel{v t_\beta} = \cancel{v t_\beta} + c t_\beta + (v_\beta + c_\beta) t + \xi_\beta$

$-\xi_\beta = c t_\beta + (v_\beta + c_\beta) t$

Any flow
Here is
adjacent to
a constant
flow d !
Can only
be a simple
wave

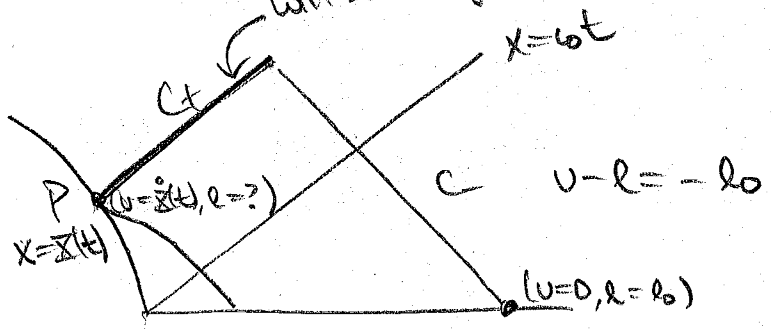




Along $C-$ characteristic $u - l$ is constant
evaluating in the constant state Region gives

$u - l = 0 - l_0$

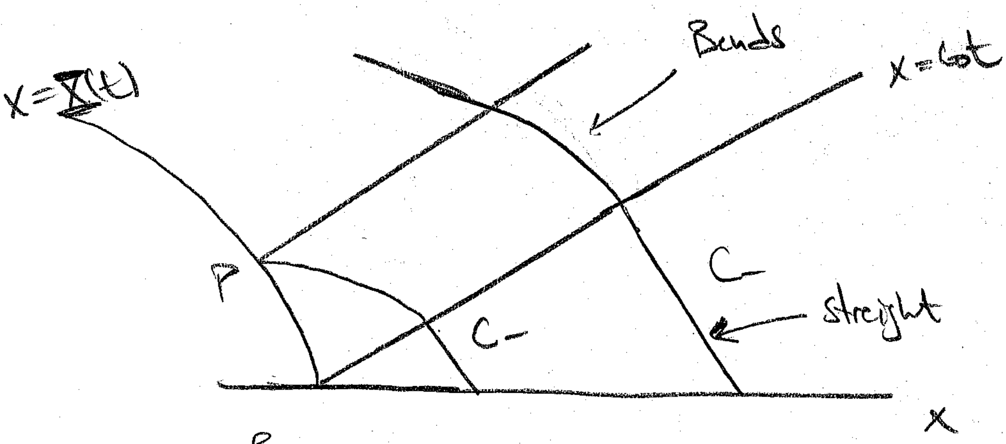
will be a straight line as $\exists$ one value of ρ at beginning of this line. But then moving along the C^- characteristic that intersects this point $\hat{p}$ requires that



$u - l = 0 - l_0$
$\|$
$u_p(t) - l = -l_0$
$\|$
velocity of piston

$\Rightarrow -u_p(t) + l = l_0$

$l = l_0 + u_p(t)$



$$l = \int_{\rho'}^{\rho} \frac{c\, d\rho}{\rho} \quad + : \quad \frac{dl}{d\rho} = \frac{c}{\rho} > 0$$

Thus given l we can invert to get ρ from

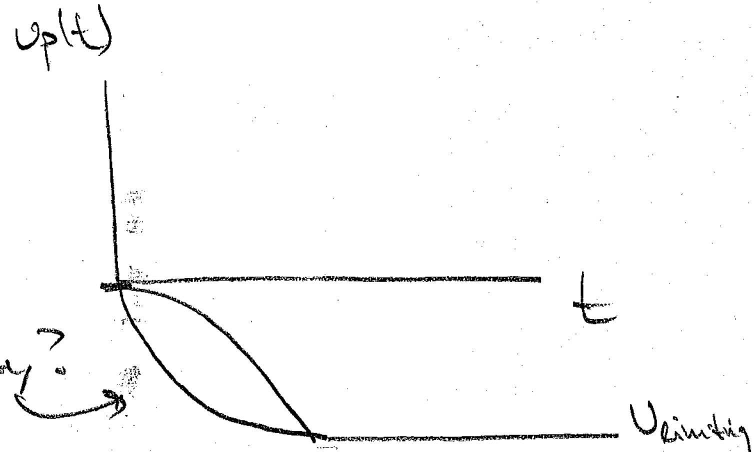
$l(\rho) = l$

Value of ℓ along piston $\ell = \ell_0 + u_p(t)$

Write code to compute simple wave that results from pulling back piston like this

$$\ell(p) = \ell_0 + u_p(t)$$

$$p = \ell^{-1}(\ell_0 + u_p(t))$$



$\frac{du_p}{dt} < 0$

$\therefore$ $\ell_0 + u_p(t)$ is decreasing as we move through the fan

$\Rightarrow$ ℓ is decreasing $\Rightarrow$ p & ρ are decreasing

$\left\{ \text{As } \frac{d\ell}{dp} > 0 \quad \underbrace{\frac{d\rho}{dp} > 0} \right\}$

From C_- characteristic from rest state to piston

$$u - \ell = 0 - \ell_0$$

$$\underset{\substack{\| \\ u_p(t)}}{u} = \ell - \ell_0 \Rightarrow u + \ell_0 = \ell$$

if $u < -\ell_0$ $\quad \{ \underbrace{|u| > \ell_0} \}$

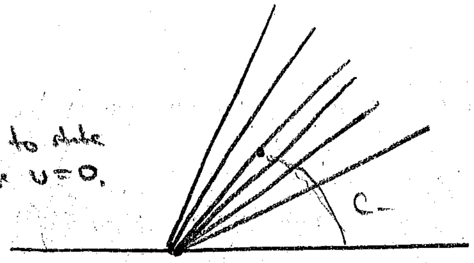
$u + \ell_0 < 0$

$\rightarrow\leftarrow$ As $\ell \geq 0$

(1) $x = (U+c)t$ eq for slopes in fan.

$U - \ell = -\ell_0$ conservation of invariant to state ahead of fan rest state $U=0$.

(2) $U = \ell - \ell_0$ known function of c



$U - \ell = 0 - \ell_0$

2 eqs + 2 unknown c + U. $\{\ell = \ell(c)\}$ $\rightarrow \ell = \frac{2c}{\gamma-1}$

$$\frac{x}{t} = U + c$$

$$-\frac{2c_0}{\gamma-1} = -\ell_0 = U - \ell = U - \frac{2c}{\gamma-1}$$

$$U + c = \frac{x}{t} \quad ✓ \qquad \Delta = \begin{vmatrix} 1 & 1 \\ 1 & -\frac{2}{\gamma-1} \end{vmatrix} = -\frac{2}{\gamma-1} - 1$$

$$\Rightarrow U - \frac{2}{\gamma-1}c = -\frac{2c_0}{\gamma-1}$$

$$= -\frac{2}{\gamma-1} - \frac{(\gamma-1)}{\gamma-1} = \frac{-2-\gamma+1}{\gamma-1} = \frac{-1-\gamma}{\gamma-1} = -\left(\frac{\gamma+1}{\gamma-1}\right) ✓ = -\frac{1}{\mu^2} ✓$$

$$U = \frac{\begin{vmatrix} \frac{x}{t} & +1 \\ -\frac{2c_0}{\gamma-1} & -\frac{2}{\gamma-1} \end{vmatrix}}{\left(-\frac{1}{\mu^2}\right)} = \frac{-\frac{2(x/t)}{\gamma-1} + \frac{2c_0}{\gamma-1}}{\left(-\frac{1}{\mu^2}\right)}$$

$$\Rightarrow U = -\left(\frac{2}{\gamma-1}\right)\left(\frac{x}{t} - c_0\right)\left(-\mu^2\right) \quad ✓$$

$$= \frac{2}{\cancel{\gamma-1}} \cdot \frac{\cancel{\gamma-1}}{\gamma+1}\left(\frac{x}{t} - c_0\right) = \frac{2}{\gamma+1}\left(\frac{x}{t} - c_0\right) \quad ✓$$

$$+ \quad c = \frac{\begin{vmatrix} 1 & x/t \\ 1 & -\frac{2c_0}{\gamma-1} \end{vmatrix}}{\left(-\frac{1}{\mu^2}\right)} = -\mu^2\left(-\frac{2c_0}{\gamma-1} - \frac{x}{t}\right) \quad ✓$$

$$\Rightarrow \quad v = \frac{2}{\gamma-1}\left(\frac{x}{t} - c_0\right)$$

$$c = \mu^2 \frac{x}{t} + \frac{2\mu^2}{\gamma-1} c_0$$

$\therefore \quad \gamma > 1$

$0 < \mu^2 < \frac{1}{2}$

Now definition of μ is

$$\mu^2 = \frac{\gamma-1}{\gamma+1} \Rightarrow \text{inverse definition is?}$$

$$(\gamma+1)\mu^2 = \gamma - 1$$

$$\gamma(\mu^2 - 1) = -\mu^2 - 1$$

$$\gamma = -\frac{(\mu^2+1)}{\mu^2-1} = \frac{1+\mu^2}{1-\mu^2}$$

$$\gamma > 1 \Rightarrow \gamma - 1 > 0$$
$$\gamma + 1 > 2$$

From the calculation to the Right

$$\frac{2}{\gamma-1} = \frac{2}{\frac{1+\mu^2}{1-\mu^2} - 1} = \frac{2}{\frac{1+\mu^2}{1-\mu^2} - \frac{(1-\mu^2)}{(1-\mu^2)}} \qquad 0 < \mu^2 < \frac{1}{2}$$

$$= \frac{2}{\frac{2\mu^2}{1-\mu^2}} = \frac{1-\mu^2}{\mu^2}$$

$$\therefore \quad c = \mu^2 \frac{x}{t} + (1-\mu^2) c_0$$

$$\& \quad v = \left(\frac{1-\mu^2}{\mu^2}\right)\left(\frac{x}{t} - c_0\right)$$

(4205) is

$$t = -c^{-\mu^{-2}} \left\{ \int c^{\mu^{-2}-1} \cancelto{0}{p_\beta}\, dB + \text{constant} \right\}$$

$$\Rightarrow \quad t = -c^{-\mu^{-2}} A \quad \text{evaluate at time 0.}$$

$$t_0 = -c_0^{-\mu^{-2}} A \quad \Rightarrow \quad A = -t_0 c_0^{\mu^{-2}}$$

$$\Rightarrow t = t_0 c_0^{\mu^{-2}} c^{-\mu^{-2}}$$

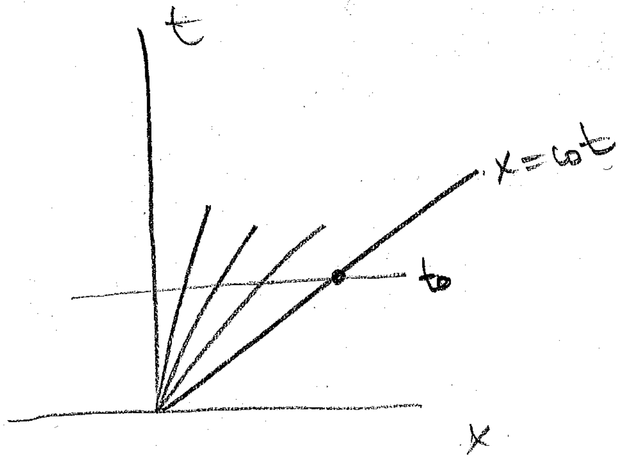
$$t = t_0\left(\frac{c}{c_0}\right)^{-\mu^{-2}}$$

Eq 42.06 is $t = \frac{1}{2}c^{-2\mu^{-2}}\left\{\int \cancel{c^{2\mu^{-2}-1}}^{\,0} \{_B\, dB + \text{constant}\right\}$

evaluating at 0

$$t_0 = \frac{1}{2}c_0^{-2\mu^{-2}}A \Rightarrow A = 2t_0 c_0^{2\mu^{-2}}$$

$$\Rightarrow t = \frac{1}{2}c^{-2\mu^{-2}}\, 2t_0 c_0^{2\mu^{-2}} \Rightarrow t = t_0\left(\frac{c}{c_0}\right)^{-2\mu^{-2}}$$



42.04 $c = \mu^2(u+c) + (1-\mu^2)c_0$

$u + c =$

$x = \xi(\beta) + \omega(\beta)t \qquad \omega(\beta) = u(\beta) + c(\beta)$

$\frac{d}{d\beta}$ w/ $\frac{dx}{d\beta} = 0$

$0 = \frac{d\xi}{d\beta} + \frac{d\omega}{d\beta}\cdot t \Rightarrow t = -\frac{d\xi/d\beta}{d\omega/d\beta} = -\frac{d\xi}{d\omega}$ ~~putting back~~

i.e. $t = -\frac{d\xi}{d\omega}$ putting into $x = \xi + \omega t$ gives

$x = \xi - \omega\frac{d\xi}{d\omega}$ eq 49.02

$\frac{d^2\xi}{d\omega^2} = -\frac{dt}{d\omega}$

$\therefore \frac{dx}{d\omega} = \frac{d\xi}{d\omega} + t + \omega\frac{dt}{d\omega}$ taking the ω derivative of $x = \xi + \omega t$

$\quad \| \qquad \| \qquad\qquad \|$

$\frac{dx}{d\omega} = -\cancel{t} + \cancel{t} + \omega\left(-\frac{d^2\xi}{d\omega^2}\right) \Rightarrow \frac{dx}{d\omega} = -\omega\frac{d^2\xi}{d\omega^2}$

$\therefore$ Both $\frac{dt}{d\omega}$ + $\frac{dx}{d\omega}$ vanish when $\frac{d^2\xi}{d\omega^2} = 0$

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$g_\tau < 0 \qquad g_\tau = -\rho^2 c^2 \qquad \tau = 1/\rho.$

$d\tau = -1/\rho^2 \, d\rho$

$g_{\tau\tau} < 0$

$$\frac{d}{d\tau} = \frac{d}{(-1/\rho^2)\,d\rho} = -\rho^2 \frac{d}{d\rho}$$

$$\frac{d^2}{d\tau^2} = \frac{d}{d\tau}\left(\frac{d}{d\tau}\right) = -\rho^2 \frac{d}{d\rho}\left(-\rho^2 \frac{d}{d\rho}\right)$$

$$= -\rho^2\left(-2\rho \frac{d}{d\rho} - \rho^2 \frac{d^2}{d\rho^2}\right)$$

$$= 2\rho^3 \frac{d}{d\rho} + \rho^4 \frac{d^2}{d\rho^2}$$

$g_\tau < 0$

$\Rightarrow -\frac{1}{\rho^2} g_\rho < 0$

$\Rightarrow g_\rho > 0$

Know $\frac{d^2 g}{d\tau^2} < 0$

$\Leftrightarrow 2\rho^3 \frac{dp}{d\rho} + \rho^4 \frac{d^2 p}{d\rho^2} < 0$ why $\frac{d^2 p}{d\rho^2} > 0$ stronger condition?

Eq 59.03

$$v_1 v_0 = \frac{p_0 - p_1}{\rho_0 - \rho_1}$$

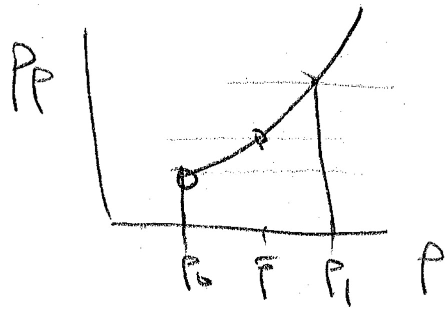
$v_i = u_i - U$

$$v_0 v_1 = \frac{p(\rho_1, S_1) - p(\rho_0, S_0)}{\rho_1 - \rho_0} > \frac{p(\rho_1, S_0) - p(\rho_0, S_0)}{\rho_1 - \rho_0}$$

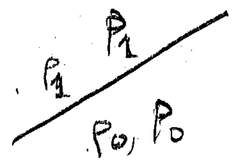
$= p_\rho(\bar\rho, s_0)$

$\bar\rho$ between ρ_0 & ρ_1

& Assuming $\frac{d^2 p}{d\rho^2} > 0 \Rightarrow$



$$p_\rho(\bar\rho, s_0) > p_\rho(\rho_0, s_0) = c^2(\rho_0, s_0) = c_0^2$$



$\rho_0 < \rho_1$ shocks are
$p_0 < p_1$ compressive.

$\Rightarrow v_0 v_1 > c_0^2$

By conservation of mass $\rho_0 v_0 = \rho_1 v_1$ & $\rho_0 < \rho_1$

Since $\rho > 0$ $v_0 > v_1$

$1 > \frac{\rho_0}{\rho_1} = \frac{v_1}{v_0} \Rightarrow$ ~~$v_0 > v_1$~~
Don't know sign of v.

$\Rightarrow 1 > \left|\frac{v_1}{v_0}\right| \Rightarrow |v_0| > |v_1|$

Then $v_0^2 > c_0^2$ from the above

$\Rightarrow |v_0| > c_0$

34.02

$I_+: \; x_\alpha - (u+c)t_\alpha = 0$

$II_+: \; u_\alpha + \frac{c}{\rho}\rho_\alpha = 0$

$I_-: \; x_\beta - (u-c)t_\beta = 0$

$II_-: \; u_\beta - \frac{c}{\rho}\rho_\beta = 0$

Now for a polytropic gas $\quad p = A\rho^\gamma \qquad p\tau = RT \Rightarrow p = \rho RT$

$$c^2 = \frac{\partial p}{\partial \rho} = A\gamma\rho^{\gamma-1} = \gamma\frac{A\rho^\gamma}{\rho}$$

$$= \gamma\frac{p}{\rho} = \gamma RT$$

Since $c^2 = A\gamma\rho^{\gamma-1}$

$$\ln c^2 = \ln(A\gamma) + (\gamma-1)\ln\rho$$

$$2\ln c = \ln(A\gamma) + (\gamma-1)\ln\rho$$

$$\frac{\partial}{\partial\square} \Rightarrow \frac{2c_\square}{c} = \frac{\gamma-1}{\rho}\rho_\square \Rightarrow \frac{c}{\rho}\rho_\square = \frac{2}{\gamma-1}c_\square$$

$\therefore$ Characteristic form of the equations become:

$I_+: \; x_\alpha - (u+c)t_\alpha = 0$

eq 83.01 $\quad II_+: \; u_\alpha + \frac{2}{\gamma-1}c_\alpha = 0$

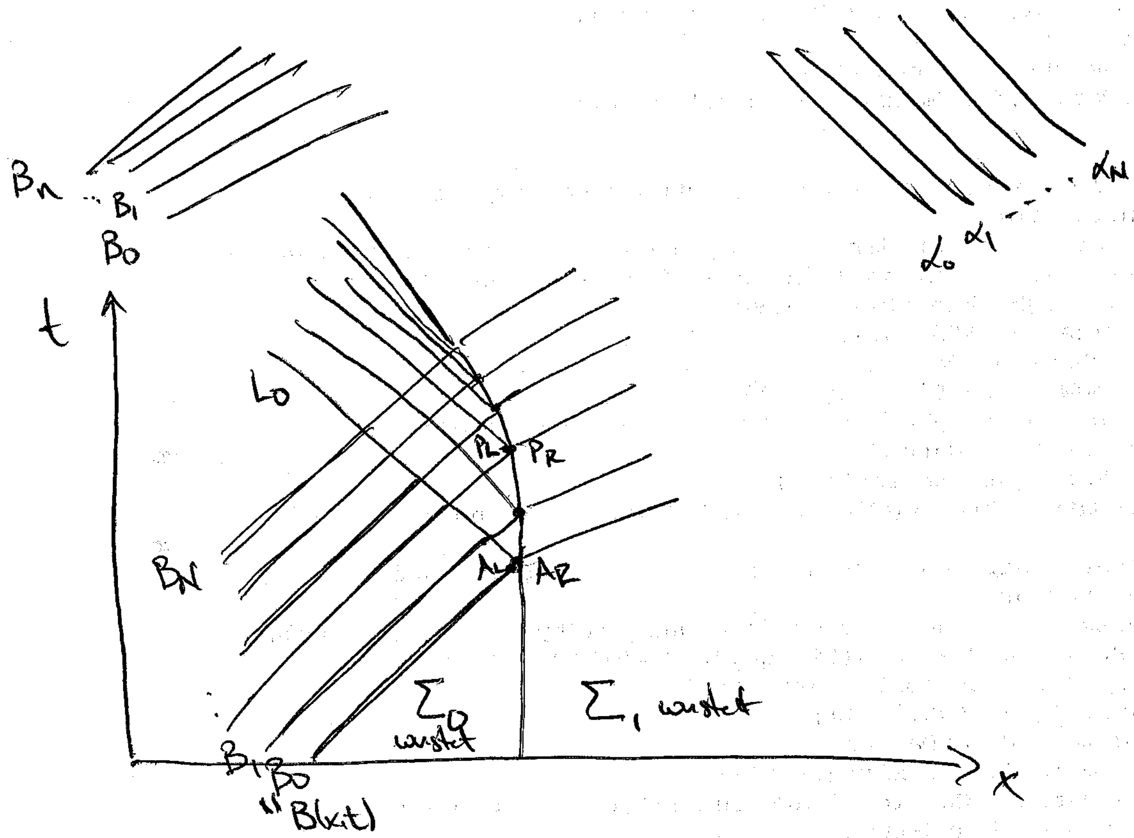
$\Rightarrow u + \frac{2}{\gamma-1}c = 2r(\beta)$ along $dx - (u+c)dt = 0$

$I_-: \; x_\beta - (u-c)t_\beta = 0$

$II_-: \; u_\beta - \frac{2}{\gamma-1}c_\beta = 0$

$\Rightarrow u - \frac{2}{\gamma-1}c = -2s(\alpha)$ along $dx - (u-c)dt = 0$

For a right facing wave



On each incoming characteristic since $B =$ const $\forall$ right going characteristic $U + \frac{2}{\gamma_0 - 1} c = 2r(B)$ is constant along each right going characteristic.

Along L_0 we have a left going characteristic so

$$U - \frac{2}{\gamma_0 - 1} c = -2s(\alpha) = \text{constant} = U_0 - \frac{2}{\gamma_0 - 1} c_0$$

Thus along L_0 Both U & c are known

I don't see how the (x,t) locations along L_0 are known

By these arguments do we know the function $B = B(x,t)$

If so how? Now since P_R is on the right is a simple wave going to the right

$$U(P_R) - \frac{2c(P_R)}{\gamma_1 - 1} = U_1 - \frac{2}{\gamma_1 - 1} c_1$$ holds

By the nature of the CD $p(P_L) = p(P_R)$ + $u(P_L) = u(P_R)$

Thus we ~~can~~ ~~express~~ get

$$u(P_L) - \frac{2c(P_R)}{\gamma_1 - 1} = \underset{\substack{\| \\ u(A_R) \\ \| \\ u(A_L)}}{U_1} - \underset{\substack{\| \\ \frac{2}{\gamma_1-1} c(A_R)}}{\frac{2}{\gamma_1 - 1} c_1}$$

$$\Rightarrow \quad u(P_L) - u(A_L) = \frac{2}{\gamma_1 - 1}\left[c(P_R) - c(A_R) \right]$$

since $c \sim p$ + p's are equal across the interface we can convert $c(P_R)$'s into $c(P_L)$'s.

$$c^2 = A\gamma \rho^{\gamma - 1} = \gamma \frac{p}{\rho}$$

$$= A\gamma \left(\frac{p}{A}\right)^{\frac{\gamma-1}{\gamma}}$$

$$c = \sqrt{A\gamma}\left(\frac{p}{A}\right)^{\frac{\gamma-1}{2\gamma}}$$

$$p = A\rho^\gamma$$

$$\rho = \left(\frac{p}{A}\right)^{1/\gamma}$$

$$\therefore \; c(P_R) = \sqrt{A_1 r_1}\left(\frac{P(P_R)}{A_1}\right)^{\frac{r_1-1}{2r_1}}$$

$$= \sqrt{A_1 r_1}\left(\frac{P(P_L)}{A_1}\right)^{\frac{r_1-1}{2r_1}}$$

$$\& \; C(A_R) = \sqrt{A_1 r_1}\left(\frac{P(A_L)}{A_1}\right)^{\frac{r_1-1}{2r_1}}$$

$$\therefore \; \frac{C(P_R)}{C(A_R)} = \left(\frac{P(P_L)}{P(A_L)}\right)^{\frac{r_1-1}{2r_1}} = \left[\left(\frac{C(P_L)}{C(A_L)}\right)^{\frac{2r_0}{r_0-1}}\right]^{\frac{r_1-1}{2r_1}}$$

$$\therefore \; \frac{C(P_R)}{c(A_R)} = \left(\frac{C(P_L)}{c(A_L)}\right)^{\frac{r_0(r_1-1)}{r_1(r_0-1)}}$$

Thus $$U(P_L) - U(A_L) = \frac{2C(A_R)}{r_1-1}\left[\frac{C(P_R)}{C(A_R)} - 1\right]$$

$$= \frac{2C(A_R)}{r_1-1}\left[\left(\frac{C(P_L)}{C(A_L)}\right)^{\frac{r_0(r_1-1)}{r_1(r_0-1)}} - 1\right] \quad \text{eq } 83.02$$

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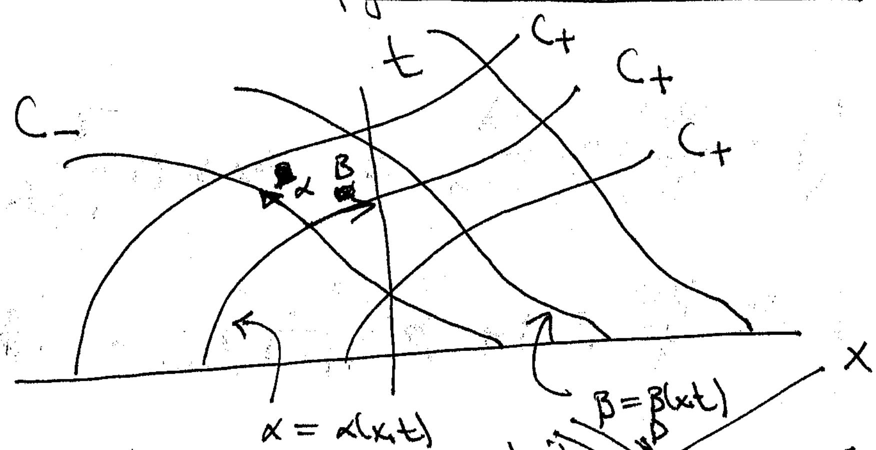
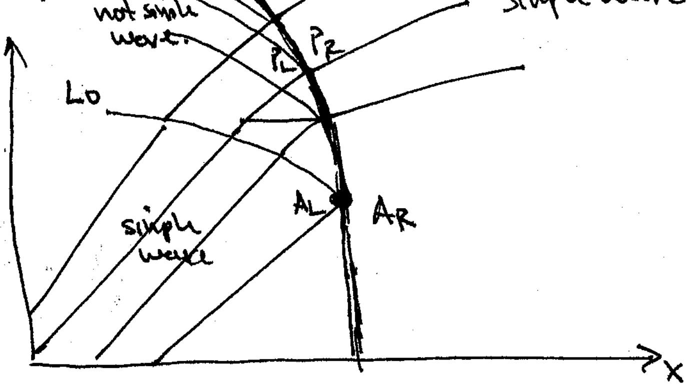


$$\frac{dx}{dt} = U + C > 0$$



P_R on D: tracing back the C_- characteristics to the x-axis gives

$$U - \frac{2}{\gamma_1 - 1} C = U_1 - \frac{2}{\gamma_1 - 1} C_1 \qquad C^2 = P'(\rho)$$

$$U(P_R) - \frac{2}{\gamma_1 - 1} C(P_R) = U_1 - \frac{2}{\gamma_1 - 1} C_1$$

$\|$ $\|$? No. Not true. But given that

$U(P_L)$ $C(P_L)$

$P_L = P_R$ crosses CD what can you determine about the equality of C's across the CD?

$$U(P_L) - \frac{2}{\gamma_1 - 1} C(P_R)$$

Attempt to determine values of P, ρ, U along Lo.

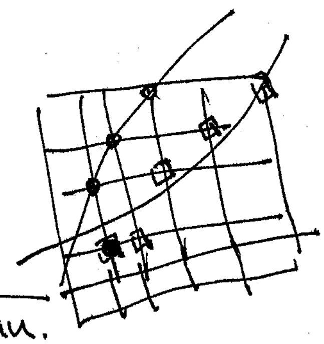
Questions: 1) How get values of p, ρ, u along Lo?

2) How get 83.02?

3) How get $\tilde{u} + \tilde{c}$ etc?

know U

4) How set up program. 2D grid but our boundary is curved?



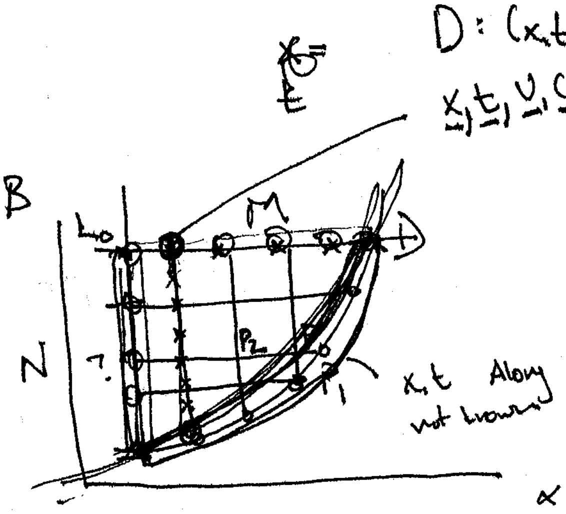
$$\frac{x - x_0}{t - t_0} = U + C$$ x, t in Fan.

D: $(x,t) \longrightarrow (\alpha, \beta)$

$\underline{x}, \underline{t}, \underline{U}, \underline{C}$

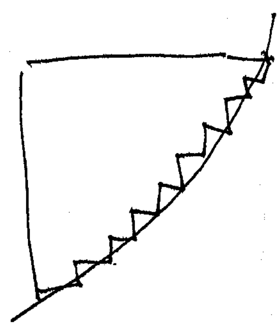
(α, β) -

Lo



$\Delta t_\alpha = ✓$

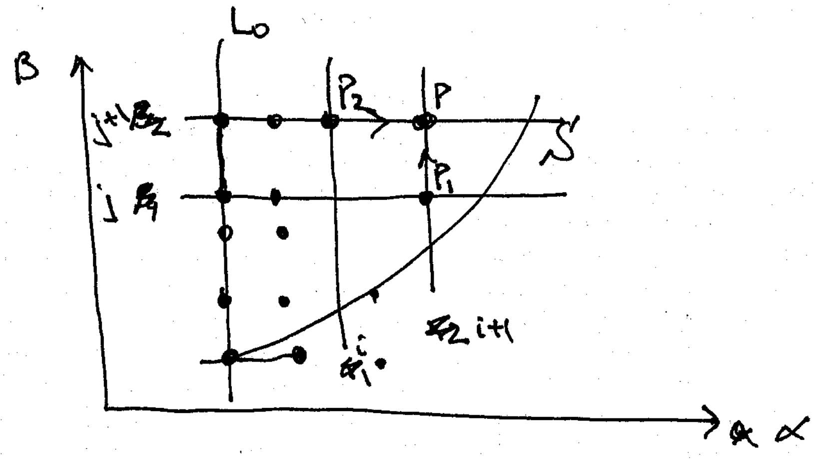
$\Delta x_\alpha = ✓$



$$u - \frac{2}{\gamma - 1} c = \text{const on } \alpha \text{ char.}$$

$$u + \frac{2}{\gamma - 1} c = \text{const on } \beta \text{ char.}$$

$$\Delta_\alpha x = \tilde{u} +$$



P_2 + P

1st $\quad X_{\alpha_1} - \overset{?}{X_{\alpha_2}} = (\hat{U}+\tilde{C})(t_{\alpha_1} - \overset{?}{t_{\alpha_2}})$ $\qquad x_{\alpha_2}, t_{\alpha_2} = ?$

3rd. $\quad U_{\alpha_1} - U_{\alpha_2} + \frac{2}{\gamma-1}(C_{\alpha_1} - C_{\alpha_2}) = \frac{\tilde{C}}{\gamma(\gamma-1)}(\eta_{\alpha_1} - \eta_{\alpha_2})$

~~B_1~~ P_1 + P. $\qquad$ How

$X_{\beta_2} - X_{\beta_1}$

<u>P_2 + P</u> :

$$X_{i,j+1} - X_{i+1,j+1} = (\hat{U}+\tilde{C})(t_{i,j+1} - t_{i+1,j+1})$$

$$U_{i,j+1} - U_{i+1,j+1} \cancel{+} \frac{2}{\gamma-1}(C_{i,j+1} - C_{i,j} \quad) = \frac{\tilde{C}}{\gamma(\gamma-1)}(\eta_{i,j+1} - \eta_{i+1,j+1})$$

$X_{i+1,j}$

5 eqs.

17.01 $\rho_t + U\rho_x + \rho U_x = 0$

17.02 $\rho(U_t + UU_x) + p_x = 0$

17.03 $S_t + US_x = 0$

~~[illegible]~~

$p = f(\rho, S)$

$$\boxed{c^2 = \frac{\partial f}{\partial \rho}\Big|_S}$$

~~Strictly~~ ~~characteristics~~ let variables be $\vec{V} = \begin{pmatrix} \rho \\ U \\ S \end{pmatrix}$

Then eqs 17.0[1-3] become

$$\begin{pmatrix} \rho \\ U \\ S \end{pmatrix}_t + \begin{pmatrix} U\rho_x + \rho U_x \\ UU_x + \frac{1}{\rho} p_x \\ US_x \end{pmatrix} = \vec{0}$$

$$p_x = p_\rho \rho_x = c^2 \rho_x$$

$$\Rightarrow \begin{pmatrix} \rho \\ U \\ S \end{pmatrix}_t + \begin{pmatrix} U\rho_x + \rho U_x \\ UU_x + \frac{c^2}{\rho} \rho_x \\ US_x \end{pmatrix} = \vec{0}$$

$$\Rightarrow \begin{pmatrix} \rho \\ U \\ S \end{pmatrix}_t + \begin{pmatrix} U & \rho & 0 \\ \frac{1}{\rho}c^2 & U & 0 \\ 0 & 0 & U \end{pmatrix} \begin{pmatrix} \rho \\ U \\ S \end{pmatrix}_x = \vec{0} \quad *$$

Define ~~[illegible]~~ $A\left(\begin{pmatrix} \rho \\ U \\ S \end{pmatrix}\right) = \begin{pmatrix} U & \rho & 0 \\ \frac{1}{\rho}c^2 & U & 0 \\ 0 & 0 & U \end{pmatrix}$

Find the left eigenvectors of $A(\bar{U})$

$\ell^T A = \lambda \ell^T$ take transpose

$A^T \ell = \lambda \ell$ ℓ is an ~~eigenvalu~~ eigenvector of A^T

$$A^T = \begin{pmatrix} U & \frac{1}{\rho}c^2 & 0 \\ \rho & U & 0 \\ 0 & 0 & U \end{pmatrix}$$ The eigenvalues/vectors of A^T are

$$\begin{vmatrix} U-\lambda & \frac{1}{\rho}c^2 & 0 \\ \rho & U-\lambda & 0 \\ 0 & 0 & U-\lambda \end{vmatrix} = 0$$

$$(U-\lambda)\left((U-\lambda)^2 - \cancel{\rho}c^2\right) = 0$$

$\lambda = U$ $(\lambda-U)^2 = \cancel{\rho}c^2$

$\lambda = U \pm c$

The left eigenvectors are: let $\lambda_1 < \lambda_2 < \lambda_3$

$$\begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda_3 \\ \| & \| & \| \\ U-C & U & U+C \end{array}$$

$$\begin{pmatrix} c & \frac{1}{\rho}c^2 & 0 \\ \rho & c & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} \ell_{11} \\ \ell_{21} \\ \ell_{31} \end{pmatrix} = \vec{0}$$

$$\Rightarrow \quad \ell_{31} = 0$$

$$\begin{pmatrix} 1 & \frac{1}{p}c & 0 \\ 1 & \frac{c}{p} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} \ell_{11} \\ \ell_{21} \\ \ell_{31} \end{pmatrix} = \vec{0} \qquad \ell_{11} = -\frac{c}{p}\ell_{21}$$

$$\vec{\ell}_1 = \begin{pmatrix} -\frac{c}{p} \\ 1 \\ 0 \end{pmatrix} \propto \begin{pmatrix} -c \\ p \\ 0 \end{pmatrix}$$

$$\vec{\ell}_3 = \begin{pmatrix} +\frac{c}{p} \\ 1 \\ 0 \end{pmatrix} \propto \begin{pmatrix} +c \\ p \\ 0 \end{pmatrix}$$

Check $$\begin{pmatrix} U & c^2/p & 0 \\ p & U & 0 \\ 0 & 0 & U \end{pmatrix}\begin{pmatrix} c/p \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} cU/p + c^2/p \\ c + U \\ 0 \end{pmatrix} = \begin{pmatrix} c/p\,(U+c) \\ (U+c) \\ 0 \end{pmatrix} \checkmark$$

$$\begin{pmatrix} 0 & \frac{1}{p}c^2 & 0 \\ p & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} \ell_{12} \\ \ell_{22} \\ \ell_{32} \end{pmatrix} = \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} \Rightarrow \quad \ell_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\therefore \text{ let } L = \begin{pmatrix} -c & 0 & +c \\ p & 0 & p \\ 0 & 1 & 0 \end{pmatrix}$$

Check Now multiply eq * by L^T

$$\begin{pmatrix} -c & \rho & 0 \\ 0 & 0 & 1 \\ +c & \rho & 0 \end{pmatrix}\begin{pmatrix} p_t \\ u_t \\ S_t \end{pmatrix} + \begin{pmatrix} -c & \rho & 0 \\ 0 & 0 & 1 \\ c & \rho & 0 \end{pmatrix}\begin{pmatrix} u & \rho & 0 \\ c^2/\rho & u & 0 \\ 0 & 0 & u \end{pmatrix}\begin{pmatrix} p_x \\ u_x \\ S_x \end{pmatrix} = \vec{0}$$

$$\begin{pmatrix} -cp_t + \rho u_t \\ S_t \\ cp_t + \rho u_t \end{pmatrix} + \begin{pmatrix} -cu + c^2 & -c\rho + \rho u & 0 \\ 0 & 0 & u \\ cu + c^2 & c\rho + u\rho & 0 \end{pmatrix}\begin{pmatrix} p_x \\ u_x \\ S_x \end{pmatrix} = \vec{0}$$

$$\Rightarrow \quad -c\underline{p_t} + \rho \underline{u_t} + c(\underline{-u+c})p_x + \rho(\underline{-c+u})u_x = 0$$

$$S_t + uS_x = 0$$

$$cp_t + \rho u_t + c(u+c)p_x + \rho(c+u)u_x = 0$$

$$\Rightarrow \quad -c(p_t + (u-c)p_x) + \rho(u_t + (u-c)u_x) = 0$$

$$S_t + uS_x = 0$$

$$+c(p_t + (u+c)p_x) + \rho(u_t + (u+c)u_x) = 0$$

Now $p_\square = c^2 \rho_\square \Rightarrow \rho_\square = \frac{1}{c^2} p_\square$ thus the eqs become

$$-\frac{c}{c^2}(P_t + (u-c)P_x) + \rho(u_t + (u-c)u_x) = 0$$

$$S_t + uS_x = 0$$

$$+\frac{c}{c^2}(P_t + (u+c)P_x) + \rho(u_t + (u+c)u_x) = 0$$

$\Rightarrow$ $p_t + (u-c)p_x - c\rho\left(u_t + (u-c)u_x\right) = 0$

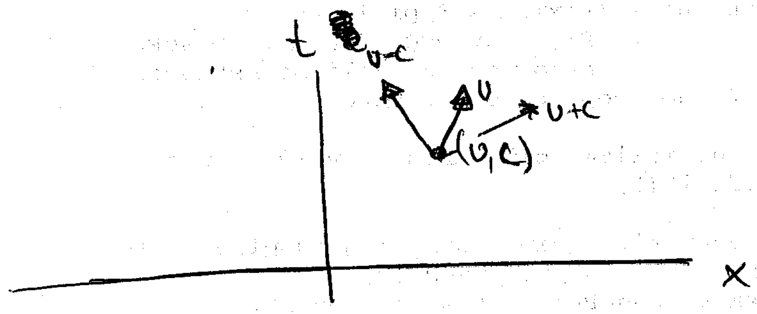
$$S_t + uS_x = 0$$

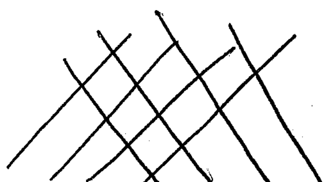
$$p_t + (u+c)p_x + c\rho\left(u_t + (u+c)u_x\right) = 0$$

$\Rightarrow$ $\dfrac{dp}{dt} - c\rho\dfrac{du}{dt} = 0$ on C_-: $\dfrac{dx}{dt} = u-c$

$\dfrac{dp}{dt} + c\rho\dfrac{du}{dt} = 0$ on C_+: $\dfrac{dx}{dt} = u+c$

$\dfrac{dS}{dt} = 0$ on P: $\dfrac{dx}{dt} = u$





β_0 β_1 $\cdots$ β_N; α_1 α_2 $\cdots$ α_N

$\alpha_0 = \alpha(x,t)$ $\quad\alpha_0$ a constant

$\Rightarrow$ $0 = \alpha_x\,dx + \alpha_t\,dt$

Now $0 = \dfrac{dx}{dt} + \dfrac{\alpha_t}{\alpha_x} \Rightarrow \left.\dfrac{dx}{dt}\right|_{\alpha=\alpha_0} = -\dfrac{\alpha_t}{\alpha_x} = u-c$

Sim. $\beta_0 = \beta(x,t)$ $\quad \left.\dfrac{dx}{dt}\right|_{\beta=\beta_0} = -\dfrac{\beta_t}{\beta_x} = u+c$

Since $\left.\frac{dx}{dt}\right|_{\alpha} = -\frac{\alpha_t}{\alpha_x} = \cancel{t_x} - \frac{1/t_\alpha}{1/x_\alpha} = u-c$

$\left.\frac{dx}{dt}\right|_{\beta} = -\frac{\beta_t}{\beta_x} = \frac{-1/t_\beta}{1/x_\beta} = u+c$

1st eq gives:

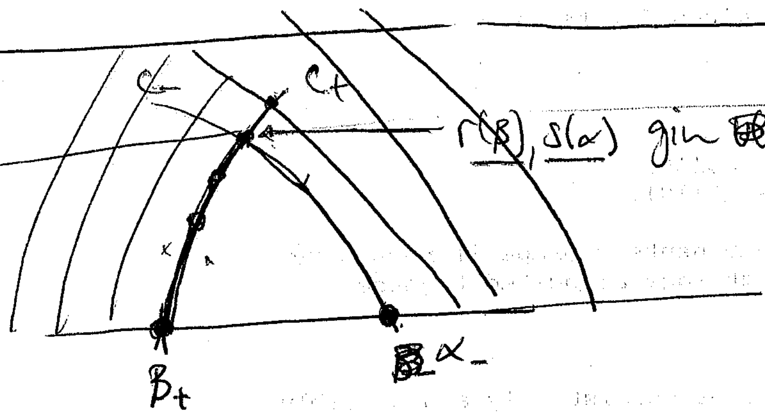
$-x_\alpha = (u-c)t_\alpha$

$= x_\alpha + (u-c)t_\alpha = 0$

2nd eq gives

$-x_\beta = (u+c)t_\beta$

$x_\beta + (u+c)t_\beta = 0$



$x_\alpha = (u+c)t_\alpha$
$x_\beta = (u-c)t_\beta$

let $x = x(\alpha, \beta)$

$x_{ij} = x(\Delta\alpha i, \Delta\beta j)$

$\frac{x_{i+1j} - x_{ij}}{\Delta\alpha} = (u+c)(t$

D.E's 83.04

$$x_\alpha = (u+c)t_\alpha$$
$$x_\beta = (u-c)t_\beta$$

$$u_\alpha + \frac{2}{r-1}c_\alpha = \frac{c\,\eta_\alpha}{r(r-1)}$$

{ $p = A\rho^r$ I think }

$$u_\beta - \frac{2}{r-1}c_\beta = -\frac{c\,\eta_\beta}{r(r-1)}$$

$$\eta_\alpha t_\beta + \eta_\beta t_\alpha = 0$$

5 eqs & 5 unknowns
x, t, u, c, η
r a constant.

w/ $\eta = (r-1)c_v S$ I think

Then discretizing in α, β space gives $x = x(\alpha,\beta)$, $t = t(\alpha,\beta)$

(1) $\Delta_\alpha x = (\tilde{u}+\tilde{c})\,\Delta t_\alpha$ along β = const

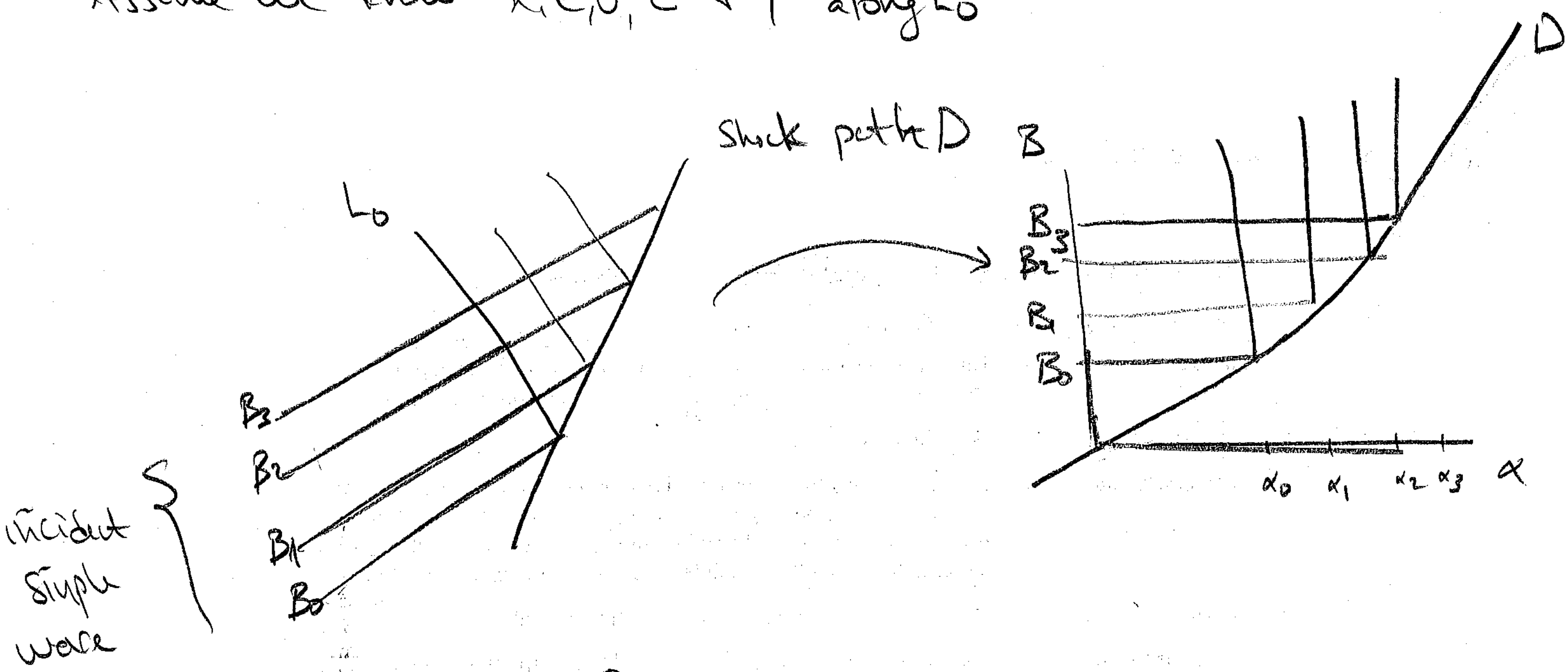
(2) $\Delta_\beta x = (\tilde{u}-\tilde{c})\,\Delta_\beta t$ along α = const $\eta = \eta(\alpha, \beta)$

(3) $\Delta_\alpha\left(u + \frac{2}{r-1}c\right) = \frac{\tilde{c}}{r(r-1)}\Delta_\alpha \eta$ along β = const

(4) $\Delta_\beta\left(u - \frac{2}{r-1}c\right) = -\frac{\tilde{c}\,\Delta_\beta \eta}{r(r-1)}$ along α = const

(5) $\Delta_\alpha \eta\, \Delta_\beta t + \Delta_\beta \eta\, \Delta_\alpha t = 0$

Assume we know x, t, u, c & η along L_0



Right going indexed by β $\quad r = r(\beta)$

left going indexed by α $\quad s = s(\alpha)$

Then eqs (1) – (5) written in terms of $\underset{\alpha = \text{index}}{\overset{\shortparallel}{i}}$ & $\underset{\beta \text{ index}}{\overset{\shortparallel}{j}}$ is

Say forward differencing

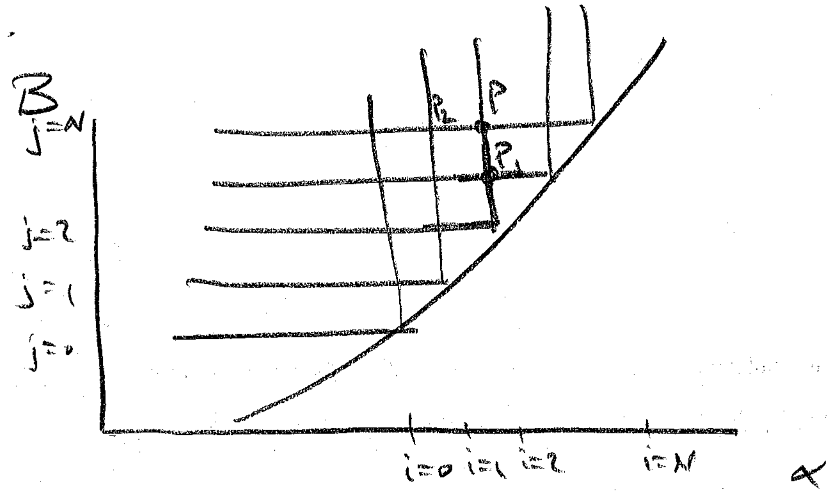
(1) $$x_{i+1,j} - x_{i,j} = (\tilde{u} + \tilde{c})(t_{i+1,j} - t_{i,j})$$

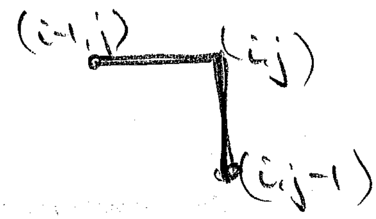
(2) $$x_{i,j+1} - x_{i,j} = (\tilde{u} - \tilde{c})(t_{i,j+1} - t_{i,j})$$

(3) $$u_{i+1,j} + \frac{2}{\gamma-1} c_{i+1,j} - u_{i,j} - \frac{2}{\gamma-1} c_{i,j} = \frac{\tilde{c}}{\gamma(\gamma-1)}(\eta_{i+1,j} - \eta_{i,j})$$

(4) $$u_{i,j+1} - \frac{2}{\gamma-1} c_{i,j+1} - u_{i,j} + \frac{2}{\gamma-1} c_{i,j} = \frac{\tilde{c}}{\gamma(\gamma-1)}(\eta_{i,j+1} - \eta_{i,j})$$

(5) $$(\eta_{i+1,j} - \eta_{i,j})(t_{i,j+1} - t_{i,j}) + (\eta_{i,j+1} - \eta_{i,j})(t_{i+1,j} - t_{i,j}) = 0$$





Determine state of P w/ states of P_1 & P_2 known

Use 1st eq between P_2 + P

3rd eq between P_2 + P

2nd eq between P_1 + P

4th eq between P_1 + P

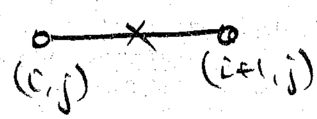
& 5th eq between P_1, P_2, & P

Update (i,j) knowing $(i,j-1)$ & $(i-1,j)$

For B constant evaluate

$$\tilde{c} = \frac{1}{2}(c_{i+1,j} + c_{i,j})$$

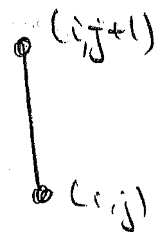
$$\tilde{v} = \frac{1}{2}(v_{i+1,j} + v_{i,j})$$



For α constant evaluate

$$\tilde{c} = \frac{1}{2}(c_{i,j} + c_{i,j+1})$$

$$\tilde{v} = \frac{1}{2}(v_{i,j} + v_{i,j+1})$$



Then 1st eq w/ $i \to i-1$ gives

$$\overset{?}{x_{i,j}} - x_{i-1,j} = \frac{1}{2}\left(\overset{?}{u_{i,j}} + u_{i-1,j} + \overset{?}{c_{i,j}} + c_{i-1,j}\right)\left(\overset{?}{t_{i,j}} - t_{i-1,j}\right)$$

3rd eq w/ $i \to i-1$ gives

$$\overset{?}{u_{i,j}} + \frac{2}{\gamma-1}\overset{?}{c_{i,j}} - u_{i-1,j} - \frac{2}{\gamma-1}c_{i-1,j} = \frac{1}{2\gamma(\gamma-1)}\left(\overset{?}{c_{i,j}} + c_{i-1,j}\right)\left(\overset{?}{\eta_{i,j}} - \eta_{i-1,j}\right)$$

2nd eq w/ $j \to j-1$ gives

$$\overset{?}{x_{i,j}} - x_{i,j-1} = \frac{1}{2}\left(u_{i,j-1} + \overset{?}{u_{i,j}} - c_{i,j-1} - \overset{?}{c_{i,j}}\right)\left(\overset{?}{t_{i,j}} - t_{i,j-1}\right)$$

4th eq w/ $j \to j-1$ gives.

$$\overset{?}{u_{i,j}} - \frac{2}{\gamma-1}\overset{?}{c_{i,j}} - u_{i,j-1} + \frac{2}{\gamma-1}c_{i,j-1} = \frac{1}{2\gamma(\gamma-1)}\left(\overset{?}{c_{i,j}} + c_{i,j-1}\right)\left(\overset{?}{\eta_{i,j}} - \eta_{i,j-1}\right)$$

5th eq written between points $(i-1,j)$ & (i,j) & (i,j) & $(i,j-1)$

$$\Rightarrow \left(\overset{?}{\eta_{i,j}} - \eta_{i-1,j}\right)\left(\overset{?}{t_{i,j}} - t_{i,j-1}\right) + \left(\overset{?}{\eta_{i,j}} - \eta_{i,j-1}\right)\left(\overset{?}{t_{i,j}} - t_{i-1,j}\right) = 0$$

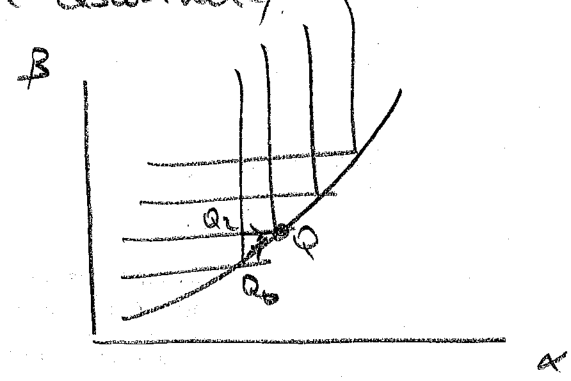
This system of difference eqs would then have to be solved.

Note that it is non linear & presents some difficulties. CNo - employ an iteration method. 1st Assume $\Delta\eta = 0$ At the point in question & then

03-31-01

If Q is on the ~~set~~ path of the discontinuity,

Then use



pg 208 Courant / Friedrichs

(86.03) $\Rightarrow$ $$E^{(1)}(\tau_1, p_1) = E^{(0)}(\tau_0, p_0)$$

But eq 85.04 $E^{(0)}(\tau_0,p_0) > E^{(1)}(\tau_0,p_0)$

$\therefore$ $E^{(1)}(\tau_1,p_1) > E^{(1)}(\tau_0,p_0)$ As $\tau_1 = \tau_0$

$\Rightarrow$ As $E_p > 0$

$\Rightarrow$ $p_1 > p_0$ process is a deflagration,

84.04 $$\frac{p_1 - p_0}{\tau_1 - \tau_0} = -\rho^2 v^2 = -\rho_0^2 v_0^2$$

$\Rightarrow$ $|v_0|$ + $|v|$ must be ∞.

? why not $|v| = \infty$.

U

u $\quad$ u_0

$v = u - U$ $\quad$ $u_0 - U = v_0$

If $p_1 = p_0$

$$I^{(1)}(\tau_1, p_1) = I^{(0)}(\tau_0, p_0) > I^{(1)}(\tau_0, p_0)$$

As $p_1 = p_0$ $\Rightarrow$ As $I_\tau > 0$ $\quad$ $\tau_1 > \tau_0$

$\frac{1}{\rho_1} > \frac{1}{\rho_0} \Rightarrow \rho_0 > \rho_1$

for ~~process~~ density decreases

process is a deflagration

84.04 $$\frac{p_1 - p_0}{\tau_1 - \tau_0} = -\rho^2 V^2 = -\rho_0^2 V_0^2$$

$$\| $$

$$0 \Rightarrow V_0 = 0.$$

$$H^{(1)}(\tau, p) = E^{(1)}(\tau, p) - E^{(1)}(\tau_0, p_0) + \tfrac{1}{2}(\tau - \tau_0)(p + p_0)$$

Then

$$H^{(1)}(\tau_1, p_1) = E^{(0)}(\tau_0, p_0) - E^{(1)}(\tau_0, p_0)$$

$$\|$$

$$E^{(1)}(\tau_1, p_1) - \cancel{E^{(1)}(\tau_0, p_0)} + \tfrac{1}{2}(\tau_1 - \tau_0)(p_1 + p_0) = \underbrace{\cancel{E^{(0)}(\tau_0, p_0)}}_{\text{zero}} - \cancel{E^{(1)}(\tau_0, p_0)}$$

$$\Rightarrow \quad \frac{p_1 - p_0}{\tau_1 - \tau_0} < 0$$

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Hugoniot fun.

$$H^{(1)}(\tau, p) = E^{(1)}(\tau, p) - E^{(1)}(\tau_0, p_0) + \frac{1}{2}(\tau - \tau_0)(p + p_0)$$

pg 211 C/F   213 - 246

$$\frac{dp}{d\tau} = \frac{p - p_0}{\tau - \tau_0}$$

eq (86.05)

$$dH^{(1)}(\tau, p) = dE^{(1)}(\tau, p) + \frac{1}{2}(d\tau)(p + p_0) + \frac{1}{2}\underline{(\tau - \tau_0)dp}$$

$$= -p\,d\tau + TdS + \frac{1}{2}d\tau(p + p_0) + \text{''}$$

$$= TdS - \frac{p}{2}d\tau + \frac{d\tau}{2}p_0 + \cdots$$

$$= TdS + \frac{1}{2}\left\{(\tau - \tau_0)dp - (p - p_0)d\tau\right\}$$

But $\dfrac{dp}{d\tau} = \dfrac{p - p_0}{\tau - \tau_0}$

$(\tau - \tau_0)dp = (p - p_0)d\tau$ At D & C.

$\therefore \quad dH^{(1)}(\tau, p) = TdS$ How know $dH^{(1)}(\tau, p) = 0$

If $\quad dH^{(1)}(\tau, p) = 0$

$\Rightarrow \quad TdS = 0 \quad$ At D & C

/

For Chapman - Jouguet process we get constant entropy. pg 212 C/F

$$\rho^2 c^2 = -\left.\frac{dp}{d\tau}\right|_s \qquad \frac{dp}{d\tau} = \frac{p-p_0}{\tau-\tau_0} = -\rho_1^2 v_1^2 = -\rho_0^2 v_0^2.$$

At D or C

$$\Rightarrow -\cancel{\rho}^2 c^2 = -\underbrace{\cancel{\rho_1}^2 v_1^2} = \cancel{-\rho_0^2 v_0^2}$$

$$\Rightarrow \quad c^2 = v_1^2 \Rightarrow \quad c = |v| \quad o|$$

---

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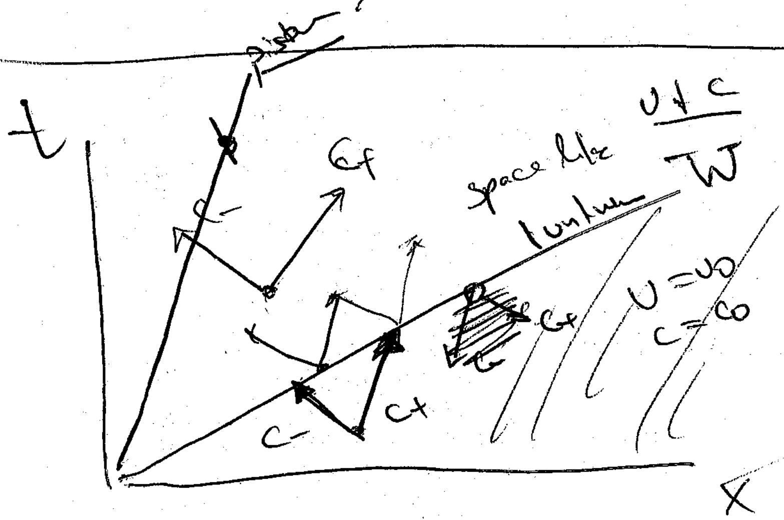
87.02 $\Rightarrow$ $dH^{(1)}($ — time like for U or $= \psi$.



flow supersonic w.r.t. gas in front.

$$\rho_0 V_0 = \rho_1 V_1$$

$$p_0 + \rho_0 V_0^2 = p_1 + \rho_1 V_1^2$$

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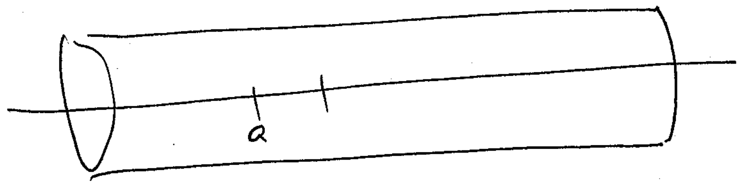
Shock waves occur in both expansive + compressive media

in contrast to



$x = \underset{\sim}{x}(a,t)$

$x_a = \frac{\partial x}{\partial a}$ $\epsilon = x_a - 1$ strain if $\Delta x = \Delta a \Rightarrow$ No change in material length

$\rho_0 da = \rho dx \iff$ conservation of mass

$\frac{\rho_0}{\rho} = 1 + \epsilon$

$\epsilon = x_a - 1 > 0 \Rightarrow$ expansion

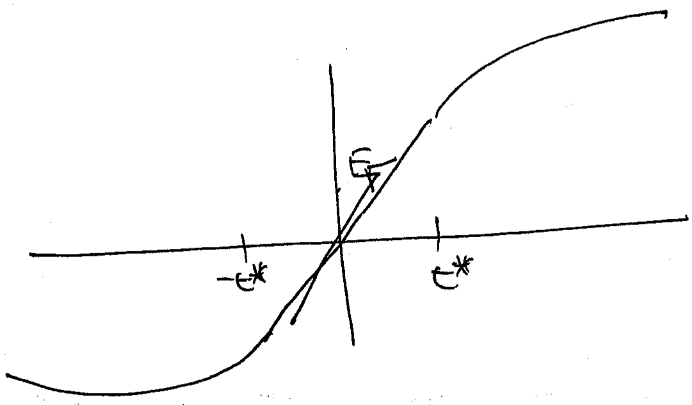
$\epsilon = x_a - 1 < 0 \Rightarrow$ ~~contraction~~ contraction.

σ = engineering stress. $\sigma = \sigma(\epsilon)$

$\sigma > 0$ for $\epsilon > 0$
expansion

$\frac{d\sigma}{d\epsilon} > 0$ elastic $\Rightarrow$ $\sigma \sim$ lin as ϵ.

$\sigma = E\epsilon$ $|\epsilon| < \epsilon^*$. E = Youngs modulus



$$p = -\sigma \qquad \rho = \frac{\rho_0}{1+\varepsilon}$$

$$\sigma = -\frac{p_0}{(1+\varepsilon)^{r}}$$

$$\rho u_t = \frac{\sigma_a}{x_a} \Leftarrow ?$$

$$\frac{\sigma_a}{1+\varepsilon}$$