Pg 25 Problems

(1.3)

(a) $\delta_{km} \epsilon_{kmn} = 0$ $\quad k = 1,2,3$ $\quad m = 1,2,3$

$\Rightarrow \sum_{k=m}$

$= \delta_{1m} \epsilon_{1mn} + \delta_{2m} \epsilon_{2mn} + \delta_{3m} \epsilon_{3mn}$

$= \epsilon_{11n} + \epsilon_{22n} + \epsilon_{33n} = 0$

(b) $\delta_{km} \delta_{kn} = \delta_{mn}$

$\delta_{1m} \delta_{1n} + \delta_{2m} \delta_{2n} + \delta_{3m} \delta_{3n}$

if $m = 1 \quad = \delta_{1n} \qquad n = 1 \quad = \delta_{1m}$

$m = 2 \quad = \delta_{2n} \qquad n = 2 \quad = \delta_{2m}$

$m = 3 \quad = \delta_{3n} \qquad n = 3 \quad = \delta_{3m}$

$\therefore \delta_{km} \delta_{kn} = \delta_{mn}$

(c) $\delta_{km} a_{kn} = a_{mn}$

Pg 25 Drumheller

(1.4) $T_{km} e_{kmn} = 0$ $\qquad (T_{km}) = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}$ 3 non diagonal elements

$$\Rightarrow T_{1m} e_{1mn} + T_{2m} e_{2mn} + T_{3m} e_{3mn} = 0$$

$$\Rightarrow T_{12} e_{12n} + T_{13} e_{13n} + T_{21} e_{21n} + T_{23} e_{23n}$$

$$+ T_{31} e_{31n} + T_{32} e_{32n} = 0$$

$$\Rightarrow (T_{12} - T_{21}) e_{12n} + (T_{13} - T_{31}) e_{13n} + (T_{23} - T_{32}) e_{23n} = 0$$

let $n = 3 \Rightarrow T_{12} = T_{21}$

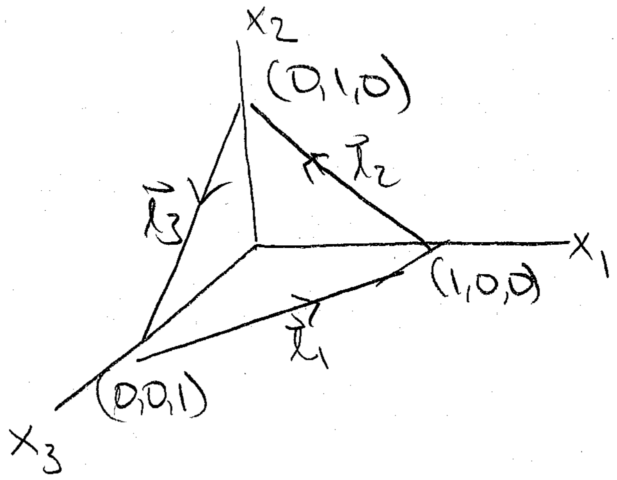
let $n = 2 \Rightarrow T_{13} = T_{31}$

let $n = 1 \Rightarrow T_{23} = T_{32}$

3 non diagonal elts are equal

$\Rightarrow T_{km} = T_{mk}$

1.10 ~~$\vec{u}\times\vec{v}$~~

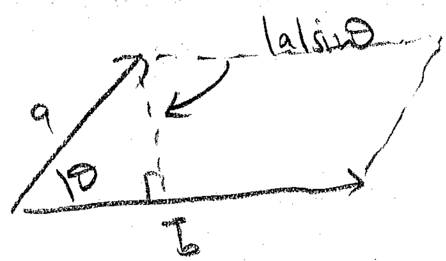


(a) $\vec{l}_1 = (1,0,0) - (0,0,1) = (1,0,-1)$

$\vec{l}_2 = (0,1,0) - (1,0,0) = (-1,1,0)$

$\vec{l}_3 = (0,0,1) - (0,1,0) = (0,-1,1)$

(b) Area of the triangle is $\frac{1}{2}|\vec{l}_1 \times \vec{l}_2|$ ← magnitude fcn



$$= \frac{1}{2}\left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} \right|$$

$$= \frac{1}{2}\left| \hat{i}(1) - \hat{j}(1) + \hat{k}(1) \right|$$

$$= \frac{1}{2}\sqrt{3}$$

(c) ~~$\hat{n} = \vec{l}_1 \times \vec{l}_2$~~

$$\hat{n} = \frac{\vec{l}_1 \times \vec{l}_2}{2\,\text{Area of } \Delta} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

pts upward
$\hat{k}$ component is positive

(1.11) $\det[c_{pq}]\, e_{rst} = c_{rk} c_{sm} c_{tn} e_{kmn}$

RHS has non zero value when kmn = 1,2,3; 2,3,1; 3,1,2 $\Big\}$ $e_{kmn} = +1$

132; 213; 321 $\Big\}$ $e_{kmn} = -1$

otherwise $= 0$.

$$= c_{r1} c_{s2} c_{t3} + c_{r2} c_{s3} c_{t1} + c_{r3} c_{s1} c_{t2} - c_{r1} c_{s3} c_{t2} - c_{r2} c_{s1} c_{t3} - c_{r3} c_{s2} c_{t1}$$

$$= c_{r1}(c_{s2} c_{t3} - c_{s3} c_{t2}) - c_{r2}(c_{s1} c_{t3} - c_{s3} c_{t1}) + c_{r3}(c_{s1} c_{t2} - c_{s2} c_{t1}) = \begin{vmatrix} c_{r1} & c_{r2} & c_{r3} \\ c_{s1} & c_{s2} & c_{s3} \\ c_{t1} & c_{t2} & c_{t3} \end{vmatrix}$$

$= \det[c_{pq}]$ if
$$\begin{array}{ccc} r & s & t \\ \hline 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{array}$$
switching 2 rows determinant are antisymmetric ~~matrix~~ vector fn.

$= -\det[c_{pq}]$ if
$$\begin{array}{ccc} r & s & t \\ \hline 1 & 3 & 2 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{array}$$

$= 0$ else

1.14

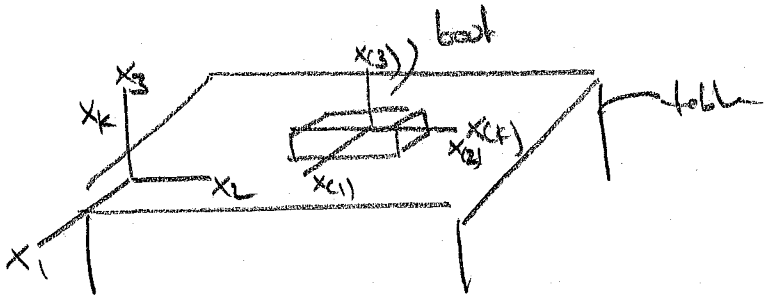
$$x_{(k)} = \alpha_{(k)m} x_m$$
$$x_m = \alpha_{m(k)} x_{(k)} = \alpha_{m(k)} (\alpha_{(k)m} x_m)$$
$$= \alpha_{m(k)} \alpha_{(k)m} x_m$$

$$\Rightarrow \alpha_{m(k)} \alpha_{(k)m} = I$$

$$\Rightarrow [\alpha_{(k)m}]^{-1} = \alpha_{m(k)} = [\alpha_{(k)m}]^T$$

Pg 26 Arambeller

(1.15)



(a) $\dot{x}_{(1)}$

$$\theta_1 \text{ rotations} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_1 \end{pmatrix}$$

$$\theta_2 \text{ rotations} = \begin{pmatrix} \cos\theta_2 & 0 & \sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{pmatrix}$$

$$\theta_3 \text{ rotations} = \begin{pmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\underbrace{\underbrace{\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\frac{\pi}{2} \text{ about } x_3} \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}}_{\frac{\pi}{2} \text{ about } x_2}}\ \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}}_{\frac{\pi}{2} \text{ about } \cancel{x}_1} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{pmatrix}}$$

$\frac{\pi}{2}$ abo θ_1 x_1

$\frac{\pi}{2}$ $x_{(2)}$

$\frac{\pi}{2}$ $x_{(3)}$

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$ <u>different matrices</u>

Pg 27 Drumheller

(1.17) $\hat{i}_{(k)} = \alpha_{(k)m} \, \hat{i}_m$

(a) $$\hat{i}_{(1)} \times \hat{i}_{(2)} \cdot \hat{i}_{(3)} = \begin{vmatrix} \alpha_{(1)1} & \alpha_{(1)2} & \alpha_{(1)3} \\ \alpha_{(2)1} & \alpha_{(2)2} & \alpha_{(2)3} \\ \alpha_{(3)1} & \alpha_{(3)2} & \alpha_{(3)3} \end{vmatrix}$$

~~[illegible]~~ Using rule for ~~[illegible]~~ ~~[illegible]~~ determinant of cross & dot product.

$$= \det[\alpha_{(k)m}]$$

(b) $\hat{i}_{(1)} \times \hat{i}_{(2)} \cdot \hat{i}_{(3)}$ ~~[illegible]~~ represents the volume spanned by 3 $\perp$ vectors of length $= 1$

$\rightarrow$ vol $= 1$

(c) (1.10) $\det[c_{pq}] = \det[a_{ij}] \det[b_{kl}]$

if $c_{pq} = a_{pm} b_{mq}$

$$\det[H_{(km)}] = \det[\alpha_{(k)r} H_{rn} \alpha_{n(m)}]$$

$$= \det[\alpha_{(k)r}]\,\det[H_{rn}]\,\det[\alpha_{n(m)}]$$

$$= \det[H_{rn}]$$

1.1B $\quad [H_{km}] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

(a) $\quad I_H = \text{trace} = H_{kk} = 1$

$$II_H = \tfrac{1}{2} H_{kr} H_{kr} = \cancel{\tfrac{1}{2}\left(H_{11}^2 + H_{23}^2 + H_{32}^2\right)}$$

$$= \tfrac{1}{2}\left(H_{1r}H_{1r} + H_{2r}H_{2r} + H_{3r}H_{3r}\right)$$

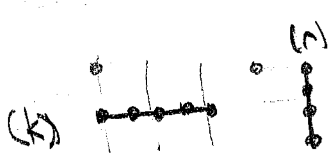
$$= \tfrac{1}{2}(1 + 1 + 1) = \tfrac{3}{2}$$

$$III_H = \det\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 1 \cdot 1 = 1$$

(b) $\quad H_{(k r)} = \alpha_{(k)i}\underbrace{H_{ij}\,\alpha_{j(r)}}$ $\quad$ if $\alpha_{n(m)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$

$$= \alpha_{(k)i}\,\tilde{H}_{i(r)}$$

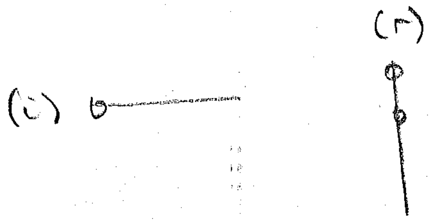
$$\alpha_{(m)n} = \alpha_{n(m)}$$



$$= \underbrace{\alpha_{(k)i} H_{ij}}_{\hat{H}_{(k)j}}$$

regular multiplication between $\alpha_{(k)i}$ + H_{ij} $[\alpha_{(k)i}]$ + $[H_{ij}]$

$H_{ij}\alpha_{j(r)}$ = reg multiplication between $[H_{ij}]$ + $[\alpha_{j(r)}]$



$$\therefore \quad H_{(kr)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin\theta & \cos\theta \\ 0 & \cos\theta & \sin\theta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{pmatrix}$$

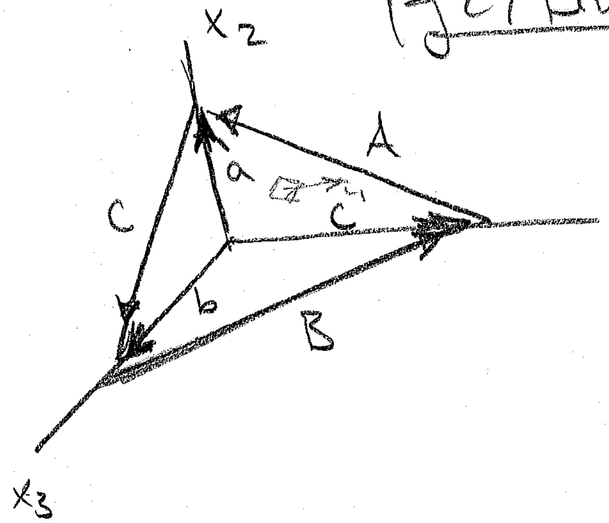
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin\theta\cos\theta + \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \\ 0 & \cos^2\theta + \sin^2\theta & 0 \end{pmatrix}$$

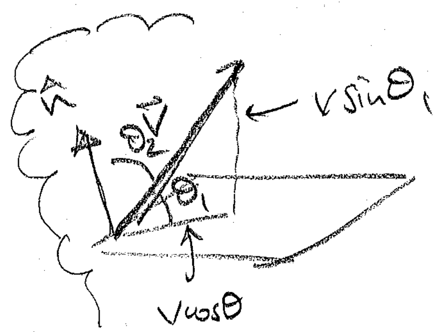
$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Same matrix?

Pg 27 Drumheller

(1.19)





$|\vec{A}\times\vec{B}\cdot\hat{e}_1|$ = Vol of parallelepiped w/ edges $\vec{A}$, $\vec{B}$ + $\hat{e}_1$

$= |\vec{A}\times\vec{B}| \cos\{\vec{A}\times\vec{B}, \hat{e}_1\}$

$= |\vec{A}|\,|\vec{B}|\,|\sin\{\vec{A},\vec{B}\}|\cos\{\vec{A}\times\vec{B}, \hat{e}_1\}$

pg 30 Drumheller

$$\hat{u}_1 = X_1 f t$$

$$u_1 = \frac{x_1 f t}{(1+ft)} \qquad X_1 = \frac{x_1}{1+ft}$$

$$\frac{\partial u_1}{\partial t} = \frac{x_1 f}{(1+ft)} + \frac{-x_1 f^2 t}{(1+ft)^2} = \frac{x_1 f(1+ft) - x_1 f^2 t}{(1+ft)^2}$$

$$\frac{\partial \hat{u}_1}{\partial t} = X_1 f = \frac{f x_1}{1+ft} \qquad = \frac{x_1 f}{(1+ft)^2}$$

pg 34 Drumheller

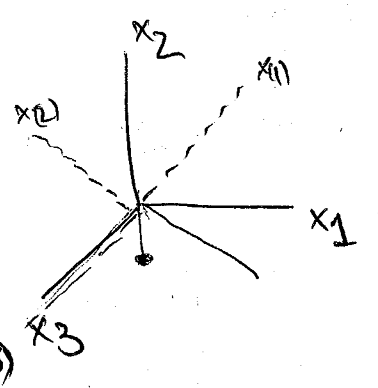
$$\vec{x} = A\vec{\underline{X}}$$

$$\underline{X} = A^T\vec{x} \quad \Rightarrow \quad A^T = A^{-1} \qquad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

Thus

$$x \cdot x = x^T x = \cancel{A^T \underline{X}^T} A\underline{X}$$

$$= \underline{X}^T A^T A \underline{X} = \underline{X}^T \underline{X}$$



Pg 34 Drumheller

(1.20) $x_1 = \frac{X_1}{(1+4t^2)}$ $\quad x_2 = X_2(1+4t^2)$ $\quad x_3 = X_3$

(a) $X_2 = \frac{x_2}{(1+4t^2)}$

$X_1 = x_1(1+4t^2)$

$X_3 = x_3$

(b) $\hat{U}_1 = \hat{x}_1(X) - X_1 = \frac{X_1}{(1+4t^2)} - X_1$

$\hat{U}_2 = X_2(1+4t^2) - X_2$

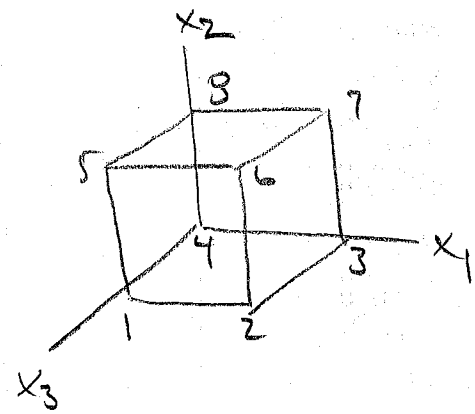
$\hat{U}_3 = X_3 - X_3 = 0$

(c) $u_1 = x_1 - X_1 = x_1 - x_1(1+4t^2) = -x_1 4t^2$

$u_2 = x_2 - X_2 = x_2 - \frac{x_2}{(1+4t^2)}$

$u_3 = x_3 - X_3 = x_3 - x_3 = 0.$

(d) material points of the 8 corners of the cube are :



		X_1	X_2	X_3
corner	1	0	0	ℓ
	2	ℓ	0	ℓ
	3	ℓ	0	0
	4	0	0	0
	5	0	ℓ	ℓ
	6	ℓ	ℓ	ℓ
	7	ℓ	ℓ	0
	8	0	ℓ	0

(d) Thus current position of each of the corners is given by the matrix

		x_1	x_2	x_3
corner #	1	0	0	ℓ
	2	$\frac{\ell}{1+4t}$	0	ℓ
	3	$\frac{\ell}{1+4t}$	0	ℓ
	4	0	0	ℓ
	5	0	$\ell(1+4t)$	ℓ
	6	$\frac{\ell}{1+4t}$	$\ell(1+4t)$	ℓ
	7	$\frac{\ell}{1+4t}$	$\ell(1+4t)$	ℓ
	8	0	$\ell(1+4t)$	ℓ

(e) ~~[illegible]~~ corner 1

Corner #	u_1	u_2	u_3
1	0	0	0
2	$\frac{\ell}{1+4} - \ell$	0	0
3	$\frac{\ell}{5} - \ell$	0	0
4	0	0	0
5	0		0
6			0
7			0
8	0		0

1.21

Pg 35 Drumheller

(1.23) $a_k = \frac{\partial}{\partial t} \hat{v}_k(X_m, t) = \frac{\partial v_k}{\partial t} + \frac{\partial v_k}{\partial x_n} v_n$ from the

discussion on the matl. derivative.

Vectors transform as $w_{(m)} = \alpha_{(m)k} w_k$

Mult BHS By $\alpha_{(m)k}$

$$= \frac{\partial(\alpha_{(m)k} v_k)}{\partial t} + \frac{\partial(\alpha_{(m)k} v_k)}{\partial x_n} v_n$$

$$= \frac{\partial v_{(k)}}{\partial t} + \frac{\partial v_{(k)}}{\partial x_n} v_n$$

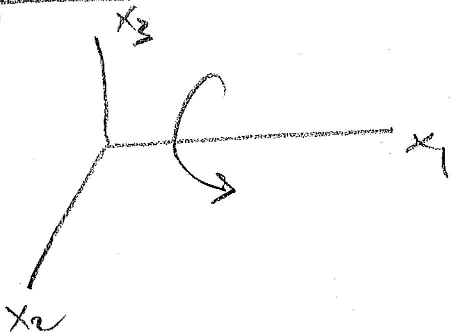
$$= \frac{\partial v_{(k)}}{\partial t} + \frac{\partial v_{(k)}}{\partial x_{(r)}} \underbrace{\frac{\partial x_{(r)}}{\partial x_n} v_n}_{\alpha_{(r)n} v_n = v_{(r)}}$$

$$= \frac{\partial v_{(k)}}{\partial t} + \frac{\partial v_{(k)}}{\partial x_{(r)}} v_{(r)}$$

Pg 35 Drumheller

(1.24)

(a) $x_1 = X_1$



Rotation about the x_1-axis is given by

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega t) & -\sin(\omega t) \\ 0 & \sin(\omega t) & \cos(\omega t) \end{pmatrix}$$

Then $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\omega t) & -\sin(\omega t) \\ 0 & \sin(\ ) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix}$ is the motion

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\ ) & \sin(\ ) \\ 0 & -\sin(\ ) & \cos(\ ) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

(b) $\vec{V}$ material $= \dfrac{\partial \hat{x}_i}{\partial t}$

material description:

$V_1 = 0$

$V_2 = -\omega \sin(\omega t) X_2 - \omega \overset{\cos}{\cancel{\sin}}(\omega t) X_3$

$V_3 = \omega \overset{\cos}{\cancel{\sin}}(\omega t) X_2 - \omega \sin(\omega t) X_3$

The material discription of the acceleration

a

$$\hat{a}_1 = \frac{\partial \hat{v}_1}{\partial t} = 0$$

$$\hat{a}_2 = \frac{\partial \hat{v}_2}{\partial t} = -\omega^2 \cos(\omega t) X_2 + \omega^2 \sin(\omega t) X_3$$

$$\hat{a}_3 = \frac{\partial \hat{v}_3}{\partial t} = -\omega^2 \sin(\omega t) X_2 - \omega^2 \cos(\omega t) X_3$$

The spatial discription of the velocity + acceleration is obtained by putting the eqs for the motion into the above eqs, Thus

$$v_1 = 0$$

$$v_2 = -\omega \sin(\omega t)\left(\cos(\omega t) x_2 + \sin(\omega t) x_3\right) - \omega \cos(\omega t)\left(-\sin(\omega t) x_2 + \cos(\omega t) x_3\right)$$

$$= -\omega x_3$$

$$v_3 = \omega \cos(\omega t)\left(\cos(\omega t) x_2 + \sin(\omega t) x_3\right) - \omega \sin(\omega t)\left(-\sin(\omega t) x_2 + \cos(\omega t) x_3\right)$$

$$= \omega x_2$$

$a_1 = 0$

$a_2 = -\omega^2 \cos(\omega t)\big(\cos(\omega t)\, x_2 + \sin(\cancel{\omega t})\, x_3\big)$

$\qquad + \omega^2 \sin(\omega t)\big(-\sin(\omega t)\, x_2 + \cos(\cancel{\omega t})\, x_3\big)$

$= -\omega^2 x_2$

$a_3 = -\omega^2 \sin(\omega t)\big(x_2 \cos(\cancel{\omega t}) + x_3 \sin(\omega t)\big)$

$\qquad - \omega^2 \cos(\omega t)\big(-\sin(\cancel{\omega t})\, x_2 + \cos(\omega t)\, x_3\big)$

$= -\omega^2 x_3$

(c) $\quad \vec{r} = x_2 \hat{i}_2 + x_3 \hat{i}_3$

$\|\vec{V}\|^2 = \hat{V}_1^2 + \hat{V}_2^2 = \omega^2\big(\sin^2(\omega t) X_2^2 + 2\sin(\omega t)\cancel{\cos(\omega t)} X_2 X_3$

material description

$\qquad + \cos^2(\omega t) X_3^2\big)$

$+ \omega^2\big(\cos^2(\omega t) X_2^2 - 2\cos(\omega t)\sin(\cancel{\omega t}) X_2 X_3$

$\qquad + \sin^2(\omega t) X_3\big)$

$= \omega^2\left(X_2^2 + X_3^2\right) = \omega^2 |\vec{r}|^2$

$\leftarrow$ By pt (d)

We Note that it is much easier to show that

$|\vec{v}|^2 = \omega^2 |\vec{r}|^2$ in the special coord system.

$$|\vec{a}|^2 = \omega^4 |\vec{r}|^2$$

(d) $|r|^2 = (\overline{X}_1^2 + \overline{X}_2^2 + \overline{X}_3^2)$

$$= (x_1^2 + x_2^2 + x_3^2)$$

Rotation Matrix opt

Pg 36 Problem

(1.25)

$x_k = \alpha_{k(m)}(t)X_m$

$x_k - \bar{x}_k = \alpha_{k(m)}(t)X_m - \alpha_{k(m)}(t)\bar{X}_m$

$= \alpha_{k(m)}(t)(X_m - \bar{X}_m)$

~~But we know~~ (We know $\alpha_{k(m)}$ is an ~~ool~~ orthogonal transform matrix & ∴ preserves distance)

$(x_k - \bar{x}_k)(x_k - \bar{x}_k) = \cancel{\alpha_{k(m)}\alpha_{k(m)}(X_m - \bar{X}_m)}$

$\alpha_{k(m_1)}(t)(X_{m_1} - \bar{X}_{m_1})$

$\alpha_{k(m_2)}(t)(X_{m_2} - \bar{X}_{m_2})$

But $\alpha_{k(m_1)}\alpha_{k(m_2)} = \alpha_{(m_1)k}\alpha_{k(m_2)} = \delta_{(m_1)(m_2)}$

∴ RHS $\rightarrow$ $(X_{m_1} - \bar{X}_{m_1})(X_{m_1} - \bar{X}_{m_1}) = \text{constant}.$

1

Pg 36 Dromheller

$$F_{km} = \frac{\partial \hat{x}_k}{\partial X_m}$$

$$x_k = \alpha_{k(m)}(t)\, x_{(m)} \qquad 1.46$$

$$= \alpha_{k(m)}(t)\, x_m^0$$

$$F_{kn} = \frac{\partial x_k}{\partial x_m^0} \frac{\partial \hat{x}_m^0}{\partial X_n} = \alpha_{k(m)}(t)\, F_{mn}^0$$

Pg 41 Dromheller

$$E_{kn} = \frac{1}{2}\left(F_{mk}F_{mn} - \delta_{kn}\right)$$

$$= \frac{1}{2}\left(\left(\frac{\partial \hat{U}_m}{\partial X_k} + \delta_{mk}\right)\left(\frac{\partial \hat{U}_m}{\partial X_n} + \delta_{mn}\right) - \delta_{kn}\right)$$

$$= \frac{1}{2}\left(\frac{\partial \hat{U}_m}{\partial X_k}\frac{\partial \hat{U}_m}{\partial X_n} + \frac{\partial \hat{U}_n}{\partial X_k} + \frac{\partial \hat{U}_k}{\partial X_n} + \cancel{\delta_{kn}} - \cancel{\delta_{kn}}\right)$$

Pg 42 Drumheller

$$E_{kn} = \frac{1}{2}\left(F_{mk}F_{mn} - \delta_{kn}\right)$$

$$= \frac{1}{2}\left(\delta_{mk}\delta_{mn} - \delta_{kn}\right) = 0.$$

$$E_{kn} =$$

$$= \frac{1}{2}\left(\alpha_{(m)p}(t)\,F^{\circ}_{pk}\,\alpha_{(m)q}(t)\,F^{\circ}_{qn} - \delta_{kn}\right)$$

$$= \frac{1}{2}\left(\alpha_{(r)m}(t)\,F^{\circ}_{mk}\,\alpha_{(r)s}(t)\,F^{\circ}_{sn} - \delta_{kn}\right)$$

$$= \frac{1}{2}\left(\alpha_{(r)m}(t)\,\alpha_{(r)s}(t)\,F^{\circ}_{mk}F^{\circ}_{sn} - \delta_{kn}\right]$$

$$\alpha_{m(r)}(t)\,\alpha_{(r)s}(t)$$

$$\alpha_{m(r)}(t)\,\alpha_{s(r)}(t) = \delta_{ms}$$

$$= \frac{1}{2}\left(\delta_{ms}\,F^{\circ}_{mk}\,F^{\circ}_{sn} - \delta_{kn}\right)$$

pg 43 Drumheller

$$\frac{|d\vec{x}|}{|d\vec{X}|} = 1 + \epsilon$$

$\Rightarrow \quad |d\vec{x}| = (1+\epsilon)|d\vec{X}|$ put into 1.68

$$\cancel{} \quad (1 + 2\epsilon + \epsilon^2) - 1 = 2E_{kn} D_k(d\vec{X}) D_n(d\vec{X})$$

$$\epsilon^2 = \frac{|d\vec{x}|^2 - 2|d\vec{X}||d\vec{x}| + |d\vec{X}|^2}{|d\vec{X}|^2} = \frac{|d\vec{x}|^2}{|d\vec{X}|^2} -$$

$$E_{kn} = \frac{1}{2}(F_{mk} F_{mn} - \delta_{kn})$$

$$F_{mk} F_{mn} = 2E_{kn} + \delta_{kn}$$

$\Rightarrow$

$$\sin\gamma = \frac{(2E_{pq} + \delta_{pq}) D_p(d\vec{A}) D_q(d\vec{B})}{\ldots}$$

$$da_1 = F_{1m} dA_m \qquad \text{if} \quad d\vec{A} = dA\,\hat{i}$$

$$= F_{11} dA$$

Becaus $d\vec{A}$ pts in the $\hat{i}$ direction in the reference coordinate

$$db_1 = F_{1m} dB_m \qquad \text{if} \quad d\vec{B} = dB\,\hat{j}$$

$$= F_{12} dB$$

etc...

pg 45 Drumheller

$$dV = \left| \det \begin{bmatrix} F_{11}\, dA & F_{12}\, dB & F_{13}\, dC \\ F_{21}\, dA & F_{22}\, dB & F_{23}\, dC \\ F_{31}\, dA & F_{32}\, dB & F_{33}\, dC \end{bmatrix} \right|$$

$$= \left| \det \begin{bmatrix} F_{11} & F_{12} & \cdots \\ \vdots & & \end{bmatrix} \right| \quad dA\, dB\, dC$$

$$e = III_F - 1 = \det \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} - 1$$

$$= \det \begin{bmatrix} \frac{\partial \hat{U}_1}{\partial X_1} + 1 & \frac{\partial \hat{U}_1}{\partial X_2} & \frac{\partial \hat{U}_1}{\partial X_3} \\ \frac{\partial \hat{U}_2}{\partial X_1} & \frac{\partial \hat{U}_2}{\partial X_2} + 1 & \frac{\partial \hat{U}_2}{\partial X_3} \\ \frac{\partial \hat{U}_3}{\partial X_1} & \frac{\partial \hat{U}_3}{\partial X_2} & \frac{\partial \hat{U}_3}{\partial X_3} + 1 \end{bmatrix} - 1$$

using equation 1.56 for $F_{kn} = \cdots$

$$= \left(\frac{\partial \hat{U}_1}{\partial X_1} + 1 \right) \begin{vmatrix} \frac{\partial \hat{U}_2}{\partial X_2} + 1 & \frac{\partial \hat{U}_2}{\partial X_3} \\ \frac{\partial \hat{U}_3}{\partial X_2} & \frac{\partial \hat{U}_3}{\partial X_3} + 1 \end{vmatrix} - 1$$

$$= \left(\frac{\partial \hat{U}_1}{\partial X_1} + 1 \right) \left(\frac{\partial \hat{U}_2}{\partial X_2} + 1 \right) \left(\frac{\partial \hat{U}_3}{\partial X_3} + 1 \right) - 1 =$$

$$= \left(\frac{\partial \hat{u}_1}{\partial \underline{X}_1} + 1\right)\left(\frac{\partial \hat{u}_2}{\partial \underline{X}_2} + \frac{\partial \hat{u}_3}{\partial \underline{X}_3} + 1\right) - 1 \quad \ldots$$

$$\sin \gamma = \frac{2 E \delta_{pq} D_p(d\vec{A}) D_q(d\vec{B})}{\left[2E\delta_{st} D_s D_q + 1\right]^{1/2}\left[2E\delta_{ij} D_i D_j + 1\right]^{1/2}}$$

$$= \quad \ldots$$

$$I_{E'} = E'_{kk} = E_{kk} - E_{kk} = 0$$

$$L_{km} = \frac{\partial}{\partial \underline{X}_r}\left(\frac{\partial}{\partial t}\hat{u}_k\right) F^{-1}_{rm}$$

$$= \frac{\partial}{\partial t}\left(\frac{\partial \hat{u}_k}{\partial \underline{X}_r}\right) F^{-1}_{rm}$$

$$= \frac{\partial \hat{F}_{kr}}{\partial t} F^{-1}_{rm}$$

$$\frac{\partial \hat{E}_{km}}{\partial t} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x_m} + \frac{\partial v_m}{\partial x_k}\right) = \frac{1}{2}\left(L_{km} + L_{mk}\right) = D_{km}$$

Eulerian definition of velocity gradient tensor

Pg 48 Fran keller

(1.4.8)

(1.26) dx_k & dX_k transform as vectors

$\Rightarrow$ From eq 1.19 $\quad x_m = x_{(r)}\alpha_{(r)m}$

& $\quad x_{(r)} = \alpha_{(r)m} x_m$

Thus $\quad dx_k = dx_{(r)}\alpha_{(r)k}$

$dx_k = F_{km}\, dX_m$ $\qquad$ multiply by $\alpha_{(i)k}$ & sum

$\alpha_{(i)k}\, dx_k = \alpha_{(i)k} F_{km}\, dX_m$

~~multiply by ... & sum~~

$dx_{(i)} = \alpha_{(i)k} F_{km}\, dX_m$ $\qquad$ But $dX_m = dX_{(r)}\alpha_{(r)m}$

$\Rightarrow \quad dx_{(i)} = \alpha_{(i)k} F_{km}\alpha_{(r)m}\, dX_{(r)}$

Now $\quad \cancel{\alpha_{(i)k} = \alpha_{k(i)}} \quad \alpha_{(r)m} = \alpha_{m(r)}$

$\Rightarrow \quad dx_{(i)} = \underbrace{\alpha_{(i)k} F_{km}\alpha_{m(r)}}_{F_{(ir)}}\, dX_{(r)}$

Thus

$$\bar{F}_{(ir)} = \alpha_{(i)k} F_{km} \alpha_{m(r)}$$

$$\Rightarrow \quad \bar{F}_{(kr)} = \alpha_{(k)m} F_{mn} \alpha_{n(r)}$$

1.27   1.61 $\quad F^{-1}_{ij} = \dfrac{\mathrm{cof}[F_{ji}]}{\det[F_{mk}]}$

$$F_{km} F^{-1}_{mr} = F_{km}$$

$$\Rightarrow \frac{F_{km}}{\det[F_{mk}]} \mathrm{cof}[F_{rm}] =$$

? How

PJ 48 Drumheller

(1.28) $F_{km} = \frac{\partial U_k}{\partial X_m} + \delta_{km}$

& $F^{-1}_{mk} = \frac{\partial X_m}{\partial x_k} = \frac{\partial}{\partial x_k}(x_m - U_m)$

From eq 1.59

(From def of displacement $U_n = x_n - X_n$

$\Rightarrow X_n = x_n - U_n$

$F^{-1}_{mk} = \delta_{km} - \frac{\partial U_m}{\partial x_k}$ Eulerian repres

$\therefore F^{-1}_{km} = \delta_{km} - \frac{\partial U_k}{\partial x_m}$

pg 48 Drankeller

1.29 eq 1.61 $F_{ij}^{-1} = \dfrac{\text{cof}[F_{ji}]}{\det[F_{mk}]}$

$$M = \begin{bmatrix} 1 & 0 & 0 \\ A & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad M^T = \begin{bmatrix} 1 & A & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det M = 1(1) - A(0) + 0 = 1$$

$\text{cof}[F_{ji}] =$

$$M^{-1} = \frac{1}{1}\begin{bmatrix} +1 & -(0) & +0 \\ -A & +1 & -0 \\ +0 & -0 & +1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -A & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Check $M^{-1}M = \begin{bmatrix} 1 & 0 & 0 \\ -A & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ A & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ✓

Pg 48 Chandler

(1.30) (a) $F_{mn} = \frac{\partial \hat{x}_m}{\partial X_n}$

$$[F_{mn}] = \begin{pmatrix} 1 & 0 & 0 \\ -X_2 ft & 1-X_1 ft & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c)

$$[F_{mn}^{-1}] = \begin{pmatrix} 1 & 0 & 0 \\ \frac{tfX_2}{1-tfX_1} & \frac{1}{1-tfX_1} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

How did I get this?

(b)

To go to ~~spatial~~ spatial description

$$X_1 = x_1 , \quad X_2 = \frac{x_2}{1-x_1 ft} \quad X_3 = x_3$$

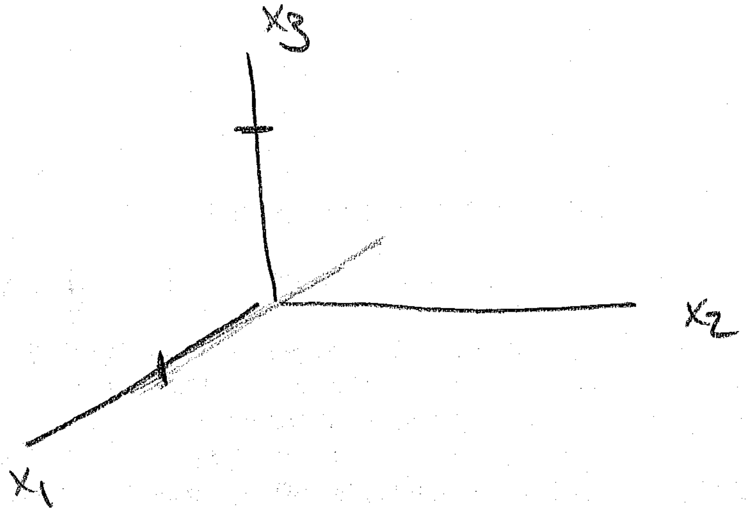
$$\therefore [F_{mn}^{-1}] = \begin{pmatrix} 1 & 0 & 0 \\ \frac{tfx_2}{(1-x_1 ft)^2} & \frac{1}{(1-tfx_1)} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$ft = 1 \qquad x_1 = X_1 , \quad x_2 = X_2(1-X_1) , \quad x_3 = X_3$

(d) F^{-1}_{ij} does not exist if $1 - t \underline{X}_1 = 0$

$$t = 1/\underline{X}_1$$

(e) $x_1 = \underline{X}_1 \quad x_2 = \underline{X}_2(1 - \underline{X}_1) \quad x_3 = \underline{X}_3$



Pg 4B Drucholler

1

(1.31)

$\frac{\partial}{\partial F_{ij}} \det[F_{rs}] = \mathrm{cof}(F_{ij})$ is obvious if you

think about it this way. knowing the object F_{ij}

one wants to ~~take~~ the derivative w.r.t.

The expand the determinant about the row or

column containing this F_{ij}. Only the

actual cofactor of F_{ij} will have F_{ij} multiplied

by it. Thus other terms will have zero derivatives

but the cofactor will remain

$$\begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix} = A_{21}(\quad) - \underbrace{A_{22}(\quad) + A_{23}(\quad)}$$

our term w/ A_{22} is 2nd

one.

Pg 48-49 Drumheller

1.33

$x_1 = X_1 + X_2\,\gamma t \qquad x_2 = X_2 \quad x_3 = X_3$

(a) $F_{ij} = ?$

$$F_{ij} = \frac{\partial \hat{x}_i}{\partial X_j}$$

$\therefore \quad F_{11} = \dfrac{\partial \hat{x}_1}{\partial X_1} = 1 \qquad F_{12} = \dfrac{\partial \hat{x}_1}{\partial X_2} = \gamma t \qquad F_{13} = \dfrac{\partial \hat{x}_1}{\partial X_3} = 0$

$F_{21} = \dfrac{\partial \hat{x}_2}{\partial X_1} = 0 \qquad F_{22} = 1 \qquad F_{23} = 0$

$F_{31} = \dfrac{\partial \hat{x}_3}{\partial X_1} = 0 \qquad F_{32} = \dfrac{\partial \hat{x}_3}{\partial X_2} = 0 \qquad F_{33} = \dfrac{\partial \hat{x}_3}{\partial X_3} = 1$

$$[F_{ij}] = \begin{pmatrix} 1 & \gamma t & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_{kn} = \frac{1}{2}(F_{mk}F_{mn} - \delta_{kn}) =$$

$$= \frac{1}{2}\left((F_{km})^T F_{mn} - \delta_{kn}\right)$$

In matrix notation

$$= \frac{1}{2}\left(F^T F - I \right)$$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 & 0 & 0 \\ ft & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & ft & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 & ft & 0 \\ ft & f^2t^2+1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{pmatrix} 0 & ft & 0 \\ ft & f^2t^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

Check w/ original definition

$$E_{11} = \frac{1}{2}(F_{m1}F_{m1} - 1)$$

$$= \frac{1}{2}(F_{11}F_{11} + F_{21}F_{21} + F_{31}F_{31} - 1$$

$$= \frac{1}{2}(1 + 0 + 0 - 1) = 0 \quad \checkmark$$

$$E_{12} = \frac{1}{2}(F_{m2}F_{m1} - 0)$$

$$= \frac{1}{2}(F_{12}F_{11} + F_{22}F_{21} + F_{32}F_{31})$$

$$= \frac{1}{2}(ft(1) + 0 + 0) = \frac{ft}{2} \quad \checkmark$$

(b) Volume strain $= e = \dfrac{dV - dV^0}{dV^0} = \mathrm{III}_F - 1$

$= \det[F_{kj}] - 1$

$= 1 - 1 = 0$

(c) Longitudinal strain in x_k direction. Then $d\vec{X} = \hat{i}_n \delta_{nk} = \hat{i}_k$

$= \epsilon = \dfrac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|}$ $\quad D_m(d\vec{X}) = \delta_{km}$

$\Rightarrow \quad 2\epsilon + \epsilon^2 = 2E_{mn} D_m(d\vec{X}) D_n(d\vec{X})$

$= 2E_{mn} \delta_{mk} \delta_{nk}$

$= 2E_{kk} = 2\left(\frac{1}{2}(\gamma^2 t^2)\right) = \gamma^2 t^2$

trace of the Lagrangian strain tensor

$\epsilon^2 + 2\epsilon - \gamma^2 t^2 = 0$

$\epsilon = \dfrac{-2 \pm \sqrt{4 + 4\gamma^2 t^2}}{2} = \dfrac{-2 \pm 2\sqrt{1 + \gamma^2 t^2}}{2}$

$= -1 \pm \sqrt{1 + \gamma^2 t^2} = \begin{cases} > 0 & \text{pos} \\ < 0 & \text{neg} \end{cases}$

How know the sign of ~~tage~~ longitudinal strain? ~~take~~ positive

If positive ⇒ material extends

" negative ⇒ " contracts.

$$\epsilon = \frac{|d\vec{x}| - |d\vec{X}|}{|d\vec{X}|}$$

(d) Shear strain $d\vec{A} = \hat{e}_1$ $d\vec{B} = \hat{e}_2$

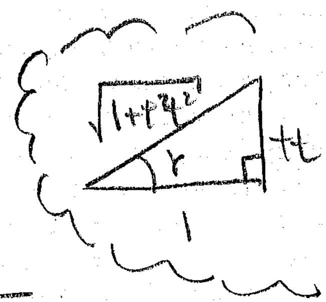
$\gamma = \ldots$

$$\sin\gamma = \frac{2E_{pq} D_p(d\vec{A})\, D_q(d\vec{B})}{(2E_{st} D_s(d\vec{A}) D_t(d\vec{A}) + 1)^{1/2} (2E_{ji} D_j(d\vec{B}) D_i(d\vec{B}) + 1)^{1/2}}$$

$D_p(d\vec{A}) = \delta_{1p}$ $D_q(d\vec{B}) = \delta_{q2}$

$$\therefore \sin\gamma = \frac{2E_{pq}\,\delta_{1p}\,\delta_{2q}}{(2E_{st}\,\delta_{1s}\,\delta_{1t} + 1)^{1/2}(2E_{ji}\,\delta_{2j}\,\delta_{2i} + 1)^{1/2}}$$

$$= \frac{2E_{12}}{(2E_{11}+1)^{1/2}(2E_{22}+1)^{1/2}}$$



$$= \frac{2(4t)/2}{(2\cdot 0+1)^{1/2}(4^2t^2+1)^{1/2}} = \frac{4t}{\sqrt{1+4^2t^2}}$$

Pg 49 Drumheller

(1.34) $\hat{x}_1 = X_1(1+4t) \quad \hat{x}_2 = X_2 \quad \hat{x}_3 = X_3$

(a) $F_{ij} = \dfrac{\partial \hat{x}_i}{\partial X_j}$

$$[F_{ij}] = \begin{pmatrix} 1+4t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$E_{ij} = \frac{1}{2}(F_{pi} F_{pj} - \delta_{ij})$$
$$= \frac{1}{2}((F_{ip})^T F_{pj} - \delta_{ij})$$

Now in matrix notation

$$[E_{ij}] = \frac{1}{2}\left[\begin{pmatrix} 1+4t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1+4t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{pmatrix} (1+4t)^2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{bmatrix} \gamma^2 t^2 + 2\gamma t & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(b) Volumn strain $= III_F - 1 = e$

$$= 1 + \gamma t - 1 = \gamma t$$

(c) longitudinal strain in x_1 direction is ε

$$2\varepsilon + \varepsilon^2 = 2E_{mn} D_m(d\vec{X}) D_n(d\vec{X})$$

$$d\vec{X} = \hat{e}_1 \rightarrow D_m(d\vec{X}) = \delta_{mk}$$

$$\therefore \ 2\varepsilon + \varepsilon^2 = 2E_{mn}\delta_{mk}\delta_{nk} = 2E_{kk} = \gamma^2 t^2 + 2\gamma t$$

one sol is trivially $\varepsilon = \gamma t$. other sol

$$\varepsilon^2 + 2\varepsilon - (\gamma^2 t^2 + 2\gamma t) = 0$$

$$\begin{array}{c|ccc} \gamma t & 1 & 2 & -(\gamma^2 t^2 + 2\gamma t) \\ & & \gamma t & \gamma^2 t^2 + 2\gamma t \\ \hline & 1 & \gamma t + 2 & \boxed{0} \end{array}$$

$$\Rightarrow \varepsilon^2 + 2\varepsilon - (\gamma^2 t^2 + 2\gamma t) = (\varepsilon - \gamma t)(\varepsilon - (\gamma t + 2))$$

$\therefore$ The other solution is $c = 4t+2$. How do you know which one to take?

(d) Shear strain between $d\vec{A} = \frac{1}{\sqrt{2}}(\hat{i}_1 - \hat{i}_2)$

$+ \ d\vec{B} = \frac{1}{\sqrt{2}}(\hat{i}_1 + \hat{i}_2)$

is γ

+

$$\sin\gamma = \frac{2E_{pq}\, D_p(d\vec{A})\, D_q(d\vec{B})}{\left(2E_{st} D_s(d\vec{A})\, D_t(d\vec{A}) + 1\right)^{1/2}\left(2E_{ji} D_j(d\vec{B})\, D_i(d\vec{B}) + 1\right)^{1/2}}$$

Now $D_p(d\vec{A}) = \frac{1}{\sqrt{2}} \quad p=1$

$-\frac{1}{\sqrt{2}} \quad p=2$

$D_q(d\vec{B}) = \frac{1}{\sqrt{2}} \quad q=1$

$+\frac{1}{\sqrt{2}} \quad q=2$

If $\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$

Then $\frac{v_x}{\|\vec{v}\|}$ = direction cosign

$\frac{v_y}{\|\vec{v}\|}$, etc.

$$= \frac{2(1/2)\{E_{11} - E_{21} + E_{12} - E_{22}\}}{\left(E_{11} - E_{21} - E_{12} + E_{22} + 1\right)^{1/2}\left(E_{11} + E_{21} + E_{12} + E_{22} + 1\right)^{1/2}}$$

$$\sin \gamma = \frac{\left[\frac{1}{2}(r^2t^2 + 2rt) - 0 + 0 - 0 \right]}{\left(\frac{1}{2}(r^2t^2 + 2rt) + 1\right)^{1/2} \left(\frac{1}{2}(r^2t^2 + 2rt) + 1\right)^{1/2}}$$

$$= \frac{\frac{1}{2}(r^2t^2 + 2rt)}{\frac{1}{2}(r^2t^2 + 2rt) + 1}$$

Pg 49 Drumheller

(1.35) For pure translation: $\bar{U}$ displacement is a fn of t only. $\bar{U} = \bar{U}(t)$

$\therefore$ From pg 39 Drumheller $F_{KM} = \delta_{KM}$

Am I to think that each motion off a given state results in a composite motion that is the 2nd motion followed by the first & that the deformation gradient tensor of the entire motion would then just be given by the product of the two deformation gradient tensors?

If so then $F_{KM}^{\text{total}} = \underbrace{F_{KM}^{\text{Pure translation}}}_{\delta_{KM}} \cdot F_{KM}^{\text{original deformation gradient tens}}$

pg 39 Drumheller

$\therefore \quad III_{F^{\text{total}}} = \underbrace{III_{I}}_{\text{Identity matrix} \; = \; 1} \cdot III_{\text{original}}$

$e = III_{F^{\text{total}}} - 1 = III_{\text{origin}} - 1$ same ✓

Total Body rotation For total body rotation the total deformation gradient becomes (pg 40 drumheller)

$$F_{kn} = \alpha_{(k)m}(t)\, F^{0}_{mn}$$ (Again see product of deformation gradients)

Then
$$III_F = \det\left[\alpha_{(k)m}(t)\right] \circ \det\left[F^{0}_{mn}\right]$$

$$= \underbrace{1}_{\text{orthogonal matrix has unit det.}} \quad III_{F^{0}_{mn}} =$$

$\therefore$ $e = III_F - 1$ is unchanged.

Pg 49 Drumheller

(1.36) $x_1 = X_1(1+f_1) \quad x_2 = X_2(1-f_2) \quad x_3 = X_3$

(a) deformation Gradient: $F_{km} = \dfrac{\partial \hat{x}_k}{\partial X_m}$

$$[F_{km}] = \begin{pmatrix} 1+f_1 & 0 & 0 \\ 0 & 1-f_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(b) $e = III_F - 1$ vol strain

$$= (1+f_1)(1-f_2) - 1$$

$$= 1 + (f_1 - f_2) - f_1 f_2 - 1 = f_1 - f_2 - f_1 f_2$$

(c) Volume strain in Linear elasticity

$$e = E_{kk} = I_E$$

$$\text{strain} = \frac{1}{2}(F_{mk} F_{mn} - \delta_{kn}) = E_{kn}$$

$$= \frac{1}{2}((F_{km})^T F_{mn} - \delta_{kn})$$

$$E_{kn} = \frac{1}{2}\left[\begin{pmatrix} 1+t_1 & 0 & 0 \\ 0 & 1-t_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1+t_1 & 0 & 0 \\ 0 & 1-t_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]$$

$$= \frac{1}{2}\left[\begin{pmatrix} (1+t_1)^2 & 0 & 0 \\ 0 & (1-t_2)^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} - I\right]$$

$$= \frac{1}{2}\begin{bmatrix} 2t_1 + t_1^2 & 0 & 0 \\ 0 & -2t_2 + t_2^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Then $E_{kk} = \frac{1}{2}\left(-2t_2 + t_2^2 + 2t_1 + t_1^2\right)$

$$= -t_2 + t_1 + \frac{1}{2}\left(t_1^2 + t_2^2\right)$$

$$= t_1 - t_2$$

Thus $e - E_{kk} = II_F - 1 - E_{kk}$

$$= -t_1 t_2 - \frac{1}{2}\left(t_1^2 + t_2^2\right) = -\frac{1}{2}\left(t_1^2 + 2t_1 t_2 + t_2^2\right)$$

$$= -\frac{1}{2}\left(t_1 + t_2\right)^2$$

Pg 49 Drumheller

1.37 Deviatoric Strain $\equiv E'_{mn} = E_{mn} - \frac{1}{3}E_{kk}\delta_{mn}$

For problem 1.33 $[E_{mn}] = \frac{1}{2}\begin{pmatrix} 0 & \Upsilon t & 0 \\ \Upsilon t & \Upsilon^2 t^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Then Deviatoric Strain becomes

$\{ E_{kk} = \frac{1}{2}\Upsilon^2 t^2$

$$[E'_{mn}] = \frac{1}{2}\begin{pmatrix} 0 & \Upsilon t & 0 \\ \Upsilon t & \Upsilon^2 t^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{1}{3}\left(\frac{1}{2}\right)\Upsilon^2 t^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{6}\Upsilon^2 t^2 & \frac{\Upsilon t}{2} & 0 \\ \frac{\Upsilon t}{2} & \Upsilon^2 t^2\left(\frac{1}{2}-\frac{1}{6}\right) & 0 \\ 0 & 0 & -\frac{1}{6}\Upsilon^2 t^2 \end{pmatrix} = \begin{pmatrix} -\frac{\Upsilon^2 t^2}{6} & \frac{\Upsilon t}{2} & 0 \\ \frac{\Upsilon t}{2} & \frac{\Upsilon^2 t^2}{3} & 0 \\ 0 & 0 & -\frac{\Upsilon^2 t^2}{6} \end{pmatrix}$$

From Problem 1.34

$$[E_{mn}] = \frac{1}{2}\begin{pmatrix} \Upsilon^2 t^2 + 2\Upsilon t & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Then the deviatoric strain tensor becomes:

$$E_{tt} = \frac{1}{2}(\gamma^2 t^2 + 2t\gamma)$$

deviatoric strain has NO affect on shear strain

$$[E'_{mn}] = \frac{1}{2}\begin{pmatrix} \gamma^2 t^2 + 2t\gamma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{1}{3}\frac{1}{2}(\gamma^2 t^2 + 2t\gamma)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \frac{1}{2}(\gamma^2 t^2 + 2t\gamma)\begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

(1.38) Spin

$$W_{km} = \frac{1}{2}(L_{km} - L_{mk})$$

w/ $$L_{km} = \frac{\partial \hat{F}_{kr}}{\partial t} F^{-1}_{rm}$$ velocity gradient tensor

From Ex 1.33 $$[\hat{F}] = \begin{pmatrix} 1 & ft & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Then $$\left[\frac{\partial \hat{F}}{\partial t}\right] = \begin{pmatrix} 0 & f & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[\hat{F}]^{-1} = \begin{pmatrix} 1 & -ft & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

So $$[L_{km}] = \begin{pmatrix} 0 & f & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & -ft & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & f(-t+1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The spin tensor

$$[W_{km}] = \frac{1}{2}\left[\begin{pmatrix} 0 & f(-t+1) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0 & 0 & 0 \\ f(-t+1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$[\omega_{km}] = \frac{1}{2}\left[\begin{pmatrix} 0 & t(-t+1) & 0 \\ t(t-1) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

For Ex 1.37 $\quad [\hat{F}_{ij}] = \begin{pmatrix} 1+4t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\frac{\partial \hat{F}}{\partial t} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad [F_{rm}^{-1}] = \begin{pmatrix} \frac{1}{1+4t} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\therefore \quad [L_{km}] = \frac{\partial \hat{F}}{\partial t}_{kr} F_{rm}^{-1}$$

$$= \begin{pmatrix} \frac{4}{1+4t} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The spin tensor

$$[\omega_{km}] = \frac{1}{2}\left([L_{km}] - [L_{km}]^T\right)$$

$$= \frac{1}{2} \cdot \underset{\approx}{0}.$$

Pg 49 Drumheller

(1.39) $W_{km} = 0$

But $W_{km} = \frac{1}{2}(L_{km} - L_{mk})$

$\Rightarrow L_{km} = L_{mk}$ $\therefore$ L is symmetric

From definition of L (velocity gradient tensor)

$$L_{km} = \frac{\partial v_k}{\partial x_m} = L_{mk} = \frac{\partial v_m}{\partial x_k}$$

$$\Rightarrow \underbrace{\frac{\partial v_k}{\partial x_m} - \frac{\partial v_m}{\partial x_k}}_{} = 0$$

$\Rightarrow$ kmj jth component of curl

$\Leftrightarrow \nabla \times \bar{v} = 0$

Pg 50 Drumheller

$$II_F = \frac{1}{2} F_{mn} F_{mn} = \frac{1}{2}\left(F_{11} F_{11} + F_{22} F_{22} + F_{33} F_{33} \right)$$

$$= \frac{1}{2} F^2 + \tilde{F}^2$$

$$F_{(km)} = \alpha_{(k)r} F_{rs} \alpha_{s(m)}$$

$$[F_{(km)}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & +\sin\theta_1 \\ 0 & -\sin\theta_1 & \cos\theta_1 \end{bmatrix} \begin{bmatrix} F & 0 & 0 \\ 0 & \tilde{F} & 0 \\ 0 & 0 & \hat{\tilde{F}} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} F & 0 & 0 \\ 0 & \tilde{F}\cos\theta_1 & \tilde{F}\sin\theta_1 \\ 0 & -\hat{F}\sin\theta_1 & \hat{F}\cos\theta_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_1 & -\sin\theta_1 \\ 0 & \sin\theta_1 & \cos\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} F & 0 & 0 \\ 0 & \tilde{F} & 0 \\ 0 & 0 & \hat{\tilde{F}} \end{bmatrix}$$

$$E_{km} = \frac{1}{2}\left(F_{\overset{\text{column sum}}{m}k} F_{\underset{\text{column sum}}{m}n} - \delta_{kn}\right) = \frac{1}{2}\left(F_{km}^{T} F_{mn}\right.$$

$$= \frac{1}{2}\left(\qquad ((F^T)_{km} F_{mn} - \delta_{kn}) \right)$$

row sum

column sum

$$= \frac{1}{2}\left(\begin{pmatrix} F & 0 & 0 \\ 0 & \hat{F} & 0 \\ 0 & 0 & \hat{F} \end{pmatrix}\begin{pmatrix} F & 0 & 0 \\ 0 & \hat{F} & 0 \\ 0 & 0 & \hat{F} \end{pmatrix} - I\right)$$

$$= \frac{1}{2}\left[\begin{pmatrix} F^2 & 0 & 0 \\ 0 & \hat{F}^2 & 0 \\ 0 & 0 & \hat{F}^2 \end{pmatrix} - I\right]$$

$$E = \frac{1}{2}(F^2 - 1) \qquad \tilde{E} = \frac{1}{2}(\tilde{F}^2 - 1)$$

$$I_E = \quad \ldots \quad \text{trace}$$

$$II_E = \frac{1}{2}\left(E_{11}^2 + 2E_{22}^2\right) = \frac{1}{2}\left(\frac{1}{4}(F^2-1)^2 + \frac{1}{2}(\hat{F}^2-1)^2\right)$$

$$= \frac{1}{8}(F^2-1)^2 + \frac{1}{4}(\hat{F}^2-1)^2 = \frac{1}{2}E_{kr}E_{kr}$$

$$III_E = E\hat{E}^2 = \frac{1}{8}(F^2-1)(\hat{F}^2-1)^2$$

$$E'_{km} = E_{km} - \frac{1}{3} E_{pp}\,\delta_{km}$$

$$\left[E'_{km}\right] = \begin{bmatrix} E & 0 & 0 \\ 0 & \hat{E} & 0 \\ 0 & 0 & \hat{E} \end{bmatrix} - \frac{1}{3}\left(E + 2\hat{E}\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{3}E - \frac{2}{3}\hat{E} & 0 & 0 \\ 0 & -\frac{1}{3}E + \frac{1}{3}\hat{E} & 0 \\ 0 & 0 & -\frac{1}{3}E + \frac{1}{3}\hat{E} \end{bmatrix}$$

$$II_{E'} = \frac{1}{2}\left(E - \hat{E}\right)^2\left[\frac{4}{9} + \frac{2}{9}\right] = \frac{1}{3}\left(E - \hat{E}\right)^2 = \frac{1}{12}\left(F^2 - \hat{F}^2\right)^2$$

$$III_{E'} = \frac{2}{27}\left(E - \hat{E}\right)^3 = \frac{\cancel{2}}{27}\,\frac{1}{\cancel{8}_{4}}\left(F^2 - \hat{F}^2\right)^3 = \frac{1}{108}(\dots)$$

$$\begin{array}{r} 27 \\ \underline{4} \\ -12 + 120 \\ = 108 \end{array}$$

$$2\epsilon + \epsilon^2 = 2E_{mn}\,D_m(d\tilde{\underline{X}})\,D_n(d\tilde{\underline{X}})$$

$$= 2E\,D_1(d\tilde{\underline{X}})\,D_1(d\tilde{\underline{X}}) + \cancel{2\hat{E}\,D} \; 2\hat{E}\left[\underbrace{D_2^2(d\tilde{\underline{X}}) + D_3^2(d\tilde{\underline{X}})}_{= 1 - D_1^2(d\tilde{\underline{X}})}\right]$$

$$2\epsilon + \epsilon^2 = 2E\,D_1^2(d\tilde{\underline{X}}) + 2\hat{E} - 2\hat{E}\,D_1^2(d\tilde{\underline{X}})$$

$$\Rightarrow \quad 2\epsilon + \epsilon^2 = 2\underbrace{(E-\hat{E})}_{\frac{1}{2}(F^2-\tilde{F}^2)} D_1^2(d\vec{X}) + 2\underbrace{\tilde{E}}_{\frac{1}{2}(\hat{F}^2-1)}$$

$\Rightarrow$

$$2\epsilon + \epsilon^2 = (F^2 - \hat{F}^2)\cos^2\theta + \hat{F}^2 - 1$$

$$(\epsilon+1)^2 = F^2\cos^2\theta - \hat{F}^2(\cos^2\theta - 1)$$

$$=$$

$$\Rightarrow \quad \frac{(1+\epsilon)^2}{\hat{F}^2} = \bar{F}^2\cos^2\theta + \sin^2\theta$$

$$\bar{\lambda} = \frac{1+\epsilon}{\tilde{F}} = \frac{\Lambda}{\tilde{F}}$$

$$\Rightarrow \quad \bar{\lambda}^2 = (\bar{F}^2\cos^2\theta + \sin^2\theta)^{1/2}$$

$$D_1(\cancel{d\vec{x}})|d\vec{x}| = D_1(d\vec{x})\sqrt{F^2 d\bar{X}_1^2 + \cancel{\hat{F}^2 d\bar{X}_2^2} + \hat{F}^2 d\bar{X}_3^2}$$

$$dx_1 = D_1(dx)\,|d\vec{x}| \qquad \text{As } dx_1 = F\,d\bar{X}_1$$

$$\& \quad d\bar{X}_1 = D_1(d\bar{X})\,|d\bar{X}_1| \quad \Rightarrow \quad D_1(dx)|d\vec{x}| = F D_1(d\vec{X})|d\vec{X}|$$

$$\cos\beta = F\cos\theta \underbrace{\frac{|d\bar{x}|}{|dx|}}_{\frac{1}{\Lambda}} = \frac{F\cos\theta}{\Lambda}$$

$$\cos\beta = \frac{\bar{F}\cos\theta}{}$$

One eq $\bar{\Lambda} = \left(\bar{F}^2\cos^2\theta + \sin^2\theta\right)^{1/2}$ v.s. x_1 & x_2 placement of end pt of $\partial(d\vec{x})$.

If $\tilde{F} = 1$ → $\bar{\Lambda} = \Lambda$

$$\bar{F} = \frac{F}{\tilde{F}} = F$$

If $F = 1$ $\Lambda = 1$ By eq 1.100

$$F = \frac{\partial\hat{x}_1}{\partial x_1} < 1$$

For β

$$\bar{\Lambda} = \left(\bar{F}^2 \frac{\bar{\Lambda}^2}{\bar{F}^2}\cos^2\beta + 1 - \frac{\bar{\Lambda}^2}{\bar{F}^2}\right.$$

$$\sin\gamma = \frac{2E\, D_1(d\vec{A})\, D_1(d\vec{B}) + 2\tilde{E}\left[D_2(d\vec{A})\, D_2(d\vec{B}) + D_3(d\vec{A})\, D_3(d\vec{B})\right]}{\left[2E\, D_1^2(d\vec{A}) \cancel{D_1(d\vec{B})} + 2\tilde{E}\left[D_2^2(d\vec{A}) + D_3^2(d\vec{A})\right] + 1\right]^{1/2} \cdot \left[2E\, D_1^2(d\vec{B}) + 2\hat{E}\left[D_2^2(d\vec{B}) + D_3^2(d\vec{B})\right] + 1\right]^{1/2}}$$

$$= \frac{2E\, D_1(d\vec{A})\, D_1(d\vec{B}) + 2\tilde{E}\left[-D_1(d\vec{A})\, D_1(d\vec{B})\right]}{\left[2E\, D_1^2(d\vec{A}) + 2\tilde{E}\left(1 - D_1^2(d\vec{A})\right) + 1\right]^{1/2} \cdot \left[2E\, D_1^2(d\vec{B}) + 2\tilde{E}\left(1 - D_1^2(d\vec{B})\right) + 1\right]^{1/2}}$$

$$= \frac{2(E - \tilde{E})\, D_1(d\vec{A})\, D_1(d\vec{B})}{\left[2(E - \tilde{E})\, D_1^2(d\vec{A}) + 2\hat{E} + 1\right]^{1/2} \cdot \left[2(E - \tilde{E})\, D_p^2(d\vec{B}) + 2\hat{E} + 1\right]^{1/2}}$$

let $\bar{E} = \dfrac{E - \tilde{E}}{\hat{F}^2} = \dfrac{1}{2}\dfrac{F^2 - \hat{F}^2}{\hat{F}^2} = \dfrac{1}{2}\left(\dfrac{F^2}{\hat{F}^2} - 1\right)$

$$= \frac{1}{2}\left(\bar{F}^2 - 1\right)$$

$$\Rightarrow \quad \sin\nu = \frac{2\bar{E}\, D_1(\partial\vec{A})\, D_1(\partial\vec{B})}{\left[2\bar{E} D_1^2(\partial\vec{A}) + 1\right]^{1/2}\left[2\bar{E} D_1^2(\partial\vec{B}) + 1\right]^{1/2}}$$

$$\sin^2\nu = \frac{4\bar{E}^2 D_1^2(\partial\vec{A})\, D_1^2(\partial\vec{B})}{\left(2\bar{E} D_1^2(\partial\vec{A}) + 1\right)\left(2\bar{E} D_1^2(\partial\vec{B}) + 1\right)}$$

$$= \frac{4\bar{E}^2 D_1^2(\partial\vec{A})\, D_1^2(\partial\vec{B})}{4\bar{E}^2 D_1^2(\partial\vec{A})\, D_1^2(\partial\vec{B}) + 2\bar{E}\left(D_1^2(\partial\vec{A}) + \cancel{\bar{E}}\, D_1^2(\partial\vec{B})\right) + 1}$$

$$= \frac{\phi}{\phi + \eta}$$

$$\eta = 1 + 2\bar{E}\left[D_1^2(\partial\vec{A}) + D_1^2(\partial\vec{B})\right]$$

$$> 1 - \left[D_1^2(\partial\vec{A}) - D_1^2(\partial\vec{B})\right] > 0$$

$$\sin^2\nu = \frac{1}{\frac{\eta}{\phi} + 1} \leq 1$$

(A)

~~Pg~~ 56 Drumheller

Max $\sin(2\theta) = 1 \Rightarrow 2\theta = \frac{\pi}{2}, \cancel{\pi\ (-\pi)} + 2n\pi.$

$$\theta = \frac{\pi}{4} + n\pi \qquad n = 0, \pm 1, \pm 2, \ldots$$

Min $\sin(2\theta) = -1 \rightarrow 2\theta = -\frac{\pi}{2} + 2n\pi \qquad n = 0, \pm 1, \pm 2, \ldots$

$$\theta = -\frac{\pi}{4} + n\pi$$

Take $n = -1, 0$ in 1st $\sin(2\theta) = 1$ w/ $\theta = \frac{\pi}{4}, -\frac{3\pi}{4}$

Take $n = +1, 0$ " 2nd $\sin(2\theta) = -1$ w/ $\theta = -\frac{\pi}{4}, \frac{3\pi}{4}$.

$$\therefore \quad \sin\gamma = \frac{-\bar{E}}{(2\bar{E}\,\frac{1}{2} + 1)^{1/2}(2\bar{E}\,\frac{1}{2} + 1)^{1/2}} \qquad \theta = \frac{\pi}{4}, -\frac{3\pi}{4}$$

$$= \frac{-\bar{E}}{(\bar{E} + 1)}$$

~~Let~~ $\gamma_{max} = \sin^{-1}\left(\frac{\bar{E}}{\bar{E}+1}\right) = \sin^{-1}\left(\frac{\frac{1}{2}(F^2 - 1)}{\frac{1}{2}F^2 - \frac{1}{2} + 1}\right)$

if $\theta =$

$$= \sin^{-1}\left(\frac{F^2 - 1}{F^2 + 1}\right)$$

pg 57 Drumheller

(1.86) $L_{km} = \frac{\partial \hat{F}_{kr}}{\partial t} F^{-1}_{rm}$ $\quad [F_{rm}] = \begin{bmatrix} F & 0 & 0 \\ 0 & \hat{F} & 0 \\ 0 & 0 & \hat{F} \end{bmatrix}$

$\left[\frac{\partial \hat{F}_{kr}}{\partial t}\right] = \ldots$ $\quad F^{-1}_{rm} = \begin{bmatrix} 1/F & 0 & 0 \\ 0 & 1/\hat{F} & 0 \\ 0 & 0 & 1/\hat{F} \end{bmatrix}$

$\frac{\partial \hat{F}_{kr}}{\partial t} \cdot F^{-1}_{rm}$

$\uparrow \qquad \uparrow$

$\rightarrow$ row summing $\quad$ column summing $\quad \ldots$

$$[L_{km}] = \begin{bmatrix} F_t & 0 & 0 \\ 0 & \hat{F}_t & 0 \\ 0 & 0 & \hat{F}_t \end{bmatrix} \begin{bmatrix} F^{-1} & 0 & 0 \\ 0 & \hat{F}^{-1} & 0 \\ 0 & 0 & \hat{F}^{-1} \end{bmatrix}$$

$$D_{km} = \frac{1}{2}(L_{km} + L_{mk})$$

$$= \ldots$$

$$W_{km} = \frac{1}{2}(L_{km} - L_{mk}) = \vec{0}$$

(1.40) $\hat{x}_k = \bar{X}_k(1+\gamma_k t)$

(a) spherical deformation $\Longrightarrow$ $[F_{km}] = \begin{pmatrix} F & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & F \end{pmatrix}$

Now $F_{km} = \dfrac{\partial \hat{x}_k}{\partial X_m} =$

$\dfrac{\partial \hat{x}_1}{\partial \bar{X}_1} = (1+\gamma_1 t)$ $\quad \dfrac{\partial \hat{x}_1}{\partial \bar{X}_j} = 0 \quad j = 2,3$

$\dfrac{\partial \hat{x}_2}{\partial \bar{X}_2} = (1+\gamma_2 t)$ $\quad \dfrac{\partial \hat{x}_2}{\partial \bar{X}_j} = 0 \quad j = 1,3$

$\dfrac{\partial \hat{x}_3}{\partial \bar{X}_3} = 1+\gamma_3 t$ $\quad \dfrac{\partial \hat{x}_3}{\partial \bar{X}_j} = 0 \quad j = 1,2$

Then $[F_{km}] = \begin{pmatrix} 1+\gamma_1 t & 0 & 0 \\ 0 & 1+\gamma_2 t & 0 \\ 0 & 0 & 1+\gamma_3 t \end{pmatrix}$

Thus for spherical deformation

$1+\gamma_1 t = 1+\gamma_2 t = 1+\gamma_3 t \rightarrow \gamma_1 = \gamma_2 = \gamma_3$

~~Uniaxial~~ Uniaxial deformation $\begin{pmatrix} F & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Then $\quad 1+f_2t = 1 = 1+f_3t$

$\Rightarrow \quad f_2 = 0 = f_3.$

f_1 arbitrary

$$[F_{km}] = \begin{pmatrix} 1+f_1t & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(c) triaxial deformation $\Rightarrow \quad [F_{km}] = \begin{pmatrix} F & 0 & 0 \\ 0 & \tilde{F} & 0 \\ 0 & 0 & \tilde{F} \end{pmatrix}$

Then $\quad 1+f_2t = 1+f_3t \quad \Rightarrow \quad f_2 = f_3$

f_1 arb

$[F_{km}]$
triaxial deformation
$$= \begin{pmatrix} 1+f_1t & 0 & 0 \\ 0 & 1+f_2t & 0 \\ 0 & 0 & 1+f_2t \end{pmatrix}$$

Pg 57 Drumheller

1.41

$$x_1 = X_1(1+f_1 t) + X_2 f_2 t \qquad x_2 = X_2(1+f_3 t)$$

$$x_3 = X_3(1+f_3 t)$$

(a) spherical deformation $\Rightarrow [F_{km}] = \begin{pmatrix} F & 0 & 0 \\ 0 & F & 0 \\ 0 & 0 & F \end{pmatrix}$

$$[F_{km}] = \begin{pmatrix} 1+f_1 t & f_2 t & 0 \\ 0 & 1+f_3 t & 0 \\ 0 & 0 & 1+f_3 t \end{pmatrix}$$

$\Rightarrow$ $f_2 = 0$ & $f_1 = f_3$ for spherical deformation

(b) ~~[illegible]~~ uniaxial deformation $\begin{pmatrix} F & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

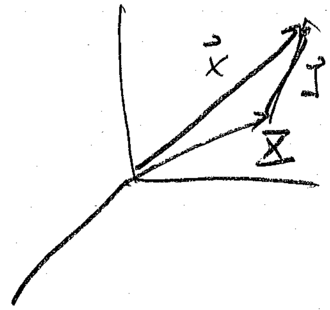
$f_3 = 0 \quad f_1$ arb $\quad f_2 = 0$

(c) Triaxial deformation $\begin{pmatrix} F & 0 & 0 \\ 0 & \hat{F} & 0 \\ 0 & 0 & \hat{F} \end{pmatrix}$

f_1 & f_3 arb $\qquad f_2 = 0$

Pg 58 Drumheller

(1.42)



(a) $\vec{x} - \vec{X} =$

$\vec{X}_B(1+4t)$ = radial ?

(b) $[F_{km}] = \begin{pmatrix} 1+4t & 0 & 0 \\ 0 & 1+4t & 0 \\ 0 & 0 & 1+4t \end{pmatrix}$

(c) ? What is asked for ? Matrix is

(d) Vol strain $e = \mathrm{III}_F - 1 = (1+4t)^3 - 1$

(1.43) see notes w/ pg 51

Pg 65 Drumheller

From ~~1.114~~ $\cancel{t_m(\vec{n}) = T_{km} n_k}$ Eq bellow 1.114.

$\cancel{\alpha_{r(m)}}$ $\qquad t^{\circ}_s(\vec{n}^{\circ}) = T^{\circ}_{rs} n^{\circ}_r$

$\cancel{\alpha_{r(m)} t_m} = \cancel{\alpha_{r(m)} T_{km} n_k}$

$\alpha_{(m)s} \longrightarrow$

$$\alpha_{(m)s} t^{\circ}_s = \alpha_{(m)s} T^{\circ}_{rs} n^{\circ}_r$$

$$t_m = \alpha_{(m)s} T^{\circ}_{rs} \alpha_{r(k)} n_k$$

$$= \alpha_{s(m)} T^{\circ}_{rs} \alpha_{r(k)} n_k$$

Think eq $n^{\circ}_r = \alpha_{r(k)}(t)\, n_k$ should be

$x_{(r)} = \alpha_{(r)m} X_m$

new $\quad$ old

$n^{\circ}_r =$

$X_m = x_{(r)} \alpha_{(r)m}$

old $\quad$ new.

$\alpha_{k(m)} = \alpha_{(m)k}$

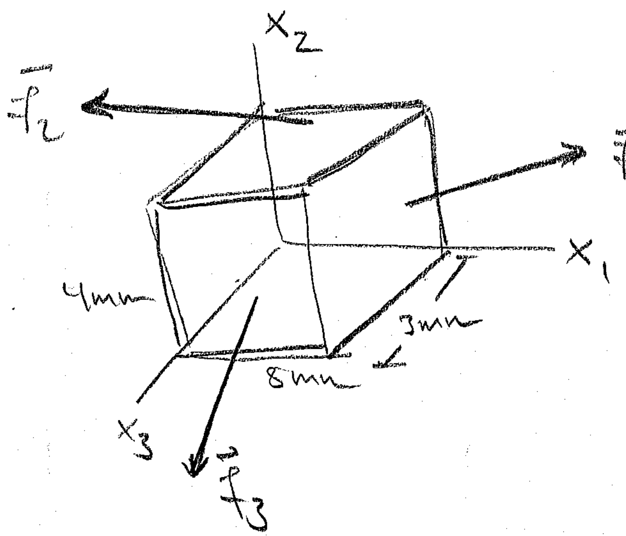
$\Rightarrow$

Pg 66 Drumheller

(1.50)

(a) Traction on a face w/ normal $\hat{n}$ is given by

$$\bar{t}(n) = \frac{\bar{f}}{\Delta S}$$



$$\bar{t}(\hat{i}_3) = \frac{\bar{f}_3}{(4mm)(5mm)}$$

$$= \frac{-20\hat{i}_2 - 40\hat{i}_3}{20\ mm^2} = 10^{+6}\left(\frac{-20\hat{i}_2 - 40\hat{i}_3}{20}\right)\frac{N}{m^2}$$

$$= 10^{+6}\left(-\hat{i}_2 - 20\hat{i}_3\right)\frac{N}{m^2}$$

$1mm = 10^{-3}m$

$$\bar{t}(\hat{i}_1) = \frac{\bar{f}_1}{12\ mm^2} = \frac{12\hat{i}_1 - 4\hat{i}_2}{12\ mm^2} = 10^6\left(\hat{i}_1 - \frac{1}{3}\hat{i}_2\right)\frac{N}{m^2}$$

$$\bar{t}(\hat{i}_2) = \frac{\bar{f}_2}{15\ mm^2} = \frac{X\hat{i}_1 + 30\hat{i}_2 + Y\hat{i}_3\ N}{15m^2} \cdot 10^6$$

From $\vec{t}(-\hat{n}) = -\vec{t}$

one gets that $\vec{t}(-\hat{i}_1) = -10^6\left(\hat{i}_1 - \frac{\hat{i}_2}{3}\right) \frac{N}{m^2}$

$$\vec{t}(-\hat{i}_2) = -\frac{(x\hat{i}_1 + 30\hat{i}_2 + y\hat{i}_3)}{15} \cdot 10^6 \frac{N}{m^2}$$

$$\vec{t}(-\hat{i}_3) = -10^6(-\hat{i}_2 - 20\hat{i}_3) \frac{N}{m^2}$$

(b) From eq $\cancel{\vec{t}(\hat{n}) = T_{km} n_k \hat{i}_m}$

$\Rightarrow$ $\cancel{t_m(\hat{n}) = T_{km} n_k}$

eq 1.110 $\vec{t}(\hat{i}_k) = T_{km}\hat{i}_m$

$\vec{t}(\hat{i}_1) = 10^6\hat{i}_1 - \frac{10^6}{3}\hat{i}_2$ $\quad T_{11} = 10^6$; $T_{12} = \frac{-10^6}{3}$; $T_{13} = 0$

$\vec{t}(\hat{i}_2) = 10^6\frac{x}{15}\hat{i}_1 + 2\cdot 10^6\hat{i}_2 + \frac{10^6 y}{15}\hat{i}_3$ $\quad T_{21} = \frac{10^6 x}{15}$ $\quad T_{22} = 2\cdot 10^6$ $\quad T_{23} = \frac{10^6 y}{15}$

$\vec{t}(\hat{i}_3) = -10^6\hat{i}_2 - 2\cdot 10^7\hat{i}_3$ $\quad T_{31} = 0$ $\quad T_{32} = -10^6$

$T_{33} = -2\cdot 10^7$

$$[T_{km}] = 10^6 \begin{pmatrix} 1 & -1/3 & 0 \\ \frac{x}{15} & 2 & \frac{y}{15} \\ 0 & -1 & -20 \end{pmatrix} = \text{cauchy stress}$$

(c) $-\frac{1}{3} = \frac{x}{15} \Rightarrow x = -5$

$+ \quad -1 = \frac{y}{15} \Rightarrow y = -15$

(d) $t(-\hat{e}_1) = -10^6\left(\hat{e}_1 - \frac{\hat{e}_2}{3}\right) = +\frac{\tilde{f}_1}{\Delta S_1} \Rightarrow \tilde{f}_1 = (3.4\,mm^2)\left\{-10^6\left(\hat{e}_1 - \frac{\hat{e}_2}{3}\right)\right\}$

etc ..

Pg 62 Drumheller

(1.51) $\hat{n} = \ldots$

(a) $\vec{t}(\hat{n}) = \vec{t}(\hat{i}_k)\, n_k$

$\Rightarrow \quad t_m(\hat{n}) = T_{km}\, n_k$

$$\vec{t}(\hat{n}) = \begin{pmatrix} \\ \\ \\ \end{pmatrix} = \underbrace{\begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}}_{\text{matrix from above}} \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix}$$

(b) $\vec{t}(\hat{n}) \cdot \hat{n}$ is component (magnitude) acting normal to this plane. $(\vec{t}(\hat{n}) \cdot \hat{n})\, \hat{n}$ is the vector

(c) $\vec{t}(\hat{n}) - (\vec{t}(\hat{n}) \cdot \hat{n})\, \hat{n}$ is the component acting tangent

(1.52)

(a) In order to have a "traction free" point

$$\Rightarrow \vec{t}(\hat{n}) = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \vec{0} \Rightarrow T_{ij}$$

Normal to the plane

$|T_{ij}| = 0$

$$|T_{ij}| = T_{11}(-4) - 2(-2) + 1\cdot 4$$

$$= -4T_{11} + 4 + 4 = 0$$

$$-4T_{11} = -8$$

$$T_{11} = 2$$

~~Then~~ Then the normal $\begin{pmatrix} 2 & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \vec{0}$

$$\begin{pmatrix} 1 & 1 & 1/2 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1/2 \\ 0 & -2 & 1 \\ 0 & 1 & -1/2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 1 & 1/2 \\ 0 & 1 & -1/2 \\ 0 & 1 & -1/2 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\Rightarrow \quad n_1 = -n_3$$

$$n_2 = \frac{n_3}{2} \qquad \vec{n} = \begin{pmatrix} -n_3 \\ \frac{n_3}{2} \\ n_3 \end{pmatrix} = n_3 \begin{pmatrix} -1 \\ 1/2 \\ 1 \end{pmatrix}$$

pg 69 Drumheller

$$II_T = \frac{1}{2} T_{km} T_{km}$$

$$= \frac{1}{2}(-p)^2 \delta_{km} \delta_{km} = \frac{3}{2} p^2$$

$$III_T = \det[T_{km}] = -p^3$$

$$-p\, \alpha_{(k)i}\, \delta_{ij}\, \alpha_{j(m)} = -p\,\delta$$

---

eq 1.112 $\quad T_{(kr)} = \alpha_{(k)m} T_{mn} \alpha_{n(r)} \qquad [T_{mn}] = \begin{bmatrix} T & 0 & 0 \\ 0 & \tilde{T} & 0 \\ 0 & 0 & \tilde{T} \end{bmatrix}$

$$= \alpha_{(k)1} T_{11} \alpha_{1(r)}$$

$$\cancel{\alpha_{(k)n}\alpha_{(m)n}} \quad \alpha_{(k)n}\,\alpha_{(m)n} = \alpha_{(k)n}\,\alpha_{n(m)} =$$

col. sum. $\qquad$ row sum

$\Rightarrow$

Here

$$T_{(km)} = \alpha_{(k)1} T \alpha_{1(m)} + \tilde{T}\,[\underbrace{\alpha_{(k)2}\alpha_{2(m)} + \alpha_{(k)3}\alpha_{3(m)}}_{=\ \delta_{km} - \alpha_{(k)1}\alpha_{1(m)}}]$$

if $\quad \alpha_{(k)i}\,\alpha_{i(m)} = \delta_{km} \; + \Rightarrow$

$$\Rightarrow T_{(km)} = (T - \tilde{T})\, \alpha_{(k)1}\, \alpha_{(m)} + \tilde{T}\, \delta_{km} .$$

$$\sigma = \frac{1}{3} T_{kk} = \frac{1}{3} I_T$$

$$[T_{km}'] = [T_{km}] - \sigma \delta_{km}$$

$$= \begin{bmatrix} T & 0 & 0 \\ 0 & \hat{T} & 0 \\ 0 & 0 & \tilde{T} \end{bmatrix} - \frac{1}{3}(T + 2\hat{T}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{pmatrix} \frac{2}{3}T - \frac{2}{3}\hat{T} & 0 & 0 \\ 0 & -\frac{1}{3}T + \frac{1}{3}\hat{T} & 0 \\ 0 & 0 & -\frac{1}{3}T + \frac{1}{3}\hat{T} \end{pmatrix}$$

$$II_{T'} = \frac{1}{2}(T - \hat{T})^2 \left[\frac{6}{9}\right] = \frac{1}{3}(\quad)^2$$

$$III_{T'} = \frac{(T - \hat{T})^3}{27}\, 2 .$$

$$\alpha_{i(2)} = \cos\left(\frac{\pi}{2}+\theta\right) = -\sin\frac{\pi}{2}\sin\theta$$

$$\frac{\partial \hat{U}}{\partial X} = \frac{\partial \hat{x}}{\partial X}\frac{\partial \hat{U}}{\partial \hat{x}} = F\frac{\partial U}{\partial x}$$

$$\frac{\partial \hat{U}}{\partial X} = \left(1+\frac{\partial \hat{U}}{\partial X}\right)\frac{\partial U}{\partial x} = \left(1+F\frac{\partial U}{\partial x}\right)\frac{\partial U}{\partial x}$$

$$= \left(1+\left(1+\frac{\partial \hat{U}}{\partial X}\right)\frac{\partial U}{\partial x}\right)\frac{\partial U}{\partial x}$$

$$L = \frac{1}{F}\frac{\partial \hat{F}}{\partial t} = \cancel{\frac{\partial \hat{x}}{\partial t}\frac{\partial \hat{F}}{\partial x}} = \frac{1}{F}\frac{\partial \hat{V}}{\partial X} \quad \text{(By 1.132)}$$

$$= \cancel{\frac{x}{F}}\ \cancel{\frac{\partial \hat{F}}{\partial t}} = \frac{1}{F}\underbrace{\frac{\partial \hat{x}}{\partial X}}_{F}\frac{\partial V}{\partial x} = \frac{\partial V}{\partial x}$$

$$-\frac{\rho_0}{\rho^2}\frac{\partial \hat{\rho}}{\partial t} = \frac{\partial F}{\partial t} =$$

$$\frac{\rho_0}{\rho^2}\frac{\partial \hat{\rho}}{\partial t} = -\frac{\partial F}{\partial t} = \underbrace{-\frac{\partial \hat{v}}{\partial X}}_{\text{By eq 1.132}}$$

Pg 73 Drumheller

(1.53)

$$[T_{kn}] = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$$ triaxial stress

(a) ~~oppos~~ upon rotating the coordinate system ~~now~~ one can get large values of τ_{max}

$$\tau_{max} = \frac{1}{2}|3+3| = 3.$$

(b) mean normall stress $\frac{1}{3}T_{kk} = \frac{1}{3}(-3) = -1$

(c) $T_{(km)} = (T - \tilde{T})\,\alpha_{(k)1}\,\alpha_{1(m)} + \tilde{T}\,\delta_{km}$

pick a rotation in around $X_{(3)}$ $\Rightarrow [\alpha_{n(m)}] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

w/ $\theta = \frac{\pi}{4}$

$$[\alpha_{m(m)}] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T_{(km)} = \alpha_{(k)k}\,T_{kn}\,\alpha_{n(m)} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}.$$

$$= \frac{1}{\sqrt{2}}\underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}_{\text{Should be transposed.}}\begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}\frac{1}{\sqrt{2}}$$

$$= \frac{1}{2}\begin{bmatrix} 3 & 3 & 0 \\ 3 & -3 & 0 \\ 0 & 0 & -3\sqrt{2} \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 6 & 0 & 0 \\ 0 & -6 & 0 \\ 0 & 0 & -3\cdot 2 \end{bmatrix} = \begin{bmatrix} 3 & & \\ & -3 & \\ & & -3 \end{bmatrix}$$

$$\alpha_{(k)n}\,\alpha_{(m)n} = \delta_{km}$$

$$T_{(km)} = (T - \tilde{T})\,\alpha_{(k)l}\,\alpha_{l(m)} + \tilde{T}\,\delta_{km} \qquad \text{eq } 1.122$$

$$= (T - \tilde{T})\begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{pmatrix}\overbrace{\begin{matrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \end{matrix}} + \tilde{T}\,\delta_{km}$$

$$= (T - \tilde{T})\begin{pmatrix} 1/2 & -1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \tilde{T}\,\delta_{km}$$

Now: $T_{(km)} =$

$$= \alpha_{(k)p} T_{pq} \alpha_{q(m)}$$

Thus $$T_{(km)} = \frac{1}{\sqrt{2}} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}_{\text{transposed matrix}} \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix}}_{\text{untransposed}} \frac{1}{\sqrt{2}}$$

$$=$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} 3 & -3 & 0 \\ -3 & -3 & 0 \\ 0 & 0 & -3\sqrt{2} \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$= \frac{1}{(\sqrt{2})^2} \begin{bmatrix} 0 & -6 & 0 \\ -6 & 0 & 0 \\ 0 & 0 & -3\cdot 2 \end{bmatrix} = \begin{bmatrix} 0 & -3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

The normal stresses then become 0, 0, −3.

Notice that after the rotation that gives maximal shear stress

$$T_{(km)} = (T - \tilde{T})\,\alpha_{(k)1}\,\alpha_{1(m)} + \tilde{T}\,\delta_{km}$$

$T_{(11)} = 6(\frac{1}{2}) - 3 = 0$

$T_{(22)} = 6(\frac{1}{2}) - 3 = 0$ same as above

$T_{(33)} = 6\,\alpha_{(3)1}\,\alpha_{1(3)} + -3 = 6\,\alpha_{1(3)}^2 - 3 = 6\cdot 0 - 3 = -3$

Pg 73 Drumheller

(1.54)

$$[T_{km}] = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

(a) $\tau_{max} = \frac{1}{2}|T - \hat{T}| = \frac{1}{2} 3$ MPa

(b) $\bar{\sigma} = \frac{1}{3} T_{kk} = \underline{1}$ invariant under coordinate transformations.

(c) Normal stresses after the coordinate transformation that yeild maximal magnitudes of shear stress are:

$$T_{(km)} = (T - \hat{T})\, \alpha_{(k)1}\, \alpha_{1(m)} + \hat{T}\, \delta_{km}$$

$\hat{T} = 0$ in this case

$$T_{(11)} = (T - \hat{T})\, \alpha_{(1)1}\, \alpha_{1(1)} + \hat{T}\, \delta_{11}$$

$$= T\, \alpha_{(1)1}\, \alpha_{1(1)} = 3\left(\tfrac{1}{2}\right) = \frac{3}{2}$$

$$T_{(22)} = 3\, \alpha_{(2)1}\, \alpha_{1(2)} = 3\left(\tfrac{1}{2}\right) = \frac{3}{2}$$

$\alpha_{(2)1} = \alpha_{1(2)} = \frac{1}{\sqrt{2}}$

$$T_{(33)} = 3\, \alpha_{(3)1}\, \alpha_{1(3)} = 3\, \alpha_{1(3)}^2 = 3 \cdot 0 = 0$$

We can check the above by computing full

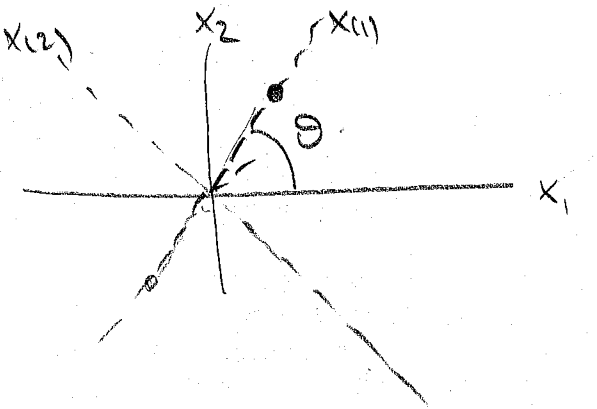
$[T_{(km)}]$

$T_{(km)} = \alpha_{(k)i} T_{ij} \alpha_{j(m)}$ w/ $\frac{\pi}{4}$ rotation in counterclockwise rotation.

$$[\alpha_{i(j)}] = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3/\sqrt{2} & 0 & 0 \\ -3/\sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$



$$= \begin{bmatrix} 3/2 & -3/2 & 0 \\ -3/2 & 3/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\alpha_{i(j)}$ transforms a vector in $x_{(1)}, x_{(2)}$ into x_1 & x_2

$\alpha_{(j)i} = \alpha_{i(j)}$

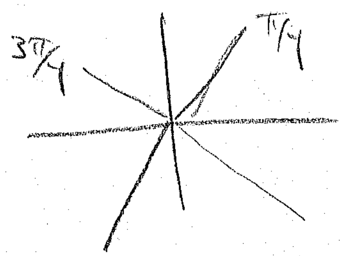
Pg 73 Drumheller

(1.56)

$$T_{(km)} = \alpha_{(k)i} T_{ij} \alpha_{j(m)}$$

(a)

$$\theta_3 = \frac{\pi}{4},\ \frac{3\pi}{4},\ \pi + \frac{\pi}{4},\ 2\pi - \frac{\pi}{4}$$



$$[\alpha_{n(m)}] = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus

$$\begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_3 & \sin\theta_3 & 0 \\ -\sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -10\cos\theta_3 & 10\sin\theta_3 & 0 \\ -2\sin\theta_3 & -2\cos\theta_3 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -10\cos^2\theta_3 - 2\sin^2\theta_3 \\ \\ \end{bmatrix}$$

(b) $I_T = T_{kk}$ independent of coordinate system

$= -14$

$II_T = \frac{1}{2} T_{km} T_{km}$ independent of coordinate system

$= \frac{1}{2}(10^2 + 4^2 + 4) = \frac{1}{2}(108) = 54$

$III_T = \det[T_{km}] =$ independent of coord system.

$= -10 \cdot 4 = -40$

(1.57) A coordinate transformation that is a rotation about $x_{(2)}$ looks like

$$[\alpha_{n(m)}] = \begin{bmatrix} \cos\theta_2 & 0 & +\sin\theta_2 \\ 0 & 1 & 0 \\ -\sin\theta_2 & 0 & \cos\theta_2 \end{bmatrix}$$

Then $[T_{km}] = \begin{bmatrix} T & 0 & 0 \\ 0 & \tilde{T} & 0 \\ 0 & 0 & \tilde{T} \end{bmatrix}$

$T_{(km)} = \quad \alpha_{(k)i} T_{ij} \alpha_{j(m)} =$

$$[T_{(km)}] = \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta_2 & 0 & +\cos\theta_2 \end{bmatrix} \begin{bmatrix} T & 0 & 0 \\ 0 & \hat{T} & 0 \\ 0 & 0 & \hat{T} \end{bmatrix} \begin{bmatrix} \cos\theta_2 & 0 & -\sin\theta_2 \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta_2 \end{bmatrix}$$

$= \ldots$

pg 79 Drumheller

$$\frac{\partial v}{\partial X} = \frac{\partial x}{\partial X}\frac{\partial v}{\partial x} = F\frac{\partial v}{\partial x}$$

$$\frac{\rho_0}{\rho^2}\frac{\partial \hat{\rho}}{\partial t} = -F\frac{\partial v}{\partial x}$$

$$\frac{\partial \hat{\rho}}{\partial t} + \underbrace{\frac{\rho^2}{\rho_0}F}\frac{\partial v}{\partial x} = 0$$

eq 4.137 $F = \frac{\rho_0}{\rho}$

$$\frac{\rho^2}{\rho_0}F = \rho$$

---

eq $\frac{\partial}{\partial t}(\hat{\rho}\hat{v}\,d\hat{x})\,d\cancel{S} = \frac{\partial T}{\partial x}\,dx\,\cancel{dS} + \rho b\,dx\,\cancel{dS}$

$\hat{\rho}\,d\hat{x} = \rho_0\,dX$ ind of time

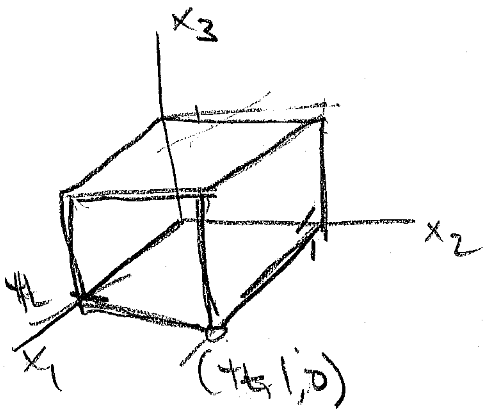
$$\frac{\partial}{\partial t}(\hat{v}) = \frac{1}{\rho}\frac{\partial T}{\partial x} + b$$

$$\rho a = \frac{\partial T}{\partial x} + \rho b$$

$$\frac{\partial T}{\partial x} = \frac{\rho}{\rho_0}\frac{\partial \hat{T}}{\partial X}$$

pg 85 Dunhellen

(1.58) $\vec{v} = \frac{x_1 4t}{1+4t}\hat{e}_1 - x_2 4t\,\hat{e}_2$



Gauss Thm $$\int_V \frac{\partial u_k}{\partial x_k}\,dV = \int_S u_k n_k\,dS$$

Vol. integral

$$\int_V \frac{\partial u_k}{\partial x_k}\,dV = \int_V \left(\frac{4t}{1+4t} - 4t\right) dV = \int_V 4t\left(\frac{1-1-4t}{1+4t}\right) dV$$

$$= -\int_V \frac{(4t)^2}{1+4t}\,dV = \frac{-(4t)^2}{1+4t}\,1^2\,(4t) = -\frac{(4t)^3}{1+4t}$$

Surface integral:

$$\int_S u_k n_k\,dS = \sum_{6\text{ faces}}\int = \cancel{\int_{\text{top, bottom}}}^{\,0} + \int_{x_1=4t} \frac{x_1 4t}{1+4t}\,dS - \int_{x_1=0} \frac{x_1 4t}{1+4t}\,dS$$

$$+ \int_{x_2=1} -x_2 4t\,dS + \int_{x_2=0} x_2 4t\,dS$$

$$= \frac{(\gamma t)^2}{1+\gamma t}\int_{x_1=\gamma t} dS - \gamma t \int_{x_2=1} dS$$

$$= \frac{(\gamma t)^2}{1+\gamma t}\cdot 1 - \gamma t(\gamma t) = (\gamma t)^2\left[\frac{1}{1+\gamma t} - \frac{1+\gamma t}{1+\gamma t}\right]$$

$$= -\frac{(\gamma t)^3}{1+\gamma t}$$ same !

(b) Leibniz rule for diff on integral.

$$\frac{df}{dt} = \frac{d}{dt}\int_{V(t)} \phi\, dV = \int_{V(t)} \phi_t\, dV + \int_{S(t)} \phi w_k n_k\, dS$$

let $f_1 = \int_{V(t)} v_1\, dV$

directly $f_1 = \int_{V(t)} \frac{x_1 \gamma t}{1+\gamma t}\, dV = \frac{\gamma t}{1+\gamma t}\int_{V(t)} x_1\, dV = \frac{\gamma t}{1+\gamma t}\, 1 \int_0^{\gamma t} x_1\, dx_1$

$$\frac{1+\gamma t}{\gamma t} f_1 = \frac{(\gamma t)^2}{2}$$

$f_1 = \frac{1}{2}\frac{(4t)^3}{1+4t}$ then $\frac{df_1}{dt} = \frac{1}{2}\left(\frac{3(4t)^2}{1+4t} - \frac{4(4t)^3}{(1+4t)^2}\right)$

using Leibniz rule $=$

$$\frac{df_1}{dt} = \int_{V(t)} \frac{\partial}{\partial t}(\varphi)\, dV + \int_{S(t)} \varphi(v_k n_k)\, dS$$

$$= \int_{V(t)} \left(\frac{x_1 4}{1+4t} - \frac{x_1 4t\, 4}{(1+4t)^2}\right) dV + \cancelto{0}{\int_{S_{\text{top, bottom}}}} + \int_{S_{x_2=1}} + \cancelto{0}{\int_{S_{x_2=0}}}$$

$$+ \int_{x_1=4t} + \cancelto{0}{\int_{x_1=0}}$$

$$= \left(\frac{4}{1+4t} - \frac{4^2 t}{(1+4t)^2}\right)\int_{V(t)} x_1\, dV + \int_{x_2=1}\left(\frac{x_1 4t}{1+4t}\right)\left(\frac{-(4t)^2}{1+4t}\right) dS$$

$$+ \int_{x_1=4t} \frac{(4t)^2}{1+4t}\left(\frac{-(4t)^2}{1+4t}\right) dS$$

$$= \frac{4}{1+4t}\Big($$

(c) $\vec{U} = \vec{x} - \vec{\underline{X}}$

$\parallel$

$\frac{x_1 + t}{1 + 4t}\hat{e}_1 - x_2 4t \hat{e}_2 + \vec{\underline{X}} = \vec{x}$

$\therefore$

$$\frac{x_1 4t}{1+4t} + \underline{X}_1 = x_1 \Rightarrow \left(\frac{\cancel{4t} - \cancel{4t} - 1}{1+4t}\right) x_1 = -\underline{X}_1$$

$$-x_2 4t + \underline{X}_2 = x_2 \Rightarrow (-4t-1)\, x_2 = -\underline{X}_2$$

$$\underline{X}_3 = x_3$$

$\therefore$ $\frac{-1}{1+4t} x_1 = -\underline{X}_1 \Rightarrow \hat{x}_1 = \underline{X}_1(1+4t)$

$\&$ $\hat{x}_2 = \frac{\underline{X}_2}{1+4t}$

$\hat{x}_3 = \underline{X}_3$ is the motion

Velocity of particles $\Rightarrow$ $\hat{v}_i = \frac{\partial \hat{x}_i}{\partial t} = \ldots$

(1.59)

$$\underbrace{\frac{d}{dt}\int_V \rho v_m \, dV}_{\text{material volume (moving w/ fluid)}} \underset{\substack{\uparrow \\ \text{transport thm}}}{=} \int_V \left(\frac{\partial}{\partial t}(\rho v_m) + \frac{\partial}{\partial x_k}(\rho v_m v_k)\right) dV$$

$$= \int_V \frac{\partial}{\partial x_k}(T_{km}) \, dV + \int_V \rho b_m \, dV$$

↑ Using Gauss's (Divergence thm) on Stress terms.

$$\Rightarrow \underline{\left(\frac{\partial \rho}{\partial t}\right)} v_m + \rho \frac{\partial v_m}{\partial t} + v_m \underline{\frac{\partial}{\partial x_k}(\rho v_k)} + \rho v_k \frac{\partial (v_m)}{\partial x_k} = \frac{\partial T_{km}}{\partial x_k} + \rho b_m$$

$$\rho \underbrace{\left(\frac{\partial v_m}{\partial t} + v_k \frac{\partial (v_m)}{\partial x_k}\right)}_{a_m} = \frac{\partial T_{km}}{\partial x_k} + \rho b_m$$

pg 86 Drumheller 1

(1.60) $\vec{x} \times \frac{d}{dt}(m\vec{v}) =$

RHS

$\frac{d}{dt}(\vec{x} \times m\vec{v})$ $\qquad \frac{d}{dt}$ = material time derivative i.e.

$\vec{x} = \vec{x}(\vec{X}, t)$

$\vec{v} = \vec{v}(\vec{X}, t)$

$= \underbrace{\frac{d\vec{x}}{dt} \times m\vec{v}}_{0} + \vec{x} \times \frac{d}{dt}(m\vec{v})$

(1.61) $\measuredangle$ mmt

$$\frac{d}{dt}\int_V (\vec{x} \times \rho\vec{v})\,dV = \int_S (\vec{x} \times \vec{t})\,dS + \int_V (\vec{x} \times \rho\vec{b})\,dV$$

LHS

looking at m th componet

~~Transport~~ th $\quad (\vec{x} \times \rho\vec{v})_m = x_i \rho v_j e_{ijm}$

$\Rightarrow$

$$\frac{d}{dt}\int_V x_i \rho v_j e_{ijm}\,dV = \int_S x_i t_j e_{ijm}\,dS + \int_V x_i \rho b_j e_{ijm}\,dV$$

Using the transport thm on 1st integral & ~~the div thm~~

~~(...)~~

$$\int_V \left(\partial_t(x_i \rho v_j e_{ijm}) + \partial_{x_k}(x_i \rho v_j e_{ijm} v_k)\right) dV$$

$$t_j = T_{kj} n_k$$

$$= \int_S x_i (T_{kj} n_k) e_{ijm} \, dS + \int_V x_i \rho b_j e_{ijm} \, dV$$

$$\Rightarrow \int_V \left(\partial_t(x_i \rho v_j e_{ijm}) + \partial_{x_k}(x_i \rho v_j e_{ijm} v_k)\right) dV$$

$$= \int_V \partial_{x_k}(x_i T_{kj}) e_{ijm} \, dV + \int_V x_i \rho b_j e_{ijm} \, dV$$

$$\Rightarrow \partial_t(x_i \rho v_j e_{ijm}) + \partial_{x_k}(x_i \rho v_j e_{ijm} v_k)$$

$$= \partial_{x_k}(x_i T_{kj}) e_{ijm} + x_i \rho b_j e_{ijm}$$

mmt conservation is

$$\boxed{\rho(\partial_t v_m + \partial_{x_k} v_m v_k) = \frac{\partial T_{km}}{\partial x_k} + \rho b_m}$$ or

~~eff [illegible]~~ cons of ∢ mmt becomes

$$x_i e_{ijm}\{\partial_t(\rho v_j)\} + e_{ijm}\{\delta_{ki}\rho v_j v_k + x_i \partial_{x_k}(\rho v_j v_k)\}$$

$$= \delta_{ki} T_{kj} e_{ijm} + x_i \partial_{x_k}(T_{kj}) e_{ijm}$$

$$+ x_i \rho b_j e_{ijm}$$

$$\Rightarrow x_i e_{ijm}\{\underline{\rho_t} v_j + \rho \partial_t v_j\} + e_{kjm}\rho v_j v_k \qquad i \to k$$

$$+ e_{ijm} x_i \{\underbrace{\partial_{x_k}(\rho v_k)}_{=0 \text{ cons of mass}} v_j + \rho v_k \partial_{x_k} v_j\}$$

$$= T_{kj} e_{kjm} + x_i \partial_{x_k}(T_{kj}) e_{ijm} + x_i \rho b_j e_{ijm}$$

$$\Rightarrow \quad x_i e_{ijm} \left\{ \underline{\rho \partial_t v_j + \rho v_k \partial_{x_k} v_j} \right\} + e_{kjm} \rho v_j v_k$$

$$= T_{kj} e_{kjm} + \underline{x_i e_{ijm} \frac{\partial}{\partial x_k}(T_{kj})} + \underline{x_i e_{ijm} \rho b_j}$$

$= 0$ By cons. of mmt

$$\Rightarrow \quad \underline{e_{kjm} \rho v_j v_k} = T_{kj} e_{kjm}$$

$\rho(e_{kjm} v_j v_k)$ = cross product of $\vec{v} \times \vec{v}$

$=$

$$kjm = \begin{array}{c} m \\ \left.\begin{matrix} 123 \\ 231 \\ 312 \end{matrix}\right\} +1 \end{array}$$

$$kjm = \left.\begin{matrix} 213 \\ 132 \\ 321 \end{matrix}\right\} = -1$$

$=$ look at fixed m say 3 v_1 + v_2 occur w/ opposite signs $= 0$

$$\Rightarrow T_{kj} e_{kjm} = 0$$

let $m = 1$

$$T_{kj} e_{kj1} = 0$$

$$\Rightarrow \left(T_{23} \underbrace{e_{231}}_{1} + T_{32} \underbrace{e_{321}}_{-1} \right) = 0$$

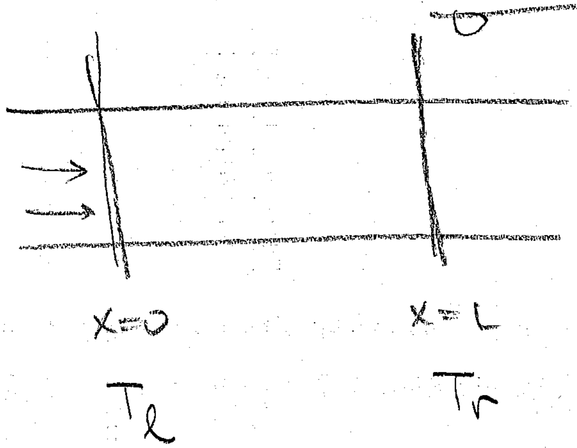
$$T_{23} = T_{32}$$ sim for the others

(1.62)

$$\frac{d}{dt}\int_0^L \rho V_m \, dx = \int_0^L \frac{\partial}{\partial t}(\rho V_m)\, dx + \int_{S} (\rho V_m) v_k (n_k)\, ds$$

~~material volume~~? spacial volume

$$= \int_0^L \frac{\partial}{\partial t}(\rho V_m)\, dx + \Big| \rho V_m$$



pg 228 Drumheller

eq (2.83) $\hat{\phi} = \bar{X} - c_s t$

$$\frac{\partial \hat{v}}{\partial \bar{X}} = \frac{d\hat{\phi}}{d\bar{X}} \frac{d\hat{v}}{d\hat{\phi}} = \frac{d\hat{v}}{d\hat{\phi}}$$

$$\frac{\partial \hat{F}}{\partial t} = \frac{d\hat{\phi}}{dt} \cdot \frac{d\hat{F}}{d\hat{\phi}} = -c_s \frac{d\hat{F}}{d\hat{\phi}}$$

$\therefore$ eq (2.33) $\Rightarrow$ $\frac{d\hat{v}}{d\hat{\phi}} = -c_s \frac{d\hat{F}}{d\hat{\phi}}$.

$$\rho_0 \left(-c_s \frac{dv}{d\hat{\phi}}\right) = \frac{dT}{d\phi}$$

---

eq 2.191 $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$

$\parallel$

$$-c_s \frac{d\rho}{d\phi} + \frac{d}{d\phi}(\rho v) = 0$$

$$\Rightarrow \frac{d}{d\phi}(-c_s \rho + \rho v) = 0 \Rightarrow \frac{d}{d\phi}\left(\rho(c_s - v)\right) = 0$$

Lin mmt: $\quad -c_s \frac{d}{d\phi}(\rho v) = \frac{d}{d\phi}(T - \rho v^2)$

$$\Rightarrow \frac{d}{d\phi}\left(-c_s \rho v - T + \rho v^2\right) = 0$$

$$\Rightarrow \frac{d}{d\phi}\left(\rho v (c_s - v) + T\right) = 0$$

pg 271 Dreimheller

$$\frac{\partial \hat{\eta}}{\partial t} = -\frac{c_v \Gamma}{\rho}\frac{\partial \hat{\rho}}{\partial t}$$

$$\frac{\partial \hat{E}_e}{\partial t} = -\frac{(\lambda + \frac{2}{3}\mu)}{\rho}\frac{\partial \hat{\rho}}{\partial t} + c_v \Theta \Gamma^2 \frac{\partial \hat{\rho}}{\partial t}$$

$$= -\frac{1}{\rho}\underbrace{\left[\underbrace{(\lambda+\tfrac{2}{3}\mu)}_{k^\eta \text{ eq } 3.72} - c_v \rho \Theta \Gamma^2\right]}_{k^\Theta}\frac{\partial \hat{\rho}}{\partial t}$$

$$\therefore k^\Theta = k^\eta - c_v \rho \Theta \Gamma^2$$

eq 3.69 $\quad \frac{1}{\rho}\frac{\partial \hat{\rho}}{\partial t} = \frac{-\rho \Theta \Gamma}{(\lambda + \frac{2}{3}\mu)}\frac{\partial \hat{\eta}}{\partial t}$

~~eq 3.70~~ $\quad \frac{1}{\rho}\frac{\partial \hat{\rho}}{\partial t} + \frac{\rho \Theta \Gamma}{(\lambda+\frac{2}{3}\mu)}\frac{\partial \hat{\eta}}{\partial t} = 0$

(eq 3.70) $\quad \frac{\Theta \Gamma}{\rho}\frac{\partial \hat{\rho}}{\partial t} + \frac{\Theta}{c_v}\frac{\partial \hat{\eta}}{\partial t} = +\frac{\partial \hat{\Theta}}{\partial t}$

2 eq 2 unk $\hat{\rho}_t, \hat{\eta}_t$.

$$\begin{pmatrix} \rho^{-1} & \rho\Theta\Gamma/k^\eta \\ \Theta\Gamma\rho^{-1} & \Theta/c_v \end{pmatrix}\begin{pmatrix} \hat{\rho}_t \\ \hat{\eta}_t \end{pmatrix} = \begin{pmatrix} 0 \\ \hat{\Theta}_t \end{pmatrix}$$

$$\begin{pmatrix} \hat{\rho}_t \\ \hat{\eta}_t \end{pmatrix} = \frac{1}{\left(\frac{\theta}{\rho c_v} - \frac{\theta^2\Gamma^2}{\kappa^\eta}\right)} \begin{pmatrix} \theta/c_v & -\rho\theta\Gamma/\kappa^\eta \\ -\theta\Gamma\rho^{-1} & \rho^{-1} \end{pmatrix} \begin{pmatrix} 0 \\ \hat{\theta}_t \end{pmatrix}$$

$$= \begin{pmatrix} \dfrac{-\rho\theta\Gamma}{\kappa^\eta\left(\frac{\theta}{\rho c_v} - \frac{\theta^2\Gamma^2}{\kappa^\eta}\right)} \dfrac{\partial\hat{\theta}}{\partial t} \\[2ex] \dfrac{1}{\rho} \dfrac{1}{\left(\frac{\theta}{\rho c_v} - \frac{\theta^2\Gamma^2}{\kappa^\eta}\right)} \dfrac{\partial\hat{\theta}}{\partial t} \end{pmatrix}$$

$$\Rightarrow \quad \frac{1}{\rho}\frac{\partial\hat{\rho}}{\partial t} = \frac{-\Gamma}{\frac{\kappa^\eta}{\rho c_v} - \theta\Gamma^2} \frac{\partial\hat{\theta}}{\partial t}$$

$$= \frac{-\rho c_v \Gamma}{\kappa^\eta - \theta\Gamma^2\rho c_v} \frac{\partial\hat{\theta}}{\partial t} = -\frac{\rho c_v \Gamma}{\kappa^\theta} \frac{\partial\hat{\theta}}{\partial t}$$

$$\& \quad \frac{\partial\hat{\eta}}{\partial t} = \frac{1}{\cancel{\rho}\theta(\cancel{\rho} c_v \kappa^\eta)^{-1}(\underbrace{\kappa^\eta - \Gamma^2\rho c_v\theta}_{\kappa^\theta})} \frac{\partial\hat{\theta}}{\partial t}$$

$$= \frac{c_v \kappa^\eta}{\theta\kappa^\theta} \frac{\partial\hat{\theta}}{\partial t}$$

$$C^\theta = \underbrace{\underset{\substack{\| \\ \kappa^\eta}}{C^\eta} - \rho c_v \theta \Gamma^2}_{\kappa^\theta}$$

Pg 272 Drumheller

$$\frac{\partial\hat{\varepsilon}}{\partial t} = \left(-\frac{b_e}{\rho^2}\left(-\beta\rho\right) + \frac{\cancel{\theta}\,\gamma^{1}}{\gamma^{\theta}}\frac{c_v}{\cancel{\theta}}\right)\frac{\partial\hat{\theta}}{\partial t}$$

$$= \left(\frac{c_v\gamma^{1}}{\gamma^{\theta}} + \frac{b_e\beta}{\rho}\right)\frac{\partial\hat{\theta}}{\partial t}$$

eq 3.20

$$\rho\frac{\partial\hat{\varepsilon}}{\partial t} = T_{km}D_{mk} - \cancel{\frac{\partial q_k}{\partial x_k}}^{\,0} + \cancel{\rho r}^{\,?}$$

$$\Vert \qquad \downarrow \text{ in 1D eq 3.10} \qquad \text{keep } r.$$

$$\rho\left(\frac{c_v\gamma^{1}}{\gamma^{\theta}} + \frac{b_e\beta}{\rho}\right)\frac{\partial\hat{\theta}}{\partial t} = T\frac{\partial\hat{F}}{\partial t} + \rho r$$

eq 3.60 is $c^\theta = c^\eta - \rho c_v \Theta \Gamma^2$ is this c_v same as defined here? Yes eq 4.9 is obtained from the def.

$$\| $$

$$\gamma p e - \underbrace{\rho c_v \Theta \Gamma}_{\text{eq 4.14 only if } c_v \text{ constant}} \cdot (\gamma - 1)$$

don't need to use

$$\gamma p e - \rho c_v \Theta (\gamma - 1)^2$$

$\|$ eq 4.8

$$\gamma N R \rho \Theta - \rho \Theta \underset{\underset{(\gamma-1)^{-1} N R}{\|}}{c_v} (\gamma - 1)^2$$

$$= \gamma N R \rho \Theta - N R \rho \Theta (\cancel{\gamma} - 1) = N R \rho \Theta = p e$$

pg 271 gives $c^\eta = k^\eta$

$$c_T = c_v \frac{c^\eta}{c^\theta}$$

look up derivation of c_T

eq (3.10) is $\rho_0 \frac{\partial \hat{\mathcal{E}}}{\partial t} = T \frac{\partial \hat{F}}{\partial t} - \cancelto{0}{\frac{\partial q}{\partial X}} + \rho_0 r$

Remember $D = F^{-1} \frac{\partial \hat{F}}{\partial t} \Rightarrow \frac{\partial \hat{F}}{\partial t} = F D = F \frac{\beta}{3} \frac{\partial \hat{\theta}}{\partial t}$

put in $\frac{\partial \hat{\mathcal{E}}}{\partial t}$ & $\frac{\partial \hat{\theta}}{\partial t}$ & $\frac{\partial \hat{F}}{\partial t}$

{ $T_e = 3 b_e$ Think }

$$\Rightarrow \rho_0 \left(\frac{c_v k^\eta}{k^\theta} + \frac{b_e \beta}{\rho} \right) \frac{\partial \hat{\theta}}{\partial t} = T_e F \frac{\beta}{3} \frac{\partial \hat{\theta}}{\partial t} + \rho_0 r$$

{ $F = \rho_0 / \rho$; F is $\propto$ to specific volume }

$$\Rightarrow \left(\frac{\rho_0 c_v k^\eta}{k^\theta} + F \cancel{b_e} \beta \right) \frac{\partial \hat{\theta}}{\partial t} = \frac{F}{3} \cancel{T_e} \cancel{\beta} \frac{\partial \hat{\theta}}{\partial t} \quad \rho_0 r$$

$$\Rightarrow \quad \underbrace{\frac{\cancel{\rho_0} c_v k^{\eta}}{k^{\theta}}}_{c_p} \frac{\partial \hat{\theta}}{\partial t} = \cancel{\rho_0} r$$

$$\frac{c_p}{c_v} = \frac{k^{\eta}}{k^{\theta}}$$

Pg 285 Drumheller

1

$\mathcal{E} - \mathcal{E}_- =$

$$T_r = \rho_0 \frac{\partial \mathcal{E}_r}{\partial F}$$

$$\rho D = \cancel{\quad} \geq 0$$

$$(T - T_e) D \geq 0$$

$$\Rightarrow T_r < T_e$$

$$T_r = \rho_0 \frac{\partial \mathcal{E}_r}{\partial F}$$

$$T_e = \rho_0 \frac{\partial \mathcal{E}_e}{\partial F}$$

e stands for equilibrium + means isentropes.

$$\frac{\partial \mathcal{E}_r}{\partial F} < \frac{\partial \mathcal{E}_e}{\partial F}$$

$$-\frac{\partial \mathcal{E}_r}{\partial F} \geq -\frac{\partial \mathcal{E}_e}{\partial F}$$ for compression.

Pg 287 Drum

3107 $$-\frac{\partial \mathcal{E}_r}{\partial F} \geq -\frac{\partial \mathcal{E}}{\partial F}$$

$\parallel$

$$-\cancel{\left(U_s^2 (F - F_-) + \frac{T_-}{\rho_0} \right) \geq -\frac{\partial \mathcal{E}}{\partial F}}$$

$$F_- = 1$$

$$\boxed{\cancel{T_- = 0}}$$

$$\Rightarrow \cancel{+U_s^2 (F-1) \geq + \frac{T_-}{\rho_0} - \frac{\partial \mathcal{E}}{\partial F}}$$

Now:

$$\frac{\partial \mathcal{E}_r}{\partial F} = U_s^2 (F - F_-) + \frac{T_-}{\rho_0}$$

$\therefore$ Above becomes w/ $F_- = 1$

$$-U_s^2 (F - 1) - \frac{T_-}{\rho_0} \geq -\frac{\partial \mathcal{E}}{\partial F}$$

$$-U_s^2 (F - 1) \geq -\frac{\partial \mathcal{E}}{\partial F} + T_-/\rho_0$$

$$U_s^2 \geq \frac{\partial \mathcal{E}/\partial F - T_-/\rho_0}{F - 1}$$ w/ $F < 1$

$$\Rightarrow \quad U_S^2 \geq \frac{-T - (\rho_0 \pm \partial \mathcal{E}/\partial F)}{F-1} \qquad \cancel{BE}$$

$$C^1 = \rho_0 F \frac{\partial^2 \mathcal{E}}{\partial F^2}$$

$$U_S^2 \geq \frac{\partial^2 \mathcal{E}/\partial F^2}{1} = \frac{C^1}{\rho_0}$$

$$C_0^2 \rho_0 = C_0^1$$

$$\underline{U_S^2 \geq C_0^2} \Rightarrow U_S^2 \geq C_0$$

$$C^2 \geq U_S^2 \Rightarrow C \geq U_S \qquad \text{eq } 3.109$$

Pg 289 Drom

$$\underline{T_R V_R} - \underline{T_L V_L} = \frac{d}{dt} \int_{X_L}^{X_R} \rho_0 (\mathcal{E} + K)\, dX \qquad +$$

2.144

$$\frac{d}{dt} \int_{X_L}^{X_R} \phi(X,t)\, dX = [[\phi]] C_S \qquad [[\phi]] = \phi_+ - \phi_-$$

$$C[[T]] = [[\rho_0$$

$$\rightarrow T_- V_- - T_+ V_+ = -[[TV]] = \rho_0 C_S [[\mathcal{E} - K]]$$

$$\rho_0 C_S [[\mathcal{E}]] = -[[TV]] + \rho_0 C_S [[K]] = -[[TV]] + \rho_0 C_S [[\tfrac{V^2}{2}]]$$

$$\boxed{K = \frac{V^2}{2}} \quad = -(T_+ V_+ - T_- V_-) + \frac{\rho_0 C_S}{2} (V_+^2 - V_-^2)$$

$$= -(T_+ V_+ - T_- V_+) - T_- V_+ + T_- V_- + \frac{\rho_0 C_S}{2} (V_+^2 - V_-^2)$$

$$\rho C_s [[E]] = -[[T]]v_+ - T_-[[v]] + \frac{\rho_0 C_s}{2}(v_+ - v_-)(v_+ + v_-)$$

sign wrong

$$= -[[T]]v_+ - T_-[[v]] + \frac{\rho_0 C_s}{2}[[v]](v_+ - \cancel{v_-} + 2v_-)$$

Try $(-v_+ + 2v_+ + v_-)$ instead

$$= \quad '' \quad '' \quad + \frac{\rho_0 C_s}{2}[[v]]^2 \qquad \underbrace{}_{-(v_+ - v_- - 2v_+)} \leftarrow \text{this trick.}$$

$$-([[v]] - 2v_+)$$

$$-\frac{\rho_0 C_s [[v]]^2}{2} + \underline{\rho_0 C_s [[v]] v_+} \qquad \text{try this one}$$

$$\rho_0 C_s [[E]] = +\rho_0 C_s [[v]] v_+ + T_- [[F]] C_s +$$

~~try~~ again:

$$-\rho_0 C_s [[E]] - \rho_0 C_s [[K]] = [[Tv]]$$

$$\Rightarrow \rho_0 C_s [[E]] = -[[Tv]] - \rho_0 C_s [[v^2/2]] \qquad \text{(As Above)}$$

$$= \underbrace{-[[T]]v_+}_{\substack{\text{for } [[T]] \\ -\rho_0 C_s [[v]]}} - \underbrace{T_-[[v]]}_{\substack{\text{for } [[v]] \\ -C_s[[F]]}} + \underbrace{\frac{\rho_0 C_s}{2}[[v]]^2}_{\substack{\text{for } [[v]] \\ = -C_s[[F]]}} - \underbrace{\rho_0 C_s [[v]] v_+}_{\substack{\cancel{\text{for } [[v]]} \\ \cancel{= -C_s [[F]]} \\ \text{leave alone}}}$$

$$= +\cancel{\rho_0 C_s [[v]] v_+} + T_- C_s [[F]] + \frac{\rho_0 C_s}{2} C_s^2 [[F]]^2 + \cancel{\rho_0 C_s^2 [[F]]}.$$

$$\cancel{-\rho_0 C_s [[v]] v_+}$$

$$\Rightarrow \rho_0 [\![\mathcal{E}]\!] = T_- [\![F]\!] + \frac{c_s^2}{2} \rho_0 [\![F]\!]^2$$

$\cancel{\rho_0 [\![\mathcal{E}]\!]} =$

$T_- + \rho_0 c_s^2 [\![F]\!] = ?$ From $[\![T]\!] = -\rho_0 c_s [\![v]\!]$

$+ \quad [\![F]\!] c_s + [\![v]\!] = 0$

ell: $[\![v]\!]$ we get

$$[\![T]\!] = +\rho_0 c_s^2 [\![F]\!]$$

$$\Rightarrow T_+ = T_- + \rho_0 c_s^2 [\![F]\!]$$

eq 2.145

$$\frac{d}{dt} \int_{x_L(t)}^{x_R(t)} \phi(x,t)\, dx = [\![\phi (c_s - v)]\!]$$

$$[\![\rho(c_s - v)(\mathcal{E} + k)]\!] = -[\![Tv]\!]$$

Mass $[\![\rho(c_s - v)]\!] = 0.$

$$[\![F]\!] c_s + [\![v]\!] = 0$$

pg 292 Dom

$$[\![\phi]\!] = \phi_+ - \phi_-$$

$$\eta = \check{\eta}(F)$$

$$[\![\eta]\!] = \check{\eta}(F_+) - \check{\eta}(\underbrace{F_-}_{\text{initial state}}) =$$

expand $\cancel{\ }$ $\quad \check{\eta}(F) = \sum_{k=0}^{\infty} \frac{\check{\eta}^{(k)}(F_0)}{k!} (F - F_0)^k$

$$\Rightarrow \therefore \quad \breve{\eta}(F_+) - \breve{\eta}(F_-) = \sum_{k=0}^{\infty} \frac{\breve{\eta}^{(k)}(F_0)}{k!} \left\{ (F_+ - F_0)^k - (F_- - F_0)^k \right\}$$

do $k = 1, 2, 3, \ldots$ expand about $F_0 = F_-$

$$\Rightarrow \quad [\![\eta]\!] = \sum_{k=0}^{\infty} \frac{\breve{\eta}^{(k)}(F_-)}{k!} [\![F]\!]^k .$$

$$\rho_0 [\![\mathcal{E}]\!] = \frac{1}{2}(T_+ + T_-)[\![F]\!] \qquad \mathcal{E} = \mathcal{E}(F, \eta)$$

$$\rho_0 \Big(\underbrace{\frac{\partial \mathcal{E}}{\partial \eta}}_{\Theta} \frac{\partial \eta}{\partial F} + \frac{\partial \mathcal{E}}{\partial F} \Big)_+ = \frac{1}{2}\left(\frac{d\breve{T}}{dF}\right)_+ (F_+ - F_-) + \frac{1}{2}(T_+ + T_-)$$

$$\Rightarrow \quad \rho_0 \left(\Theta \frac{\partial \eta}{\partial F}\right)_+ = \frac{1}{2}\left(\frac{d\breve{T}}{dF}\right)_+ (F_+ - F_-) - \underbrace{\left(\rho_0 \frac{\partial \mathcal{E}}{\partial F}\right)_+}_{T_+} + \frac{1}{2}(T_+ + T_-)$$

$$= \quad \text{''} \quad - \frac{1}{2} T_+ + \frac{1}{2} T_-$$

evaluate at $F_+ = F_-$

$$\Rightarrow \quad \rho_0 \Theta \left(\frac{\partial \eta}{\partial F}\right)_- =$$

$$\rho_0 \left[\frac{d\Theta}{dF} \cdot \frac{d\eta}{dF} + \Theta \frac{d^2\eta}{dF^2} \right]_+ = \frac{1}{2}\left(\frac{d^2 T}{dF^2}\right)(F_+ - F_-) + \cancel{\frac{1}{2}\left(\frac{dT}{dF}\right)} - \cancel{\frac{1}{2} \frac{dT}{dF}}$$

$$\Rightarrow \quad \rho_0 \theta \frac{d^2\eta}{dF^2} = -\rho_0 \cancelto{0}{\frac{d\theta}{dF}\frac{d\eta}{dF}} + O \ldots$$

$$\rho_0 \Big[\underbrace{\cancel{\frac{d^2\theta}{dF^2}\frac{d\eta}{dF}}}_{0} + \underbrace{\frac{d\theta}{dF}\frac{d^2\eta}{dF^2} + \frac{d\theta}{dF}\frac{d^2\eta}{dF^2}}_{\cancelto{0}{2\frac{d\theta}{dF}\frac{d^2\eta}{dF^2}}} + \theta\frac{d^3\eta}{dF^3}\Big] = \frac{1}{2}\left(\frac{d^3T}{dF^3}\right)(F_f - F) + \frac{1}{2}\left(\frac{d^2T}{dF^2}\right)$$

evaluating at $F_f = F_-$

$$\rho_0 \theta \frac{d^3\eta}{dF^3} = \frac{1}{2}\frac{d^2T}{dF^2}$$

$$\therefore \quad [[\eta]] = \frac{1}{6}[[F]]^3 \cdot \frac{1}{\rho_0\theta}\frac{1}{2}\frac{d^2T}{dF^2} = \frac{1}{12\rho_0}\left(\frac{1}{\theta}\frac{d^2\tilde{T}}{dF^2}\right)_- [[F]]^3.$$

$$\tilde{T}(F) = T(F, \tilde{\eta}(F))$$

$$\frac{d\tilde{T}}{dF} = \frac{\partial T}{\partial F} + \frac{\partial T}{\partial \eta}\frac{\partial \tilde{\eta}}{\partial F}$$

$$\frac{d^2\tilde{T}}{dF^2} = \frac{\partial^2 T}{\partial F^2} + \underline{\frac{\partial^2 T}{\partial \eta \partial F}}\cdot\frac{\partial \eta}{\partial F} + \underline{\frac{\partial^2 T}{\partial F \partial \eta}\frac{\partial \tilde{\eta}}{dF}} + \frac{\partial^2 T}{\partial \eta^2}\left(\frac{d\eta}{dF}\right)^2 + \frac{\partial T}{\partial \eta}\frac{\partial^2 \eta}{\partial F^2}$$

$$= \frac{\partial^2 T}{\partial F^2} + 2\frac{\partial^2 T}{\partial F \partial \eta}\frac{d\eta}{dF} + \frac{\partial^2 T}{\partial \eta^2}\left(\frac{d\eta}{dF}\right)^2 + \frac{\partial T}{\partial \eta}\frac{d^2\eta}{dF^2}$$

pg 294 Drumheller

$$\frac{dT}{dF} = \rho_0 c_0^2 + 2\rho_0 c_0 \left(\frac{dc}{dF}\right)_- (F-1)$$

$$\frac{d^2T}{dF^2} = 2\rho_0 c_0 \left(\frac{dc}{dF}\right)_-$$

$$\therefore \; [[\eta]] = \frac{1}{6\rho_0 \cancel{\theta}} \left(\frac{1}{\theta} 2\cancel{\rho_0} c_0 \left(\frac{dc}{dF}\right)_-\right) [[F]]^3$$

$$= \frac{1}{6\theta} c_0 \left(\frac{dc}{dF}\right)_{F=1} [[F]]^3$$

$c' =$

for elastic material.

Now $\quad c' = \left(\frac{1}{\rho_0} \frac{dT}{dF}\right)^{1/2}$

$$\therefore \; \frac{dc'}{dF} = \frac{1}{2}\left(\frac{1}{\rho_0}\frac{dT}{dF}\right)^{-1/2} \frac{1}{\rho_0}\frac{d^2T}{dF^2} \qquad ?$$

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pg 295 Drum

(3.53) $\quad \Gamma = \frac{-F}{\theta} \frac{\partial^2 \varepsilon}{\partial F \partial \eta}$

$$= \frac{-F}{\theta} \cdot \; = \; 0 \qquad \text{If } \; \varepsilon = \varepsilon_F(F) + \varepsilon_\eta(\eta).$$

(3.45) $\quad \alpha = -\frac{\rho_0 c_v}{\theta c^\theta} \frac{\partial \theta}{\partial F} = -\frac{\rho_0 c_v}{\theta c^\theta} \frac{\partial}{\partial F} \frac{\partial \varepsilon}{\partial \eta} = 0$

pg 321 from

$$\int_{\varepsilon_0}^{\varepsilon} \frac{d\varepsilon'}{\theta(\varepsilon')} = c_v \int_{\varepsilon_0}^{\varepsilon} \frac{d\varepsilon'}{\varepsilon'} = c_v \ln(\varepsilon/\varepsilon_0) = c_v \ln(\theta/\theta_0)$$

$$\theta(\varepsilon') = \frac{\varepsilon'}{c_v}$$

$$= c_v \ln($$

$$\theta = \frac{p_e}{NR\rho}$$

$$\frac{\partial}{\partial \eta} \Rightarrow \qquad \frac{1}{\theta(\varepsilon)} \frac{\partial \varepsilon}{\partial \eta} = 1$$

$$\frac{\partial \varepsilon}{\partial \eta} = \theta(\varepsilon)$$

$$\frac{\partial}{\partial F} \Rightarrow$$

$$\frac{1}{\theta(\varepsilon)} \frac{\partial \varepsilon}{\partial F} = \frac{\partial}{\partial(\rho_0/\rho)} = \frac{1}{\rho_0} \frac{\partial}{(-\rho^{-2})\partial\rho} = -\frac{\rho^2}{\rho_0} \frac{\partial}{\partial \rho}$$

$$\Rightarrow \frac{1}{\theta(\varepsilon)} \frac{\partial \varepsilon}{\partial F} = -\frac{\rho^2}{\cancel{\rho_0}} NR \frac{1}{(\rho/\cancel{\rho_0})} \frac{1}{\rho_0} = -\frac{NR\rho}{\rho_0} = -\cancel{NRF}$$

$$= -NRF^{-1}$$

eq (3.40) $\quad T_e = \rho_0 \frac{\partial \varepsilon}{\partial F}$

$$p_e = -T_e = -\rho_0 \frac{\partial \varepsilon}{\partial F} = \rho_0 NRF^{-1} = NR\cancel{\rho_0}\left(\frac{\rho}{\cancel{\rho_0}}\right)\theta$$

$$\frac{\partial}{\partial \eta}\left(\frac{1}{\Theta(\varepsilon)}\frac{\partial \varepsilon}{\partial \eta}\right) = \frac{\partial}{\partial \eta}(1) = 0$$

$$-\frac{\Theta'(\varepsilon)}{\Theta^2(\varepsilon)}\left(\frac{\partial \varepsilon}{\partial \eta}\right)^2 + \frac{1}{\Theta(\varepsilon)}\frac{\partial^2 \varepsilon}{\partial \eta^2} = 0$$

$$\Rightarrow \quad \frac{\partial^2 \varepsilon}{\partial \eta^2} = \frac{\Theta'(\varepsilon)}{\Theta(\varepsilon)}\left(\frac{\partial \varepsilon}{\partial \eta}\right)^2 = \underbrace{\frac{\Theta'(\varepsilon)}{\Theta(\varepsilon)}\frac{\partial \varepsilon}{\partial \eta}}_{1 \cdot \Theta'(\varepsilon)} \cdot \frac{\partial \varepsilon}{\partial \eta} = \frac{\partial \Theta(\varepsilon)}{\partial \varepsilon} \cdot \frac{\partial \varepsilon}{\partial \eta}$$

$$\Rightarrow \frac{\partial \Theta}{\partial \varepsilon} = \frac{1}{\Theta}\frac{\partial^2 \varepsilon}{\partial \eta^2} \underset{\uparrow \text{By } 3.55}{=} \frac{1}{\Theta}\cdot\frac{\Theta}{C_V} = \frac{1}{C_V}.$$

3.55 $\quad \frac{\Theta}{C_V} = \frac{\partial \Theta}{\partial \eta} = \frac{\partial^2 \varepsilon}{\partial \eta^2}$

$$\Theta = \frac{1}{C_V}\varepsilon.$$

---

eq 4.7 $\quad \frac{1}{\Theta(\varepsilon)}\frac{\partial \varepsilon}{\partial F} = -\frac{NR}{F}$

$\frac{\partial}{\partial F} \quad -\frac{1}{\Theta^2(\varepsilon)}\frac{\partial \Theta}{\partial \varepsilon}\cdot\left(\frac{\partial \varepsilon}{\partial F}\right)^2 + \frac{1}{\Theta(\varepsilon)}\frac{\partial^2 \varepsilon}{\partial F^2} = +\frac{NR}{F^2}.$

$$\frac{\partial^2 \varepsilon}{\partial F^2} = \frac{NR\Theta}{F^2} + \frac{1}{\Theta(\varepsilon)}\frac{\partial \Theta}{\partial \varepsilon}\left(\frac{\partial \varepsilon}{\partial F}\right)^2$$

$$= \frac{NR\Theta}{F^2} + \underbrace{\left(\frac{1}{\Theta(\varepsilon)}\frac{\partial \varepsilon}{\partial F}\right)}_{\text{eq } 4.7}\frac{\partial \Theta}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial F}$$

$$\frac{\partial^2 \mathcal{E}}{\partial F^2} = \frac{NR\theta}{F^2} - \frac{NR}{F}\frac{\partial \theta}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial F}$$

|| 3.50 || eq 4.8

$$\frac{c^2}{\rho_0 F} = \frac{p_e}{\rho F^2} - \frac{p_e}{\rho F}\frac{1}{c_v}\left(-\frac{NR}{F}\right)$$

$$= \frac{p_e}{\rho F}\underbrace{\left(1 + \frac{NR}{c_v}\right)}_{\gamma} \qquad F = \rho_0/\rho$$

$$\Rightarrow \quad c^2 = \gamma p_e$$

$$\frac{\partial \varepsilon}{\partial \eta} = \theta(\varepsilon)$$

$$\frac{\partial^2 \varepsilon}{\partial \eta \partial F} = \frac{\partial \theta}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial F} = \frac{\partial \theta}{\partial \varepsilon}\left(-\frac{NR\theta}{F}\right) = -\frac{NR\theta}{F c_v}$$

$$\Uparrow$$

$$\frac{\partial^2 \varepsilon}{\partial F \eta} = -\frac{\theta \Gamma}{F} \quad (\text{eq } 3.53)$$

$$\Rightarrow \quad \Gamma = \frac{NR}{c_v} = \gamma - 1$$

$$p_e = c_v \Gamma \rho \theta = \Gamma \rho \varepsilon$$

---

eq (3.60) is $\quad c^2 = c^\theta + \rho c_v \theta \Gamma^2 \qquad$ Same eq?

$$\begin{array}{c} \| \\ \gamma p_e = c^\theta + \rho c_v \theta \Gamma^2 \end{array}$$

Pg 324 Drumheller Ideal Gas s

eq (3.63) $\frac{c_T}{c_v} = \frac{c^\eta}{c^\theta}$ 4.11 $c^\eta = \gamma p e$

$\Rightarrow \frac{c_p}{c_v} = \frac{c^\eta}{c^\theta}$ As $c_T = c_p$ for fluids

$\Rightarrow \frac{\gamma p e}{c^\theta} = \gamma$ By eq 4.15

$\Rightarrow \frac{NR}{c_v} + 1$ By 4.13

$\Rightarrow \frac{c_p}{c_v} = \frac{NR}{c_v} + 1$

$c_p = NR + c_v$

From 4.14

3.66 $\alpha = \frac{\rho c_v \Gamma}{c^\theta} = \cancel{\frac{\rho c_v(\gamma-1)}{\rho e}}$

$= \frac{\rho e}{\theta c^\theta} = \frac{1}{\theta}$

Pg 359 Drumheller

define $A(\rho)$ + $B(\eta)$ +

$$\ln A(\rho) = \int_{\rho_0}^{\rho} \frac{\Gamma(\rho')}{\rho'}\, d\rho' \quad + \quad \ln B(\eta) = \int_{\eta_0}^{\eta} \frac{d\eta'}{c_\eta(\eta')}$$

Ends

$$\Gamma_0 \ln(\rho/\rho_0) + \frac{1}{c_v}(\eta - \eta_0)$$

$$\Rightarrow \quad c_v \ln\left(\frac{\varepsilon - \varepsilon_c}{\varepsilon_0}\right) = \underbrace{c_v \Gamma_0}_{} \ln(\rho/\rho_0) + \eta - \eta_0$$

$$NR \ln(\rho/\rho_0)$$

$c_\eta \to \infty$

$$\varepsilon - \varepsilon_c = \cancel{0} \Rightarrow \varepsilon = \varepsilon_c$$

$$\frac{1}{\varepsilon - \varepsilon_c} \underset{\theta}{\underset{\parallel}{\frac{\partial \varepsilon}{\partial \eta}}} = \frac{1}{c_\eta(\eta)}$$

$$\varepsilon - \varepsilon_c = c_\eta \theta = \varepsilon_\theta \qquad \text{By 4.67}$$

$$\frac{1}{\varepsilon - \varepsilon_c}\left(\frac{\partial \varepsilon}{\delta F} - \frac{\partial \varepsilon_c}{\delta F}\right) = \frac{\partial}{\delta(\rho_0/\rho)} \int_{\rho_0}^{\rho} \frac{\Gamma(\rho')}{\rho'}\, d\rho'$$

$$= \frac{1}{-\frac{\rho_0}{\rho^2}} \frac{\partial}{\delta \rho} \int_{\rho_0}^{\cdots} \cdots$$

$$= -\frac{\rho^2}{\rho_0}\frac{\Gamma(\rho)}{\rho} = -\frac{\rho}{\rho_0}\Gamma(\rho) = \cancel{\frac{\rho}{\rho}\left(\frac{-1}{\rho_0}\right)} = \frac{\Gamma}{\rho}\left(\frac{-\rho^2}{\rho_0}\right)$$

$$F = \rho_0/\rho \qquad = \frac{\Gamma}{\rho}\frac{\partial F}{\partial \rho}$$

$$\left[\frac{\partial}{\partial \rho}\int \cdots \quad \frac{\partial \rho}{\partial F}\right.$$

$$\rho = \rho_0/F$$

$$\frac{\partial \rho}{\partial F} = -\frac{\rho_0}{F^2} = -\rho_0/(\rho_0/\rho)^2 = -\frac{\rho^2}{\rho_0}$$

$$\Rightarrow \quad \frac{\partial \varepsilon}{\partial F} - \frac{\partial \varepsilon_c}{\partial F} = \frac{-\rho}{\rho_0}\Gamma(\varepsilon - \varepsilon_c) = -\frac{\Gamma\rho}{\rho_0}c_\eta\theta = -\frac{\Gamma c_\eta \theta}{F}$$

$$p_e = p_c + \frac{\rho_0 \Gamma c_\eta \theta}{F}$$

$$= p_c + \Gamma\rho\,(\underbrace{+c_\eta\theta}) = p_c + \Gamma\rho\,(\varepsilon - \varepsilon_c)$$

$$\underbrace{\varepsilon - \varepsilon_c \quad \text{eq } 4.70}$$

$$p_\theta = \rho\Gamma c_\eta \theta = \rho\Gamma\varepsilon_\theta$$

$$\frac{\partial^2 \varepsilon}{\partial\eta\,\partial F} = -\frac{\Gamma(\rho)}{F}\frac{\partial(c_\eta\theta)}{\partial\eta} \qquad \frac{\partial}{\partial\eta}$$

$$= -\frac{\Gamma}{F}\frac{\partial}{\partial\eta}(\varepsilon - \varepsilon_c) = -\frac{\Gamma}{F}\underbrace{\left[\frac{\partial\varepsilon}{\partial\eta}\right]}_{\theta} = -\frac{\Gamma}{F}\theta$$

$$\theta = \frac{\partial}{\partial\eta}(c_\eta\theta) = \frac{\partial c_\eta}{\partial\eta}\theta + c_\eta\frac{\partial\theta}{\partial\eta}$$

$$\Rightarrow \frac{\partial^2\varepsilon}{\partial\eta^2} = \frac{\partial\theta}{\partial\eta} = \frac{1}{c_\eta}\left(\theta - \frac{\partial c_\eta}{\partial\eta}\theta\right) = \frac{\theta}{c_\eta}\left(1 - \frac{\partial c_\eta}{\partial\eta}\right) =: \frac{\theta}{c_v}$$

$$\Rightarrow \quad c_r = \frac{c_\eta}{\left(1 - \frac{dc_\eta}{d\eta}\right)}$$

$$\frac{\partial^2 \varepsilon}{\partial F^2} = \frac{\partial^2 \varepsilon_c}{\partial F^2} - c_\eta \frac{d}{dF}\left(\frac{\Gamma \theta}{F}\right)$$

$$= \frac{\partial^2 \varepsilon_c}{\partial F^2} - c_\eta \left[\frac{\theta}{F}\frac{d\Gamma}{dF} + \Gamma\theta \frac{d}{dF}\left(\frac{1}{F}\right) + \frac{\Gamma}{F}\frac{d\theta}{dF}\right]$$

$$= \frac{\partial^2 \varepsilon_c}{\partial F^2} - c_\eta \left[\frac{\theta}{F}\frac{d\Gamma}{d\rho}\cdot\frac{d\rho}{dF} - \frac{\Gamma\theta}{F^2} + \frac{\Gamma}{F}\left(\frac{-\Gamma\theta}{F}\right)\right]$$

$F = \rho_0/\rho$

$\rho = \rho_0/F$

$$\frac{\partial \rho}{\partial F} = \frac{-\rho_0}{F^2}$$

$$= \frac{\partial^2 \varepsilon_c}{\partial F^2} - c_\eta \left[\frac{\theta}{F}\Gamma'(\rho)\left(\frac{-\rho_0}{F^2}\right) - \frac{\Gamma\theta}{F^2} - \frac{\Gamma^2\theta}{F^2}\right]$$

$$= \frac{\partial^2 \varepsilon_c}{\partial F^2} + \frac{c_\eta \theta}{F^2}\left[\frac{\Gamma' \rho_0}{F} + \Gamma^2 + \Gamma\right]$$

$$= \text{''} + \text{''} \underbrace{\left[\Gamma'\rho + \Gamma + \Gamma^2\right]}_{\frac{d}{d\rho}(\rho\Gamma) + \Gamma^2}$$

$$c^\eta = \rho_0 F \frac{\partial^2 \varepsilon}{\partial F^2} = \underbrace{\rho_0 F \frac{\partial^2 \varepsilon_c}{\partial F^2}}_{C^c} + \frac{\rho_0 c_\eta \theta}{F}\left[\Gamma^2 + \frac{d}{d\rho}(\rho\Gamma)\right]$$

$$+ \underbrace{\Gamma \rho}_{\rho_0} c_\eta \theta \left[\Gamma + \frac{1}{\Gamma}\frac{d}{d\rho}(\rho\Gamma)\right]$$

$\rho_0/F = \rho$

$$c^\eta = c^c + \rho\theta\,\Phi$$

eq 360 $\quad c^\eta = c^\theta + \rho c_v \theta \Gamma^2$

$$c^\theta = c^\eta - \rho c_v \theta \Gamma^2$$

$$= c^c + \rho\theta\left[\Gamma + \frac{1}{\Gamma}\frac{d}{d\rho}(\rho\Gamma)\right] - \overbrace{\rho\frac{c_\eta}{c_\eta}\theta}^{\rho\theta}\Gamma\,(\Gamma c_v)$$

$$= c^c + \rho\theta\underbrace{\left[\Gamma + \frac{1}{\Gamma}\frac{d}{d\rho}(\rho\Gamma)\right]}_{\Phi} - \Gamma\frac{c_v}{c_\eta}\rho\theta$$

$$= c^c + \left(\Phi - \Gamma\frac{c_v}{c_\eta}\right)\rho\theta$$

c_v & Γ constant.

$$c^\eta = c^c + \rho\theta\left(\Gamma_0 + \frac{\Gamma_0}{\Gamma_0}(1)\right) = c^c + \rho\theta(\Gamma_0 + 1)$$

$$c^\theta = c^c + (\Gamma_0 + 1 - \Gamma_0)\,\rho\theta = c^c + \rho\theta$$

c_v & $\rho\Gamma$ constant $= \rho_0\Gamma_0$

$$c^\eta = c^c + \rho\theta\left(\Gamma + \frac{1}{\Gamma}\cdot 0\right) = c^c + \Gamma\rho\theta$$

$$c^\theta = c^c + \rho\theta(\Gamma - \Gamma) = c^c$$

$$F = \frac{\Delta x}{\Delta X} = \frac{\Delta X - 2\delta}{\Delta X} = 1 - \ldots$$

$$\Delta x = \Delta X - 2\delta$$

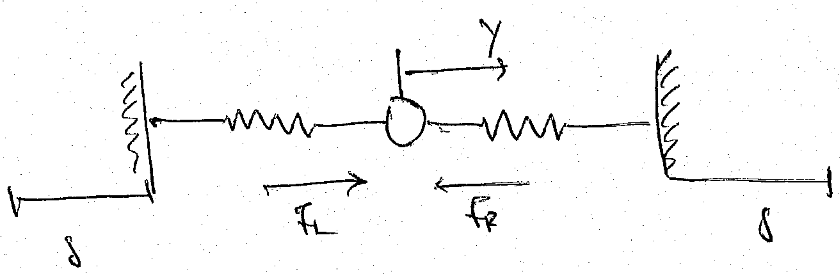
eq 2.22 $\quad \frac{dT}{dF} > 0 \qquad \frac{d^2T}{dF^2} \leq 0$

$$\frac{d(-T)}{dF} < 0 \qquad \frac{d^2(-T)}{dF^2} \geq 0 \Rightarrow \frac{d^2(-T)}{d\delta^2} \geq 0.$$

$$\frac{d(-T)}{d\delta} \cdot \underbrace{\frac{d\delta}{dF}}_{= -\frac{\Delta X}{2}} \qquad \frac{dF}{d\delta} = \frac{-2}{\Delta X}$$

$$f = f_R = (k_0\delta + k_1\delta^2)A$$

$$\Rightarrow \frac{d(-T)}{d\delta} > 0.$$



$$f_L = A[k_0(\delta - y) + k_1(\delta - y)^2]$$

$$f_R = A[k_0(\delta + y) + k_1(\delta + y)^2]$$

$$f_L - f_R = A[-2k_0 y + k_1(\cancel{\delta^2} - 2\delta y \cancel{+y^2} - \cancel{\delta^2} - 2\delta y - \cancel{y^2})]$$

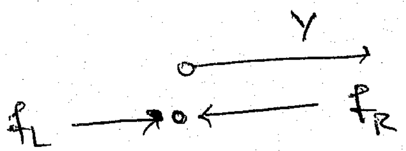
$$= A[-2k_0 y - 4\delta y] = -2A[k_0 + 2\delta]y$$

$$\langle f_L\rangle = \frac{A}{T}\int_0^T \left[k_0\delta - k_0 y + k_1(\delta^2 - 2\delta y + y^2)\right] dt$$

$$= \frac{A}{T}\left[k_0\delta T + k_1\delta^2 T + (-k_0 - 2k_1\delta)\cancelto{0}{\int_0^T y\,dt} + k_1\int_0^T y^2(t)\,dt\right]$$

$$= A\left[k_0\delta + k_1\delta^2 + \frac{k_1 y_0^2}{2}\right]$$

$$\int 2y\,dy = y^2$$



$$\int_0^Y 2A(k_0 + 2k_1\delta)\,y\,dy = \underbrace{A(k_0 + 2k_1\delta)}_{\frac{\omega^2}{2}m} Y^2$$

$$\therefore \quad E = \frac{m}{2}\left(\frac{dy}{dt}\right)^2 + \frac{\omega^2 y^2 m}{2} = \frac{m}{2}\left[\left(\frac{dy}{dt}\right)^2 + \omega^2 y^2\right]$$

$$= \frac{m}{2}\left[Y_0^2\right]\left[\omega^2\right](1)$$

$\cancel{\varepsilon_0} =$

Now

$$Y_0^2 = \frac{2\varepsilon_0}{\omega^2}$$

$$p_e = p_c + \frac{1}{2}k_1 Y_0^2$$

$$\Rightarrow p_e = p_c + (k_1/\omega^2)\,\varepsilon_0$$

eq (4.83) $\cancel{p_e = p_c + \frac{1}{2} k_1 \gamma_0^2}$

B. $p_e = p_c + \frac{k_1}{\omega^2} \varepsilon_0$ Now if $\rho = \frac{m}{A \Delta x}$

& $\Gamma = \frac{A k_1 \Delta x}{m \omega^2}$

Then $\rho \Gamma = k_1 / \omega^2$

$\therefore \; p_e = p_c + \Gamma \rho \, \varepsilon_0$

w/ defs of k_1 & k_0 eq 4.80 into

$$\omega^2 m = 2A \left(\frac{-2}{\Delta X} \frac{dp_c}{dF} + \frac{2^2}{\Delta X^2} \frac{d^2 p_c}{dF^2} \delta \right)$$

Then $$\Gamma = \frac{A \Delta x \, 4/(\Delta X)^2 \, \frac{d^2 p_c}{dF^2}}{2A \left(\frac{-2}{\Delta X} \frac{dp_c}{dF} \right)}$$ take $\delta \approx 0$ undeformed state

$$= \frac{-\Delta x}{2 \Delta X} \frac{\frac{d^2 p_c}{dF^2}}{\frac{dp_c}{dF}} = -\frac{F}{2} \frac{d^2 p_c / dF^2}{dp_c / dF}$$

$$\frac{d\omega}{d\delta} = \frac{1}{2} \left[2A (k_0 + 2 k_1 \delta) / m \right]^{-1/2} \cdot 2^2 A k_1 / m = \frac{2 k_1 A}{m \omega}$$

$$\frac{F}{\omega} \frac{d\omega}{d\delta} \frac{d\delta}{dF} = \frac{F}{\omega} \left(\frac{2 k_1 A}{m \omega} \right) \left(-\Delta X \right) = -\frac{\Delta x}{m \omega^2} \frac{2}{2} k_1 A$$

$$\Rightarrow \quad \frac{F}{\omega}\cdot\frac{d\omega}{dF} = \frac{-k_1 A\,\Delta x}{m\omega^2}$$

$$\frac{d\ln\omega}{d\ln F} = -\Gamma$$

$$P_C = -\rho_0 \frac{d\mathcal{E}_c}{dF} = -\rho_0\left(\frac{c_0}{s}\right)^2 \left\{ \frac{(s)}{1-s(1-F)} + \frac{-s}{(1-s(1-F))^2} \right\}$$

$$= -\frac{\rho_0 c_0^2}{s}\left\{ \frac{\cancel{1}-s(1-F) - \cancel{1}}{(1-s(1-F))^2} \right\} = \frac{\rho_0 c_0^2 (1-F)}{\cdots}$$

$$\frac{dp_c}{dF} = \frac{\rho_0 c_0^2(-1)}{[\quad]^2} - \frac{2\rho_0 c_0^2 s(1-F)}{[\quad]^3}$$

$$= \frac{\rho_0 c_0^2}{[\quad]^3}\left\{-1 + s(1-F) - 2s(1-F)\right\}$$

$$= \frac{''}{\cdots}\left(-1 - s(1-F)\right)$$

$$-1\left(1 + s(1-F)\right)$$

$$\frac{d^2p_c}{dF^2} = -\rho_0 c_0^2\left[\frac{-s}{(1-s(1-F))^3} - \frac{3(1+s(1-F))s}{[\quad]^4}\right]$$

$$= -\frac{\rho_0 c_0^2(-s)}{[1-s(1-F)]^4}\left[1 - s(1-F) + 3(1+s(1-F))\right]$$

$$= \cancel{\frac{4\rho_0 c_0^2 s\,(1-s(1-F))}{(1-s(1-F))^4}} \qquad \cancel{=} \quad \cancel{4\rho_0 c_0^2 s}$$

pg 271 gms

$k^? = \lambda + \frac{2}{3}\mu$ Th 2.25

$T \approx \mathfrak{b}(F)$ +

$\cancel{T_0} + \frac{dT}{dF}\Big|_1 (F-1) + \cdots = \mathfrak{b}(F)$

$\therefore \underline{\mathfrak{b}(F)} = k^?(F-1)$

$-\underline{\mathfrak{b}(F)} = -k^?(F-1)$ How get rid of minus sign

$p_e = k^?(F-1)$

$C_T \approx C_p = C_V \frac{k^?}{k^\theta} = \frac{C_V}{k^\theta}(\lambda + \frac{2}{3}\mu)$ $\beta = \frac{\rho C_V \Gamma}{k^\theta}$

for fluids $\cancel{C^\theta \approx C^\theta}$

$\cancel{C^\theta} = \quad C^? \approx k^?$

$\frac{1}{C^\theta} = \frac{1}{C^?} + \frac{\theta \alpha^2}{\rho C_T}$

eq 366. $\Gamma = \frac{\alpha C^\theta}{\rho C_V} \approx \frac{\alpha C^?}{\rho C_V} = \frac{3\beta C^?}{\rho_0 C_V} \approx C_p$

eq $[\![T]\!] = \rho_0 C_S^2 [\![F]\!]$

In 1D

$F = \frac{\rho_0}{\rho}$

$F < 1 \Rightarrow \rho > \rho_0$ expansion.

$F > 1 \Rightarrow \rho < \rho_0$ compression

If $\quad p_+ = p_c + \rho\Gamma[(\varepsilon_+ - \varepsilon_-) - (\varepsilon_c - \varepsilon_-)]$

$$p_c - \rho\Gamma(\varepsilon_c - \varepsilon_-) = \underbrace{p_+} - \rho\Gamma(\varepsilon_+ - \varepsilon_-)$$

$$= p_H(F_+) - \rho\Gamma(\varepsilon_H(F_+) - \varepsilon_-)$$

$$= p_H(F_+) - \rho\Gamma\left[\frac{p_H}{2\rho_0}(1 - F_+)\right]$$

$$= p_H\left[1 - \frac{\Gamma}{2}\frac{\rho}{\rho_0}(1 - F)\right]$$

$$= p_H\left[1 - \frac{\Gamma}{2}\frac{1-F}{F}\right]$$

$$p_e = \cancel{\quad} = p_H\left[1 - \frac{\Gamma}{2}\left(\frac{1-F}{F}\right)\right] + \rho\Gamma(\varepsilon - \varepsilon_-)$$

$$\Rightarrow \quad = p_H - \frac{p_H\Gamma}{2}\left(\frac{1}{F} - 1\right) + \rho\Gamma(\varepsilon - \varepsilon_-)$$

$$= p_H - \frac{p_H\Gamma}{2}\frac{(1-F)}{F} + \rho\Gamma\varepsilon - \rho\Gamma\varepsilon_-$$

$$P_e = p_H + \Gamma\rho\left[-\frac{p_H}{2\rho_0}(1-F) + \mathcal{E} - \mathcal{E}_-\right]$$

$$= p_H + \rho\Gamma\left[\mathcal{E} - \mathcal{E}_H\right]$$

Pg 375 Drumheller

2.139 $\quad [[F]]C_s + [[v]] = 0 \qquad F_- = 1 \qquad v_- = 0$

2.146 $\quad [[T]] = -\rho_0 C_s [[v]] \qquad T_- = 0$

$T \approx -p$

$$\Rightarrow (F_+ - 1)C_s + V_+ = 0$$

if $C_s = U_s$

$$F_+ = 1 - \frac{V_+}{C_s}$$

$$F_+ = 1 - \frac{V_+}{U_s}$$

$$T_+ = -\rho_0 U_s V_+$$

$$P_+ = \rho_0 U_s V_+$$

eq 2.177 $\quad T_+ = -\dfrac{\rho_0 c_0^2(1-F_+)}{[1-s(1-F_+)]^2}$

pg 325 Prowheller

$p_e = p_H + \rho\Gamma(\varepsilon - \varepsilon_H)$

4.100 $\Rightarrow$ $p_H(F) = \rho_0 U_s^2 (1-F)$

$$\varepsilon_H(F) = \frac{p_H}{2\rho_0}(1-F) + \varepsilon_-$$

But want U_s in terms of F (deformation gradient after shock)

Then taking as at def of $\Gamma = \frac{-F}{2}\frac{d^2p_H/dF^2}{dp_H/dF} - 1$

w/ $p_H = \frac{\rho_0 C_b^2 (1-F)}{(1-s(1-F))^2}$

$$\frac{dp_H}{dF} = \frac{\rho_0 C_b^2(-1)}{(1-s(1-F))^2} + \frac{-2\rho_0 C_b^2 s(1-F)}{(1-s(1-F))^3}$$

$$= \rho_0 C_b^2 \frac{(-1+s(1-F) - 2s(1-F))}{(1-s(1-F))^2}$$

$$= \rho_0 C_b^2 \frac{(-1-s(1-F))}{(1-s(1-F))^3} = \frac{-\rho_0 C_b^2(1+s(1-F))}{(1-s(1-F))^3}$$

$$\frac{d^2p_H}{dF^2} = \frac{-\rho_0 C_b^2[-s]}{(1-s(1-F))^3} - \frac{\rho_0 C_b^2(-3)s(1+s(1-F))}{(1-s(1-F))^4}$$

$$\frac{d^2 p_H}{dF^2} = \frac{-\rho_0 c_b^2\left[(-1+s(1-F))s - 3s(1+s(1-F))\right]}{(1-s(1-F))^4}$$

$$= \frac{-\rho_0 c_b^2\left[-s + s^2(1-F) - 3s - 3s^2(1-F)\right]}{(1-s(1-F))^4}$$

$$= \frac{-\rho_0 c_b^2\left[-4s - 2s^2(1-F)\right]}{(1-s(1-F))^4}$$

$$= \frac{2\rho_0 c_b^2 s(2 - s(1-F))}{(1-s(1-F))^4}$$

Then using Rice/McQueen/Walsh

$$\Gamma = \frac{\cancel{+}F}{2}\;\frac{\dfrac{2\rho_0 c_b^2 s(2-s(1-F))}{(1-s(1-F))^{\cancel{4}}}}{\dfrac{-\rho_0 c_b^2(1+s(1-F))}{(1-\cancel{s(1-F)})^3}} - 1$$

$$= \frac{F\cancel{\rho_0 c_b^2} s(2-s(1-F))}{(1-s(1-F))\cancel{\rho_0 c_b^2}(1+s(1-F))} - 1$$

$$= \frac{2sF\left(1 - \frac{s}{2}(1-F)\right)}{1-s^2(1-F)^2} - 1$$

let $F_+ = 1 - \frac{1}{s}$ $\qquad \rho_+ \longrightarrow \infty$

$1 - F_+ = \frac{1}{s}$

If $p_H \to \infty$ As $F_+ \to 1 - \frac{1}{s}$

+ need $\frac{\Gamma}{2}\left(\frac{1-F}{F}\right) = 1$ when $F = 1 - \frac{1}{s}$

$$\Rightarrow \Gamma = \frac{2F}{1-F} = \frac{2(1-1/s)}{\cancel{1-1} + 1/s} = 2(s-1)$$

eq 4.73 $p_c = -\rho_0 \frac{d\mathcal{E}_c}{dF}$

eq 4.101 $\cancel{p_c = \rho F}$

$$\Rightarrow -\rho_0 \frac{d\mathcal{E}_c}{dF} - \rho\Gamma(\mathcal{E}_c - \mathcal{E}_-) = \frac{p_H}{\rho_0}\left[1 - \frac{1}{2}\left(\frac{\Gamma}{F}\right)(1-F)\right]$$

$$\Rightarrow \frac{d\mathcal{E}_c}{dF} + \frac{\Gamma}{F}(\mathcal{E}_c - \mathcal{E}_-) = -\frac{p_H}{\rho_0}\left[1 - \frac{1}{2}\left(\frac{\Gamma}{F}\right)(1-F)\right]$$

pg 379 Drumheller

$$\left|\frac{\partial \hat{F}}{\partial t}\right|_M \propto P_+^4$$

$$p_e = p_H$$

$$p_e = \frac{\rho_0 c_b^2(1-F)}{\left(1-s(1-F)\right)^2} = \rho_0 c_b^2(1-F)\left\{1 + s(1-F) + O\left((1-F)^2\right)\right\}$$

$$= \qquad \frac{1}{(1-x)^2} = \left(\sum_{k=0}^{\infty} x^k\right)^2 =$$

or $$\stackrel{?}{=} \frac{d}{dx}\frac{1}{(1-x)} = (-1)\frac{(-1)}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\therefore \quad \frac{1}{(1-x)^2} = \sum_{k=1}^{\infty} k x^{k-1}$$

$$\therefore \quad p_e = \rho_0 c_b^2(1-F)\left\{1 + 2\left(s(1-F)\right) + O\left((1-F)^2\right)\right\}$$

$$= \rho_0 c_b^2(1-F) + 2s\rho_0 c_b^2(1-F)^2 + O\left((1-F)^3\right)$$

eq (3.10) $$\rho_0 \frac{\partial \hat{\varepsilon}}{\partial t} = T\frac{\partial \hat{F}}{\partial t} - \cancelto{0}{\frac{\partial \hat{q}}{\partial X}} + \cancelto{0}{\rho_0 r}$$

$$\rho_0 \frac{\partial \hat{\varepsilon}}{\partial t} = -p_e \frac{\partial \hat{F}}{\partial t}$$

~~Int~~

$$\to \rho_0 \frac{\partial \hat{\mathcal{E}}}{\partial t} = -\left(\rho_0 c_b^2 (1-\hat{F}) + 2s\rho_0 c_b^2 (1-\hat{F})^2 + O((1-\hat{F})^3)\right) \frac{\partial \hat{F}}{\partial t}$$

$$\int \to$$

$$\rho_0(\mathcal{E}_S - \mathcal{E}_-) = -\left(\rho_0 c_b^2 \frac{(1-\hat{F})^2}{(-2)} + 2s\rho_0 c_b^2 \frac{(1-\hat{F})^3}{(-3)} + O((1-\hat{F})^4)\right)$$

$$= \frac{1}{2}\rho_0 c_b^2 (1-F_+)^2 + \frac{2}{3} s\rho_0 c_b^2 (1-F_+)^3 + O((1-F_+)^4)$$

$$\rho_0 [[\mathcal{E}]] = \frac{1}{2}(T_+ + T_-)[[F]] \quad \text{eq } 3.112$$

$$\rho_0(\mathcal{E}_+ - \mathcal{E}_-) = \cancel{\tfrac{1}{2}(-P_e + \ldots)}$$

$$= \frac{1}{2}(-P_e + 0)(F_+ - 1) = \frac{1}{2} P_e (1-F_+)$$

put in eq 4.114 $\Rightarrow$

$$= \frac{1}{2}\left(\rho_0 c_b^2 (1-F_+)^2 + 2s\rho_0 c_b^2 (1-F_+)^3 + \ldots\right)$$

$$= \frac{\rho_0 c_b^2}{2}(1-F_+)^2 + s\rho_0 c_b^2 (1-F_+)^3 + O((1-F_+)^4)$$

$$\rho_0\,\delta\mathcal{E} = \rho_0(\mathcal{E}_+ - \mathcal{E}_s) = \rho_0(\mathcal{E}_+ - \mathcal{E}_-) - \rho_0(\mathcal{E}_s - \mathcal{E}_-)$$

$$= \frac{1}{3}\rho_0 c_b^2 (1-F_+)^3 + O((1-F_+)^4)$$

Steady wave dissipates energy
Structured wave does not dissipate energy. } look up.

$$A = \rho_0\,\delta\mathcal{E}\,\delta t \qquad \text{(shock invariant)}$$

$$= \frac{1}{3} s\rho_0 c_b^2 \frac{(1-F_+)^{3+1}}{|\partial\hat{F}/\partial t|_M}$$

or $$\left|\frac{\partial\hat{F}}{\partial t}\right|_M = \frac{s\rho_0 c_b^2 (1-F_+)^4}{3A}$$

4,114 $\Rightarrow$

$$P_e = \rho_0 c_b^2(1-F) + 2s\rho_0 c_b^2(1-F)^2 + \cdots$$

$$\Rightarrow (1-F) = \frac{P_e}{\rho_0 c_b^2} - 2s(1-F)^2 + \cdots$$

$$\Rightarrow 1-F = \frac{P_e}{\rho_0 c_b^2} + O((1-F)^2)$$

$$\Rightarrow 1-F = \frac{P_e}{\rho_0 c_b^2} + O\left(\frac{P_e^2}{\rho_0^2 c_b^4}\right) \cdots$$

$$\Rightarrow \left|\frac{\partial\hat{F}}{\partial t}\right|_M = \frac{s\rho_0 c_b^2}{3A}\,\frac{P_e^4}{\rho_0^4 c_b^8} = \frac{sP_e^4}{3A(\rho_0 c_b^2)^3}$$

Y

Steady wave : structure of wave does not change w/ time

{shock wave ⟺ steady wave}

vs.

smeared shock wave

structured wave: ?

2.188 is $T - \cancel{T_-}^{0} = -\rho_0 C_s (v - \cancel{v_-})^{0}$ velocity

$\Uparrow$

lin Hug.

$$T = -\rho_0 (\underbrace{C_0 + S v_+}) (\cancel{v_+} - \cancel{v_-})^{0}$$ full stress

valid after shock has passed

~~$P_e = \rho_0 (C_0 + S v) v$~~ $\quad P_e = \rho_0 (C_0 + S v_+) v$

$$T = -P_e + J$$

$$\Rightarrow \quad J = T + P_e =$$

If $\quad J = S\rho_0 (v^2 - v_+ v)$

$$\frac{dJ}{dv} = S\rho_0 (2v - v_+) = 0 \qquad v = \frac{v_+}{2}$$

$$|\tau|_m = S\rho_0 \left(\frac{v_+^2}{4} - \frac{v_+^2}{2} \right) = -\frac{S\rho_0 v_+^2}{4}$$

$$|T|_m \approx \frac{P_+^2 S\rho_0}{\rho_0^2 C_0^2 4} = \frac{S P_+^2}{4 \rho_0 C_0^2}$$

(9.115) pg 382 Drumheller

$$\left|\frac{\partial \hat{F}}{\partial t}\right|_M = A' p_+^4$$

$$= A' \frac{16 \rho_0^2 C_b^4}{S^2} |\tau|_M^2$$

$$\tau = \left(\frac{S^2}{16\rho_0^2 C_b^4 A'}\right)^{1/2} \operatorname{sgn}\left(\frac{\partial \hat{F}}{\partial t}\right)\left(\frac{\partial \hat{F}}{\partial t}\right)^{1/2}$$

---

eq 2.139 $\quad (F_+ - 1)\underset{\underset{U_S}{\parallel}}{G_S} + (v_+ - 0) = 0$

$$v_+ = (1 - F_+) U_S$$

$$\overset{\parallel}{=} \frac{(U_S - c_0)}{S} = (1 - F_+) U_S$$

$$U_S - c_0 = S(1 - F_+) U_S$$

$$U_S = \frac{c_0}{1 - S(1 - F_+)}$$

eq 2.146 $\quad [[T]] = -\rho_0 G_s [[v]]$

$$T_+ - \cancelto{0}{T_-} = -\rho_0 U_S (v_+ - \cancelto{0}{v_-})$$

$$-p_e = -\rho_0 U_S v_+ =$$

Pg 384 Drumheller

$$\frac{d\mathcal{E}_c}{dF} +$$

If $\Gamma_p = \rho_0 \Gamma_0 \Rightarrow \underbrace{\frac{\rho}{\rho_0}\Gamma}_{} = \Gamma_0$

$$\frac{\Gamma}{F} = \Gamma_0$$

$$\Rightarrow \frac{d\mathcal{E}_c}{dF} + \Gamma_0(\mathcal{E}_c - \mathcal{E}_-) = \frac{-p_H}{\rho_0}\left[1 - \frac{1}{2}\Gamma_0(1-F)\right]$$

$p_H(F) = \rho_0 U_s V_+$ (eq Now: $V_+ U_s = U_s^2(1-F_+)$

Above 4.119)

$\therefore \quad p_H(F) = \rho_0 U_s^2(1-F)$ $\quad U_s$ still depends on F?

$$\Rightarrow \frac{d}{dF}(\mathcal{E}_c - \mathcal{E}_-) + \Gamma_0(\mathcal{E}_c - \mathcal{E}_-) = \frac{-p_H}{\rho_0}\left[1 - \frac{\Gamma_0}{2}(1-F)\right]$$

Integ factor is

$$\left\{ \frac{d}{dx} \quad v\cancel{\frac{du}{dx}} + v[\,] = \frac{d}{dx}(vu) = v\cancel{\frac{du}{dx}} + u\frac{dv}{dx}\right.$$

$$\Rightarrow u\frac{dv}{dx} = v[\,]$$

$$\Rightarrow \quad \frac{dv}{v} = \frac{\square}{U}\,dx$$

in our problem

$$\frac{dv}{v} = \frac{\Gamma_0(\mathcal{E}_c - \mathcal{E}_-)}{\mathcal{E}_c - \mathcal{E}_-} = \Gamma_0\, dF$$

$$\ln(v)\Big|_{v_-}^{v} = \Gamma_0 F\Big|_{F_-}^{F}$$

drops constants

$$\ln v - \ln v_- = \Gamma_0(F - 1)$$

$$\ln v = \Gamma_0 F.$$

$$v = e^{\Gamma_0 F}$$ check

$$e^{\Gamma_0 F}\frac{d}{dF}(\mathcal{E}_c - \mathcal{E}_-) + \Gamma_0 e^{\Gamma_0 F}(\mathcal{E}_c - \mathcal{E}_-) = \checkmark$$

$$\Rightarrow \frac{d}{dF}\left[e^{\Gamma_0 F}(\mathcal{E}_c - \mathcal{E}_-)\right] = -\frac{P_H}{\rho_0}\left[1 - \frac{\Gamma_0}{2}(1-F)\right] e^{\Gamma_0 F}$$

$$(\mathcal{E}_c - \mathcal{E}_-)e^{\Gamma_0 F}\Big| =$$

~~Pg 306 Strogatz~~

(4.70)

$$\mathcal{E} - \mathcal{E}_c = \mathcal{E}_\theta = C_\eta \theta$$

$C_v = C_\eta$ when? (when specific heat is constant.

$$\mathcal{E}_- - \mathcal{E}_c(1) = C_v \theta_-$$

$$\Rightarrow \quad (\mathcal{E}_c - \mathcal{E}_-) e^{-\Gamma_0(1-F)} = C_v \theta_H + \underbrace{(\mathcal{E}_c(1) - \mathcal{E}_-)}_{-C_v\theta_-}$$

$$= C_v(\theta_H - \theta_-)$$

$$\Rightarrow \quad \mathcal{E}_c - \mathcal{E}_- = C_v(\theta_H - \ldots \qquad \cancel{p_H}$$

eq 4.120

$$\Rightarrow \quad \rho_0 \frac{d\mathcal{E}_c}{dF} = -\rho_0 \Gamma_0 (\mathcal{E}_c - \mathcal{E}_-) - \frac{p_H}{\rho_0}\left[1 - \frac{\Gamma_0}{2}(1-F)\right]$$

$$\therefore \quad p_c = -\rho_0 \frac{d\mathcal{E}_c}{dF} = \ldots$$

$$c^c = -F\left(\rho_0 \Gamma_0 \frac{d\mathcal{E}_c}{dF} + \frac{dp_H}{dF}\left[1 - \frac{\Gamma_0}{2}(1-F)\right] + p_H\left[\frac{1}{2}\Gamma_0\right]\right)$$

$$C^c = \cancel{F \rho_0 \Gamma_0} - F \frac{dp_H}{dF}\left[1 - \frac{\Gamma_0}{2}(1-F)\right]$$

$$\underbrace{-F \rho_0 \Gamma_0 \frac{d\varepsilon_c}{dF}}_{F\Gamma_0 p_c} \quad \underbrace{- \frac{F}{2}\Gamma_0 p_H}_{-\frac{F}{2}\Gamma_0 p_H}$$

$$\Rightarrow \quad C^c = -F\left[1 - \frac{\Gamma_0}{2}(1-F)\right]\frac{dp_H}{dF} + F\Gamma_0\left(p_c - \frac{p_H}{2}\right)$$

$$p_e = p_c + p_\theta = p_c + \rho\Gamma\varepsilon_\theta$$

$$= p_c + \rho_0\Gamma_0 C_v \theta_+$$

$$C_v \Theta_H = -\int_1^F \frac{p_H}{\rho_0}\left[1 - \frac{\Gamma_0}{2}(1-F)\right] e^{-\Gamma_0(1-F)}\, dF$$

$$p_H(F) = \rho_0 U_s v_+ = \rho_0 U_s^2 (1-F)$$

$$= \frac{\rho_0 c_0^2}{\left(1 - s(1-F)\right)^2}(1-F)$$

Thus $$G_r \Theta_H = -C_0^2 \int_1^F \frac{(1-F)}{(1-s(1-F))^2}\left[1 - \frac{\Gamma_0}{2}(1-F)\right] e^{-\Gamma_0(1-F)}\, dF$$

$C_0 = 4.58$ km/s , $s = 1.49$, $\Gamma_0 = 1.94$ ~ 1.98 , $F_t = .75$.

MKS

Then $G_r \Theta_H(F_t) = .72159\ \mathrm{km}^2/s^2$

w/ $G_r = 447 \frac{J}{kg}\frac{1}{K}$ $\qquad [J] = [F \cdot d] = [Ma \cdot d]$

$$= kg \frac{m^2}{s^2}$$

$$\therefore \quad \Theta_H(F_t) = \frac{.72159\ 10^6\ \cancel{m^2/s^2}}{447\ \cancel{m^2/s^2}\ \cancel{\frac{kg}{kg}}\ \frac{1}{K}} \qquad 1\ km = 10^3\ m$$

$$= \frac{.72159}{447} \cdot 10^6\ K = 1593.98\,K$$ Assuming non constant shock speed.

Assuming A constant shock speed ? Why/Why not ?

$$P_H = \rho_0 (U_s)^2 (1-F)$$

Then $$G_r \Theta_H = -\int_1^F U_s^2 (1-F)\left[1 - \frac{\Gamma_0}{2}(1-F)\right] e^{-\Gamma_0(1-F)}\, dF$$

$$G_r \Theta_H = 1.025\ \mathrm{km}^2/s^2$$

$\Rightarrow \Theta_H(F_+) = \quad 1.6$

$$\varepsilon_c - \varepsilon_- = C_v(\Theta_H - \Theta_-)e^{\gamma_0(1-F)}$$

$$\varepsilon_c - \varepsilon_- = 447\,\tfrac{J}{kgK}\cdot(1{,}360K - 295K)\,e^{1.94(1-.75)}$$

$$= 7.54\times10^5\ J/kg = 754\ kJ/kg$$

$$p_c = \cancel{(7.89\times10^6\ g/m^3)(1.94)(773\times10^3\ J/kg)}$$

$$\cancel{= 7.}$$

$$\rho = 7.89\times10^6\ g/m^3 = 7.89\times10^3\ kg/m^3$$

$$p_c = (7.89\times10^3\ kg/m^3)(1.94)(773\times10^3\ J/kg)$$

$$+(105\ GPa)\left[1 - \frac{(1.94)}{2}(1-.75)\right]$$

$$= 79.5\ GPa + 1.18\times10^{19}\ J/m^3 =$$

$$Pa = \qquad \left[\frac{J/m}{m^2}\right] = [Pa]$$

Is Pascal an MKS unit? yes $\qquad M = 10^6$.

$1.18 \times 10^{10} = $ [illegible] . 11.8 GPa

$P_c = 9.13 \times 10^1$ GP $= 91.3$ GPa

$$\theta_+ = \frac{P_+ - P_c(F_+)}{\rho_0 \Gamma_0 C_v} = \frac{(105 - 91.3) \times 10^9 \text{ Pa}}{7890\,\cancel{\text{kg}}/\text{m}^3 (1.94)(447 \frac{\text{J}}{\cancel{\text{kg}}\,\text{K}})}$$

$$= 2.00 \times 10^{-6} \cdot 10^9 \frac{\text{Pa} \quad \text{K}}{\text{J}/\text{m}^3}$$

$$= 2000 \text{ K} \quad \frac{\text{Pa}}{\left(\frac{\text{J}/\text{m}}{\text{m}^2}\right)} = 2000 \text{ K}.$$

$$\frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_+} = e^{\Gamma_0(1-F)} \, e^{\Gamma_0(F_+ - 1)}$$

eq 4.92

$$\frac{\theta}{\theta_0} = e^{\Gamma_0(1-F)}$$

$$\cancel{\frac{\theta}{\theta_+} = e^{\Gamma_0(F}}$$

$$= e^{\Gamma_0(1-F)} \, e^{-\Gamma_0(1-F_+)}$$

$$\frac{\theta}{\theta_+} = e^{\Gamma_0(F_+ - F)}$$