pg 32 Courant / John

$$f_x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f_{xx} = \frac{1}{\sqrt{x^2+y^2}} - \frac{1}{\cancel{2}}\frac{x(\cancel{2}x)}{(x^2+y^2)^{3/2}} = \frac{x^2+y^2-x^2}{\left(\sqrt{x^2+y^2}\right)^3} = \frac{y^2}{(x^2+y^2)^{3/2}}$$

$$= \frac{r^2\sin^2\phi}{r^3}$$

$$= \frac{\sin^2\phi}{r}$$

By symm. $f_{yy} = \dfrac{x^2}{(x^2+y^2)^{3/2}} = \dfrac{\cos^2\phi}{r}$

$$f_{xy} = \frac{x(-1/2)\,2y}{(x^2+y^2)^{3/2}} = \frac{-xy}{(x^2+y^2)^{3/2}}$$

$$= -\frac{r^2\sin\phi\cos\phi}{r^3}$$

$$f_x = \frac{-2x}{2(x^2+y^2+z^2)^{3/2}} = \frac{-x}{r^3}$$

$$r = \sqrt{x^2+y^2+z^2}$$

$$\frac{\partial r}{\partial x_i} = \frac{2x_i}{2\sqrt{x^2+y^2+z^2}}$$

$$= \frac{x_i}{\sqrt{\cdots}}$$

$$f_{xx} = \frac{-1}{r^3} + \frac{3x}{r^4}\cdot\frac{\partial r}{\partial x} = \frac{-1}{r^3} + \frac{3x}{r^4}\cdot\frac{x}{r}$$

$$f_{xy} = -x(-3)\frac{1}{r^4}\frac{\partial r}{\partial y} = \frac{3x}{r^4}\frac{y}{r} = \qquad \frac{3xy}{r^5}$$

$f_{yz} =$

Pg 34 Carrut, John

$$f_x = \frac{1}{\sqrt{y}} e^{-(x-a)^2/4y} \cdot \left(\frac{-2(x-a)}{4y} \right) = -\frac{(x-a)}{2y^{3/2}} e^{-(x-a)^2/4y}$$

$$f_y =$$

Pg 38 Conrad / John

$$f_{xy}(0,0) = \lim_{k\to 0} \frac{f_x(0,k) - f_x(0,0)}{k} = \lim_{k\to 0} \frac{-k}{k} = -1$$

$\neq$

$$f_{yx}(0,0) = \lim_{h\to 0} \frac{f_y(h,0) - f_y(0,0)}{h} = \lim_{h\to 0} \frac{h}{h} = +1$$

$$f_{xy}(x,y) = ?$$

$$f_x(x,y) =$$

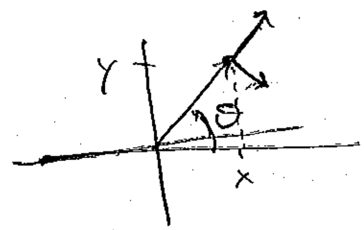
$$\epsilon_1 h + \epsilon_2 k \overset{?}{=} o\left(\sqrt{h^2+k^2}\right)$$

$$\frac{\epsilon_1 h + \epsilon_2 k}{\sqrt{h^2+k^2}} = \frac{\epsilon_1/k + \epsilon_2/h}{\sqrt{}} \cdot hk$$

$$= \frac{\epsilon_1/k + \epsilon_2/h}{\sqrt{1/k^2 + \frac{1}{h^2}}} \xrightarrow{h,k \to 0} 0 \quad \text{How slow?}$$

pg 44 Current / John

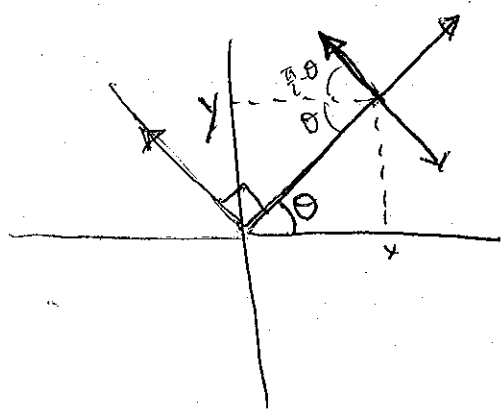
B $f(x,y) = x$



$D_{(\alpha)} f(x,y) = \cos\alpha$

$D_{(\theta+\frac{\pi}{2})} f(x,y) = \cos(\theta + \frac{\pi}{2})$

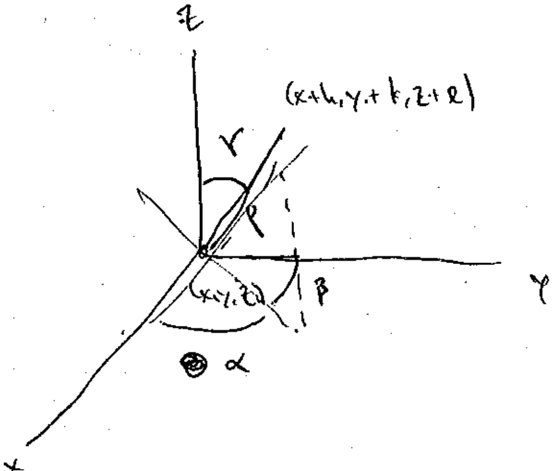
$= -\sin\theta$



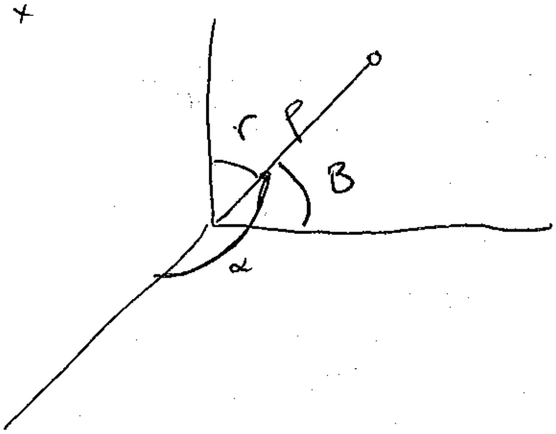
$D_{(\theta+\frac{\pi}{2})} y = \sin\alpha$

$\alpha = \theta + \frac{\pi}{2}$

$= \cos\theta$



$l = \rho\cos\gamma$



$$v = \frac{xy}{\sqrt{x^2+y^2}} = \frac{r^2 \sin\theta \cos\theta}{r} = r \sin\theta\cos\theta = \frac{r}{2}\sin 2\theta$$

$$v_x = y\left(\frac{1}{\sqrt{x^2+y^2}} - \frac{x^2}{\sqrt{(x^2+y^2)^3}}\right) = \frac{x^2+y^2-x^2}{(x^2+y^2)^{3/2}} = \frac{y^2}{(x^2+y^2)^{3/2}}$$

Pg 50

$$\frac{\partial^2 f}{\partial x^2} h + \frac{\partial^2 f}{\partial x \partial y} kh + \frac{\partial^2 f}{\partial y \partial x} hk + \frac{\partial^2 f}{\partial y^2} k^2$$

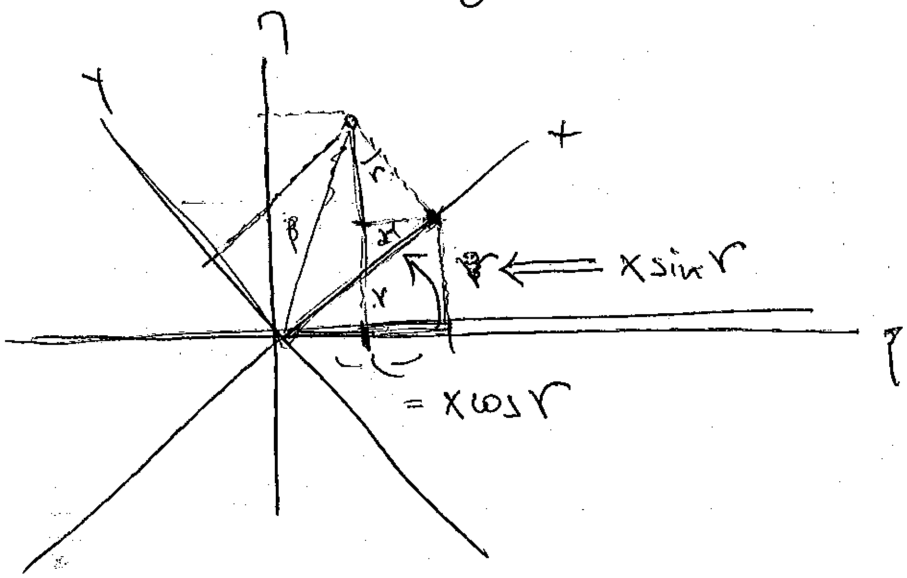
Pg 52

$$\frac{\partial s}{\partial m} = \frac{1}{(m-\bar{m})} + \frac{(-1)m}{(m-\bar{m})^2} = \frac{\cancel{m} - \bar{m} - \cancel{m}}{(m-\bar{m})^2} = \frac{-\bar{m}}{(m-\bar{m})^2}$$

$$\frac{\partial s}{\partial \bar{m}} = \frac{+m}{(m-\bar{m})^2}$$

$$ds = \frac{-\bar{m}}{(m-\bar{m})^2} dm + \frac{m}{}$$

Pg 60 Conrad / John



given (ξ, η) get (x, y)

$$x \sin \gamma + y \overset{\cos}{\cancel{\sin}} \gamma = \eta \quad \Big| \quad \beta =$$

$$\xi = x \cos \gamma - y \sin \gamma$$

$$\xi = \cos \gamma \, x - \sin \gamma \, y$$

$$\eta = \sin \gamma \, x + \cos \gamma \, y$$

~~Books γ is pos & from~~ x (Books γ is negative my γ)

pg 67 Courant (John

Bonoullis ineq:

$$(1+x)^n \geq 1+nx$$

$$\frac{|h|+|k|}{2} \overset{?}{\leq} \sqrt{h^2+k^2} = h(1+(k/h)^2)^{1/2} =$$

$$\frac{1}{4}(h^2+2hk+k^2) \overset{?}{\leq} h^2+k^2$$

$$h^2+2hk+k^2 \leq 4h^2+4k^2$$

$$-3h^2+2hk-3k^2 \overset{?}{<} 0$$

$$-(3h \quad k)(h \quad 3k)$$

$$-(3h^2-2hk+3k^2)$$

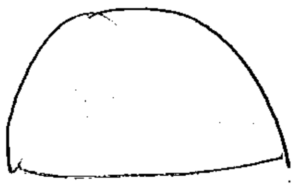
$$= -$$

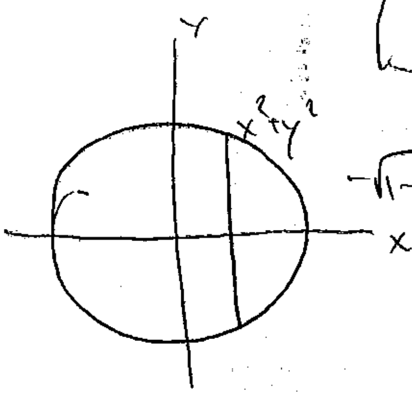
pg 72 Courant (John

$$\int_0^1 \frac{x\,dy}{\sqrt{1-x^2y^2}} = \int_0^x \frac{du}{\sqrt{1-u^2}}$$

$u = xy$

$du = x\,dy$





$-\sqrt{1-x^2} \le y \le \sqrt{1-x^2}$

Pg 79 Courant / John

$$F'(k) = \int_{-1}^{1} \frac{(-\frac{1}{2})\, dx\, (-k 2x^2)}{((1-x^2)(1-k^2x^2))^{3/2}} = \int_{-1}^{1} \frac{k x^2\, dx}{((\ \ )(\ \ ))^{3/2}}$$

$$\frac{F(k+h) - F(k)}{h} = \int_{-1}^{1} f_k(k+\theta h, x)\, dx$$

$$= \int_{-1}^{1} \frac{(k+\theta h) x^2\, dx}{\sqrt{[(1-x^2)(1-(k+\theta h)^2 x^2)]^3}}$$

consider $\dfrac{F(k+h) - F(k)}{h} - \displaystyle\int_{-1}^{1} \frac{k x^2}{\sqrt{(1-x^2)(1-k^2x^2)^3}}$

$\dfrac{k}{(1-k^2x^2)^{3/2}}$ is not uniformly cont at $x = \pm\frac{1}{k}$ $(k>0)$

$0 < k < 1$

Pg 81 Carcut / John

$$\int_0^{\pi/2} \eta \sin(\xi\eta)\,d\eta = \cancel{\sin(\xi\eta)\frac{\eta^2}{2}\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin(\xi\eta)\,d\eta}$$

$$= \cancel{\frac{\sin(\xi\pi/2)\,\pi^2}{8} - 0 + \frac{\cos(\xi\eta)}{\xi}}\Bigg|_0^{\pi/2}$$

$$= \cancel{\frac{\pi^2}{8}\sin\left(\frac{\pi}{2}\xi\right) + \frac{\cos\left(\frac{\pi}{2}\xi\right)}{\frac{\pi}{2}}} - \frac{1}{\xi}$$

$$= \cancel{\frac{\pi^2}{8}\sin\left(\frac{\pi}{2}\xi\right) + \frac{2}{\pi}\cos\left(\frac{\pi}{2}\xi\right) - \frac{1}{\xi}}$$

$$= -\eta\frac{\cos(\xi\eta)}{\xi}\Bigg|_0^{\pi/2} + \frac{1}{\xi}\int_0^{\pi/2}\cos(\xi\eta)\,d\eta$$

$$\int v\,du = vu - \int u\,dv$$

$$= -\frac{1}{\xi}\left(\frac{\pi}{2}\cos\left(\frac{\pi}{2}\xi\right)\right) + \frac{\sin(\xi\eta)}{\xi^2}\Bigg|_0^{\pi/2}$$

$$= -\frac{\pi}{2\xi}\cos\left(\frac{\pi}{2}\xi\right) + \frac{\sin\left(\frac{\pi}{2}\xi\right)}{\xi^2}$$

$$\therefore\ I = \int_0^1 \text{''}\ d\xi = ?$$

$$-\frac{\pi}{2}\underbrace{\int_0^1 \frac{\cos(\frac{\pi}{2}\xi)}{\xi}\,d\xi}_{\text{div. at } \xi=0}$$

$$= \int_0^1 \underbrace{\left(-\frac{\pi}{2}\frac{\cos(\frac{\pi}{2}\xi)}{\xi} + \frac{\sin(\frac{\pi}{2}\xi)}{\xi^2}\right)}_{\text{together poss not div.}} d\xi$$

$$= \int_0^1 \left(\frac{\sin(\frac{\pi}{2}\xi)}{\xi^2} - \frac{\pi}{2}\frac{\cos(\frac{\pi}{2}\xi)}{\xi}\right) d\xi \qquad \text{How integrate?}$$

$$= \oint \quad \text{let } v = \frac{\pi}{2}\xi \qquad \xi = \frac{2}{\pi}v$$

$$dv = \frac{\pi}{2}\,d\xi$$

$$= \int_0^{\pi/2} \left(\frac{\sin(v)}{(\frac{2}{\pi})^2 v^2} - \frac{\pi}{2}\frac{\cos(v)}{\frac{2}{\pi}v}\right)\frac{2}{\pi}\,dv$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\left(\frac{\pi^2}{4}\frac{\sin(v)}{v^2} - \frac{\cos(v)}{v}\right) dv$$

$$= \frac{2}{\pi}\int_0^{\pi/2}\left(\frac{\pi^2}{4}\frac{1}{v^2}\left(\frac{e^{iv}-e^{-iv}}{2i}\right) - \frac{1}{v}\left(\frac{e^{iv}+e^{-iv}}{2}\right)\right) dv$$

$$= \frac{2}{\pi}\int_0^{\pi/2} \underbrace{\frac{\pi^2}{8i}\frac{(e^{iv}-e^{-iv})}{v^2} - \frac{1}{2v}(e^{iv}+e^{-iv})}_{F(v)} \cdot dv$$

$$= \frac{1}{\pi}\int_0^{\pi/2} -\frac{\pi^2}{4}\frac{i}{v^2}($$

where are the singularities of F(v)?

$$F(v) = -\frac{\pi^2 i}{4v^2}(e^{iv}-e^{-iv}) - \frac{(e^{iv}+e^{-iv})}{v}$$

~~Trouble is — If I were to work w/ F(v) like this I won't have an analytic form to~~

$$-\frac{\pi^2 i}{4v^2}e^{iv} - \frac{1}{v}e^{iv} + \frac{\pi^2 i}{4v^2}e^{-iv} - \frac{1}{v}e^{-iv}$$

$$= \left(-\frac{\pi^2 i}{4v^2} - \frac{1}{v}\right)e^{iv} + \left(\frac{\pi^2 i}{4v^2} - \frac{1}{v}\right)e^{-iv}$$

$$= \left(\frac{-i}{(\frac{2}{\pi}v)^2} - \frac{1}{v}\right)e^{iv}$$

$$= -\left(\frac{i}{(\frac{2}{\pi}v)^2} + \frac{1}{v}\right)e^{iv}$$

$$= \cancel{\left(\frac{i}{(\frac{2}{\pi}v)^2} + \frac{1}{v}\right)} - \left(\frac{i}{(\frac{2}{\pi}v)^2} + \frac{(\frac{2}{\pi})^2 v}{(\frac{2}{\pi}v)^2}\right)$$

Pg 81 Caract / John

$$\int_0^{\frac{\pi}{2}} d\eta \int_0^1 \eta \sin(\xi\eta)\, d\xi$$

$$= \int_0^{\frac{\pi}{2}} \eta\, d\eta \left(-\frac{\cos(\xi\eta)}{\eta}\right|_0^1$$

$$= -\int_0^{\pi/2} (\cos(\eta) - 1)\, d\eta = \frac{\pi}{2} - \sin\eta \Big|_0^{\pi/2} = \frac{\pi}{2} - 1$$

Pg 82 Caract / John

F_h

$$u(x,y) = \int_a^x v(\xi, y)\, d\xi = \int_\alpha^y \int_a^x \cancel{} f(\xi,\eta)\, d\xi\, d\eta$$

$$\frac{1}{4h^2} \int_{x-h}^{x+h} d\xi \int_{y-h}^{y+h} f(\xi,\eta)\, d\eta$$

α $y-h$ $y+h$

$$= \frac{1}{4h^2} \Big(\underbrace{\int_\alpha^{y+h} \int_a^{x+h} \cdots - \int_\alpha^{y-h} \int_a^{x+h}} - \int_\alpha^{y+h} \int_a^{x-h} + \int_\alpha^{y-h} \int_a^{x-h}$$

$$\int_{y-h}^{y+h} \int_a^{x+h} - \left(\int_{y-h}^{y+h} \int_a^{x-h} \right) = \int_{y-h}^{y+h} \int_{x-h}^{x+h} \cdots$$

Pg 84 Carat / Thm

$$B = \frac{\partial f}{\partial y} \; ; \; C = \frac{\partial f}{\partial z} \; ; \; A = \frac{\partial f}{\partial x}$$

If $L = yz\,dx + zx\,dy + xy\,dz$

$$\frac{\partial B}{\partial z} - \frac{\partial C}{\partial y} = \frac{\partial (zx)}{\partial z} - \frac{\partial (xy)}{\partial y} = x - x = 0$$

$$\frac{\partial C}{\partial x} - \frac{\partial A}{\partial z} = y - y = 0$$

$$\frac{\partial A}{\partial y} - \frac{\partial B}{\partial x} = z - z = 0.$$

$$L = A\,dx + B\,dy + C\,dz + D\,dp \left\{ = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz + \frac{\partial F}{\partial p}dp \right.$$

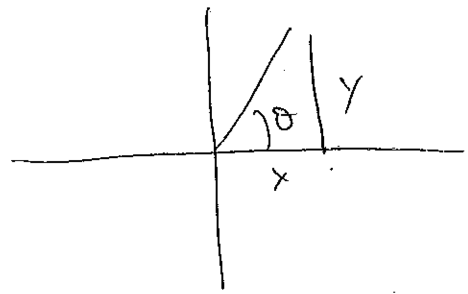
The a ness. conditi for L to be an exact differential form is

$$\left.\begin{array}{lll}
\dfrac{\partial A}{\partial y} = \dfrac{\partial B}{\partial x} & \dfrac{\partial B}{\partial z} = \dfrac{\partial C}{\partial y} & \dfrac{\partial C}{\partial p} = \dfrac{\partial D}{\partial z} \\[2ex]
\dfrac{\partial A}{\partial z} = \dfrac{\partial C}{\partial x} & \dfrac{\partial B}{\partial p} = \dfrac{\partial D}{\partial y} & \\[2ex]
\dfrac{\partial A}{\partial p} = \dfrac{\partial D}{\partial x} & &
\end{array}\right\} \text{6 conditions for integrability of the Linear differential form}$$

- - - -

By integrating an Linear diff form we obtain a fn whose total derivative is the giv Linear differential form

Pg 99 Carroll (John



$$d\theta = d\arctan\left(\frac{y}{x}\right)$$

$$= \frac{1}{1+(y/x)^2}\cdot d\left(\frac{y}{x}\right)$$

$$= \frac{x^2}{x^2+y^2}\left(-\frac{y}{x^2}\,dx + \frac{dy}{x}\right)$$

$$= \frac{-y\,dx + x\,dy}{x^2+y^2} \quad \text{eq}(71)$$

pg 99 Courant / John

$$A = \frac{-y}{x^2+y^2} \qquad B = \frac{x}{x^2+y^2} \qquad C = 0$$

(69) $\dfrac{\partial B}{\partial z} - \dfrac{\partial C}{\partial y} = 0$ ✓ $\qquad \dfrac{\partial A}{\partial y} - \dfrac{\partial B}{\partial x} = 0$

$\dfrac{\partial C}{\partial x} - \dfrac{\partial A}{\partial z} = 0$ ✓ $\qquad \dfrac{-1}{x^2+y^2} + \dfrac{y\,2y}{(x^2+y^2)^2} - \left(\dfrac{1}{x^2+y^2} - \dfrac{x(2x)}{(x^2+y^2)^2}\right) \stackrel{?}{=} 0$

$$\frac{-2}{x^2+y^2} + \frac{2(x^2+y^2)}{(x^2+y^2)^2} = 0 \quad ✓$$

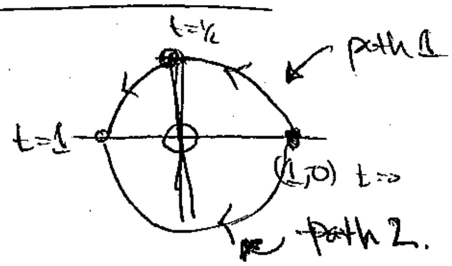
— — —

$$\int_0^{2\pi}\left(A\frac{dx}{dt} + B\frac{dy}{dt}\right)dt = \int_0^{2\pi}\left(\left(\frac{-\sin t}{1}\right)(-\sin t) + \cos t(\cos t)\right)dt$$

$$= 2\pi$$

$$d\theta = d\left(\arctan\frac{y}{x}\right) = L$$

$x = \cos \pi t \qquad y = \sin \pi t$



pg 105 Courant (John

$$(Ax_t + By_t + Cz_t)_\lambda$$

$$= A_\lambda x_t + B_\lambda y_t + C_\lambda z_t + A x_{\lambda t} + B y_{\lambda t} + C z_{\lambda t}$$

$$= \underline{A_x x_\lambda x_t} + A_y y_\lambda x_t + A_z z_\lambda x_t$$

$$+ B_x x_\lambda y_t + \underline{B_y y_\lambda y_t} + B_z z_\lambda y_t$$

$$+ C_x x_\lambda z_t + C_y y_\lambda z_t + \underline{C_z z_\lambda z_t} + \underline{A x_{\lambda t} + B y_{\lambda t} + C z_{\lambda t}}$$

$$= (A x_\lambda + B y_\lambda + C z_\lambda)_t - A_t x_\lambda - B_t y_\lambda - C_t z_\lambda$$

$$+ A_y \underline{y_\lambda x_t} + A_z \underline{z_\lambda x_t} + A_x x_\lambda x_t$$

$$+ B_x \underline{x_\lambda y_t} + B_z \underline{z_\lambda y_t} + B_y y_\lambda y_t$$

$$+ C_x \underline{x_\lambda z_t} + C_y \underline{y_\lambda z_t} + C_z z_\lambda z_t$$

$$= \cancel{(A x_\lambda + B y_\lambda + C z_\lambda)_t} \; x_\lambda ($$

$$+ A_y \cancel{(y_\lambda x_t + x_\lambda y_t)} + C_x \cancel{(x_\lambda z_t + z_\lambda x_t)}$$

$$+ \cancel{B_z (z_\lambda y_t + y_\lambda z_t)}$$

$$= (A x_s + B y_s + C z_s)_t$$

$$- x_s(A_{\cancel{x}} x_t + A_{\cancel{y}} y_t + A_z z_t)$$

$$- y_s(B_x x_t + B_{\cancel{y}} y_t + B_{\cancel{z}} z_t)$$

$$- z_s(C_{\cancel{x}} x_t + C_y y_t + C_{\cancel{z}} z_t) + A_y(y_s x_t + x_{\cancel{s}} y_t)$$ } using eq ~~16a~~ 75a.

$$+ C_x(x_s z_t + z_{\cancel{s}} x_t) + B_z(z_s y_t + y_{\cancel{s}} z_t)$$

$$+ A_x \cancel{x_s} x_t + B_y \cancel{y_s} y_t + C_z \cancel{z_s} z_t$$ ← part I did incorrectly before

$$= (A x_s + B y_s + C z_s)_t - \underline{A_z x_s z_t} - \underline{B_x y_s x_t} - \underline{C_y z_s y_t}$$

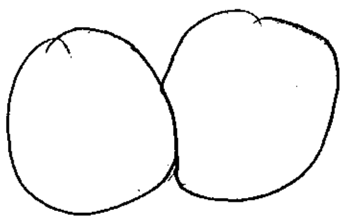
$$+ \underbrace{A_y y_s x_t}_{=0} + \underbrace{C_x x_s z_t}_{=0} + \underbrace{B_z z_s y_t}_{=0}$$

PS 118 Coorant (John 1

2a) 1) $S^\circ \cup T^\circ \subset (S \cup T)^\circ$

Example: 2 sets who share boundaries i.e. S & T include their boundaries

The $S \cup T$ includes the boundary in between & these points would be in the interior of $S \cup T$.



2a) 2) $\overline{S \cup T} \subset \bar{S} \cup \bar{T}$

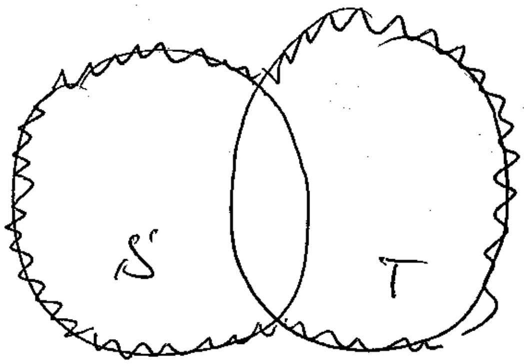
~~Let S be a set & T = ∂S its boundary.~~

~~Th~~ ~~$\overline{S \cup T} = \overline{S} = S$~~

~~But~~

2(b) $\partial(S \cup T) \subset \partial S \cup \partial T$

Eg:



sets who overlap

Then $\partial(S \cup T)$ is shown in ~~

But ∂S & ∂T would include inside points.

pg 120 Courant/John

$$f(tx, ty, \dots) = t^n f(x, y, \dots)$$

$$x f_x + y f_y + \cdots = n t^{n-1} f(x, y, \dots)$$

$$g = \frac{1}{\xi^n} f(\xi, \eta\xi, \zeta\xi, \dots)$$

$$g_\xi = \frac{-n}{\xi^{n+1}} f(\xi, \eta\xi, \zeta\xi, \dots) + \frac{1}{\xi^n}\left(\cancel{f_x + \eta f_y + \zeta f} \cdot \frac{\partial}{\partial \xi} f(\xi, \eta\xi, \zeta\xi, \dots)\right)$$

$$= \frac{-n}{\xi^{n+1}} f(\xi, \eta\xi, \zeta\xi, \dots) + \frac{1}{\xi^n}\left(f_{x_1} + \eta f_{x_2} + \zeta f_{x_3} + \cdots\right)$$

$$= \frac{-n}{\xi^{n+1}} f + \frac{1}{\xi^n}\left(f_x + \eta f_y + \zeta f_z + \cdots\right)$$

$$= \quad '' \quad + \frac{1}{\xi^n}\left(f_x + \frac{y}{x} f_y + \frac{z}{x} f_z + \cdots\right)$$

$$= \frac{-n}{\xi^{n+1}} f + \frac{1}{\xi^{n+1}}\left(\underbrace{x f_x + y f_y + z f_z + \cdots}_{nf}\right)$$

Pg 121 Courant (3)

$$g'(t) = ht^{h-1} f(x,y,\dots) - \cancel{x f_x - y f_y - \dots}$$

$$- x f_x(tx,ty,\dots) - y f_y(tx,ty,\dots)$$

$= \cancel{ht}$

$x f_x + y f_y + \dots$

$$tx f_{xt} + ty f_{yt} + \dots = hf$$

$$\Rightarrow x f_{xt} + y f_{yt} + \dots = \frac{h}{t} f$$

$$\therefore g'(t) = ht^{h-1} f - \frac{h}{t} f$$

$$= \frac{h}{t}(t^h f - f) = \frac{h}{t} g(t)$$

— — — —

$$g(t) = r(t) t^h$$

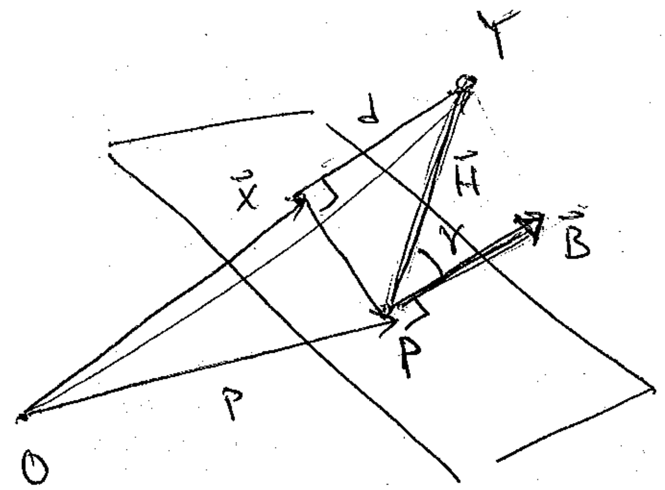
$$g'(t) = r'(t) t^h + h t^{h-1} r(t)$$

$$= r'(t) t^h + \frac{h}{t} g(t)$$

$$\Rightarrow r'(t) t^h = 0$$

$|x||B| > |B \circ x| = p$

eq $|\vec{OP}| = p$



$d = ?$

~~Box~~ at $\vec{Y} = \vec{P} + \vec{H}$

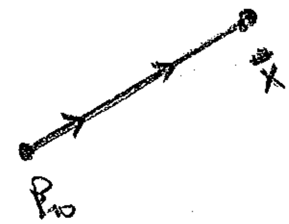
Then $\vec{B} \circ \vec{Y} = \vec{B} \circ \vec{P} + \vec{B} \circ \vec{H} = p + \vec{B} \circ \vec{H}$

Now $\vec{B} \circ \vec{H} = |B||H| \cos \gamma = |H| \cos \gamma = d$

$\Rightarrow \quad d = |\vec{B} \circ \vec{Y} - p|$

① $\vec{P} = \vec{P}_0 + t\vec{A}$

$= (-2,0,4) + t(2,1,3)$



$x = -2 + 2t$

$y = 0 + t$

$z = 4 + 3t$

② $\vec{l} = \vec{Q} - \vec{P} = (3, -5+2, 2) = (3, -3, 2)$

(a)

$\vec{P} = (3, -2, 2) + t(3, -3, 2)$

$= (3+3t, -2-3t, 2+2t)$

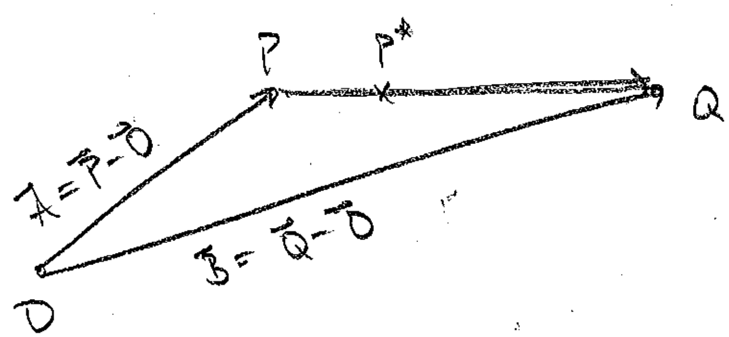
(b) $\vec{l} = (p_x - q_x, p_y - q_y, p_z - q_z)$

$\vec{P} = (p_x + t(p_x - q_x), p_y + t(p_y - q_y), p_z + t(p_z - q_z))$

ein

$= ((t+1)p_x - tq_x, (t+1)p_y - tq_y, (t+1)p_z - tq_z)$

③



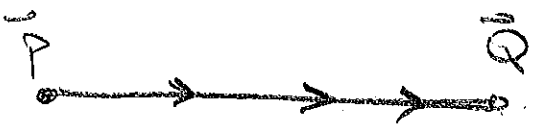
$p^* = \cancel{\vec{O}} + (\vec{P} - \cancel{\vec{O}}) + \lambda(\vec{Q} - \vec{P}) = \lambda\vec{Q} - \lambda\vec{P} + \vec{P}$

$$\vec{P}^* = \lambda(\vec{B}+\vec{O}) - \lambda(\vec{A}+\vec{O}) + (\vec{A}+\vec{O})$$

$$= \lambda\vec{B} - \lambda\vec{A} + \vec{A} + \vec{O}$$

$$= \vec{O} + (1-\lambda)\vec{A} + \lambda\vec{B}$$

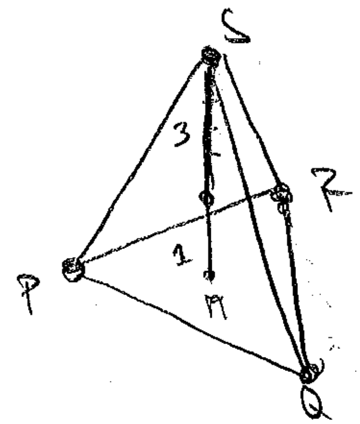
(4)



$\lambda > 0$?

$$\vec{P}^* = \vec{A} + \lambda(\vec{B} - \vec{A})$$

(5)



$\vec{M} = \frac{1}{3}(\vec{P} + \vec{R} + \vec{Q})$ Center of mass of the base

$\vec{M}_{\text{tetrahedron}} = \vec{M} + \frac{1}{4}(\vec{S} - \vec{M})$

From problem

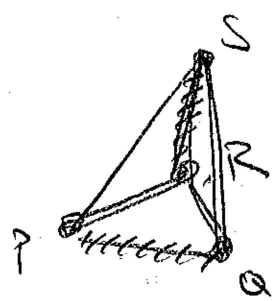
$$= \frac{\vec{S}}{4} + \left(1 - \frac{1}{4}\right)\vec{M}$$

$\vec{M}_{\text{tetrahedron}} = \dfrac{\vec{P} + \vec{Q} + \vec{R} + \vec{S}}{4}$ independent of order taken

from Vol I

$$= \frac{\vec{S}}{4} + \frac{3}{4}\vec{M} \quad ✓$$

(6)



if 2 edges are opposite then their mid pts must
Not contain a common "point"

$m_1 = \frac{1}{2}(P+Q)$ where I have used R, S, P, Q

$m_2 = \frac{1}{2}(R+S)$ as a general opposite edge

The segment passing through the midpts is

$$\lambda(\bar{m}_1) + (1-\lambda)\bar{m}_2 \qquad 0<\lambda<1$$

$$= m_2 + \lambda(m_1 - m_2)$$

$$= \frac{1}{2}(R+S) + \frac{\lambda}{2}(P+Q-R-S)$$

for some λ this must eq $\frac{1}{4}(R+S+P+Q)$

Set

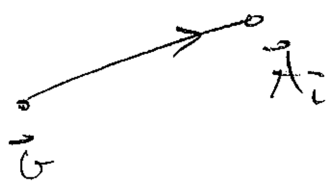
$$\frac{1}{2}(R+S) + \frac{\lambda}{2}(P+Q-R-S) = \frac{1}{4}(R+S+P+Q)$$

$$\frac{1}{2}(P+Q-R-S) = -\frac{1}{4}(R+S) + \frac{1}{4}(P+Q)$$

$$\lambda(P+Q) - \lambda(R+S) = -\frac{1}{2}(R+S) + \frac{1}{2}(P+Q)$$

pick $\lambda = \frac{1}{2}$ mid pt of midpts of opposite edges

(7) $\vec{A}_i = \vec{A}_i - \vec{G}$



this is another way of writing
the center of mass

(8) To be a vector space: Addition of objects & mult by scalars
positive real #'s

Addition of vectors is multiplication   checking req on pg 123

$$X + Y = X \cdot Y$$

Scaler multip $\quad \alpha X = ?$

Scaler multiplication must be distributive

$$\lambda(\vec{A}+\vec{B}) = \lambda\vec{A} + \lambda\vec{B}$$

consider the rule that mixes both multiplication (scalar) &
Addition every elt must have an inverse

$A + (-A) = 0$

$-A$ should be the reciprocal of A

$\Rightarrow \alpha A = A^{\alpha}$ exponentiation ~~...~~ scalar multiplication of vector A = expon

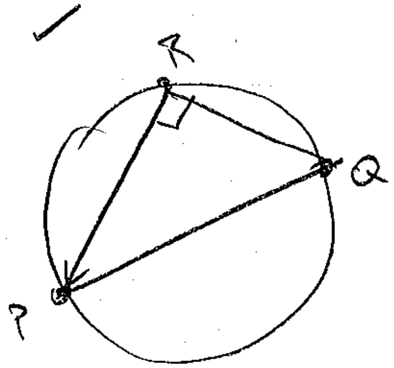
check $\alpha(A+B) = (AB)^{\alpha} = A^{\alpha}B^{\alpha} = \alpha A$

$(\lambda+\mu) A = A^{\lambda+\mu} = A^{\lambda} \cdot A^{\mu} =$

$0 = A^{-1} = -A$

(9) ✓

(10)



$\vec{PR} \circ \vec{RQ} \stackrel{?}{=} 0$

$(p_x - r_x, p_y - r_y, p_z - r_z) \circ (q_x - r_x, q_y - r_y, q_z - r_z) =$

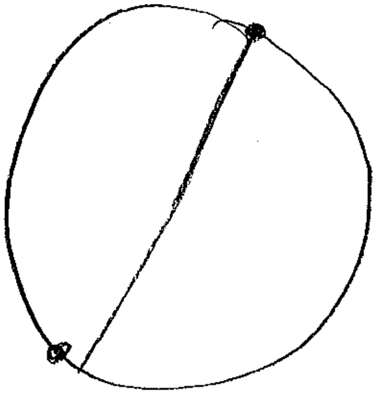
$\sum_{i=1}^{3} (p_x - r_x)(q_x - r_x) = \sum_{i=1}^{3} p_i q_i - r_i p_i - r_i q_i + r_i^2 =$

$\sum_{i=1}^{3} p_i q_i - r_i p_i - r_i q_i = -1$

diametrically opposed $\Rightarrow$ ~~[illegible]~~

~~[illegible]~~

~~$p_i = -q_i$~~



$p_i = -q_i$ reflect this point q_i through the origin

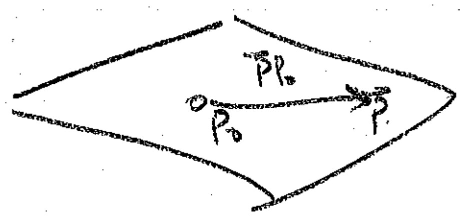
$$\Rightarrow \underbrace{\sum_{i=1}^{3} -q_i^2 + \cancel{r_i q_i} - \cancel{r_i q_i}}_{=0} = -1$$

~~$\overline{PR} = R - P$~~

$(R-P)\circ(Q-R)$ ~~$= PQ - R^2 - PQ + RP$~~

$=$

(11) (a) $\vec{A} = (1,2,-2)$



$\vec{A}\circ(\vec{P}-\vec{P}_0) = 0$

$(1,2,-2)$

$\vec{A}\vec{P} = \vec{A}\vec{P}_0$

$\Rightarrow x + 2y - 2z = -3 + 4 + -2 = -1$

(b) Form line from $\vec{Q}$ in direction of normal $\vec{n}$

$P = \vec{Q} + \lambda \vec{n}$

$\vec{P} = (1+\lambda(1), -1+2\lambda, -1+-2\lambda)$

when this line intersects the plane put into eqs for plane

$$1+\lambda + 2(-1+2\lambda) + -2(-1-2\lambda) = -1$$

$$1+\lambda - \cancel{2} + 4\lambda + \cancel{2} + 4\lambda = -1$$

$$9\lambda = -2$$

$$\lambda = -2/9$$

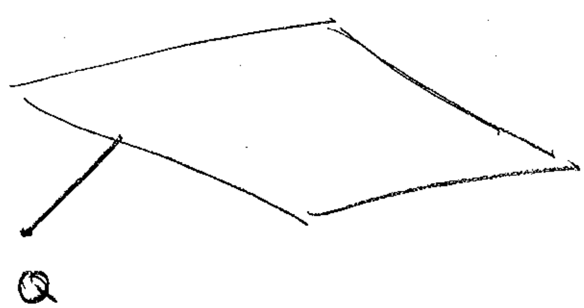
$\Rightarrow \quad P = (7/9, \frac{-13}{9}, 5/9)$

↑ pt where intersection takes place

The distance is $|\vec{PQ}| = \sqrt{(\frac{7}{9}-1)^2 + (\frac{-13}{9}+1)^2 + (\frac{5}{9}+1)^2}$

error somewhere

(c)



$\lambda < 0$ to get to plane from Q.

$\lambda = ?$ to get to plane from O

$$\lambda + 2(2\lambda) - 2(-2\lambda) = -1$$

$$9\lambda = -1$$

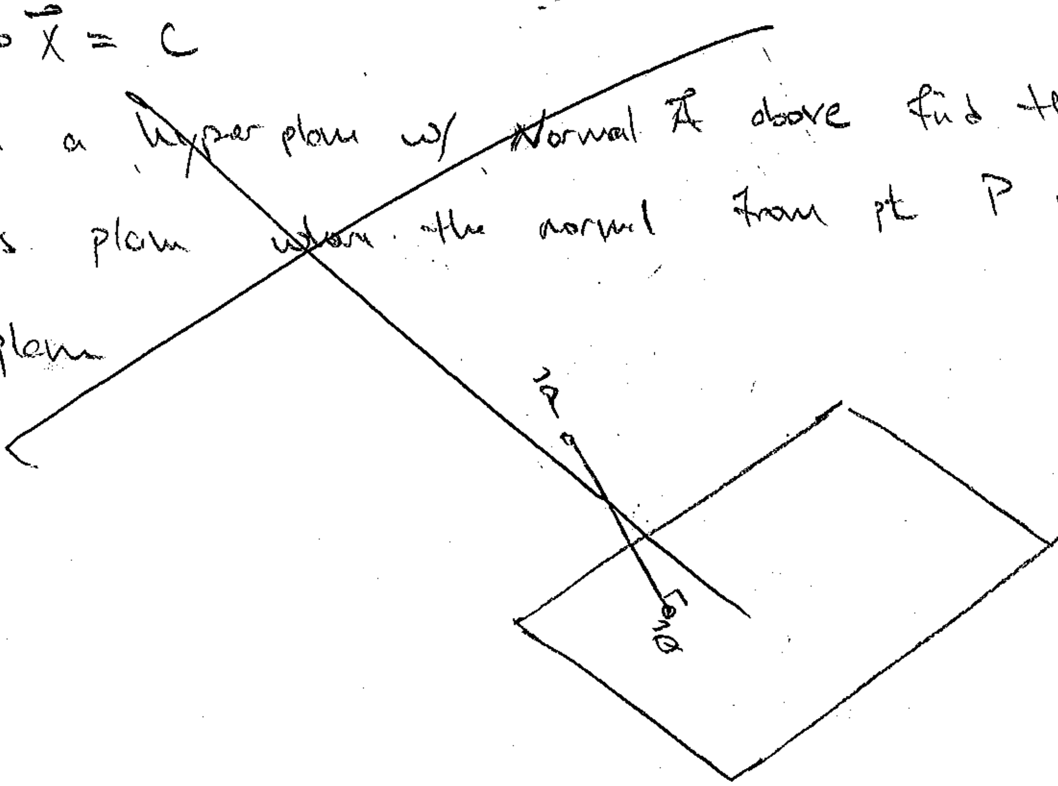
$$\lambda = -1/9$$ Same sign = same side of plane

(12)

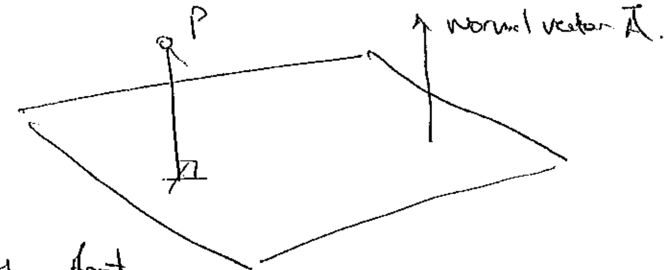
(a) $\vec{A} \circ \vec{X} = C$

Given a hyper plane w/ Normal $\vec{A}$ above find the point on this plane where the normal from pt P would intersect This plane

$\vec{Q} = ?$



(12) (a) $\vec{A} \circ \vec{X} = C$



The line through P & in direction normal to the plane intersects the plane at the foot.

$\vec{r}(t) = \vec{P} + \vec{A}t$ eq of the line.

If this is also to intersect the plane then

$\vec{r}(t^*) = \vec{X}$ for $t^* \in \mathbb{R}$

$\Rightarrow \vec{A} \circ (\vec{P} + \vec{A}t^*) = C$

$\vec{A} \circ \vec{P} + |\vec{A}|^2 t^* = C$

$\Rightarrow t^* = \dfrac{C - \vec{A} \circ \vec{P}}{|\vec{A}|^2}$

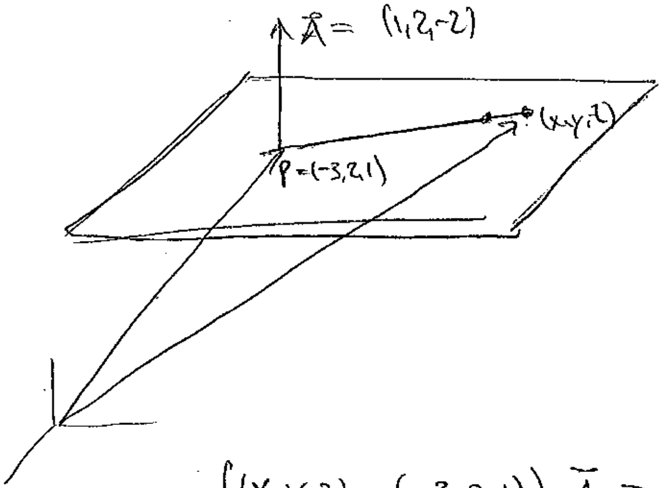
Thus the coordinate of the foot of this plane is given by

$\vec{r}(t^*) = \vec{P} + \dfrac{\vec{A}}{|\vec{A}|^2}(C - \vec{A} \circ \vec{P})$

$= \vec{P} + \left(\dfrac{C - \vec{A} \circ \vec{P}}{|A|^2}\right)\vec{A} = \vec{P} + \dfrac{(C - \vec{A} \cdot \vec{P})}{|\vec{A}|} \dfrac{\vec{A}}{|\vec{A}|}$

(b) In problem 11. $\vec{A} = (1, 2, -2)$

Then the foot



$$((x,y,z)-(-3,2,1))\circ\vec{A} = 0$$

$$\Rightarrow \quad \vec{X}\circ\vec{A} = (-3,2,1)\circ\vec{A} = -3+4-2 = -1$$

$\therefore$ feet for $O = (0,0,0)$ is

$$\vec{r}_{feet_O} = (0,0,0) + \frac{(-1-0)}{\sqrt{1+4+4}}\,\frac{(1,2,-2)}{\sqrt{1+8}} = (0,0,0) - \frac{1}{9}(1,2,-2)$$

$$= -\frac{1}{9}(1,+2,-2)$$

$$\vec{r}_{feet_Q} = (1,-1,-1) + \frac{(-1-(1\cancel{-2}\cancel{+2}))}{3\cdot 3}(1,2,-2)$$

$\{ Q = (1,-1,-1) \}$

$$= (1,-1,-1) + \left(\frac{-2}{9}\right)(1,2,-2)$$

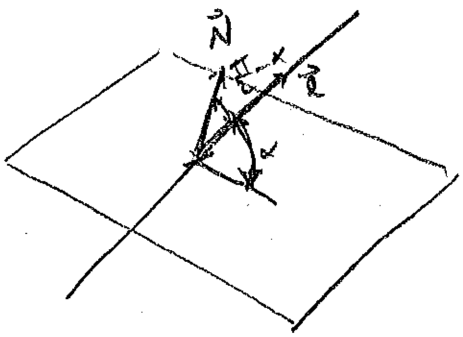
$$= \ldots$$

pg 142 cont/John

(13) $$C = A - \frac{A \circ B}{|B|^2} B$$ See other pg. $\longrightarrow$

$$\vec{C} \circ \vec{B} = A \circ B - \frac{A \circ B}{|B|^2} B \circ B = 0$$

(14)



Normal to plane is

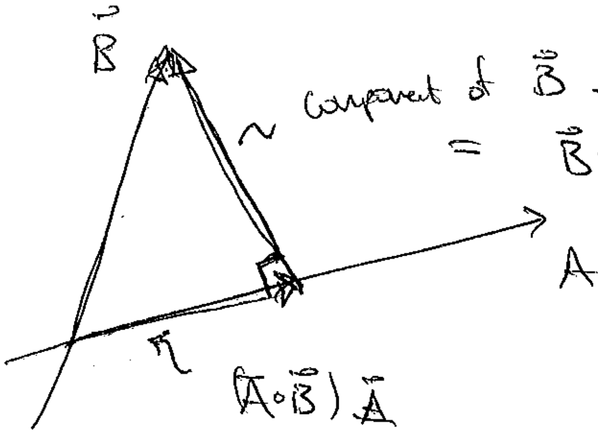
$\vec{N} = (A, B, C)$

line is $\parallel$ to $\ell = (\alpha, \beta, \gamma)$

$$\underline{\cos\left(\frac{\pi}{2} - \phi\right)} = \frac{A\alpha + B\beta + C\gamma}{\sqrt{A^2 + B^2 + C^2}\,\sqrt{\alpha^2 + \beta^2 + \gamma^2}}$$

$$\sin \phi = \quad \text{"}$$

(13)



component of $\vec{B} \perp$ to $\vec{A}$

$$= \vec{B} - \frac{(\vec{A}\circ\vec{B})}{|A|^2}\vec{A}$$

$$(\vec{A}\circ\vec{B})\,\frac{\vec{A}}{|\vec{A}|}$$

pg 164 Courant / John

$$f(\vec{A}) = 3a_2 - 27a_3$$

$$f(\mu\vec{A} + \lambda\vec{B}) = 3(\mu a_2 + \lambda b_2) - 27(\mu a_3 + \lambda b_3)$$

$$= \mu(3a_2 - 27a_3) + \lambda(3b_2 - 27b_3)$$

$$= \mu f(\vec{A}) + \lambda f(\vec{B})$$

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pg 167 Courant / John

6a $\phi(x,y) = y - x$

$$\phi(y,x) = x - y = -(y-x) = -\phi(x,y)$$

6b $\phi(x,y,z) = (z-y)(z-x)(y-x)$

$$\phi(x,z,y) = (y-z)(y-x)(z-x) = -(z-y)(z-x)(y-x) = -\phi(x,y,z)$$

$$\phi(y,x,z) = (z-x)(z-y)(x-y) = -(z-y)(z-x)(y-x) = -\phi(x,y,z)$$

$$\phi(z,y,x) = (x-y)(x-z)(y-z) = -(z-y)(-1)(z-x)(-1)(y-x)$$

$$= -\phi(x,y,z)$$

$$f(\vec{A}_1, \vec{A}_2) =$$

$$f(a_{11}\vec{E}_1 + a_{21}\vec{E}_2, a_{12}\vec{E}_1 + a_{22}\vec{E}_2) = \sum_{j,k=1}^{2} c_{jk} a_j b_k$$

$$c_{jk} = f(E_j, E_k)$$

$$\therefore \quad c_{11} = 0 \qquad c_{22} = 0$$

$$c_{12} = -c_{21}$$
$$\parallel$$
$$c = f(E_1, E_2)$$

$$\cancel{c_{12} a_1 b_2}$$

$$\therefore \quad f(\vec{A}_1, \vec{A}_2) = c_{12} a_1 b_2 + c_{21} a_2 b_1$$

$$= c_{12} a_1 b_2 - c_{12} a_2 b_1$$

$$= c_{12}(a_1 b_2 - a_2 b_1)$$

$$= c\,(a_{11} a_{22} - a_{21} a_{12})$$

$$\vec{a} = \begin{pmatrix} a_{11} \\ a_{21} \end{pmatrix}$$

$$\vec{b} = \begin{pmatrix} a_{12} \\ a_{22} \end{pmatrix}$$

If

$$f(\vec{A}_1, \vec{A}_2) = \sum_{j,k=1}^{n} c_{jk} a_{j1} a_{k2}$$

Then

$$f(\vec{A}_2, \vec{A}_1) = \sum_{j,k=1}^{n} c_{jk} a_{j2} a_{k1}$$

$$\|$$

$$-f(\vec{A}_1, \vec{A}_2) = \sum_{j,k=1}^{n} c_{jk} a_{k1} a_{j2} = -\sum_{j,k=1}^{n} (-c_{jk}) a_{k1} a_{j2}$$

put $\vec{A}_1$ 1st + $\vec{A}_2$ second

If $-c_{jk} = c_{kj}$

Then $f(\vec{A}_2, \vec{A}_1) = -\sum_{j,k=1}^{n} c_{kj} a_{k1} a_{j2} = -f(\vec{A}_1, \vec{A}_2)$.

$f(E_j, E_k,$

$\epsilon_{jkr} = \operatorname{sgn} (r-k)(r-j)(k-j)$

$\epsilon_{123} = \operatorname{sgn} (3-2)(3-1)(2-1) = +1$

$\epsilon_{213} = \operatorname{sgn} (3-1)(3-2)(1-2) = -1$