Solutions for the Book:
Fantastic Book of Math Puzzles
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Slaying Glubs

Letting h be the number of glubs killed by Hylar and g the number of glubs killed by Grabus, then the statements for this problem require

$$\begin{aligned} g + h &= 24 \\ g &= h + 4 \,. \end{aligned}$$

This gives $h = 10$ and $g = 14$.

How Much Did Alaranthus Weigh?

$$w = 1000 + \frac{2}{3}w \quad \text{or} \quad w = 3000 \,.$$

How Much Did a Fully Grown Alaranthus Weigh?

$$w = \frac{1}{5}w + m = \frac{1}{5}w + (w - 800) \,.$$

This gives $w = 4000$.

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How Many Winged Cats?

$$w = \frac{2}{3}w + \frac{2}{3}\,.$$

This gives $w = 2$.

The Cost of Cider

Let d be the cost of the drink (cider) and c be the cost of the container. Then we are told that

$$\begin{aligned} d + c &= 10 \\ d &= c + 4\,. \end{aligned}$$

This gives $d = 7$ and $c = 3$.

Apples for All

Let n be the number of boys and a be the number of apples initially. Then we are told that $a = 3n$. After three more boys arrive we are told that

$$2(n+3) = a + 1\,.$$

These two equations give $n = 5$ and $a = 15$ initially and afterwards we have $n + 3 = 8$ boys and $a + 1 = 16$ apples.

How Many Horses?

Let m be the number of men and h be the number of horses. Then we are told by counting legs first and then heads that

$$\begin{aligned} 2m + 4h &= 120 \\ m + h &= 45\,. \end{aligned}$$

This gives $h = 15$ and $m = 30$.

Wooden Swords

We are told the relationships

$$\begin{aligned} j - 1 &= k \\ j + 1 &= 2k\,. \end{aligned}$$

These two give $k = 2$ and $j = 3$.

How Old is Mongo's Son?

If we let a be the age of Mongo's son and n the unknown positive integral number's square root, then the description given can be written as

$$a + 6 = n^2$$
$$a - 6 = n\,.$$

When we put the second equation into the first and solve the resulting quadratic equation for a we get

$$a = \frac{13 \pm \sqrt{169 - 120}}{2}\,.$$

This has two values $\frac{5}{2}$ and 10. The boys age must be 10.

Two Riders

Let the distance between the two towns be denoted by D, then this distance is shrinking at a rate of $30 + 28 = 58$ miles per hour. The distance between the two riders as a function of time is given by

$$d(t) = D - 58t\,.$$

The two riders meet at at time t_m when $d = 0$ or $t_m = \frac{D}{58}$ hours. They will have a distance between them given by $d(t_m - 1)$ one hour before they meet. This can be evaluated as

$$d(t_m - 1) = D - 58t_m + 58 = 58\,,$$

miles since $D - 58t_m = 0$ by the definition of t_m.

A Lame Horse

Let D be the total distance the knight travels. Let t_{ride} the the time taken on the first part of the journey (on horseback) and v_{ride} the velocity on this segment of the journey. Then

$$t_{\text{ride}} = \frac{\frac{1}{3}D}{v_{\text{ride}}}\,.$$

Let t_{walk} be the time to walk on the second half of the journey (on foot) and v_{walk} his velocity. Then we have

$$t_{\text{walk}} = \frac{\frac{2}{3}D}{v_{\text{walk}}}\,.$$

Since we are told that $t_{\text{walk}} = 20t_{\text{ride}}$ this means that

$$\frac{\frac{2}{3}D}{v_{\text{walk}}} = 20\frac{\frac{1}{3}D}{v_{\text{ride}}}\,.$$

If we then divide by $\frac{2D}{3}$ we get

$$\frac{1}{v_{\text{walk}}} = \frac{10}{v_{\text{ride}}},$$

or

$$\frac{v_{\text{ride}}}{v_{\text{walk}}} = 10.$$

The Messenger

The time to run the first sixteen miles was $\frac{16}{8} = 2$ hours. The time to run the last eight miles if we run at a rate of r will be $\frac{8}{r}$. The total time to run the full 24 miles is then $2 + \frac{8}{r}$. The average rate is then $\frac{24}{2+\frac{8}{r}}$. To have this equal to 12 will require that

$$12 = \frac{24}{2 + \frac{8}{r}},$$

or

$$2 = 2 + \frac{8}{r}.$$

This would mean that $\frac{8}{r} = 0$ or that $r = +\infty$ which is not possible.

Jousting Tournament Number

To better explain this problem assume that our knights number was three. Then the numbers that are less than this number are one and two (so there are two of them) and the numbers that are greater than this number are

$$4, 5, 6, 7, 8, 9, 10, 11,$$

which are eight numbers. The product is then $2 \cdot 8 = 16$. If we make a table of all the possible tournament numbers, the numbers less than the given tournament number, the numbers greater than the given tournament number, and the product of these two we would get the following

tournament_number	1	2	3	4	5	6	7	8	9	10	11
numbers_less_than	0	1	2	3	4	5	6	7	8	9	10
numbers_greater_than	10	9	8	7	6	5	4	3	2	1	0
number_product	0	9	16	21	24	25	24	21	16	9	0

Thus we see if we are number five then two plus that number is seven and we get the same product of 24.

Inspecting the Troops

To get to the 30th man we have to pass by 29 segments (the ten feet between any two men). Thus the general is passing one segment per second. To get to the sixtieth man the general would have to pass by 59 segments thus the total time for the full inspection would be 59 seconds.

How much for a Knife, Sword, or Arrow

From the problem we are told that

$$k + s + 9a = 36 \quad (1)$$
$$2s = k + 4a \,. \quad (2)$$

From Equation 1 by multiplying by two we get $2k + 2s + 18a = 72$. Putting this into Equation 2 we get

$$3k + 22a = 72 \,.$$

If we assume that a knife and an arrow must be positive whole numbers we can try to find values of k and a that will satisfy the above equation. From the above we have that the limits of a are bounded as

$$1 \leq a \leq \lfloor \frac{72}{22} \rfloor = 3 \,.$$

Thus there are only three possible values for a that of $1, 2, 3$. Once a is specified we know that k must be given by

$$k = \frac{72 - 22a}{3} \,.$$

For this specification we get the simple table

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arrows   1        2           3
knives 16.66667 9.333333    2
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Thus we get positive whole numbers if $a = 3$, so that $k = 2$ and $s = 36 - 9a - k = 7$.

Strange Rabbits

The location of a rabbit after x jumps of five and y jumps of seven is given by

$$5x + 7y \,.$$

Here x and y are integers can be negative if needed. We want to have this expression equal 13 in the smallest number of jumps. That is the values of $|x|$ and $|y|$ which count the number of jumps should be as small as possible. We can find possible values of x and y that satisfy the above by taking integer values of x in some range (say $x \in [-20, +20]$) and then computing the y needed to make the above equal to 13 using

$$y = \frac{1}{7}(13 - 5x) .$$

We will only consider values of y computed in the above that are integers. When we do the above computations we find that some possible values for x and y are given by

X	-17	-10	-3	4	11	18
Y	14	9	4	-1	-6	-11
Num. Jumps	31	19	7	5	17	29

We see that the smallest number of jumps happens when $x = 4$ and $y = -1$, meaning that we jump to 20 using four jumps of five and then jump to 13 from 20 using one jump of seven backwards. This takes five total jumps.

What is the Human Population of South Primm?

Let N be the population of north Primm. We are told that all of

$$\frac{1}{3}N \, , \frac{1}{4}N \, , \frac{1}{5}N \, , \frac{1}{7}N \, ,$$

are all whole numbers. Thus the number N must be a multiple of another number N' as

$$N = 3(4)(5)(7)N' = 420N' .$$

Since we are told that $N < 500$ we must have $N' = 1$. Thus $N = 420$. The population of South Primm is then given by summing the above fractions of N. When we do that we get the value of 389.

Measuring Two Gallons of Cider

One way would be to fill the four gallon container and then poor what you can into the three gallon container. This will yield one gallon of liquid in the four gallon container. Transfer this liquid to the container that Mongo will leave with. Doing this procedure a second time will give Mongo two gallons of cider.

Another way would be to first fill the three gallon container and poor this into the four gallon container. Fill the three gallon container a second time and poor what can be poured into the four gallon container. Only one gallon will be able to be poured into the four gallon container what remains in the three gallon container is two gallons.

How Many Dwarf's?

From the problem statement, one human requires $\frac{1}{120}$ of an amphitheater and one dwarf requires $\frac{1}{144}$ of an amphitheater. Thus if we have 90 humans in the amphitheater we would like to find the number of dwarfs d such that the amphitheater is full. In equations, this is

$$90\frac{1}{120} + d\frac{1}{144} = 1\,.$$

Solving for d we find $d = 36$.

Chased by a Glub

Note that as Phipos ran for 1/2 of the *time* it took him to reach the fort and then went to walking he had to have run more than 1/2 the distance to the fort when he changed to walking. This is because if he had not run that far when he switched to walking (and slowed down) the second half of his journey would have taken him *longer* to travel than the first half making the two parts of the journey of different lengths. Thus then the glub stopped running (at 1/2 of the distance) Phipos was still running and created an even larger separation between him and the glub than five feet.

A Dragon Story

Let h_m, b_m, and t_m represent the lengths of the head, the body, and the tail of the mother and h_o, b_o, and t_o represent the same thing for the dragons offspring. Then the various statements given imply the following equations

$$b_m = 3h_m \tag{3}$$
$$h_m = 2h_o \tag{4}$$
$$h_m = b_m - t_m \tag{5}$$
$$b_o = h_o + 2 \tag{6}$$
$$t_o = \frac{1}{3}t_m \tag{7}$$
$$h_m + b_m + t_m + h_0 + b_o + t_o = 48\,. \tag{8}$$

This is a system of six equation in six unknowns and thus can possibly have a unique solution. To solve this we perform the following manipulations. Our goal is two write every variable in Equation 8 in terms of the one variable h_m. To start we put Equation 3 into 5 to get $h_m = 3h_m - t_m$ or

$$2h_m = t_m\,, \tag{9}$$

which gives t_m in terms of h_m. Thus in Equation 8

- To replace b_m use Equation 3

- To replace t_m use Equation 9
- To replace h_o use Equation 4 as $h_o = \frac{1}{2}h_m$
- To replace b_o use Equation 6 and 4 as $b_o = h_o + 2 = \frac{1}{2}h_m + 2$
- To replace t_o use Equation 7 with 9 as $t_o = \frac{1}{3}t_m = \frac{1}{3}(2h_m)$

When we put all of these into Equation 8 we get

$$h_m + 3h_m + 2h_m + \frac{1}{2}h_m + \frac{1}{2}h_m + 2 + \frac{2}{3}h_m = 48\,.$$

When we solve for h_m we get $h_m = 6$. We can then put this back into the above expressions to find values for all the other unknowns.

How Far Apart were the Dragons?

Let D be the distance between the two dragons initially. The distance between the dragons as a function of time is given by

$$d(t) = D - (24 + 36)t = D - 60t\,,$$

for $0 \leq t \leq t_{\text{meet}}$. The time the meet will be when $d(t) = 0$ or $t_{\text{meet}} = \frac{D}{60}$. When the time is 5 minutes before they meet (converting 5 minutes into hours means we need to consider $\frac{5}{60} = \frac{1}{12}$ hours) the distance between the two dragons is given by

$$d = D - 60\left(t_{\text{meet}} - \frac{1}{12}\right) = D - 60t_{\text{meet}} + \frac{60}{12} = 5\,,$$

miles.

The Farmer and the Hobgoblin

Let M be the number of coins the farmer starts with initially. On the first crossing he will have $2M - 16$, after he has payed the hobgoblin his 16 coins. On the next crossings we find that the farmer has

$$\begin{aligned} \text{Second crossing:} &\quad 2(2M - 16) - 16 = 4M - 48 \\ \text{Third crossing:} &\quad 2(4M - 48) - 16 = 8M - 112 \\ \text{Fourth crossing:} &\quad 2(8M - 112) - 16 = 16M - 240\,. \end{aligned}$$

On the fourth crossing the farmer had no more money. This means that

$$16M - 240 = 0\,.$$

Solving for M we find $M = 15$.

The Wizards Dragon Spells

From the problem statement we have the relationships that

$$S = \sqrt{T} \tag{10}$$

$$S = \frac{1}{2}\sqrt{M} \tag{11}$$

$$T = 2\sqrt{M}\,. \tag{12}$$

These are three equations and three unknowns and thus there can be a unique solution. Given Equations 10 and 11 we have $\sqrt{T} = \frac{1}{2}\sqrt{M}$ or by squaring both sides that $T = \frac{1}{4}M$. Putting this into Equation 12 means that

$$2\sqrt{M} = \frac{1}{4}M \quad \text{or} \quad 8 = \sqrt{M}\,.$$

Thus $M = 64$. Since we now know M we have that $T = 2(8) = 16$ and $S = \sqrt{16} = 4$.

Wizard Rankings

Let the letters A_1, B_1, C_1, D_1, E_1, and F_1 stand for the various wizards rankings on the first week. Thus we are told that $A_1 = 1$, $B_1 = 2$, $C_1 = 3$, $D_1 = 4$, $E_1 = 5$, and $F_1 = 6$. Let A_2, B_2, C_2, D_2, E_2, and F_2 stand for the various wizards rankings on the second week. The second fact given in the problem (about the product of the rankings) can be written as

$$D_1 D_2 = F_1 F_2 \quad \text{or} \quad 4D_2 = 6F_2 \quad \text{or} \quad 2D_2 = 3F_2\,.$$

We now need to consider only values of D_2 and F_2 in the range $[1, 6]$ that satisfy the above condition. Note that for $1 \le D_2 \le 6$ we have that

$$2D_2 \in \{2, 4, 6, 8, 10, 12\}\,,$$

and for $1 \le F_2 \le 6$ we have that

$$3F_2 \in \{3, 6, 9, 12, 15, 18\}\,.$$

Values of $2D_2$ and $3F_2$ are only equal at the values of 6 and 12, which would correspond to the rankings

$$D_2 = 3 \quad \text{and} \quad F_2 = 2 \quad \text{or}$$
$$D_2 = 6 \quad \text{and} \quad F_2 = 4\,.$$

Consider the first of the possible second week rankings above i.e. the one where $D_2 = 3$ and $F_2 = 2$. To see if we can form a valid ranking with this assignment we have to attach rankings $1, 4, 5, 6$ to the wizards A_2, B_2, C_2, and E_2 such that Bogara's change in ranking from week one to week two is the largest. Since Bogara's ranking in the first week was 2 his change in ranking will be 1, 2, 3, or 4 under his various possible new rankings. Note that since the first week Fortuna was ranked sixth i.e. $F_1 = 6$ the change in rankings of Fortuna is four. Thus the largest change in ranking is not Bogara and thus we cannot use the assignment where $D_2 = 3$ and $F_2 = 2$.

From the arguments above it must be true that $D_2 = 6$ and $F_2 = 4$ and we have to now attach rankings $1, 2, 3, 5$ to wizards A_2, B_2, C_2, and E_2 again such that Bogara's change in ranking from week one to week two is the largest. To enforce this last constraint we have that $B_2 = 5$ to give Bogara's ranking change of three We can't assign the rank of one or two for E_2 or otherwise his ranking change will be equal or larger than Bogara's. Thus $E_2 = 3$. That leaves $A_2 = 1$ or $A_2 = 2$. To have each wizard have a different ranking than last week we need $A_2 = 2$. Thus the final ranking are

$$A_2 = 2\,, B_2 = 5\,, C_2 = 1\,, D_2 = 6\,, E_2 = 3\,, F_2 = 4\,.$$

How Long are the Glubs?

We start by denoting the length of the five glubs to be given by L_1, L_2, L_3, L_4, and L_5. Then from the fact about the total length of the glubs we have

$$L_1 + 5 + L_2 + 5 + L_3 + 5 + L_4 + 5 + L_5 = 200\,. \tag{13}$$

Here I have inserted the five feet between each glub. Next we are told that $L_1 = L_3 = L_5$ and $L_2 = L_4$. The common value of L_1, L_3, and L_5 we will denote as l_1 and the common value of L_2 and L_4 we will denote as l_2. With this Equation 13 becomes

$$3l_1 + 2l_2 = 180\,.$$

In addition, we are told that both l_1 and l_2 are multiples of 10 so we can write $l_1 = 10n_1$ and $l_2 = 10n_2$ and get

$$30n_1 = 20n_2 = 180 \quad \text{so} \quad 3n_1 + 2n_2 = 18\,.$$

For the neighboring glubs to be 10 feet shorter or longer than each other we need $n_1 = n_2 \pm 1$. Here the $+1$ means that the first, third, and fifth glubs are 10 feet longer than the second and the fourth, while the -1 means that the first, third, and fifth glubs are 10 feet shorter than the second and the fourth. If we entertain both choices we would get

$$3(n_2 \pm 1) + 2n_2 = 18\,.$$

or

$$5n_2 \pm 3 = 18\,.$$

The two solutions would be $n_2 = 3$ (for the $+1$) and $n_2 = \frac{21}{5}$ (for the -1). Since we want n_2 to be an integer we must take $n_2 = 3$. This means that $n_1 = n_2 + 1 = 4$ and the lengths of the glubs are given by

$$L_1 = 40\,, L_2 = 30\,, L_3 = 40\,, L_4 = 30\,, L_5 = 40\,.$$

Did the Dragon Catch Pryor?

To solve this problem we will compute how long it will take Pryor to get to the ocean and at the same time how long it will take the dragon to get to the ocean. If the dragon gets to the ocean *before* Pryor he must pass him in the meantime and would therefore catch him.

The calculation for Pryor is easy since he runs at a fixed rate of 20 miles per hour. Thus the time it takes him to run the 2 miles is given by $\frac{2}{20} = \frac{1}{10}$ of an hour. Thus it take Pryor is 6 minuets to reach the ocean.

Once the dragon starts it will take him $\frac{1}{20}$ of an hour to run the first mile, $\frac{1}{40}$ of an hour to run the second mile, $\frac{1}{80}$ of an hour to run the third mile etc. Thus to run the total seven miles will take the dragon an amount of time (in hours) given by

$$\frac{1}{20} + \frac{1}{40} + \frac{1}{80} + \frac{1}{160} + \frac{1}{320} + \frac{1}{640} + \frac{1}{1280}\,.$$

There are seven terms (for the seven miles) in the above sum. We can write the above as

$$\frac{1}{20}\left(\sum_{k=0}^{6}\left(\frac{1}{2}\right)^k\right) = \frac{1}{20}\left(\frac{1-\left(\frac{1}{2}\right)^7}{1-\frac{1}{2}}\right) = \frac{1}{10}\left(1-\left(\frac{1}{2}\right)^7\right).$$

Since the dragon leaves six seconds after Pryor does we need to add six seconds to the time the dragon takes to get to the ocean (to properly compare the two times). As six seconds is $\frac{6}{3600} = \frac{1}{600}$ of an hour, the dragon will catch Pryor if the time for him to get to the ocean is less than the time for Pryor to get to the ocean or

$$\frac{1}{10}\left(1-\left(\frac{1}{2}\right)^7\right) + \frac{1}{600} < \frac{1}{10}\,,$$

or

$$1-\left(\frac{1}{2}\right)^7 + \frac{1}{60} < 1\,,$$

or

$$-\frac{1}{128} + \frac{1}{60} < 0\,.$$

Since the above expression is false, the dragon does not catch Pryor.

How Old is the Wizard Alchemerion?

Using the letters A, S, and F to represent the ages of Alchemerion, his son, and his father respectively the problem statement tells us that

$$A = 3S \tag{14}$$
$$F = 2A + 40 \tag{15}$$
$$S + A + F = 1240\,. \tag{16}$$

	Logi	Magnus	Nepo
Initial State	L	M	N
First loaning	$L - M - N$	$2M$	$2N$
Second loaning	$2(L - M - N)$	$3M - L - N$	$4N$
Third loaning	$4(L - M - N)$	$2(3M - L - N)$	$7N - L - M$

Table 1: The Minotaur Fighters Problem

Put Equation 14 into Equation 15 we obtain

$$F = 6S + 40\,. \tag{17}$$

If we next put Equation 14 and Equation 17 into 16 we get

$$S + 3S + 6S + 40 = 1240\,.$$

Solving for S we get $S = 120$. Thus Equation 14 then tells us that $A = 360$.

Minotaur Fighters

Let L, M, and N be the number of fighters each of Logi, Magnus, and Nepo originally have. Then the statements given in the problem imply that after the first three loans we would have

$$L - M - N\,, 2M\,, 2N\,.$$

These values are shown in the appropriate line in Table 1. After the second loaning we take fighters from Magnus and give them to Logi and Nepo such that we double the number of fighters that Logi and Nepo have. This means that we need to take $L - M - N$ and $2N$ from Magnus. This gives

$$2M - (L - M - N) - 2N = 3M - L - N\,,$$

fighters for Magnus. These values are shown in the appropriate line in Table 1. After the third loaning we take fighters from Nepo to double the number of fighters that Logi and Magnus that now have. This gives

$$4N - 2(L - M - N) - (3M - L - N) = 7N - L - M\,,$$

for Nepo again see Table 1. At the end of these exchanges we are told that each of Logi, Magnus, and Nepo have 48 fighters. This gives the system of equations given by

$$\begin{aligned} 4L - 4M - 4N &= 48 \\ 6M - 2L - 2N &= 48 \\ 7N - L - M &= 48\,. \end{aligned}$$

When we solve this system for L, M, and N we get $L = 78$, $M = 42$, and $N = 24$.

Contests of Skill

Let E, K, and M be the number of elves, knights, and minotaur contestants. The number of pairs of contestants that must played against each other is then

$$EK + EM + KM\,,$$

and then since each pair must play three games (for the three skills) the total number of games that must be played is

$$3(EK + EM + KM)\,. \tag{18}$$

We are next told that the number of elf vs. knight competitions was 72. This means that

$$3KE = 72 \quad \text{so} \quad KE = 24\,.$$

Since $K > E$ when we list the factors of 24 in knight elf ordering we would get the possible pairs of

$$(24, 1)\,, (12, 2)\,, (8, 3)\,, (6, 4)\,.$$

Since we are told that $M = E - 1$ we know the number of minotaurs in each of the cases above. Thus the possible knight, elf, minotaur fighters could be

$$(K, E, M) \in \{(24, 1, 0)\,, (12, 2, 1)\,, (8, 3, 2)\,, (6, 4, 3)\}\,.$$

We now check each of these options and see if it has nine contestants that are either knights or minotaurs. The only set in the above where that is true is the last one. Thus the number of games from Equation 18 is given by

$$3(4(6) + 4(3) + 6(3)) = 162\,.$$

If we divide this number by 27 the number of days the tournament ran was six.

Going to the Fair

Let B_{rode}, B_{rested}, C_{rode}, and C_{rested} be the amount of time that Bopp rode, rested and Clack rode and rested respectively. We are told in the problem that

$$12 = B_{\text{rested}} + B_{\text{rode}} \tag{19}$$

$$12 = C_{\text{rested}} + C_{\text{rode}} \tag{20}$$

$$C_{\text{rested}} = B_{\text{rode}} \tag{21}$$

$$B_{\text{rested}} = C_{\text{rode}}\,. \tag{22}$$

Because Clack rode twice as fast as Bopp we would have that $C_{\text{rode}} = \frac{1}{2} B_{\text{rode}}$. Then using Equation 21 and the previous expression for C_{rode} in terms of B_{rode} in Equation 20 we get

$$12 = B_{\text{rode}} + \frac{1}{2} B_{\text{rode}}\,.$$

This means that $B_{\text{rode}} = 8$ so that $C_{\text{rode}} = 4$.

	Dravid	Gopar
Initial State	D	G
First game	$\frac{1}{2}D$	$G + \frac{1}{2}D$
Second game	$\frac{1}{2}D + \frac{3}{4}(G + \frac{1}{2}D) = 30$	$\frac{1}{4}(G + \frac{1}{2}D)$
Third game	$\frac{3}{4}G + \frac{7}{8}D - M = D$	$\frac{1}{4}G + \frac{1}{8}D + M = G$

Table 2: The Winning at Marbles Problem. On the third game M is the unknown number of marbles Gopar won from Dravid on the third game.

How Many Cakes?

Let N be the number of cakes originally brought on the platter and consider the number of cakes left after each person takes their share. After Hearnik took his there will be $\frac{3}{4}N$ cakes remaining. After Scowler took his there will be $\frac{2}{3}\left(\frac{3}{4}N\right)$ cakes left. After Goodin took his cakes there would be

$$\frac{1}{2}\left(\frac{2}{3}\left(\frac{3}{4}N\right)\right) = \frac{1}{4}N\,.$$

We are told that this was equal to six. Thus $N = 24$. From this initial number we can work out how many each person took. For example, Heartnik would have taken $\frac{1}{4}(24) = 6$. The remaining people are calculated the same way.

Winning at Marbles

Let D and G be the number of marbles that Dravid and Gopar have initially. The number of marbles that each has is given in Table 2. After the second game we are told that the number of marbles that Dravid has is 30. This means (when we simplify the given expression) that

$$\frac{3}{4}G + \frac{7}{8}D = 30\,. \tag{23}$$

The last piece of information is that on the last (and initial) game we have $D = 2G$. When we put that into Equation 23 we get $G = 12$ and $D = 24$. Once we know G and D we can use the last row in Table 2 to compute M in that

$$\frac{3}{4}G + \frac{7}{8}D - M = D \quad \text{so} \quad \frac{3}{4}(12) + \frac{7}{8}(24) - M = 24 \quad \text{so} \quad M = 6\,.$$

The Fools' Show

Let C, K, and A be the number of children, knights, and other adults that went to the show. Then the problem tells us that

$$30 = C + K + A \tag{24}$$

$$1000 = 5C + 25K + 50A\,. \tag{25}$$

In addition, we have the following inequalities between the number of children, knights, and other adults

$$2C > A > C \tag{26}$$
$$K < C\,. \tag{27}$$

From Equation 25 since 1000, $25K$, and $50A$ are all dividable by 25 so must the term $5C$. Thus for some value of n we must have

$$5C = 25n \quad \text{so} \quad C = 5n\,.$$

We know that because of Equation 24 that possible values for n are such that $C < 30$ thus $n = 1, 2, 3, 4, 5$. If $n \geq 3$ then $C = 5n \geq 15$ and $A + C \geq 2C \geq 30$ as $A > C$ by Equation 26. But this large a value of C would then cause Equation 24 to be violated and we must have that $n \leq 2$.

Note that the we can bound the right-hand-side of Equation 25 as

$$5C + 25K + 50A < 5C + 25C + 100C = 130C\,.$$

If $n = 1$ then $C = 5$ and the above is bounded by 650. Since this is below 1000 we cannot have $C = 5$. If $C = 10$ then the above is bounded by 1300 and this must be the correct value for C. Now that we know $C = 10$ we can put this value into Equation 24 and 25 to get

$$\begin{aligned} 20 &= K + A \\ 1000 &= 50 + 25K + 50A \quad \text{so} \quad 38 = K + 2A\,. \end{aligned}$$

Solving the two equations for A and K give $A = 18$ and $K = 2$. Thus we have $C = 10$, $K = 2$, and $A = 18$.

To the King's Castle

Let the distance from the start of the journey (at 11:00 AM) to the wife's first question at 11:30 be D_1, the distance from the first question to the inn be D_2, the distance from the inn to the second question be D_3, and the distance from second question to the destination be D_4. We are told in the problem that

$$\begin{aligned} D_1 &= 3D_2 \\ D_4 &= 3D_3 \\ D_2 + D_3 &= 2\,. \end{aligned}$$

The total distance from the cottage to the castle can then be written as

$$\begin{aligned} D_1 + D_2 + D_3 + D_4 &= 3D_2 + 2 + 3D_3 \\ &= 2 + 3(D_2 + D_3) = 2 + 3(2) = 8\,. \end{aligned}$$

Building A Bridge

Let D and M be the rates at which Dobbin and Mobbit can work to complete a bridge. Then from the problem we know that

$$
\begin{aligned}
D &= \frac{1}{30} \\
M &= \frac{1}{45} \,,
\end{aligned}
$$

in units of one bridge per hour. The amount of work done by Dobbit alone is

$$5\left(\frac{1}{30}\right) = \frac{1}{6} \,,$$

so the amount of the bridge still to be finished is $1 - \frac{1}{6} = \frac{5}{6}$. When the two workers are working together they work at a combined rate of $\frac{1}{30} + \frac{1}{45}$ thus if we let x be the amount of time needed to finish the bridge we get that x must solve

$$x\left(\frac{1}{30} + \frac{1}{45}\right) = \frac{5}{6} \,.$$

Solving for x we get $x = 15$ hours.

Sharing a Job

Let a, b, and c be the rates at which Abelard, Brendan, and Cullen can do the given job respectively. From the given statements for this problem, since we are given the combined rates, we have that

$$a + b = \frac{1}{10} \tag{28}$$

$$a + c = \frac{1}{15} \tag{29}$$

$$b + c = \frac{1}{30} \tag{30}$$

If we subtract Equations 29 from 30 we get

$$2a = \frac{1}{10} + \frac{1}{30} = \frac{2}{15} \,,$$

or $a = \frac{1}{15}$. Once we know a we can use Equation 29 to determine c and we find $c = 0$. With these two we can use Equation 30 to determine b and we find $b = \frac{1}{30}$. Thus it will take a 15 days to do the job, b 30 days to do the job, and c cannot do the job.

A Job Not Done On Time

Let r be the rate at which a cave dwarfs can work and let T_0 be the time to complete the job given all four are working. Then we know that

$$4rT_0 = 1\,,$$

since one job is completed. As the four only worked one day the amount of work they got done would be $4r(1) = 4r$. We are told that the remaining dwarfs finished the job but it took two more days or $T_0 + 2$ to finish the job. This means that the two dwarfs were working for $T_0 + 2 - 1 = T_0 + 1$ days. Since the job was finished in this time with two dwarfs we must have

$$4r + 2r(T_0 + 1) = 1\,.$$

Expanding the above we get

$$4r + 2rT_0 + 2r = 1\,.$$

Since we know that $2rT_0 = \frac{1}{2}$ we have that the above is

$$4r + \frac{1}{2} + 2r = 1\,.$$

Solving the above for r we get $r = \frac{1}{12}$. Putting this into $4rT_0 = 1$ we get that $T_0 = 3$ days.

At What Time does the Wagon Driver Leave the Hut?

Let the distance from the hut to the dock be given by D measured in miles. Then between 4:05 and 5:15 (or 10 minuets or $\frac{1}{6}$ of an hour) the wagon driver drives from $\frac{1}{5}D$ to $\frac{1}{3}D$ and thus has a velocity of

$$\frac{\frac{1}{3}D - \frac{1}{5}D}{\frac{1}{6}} = \frac{4}{5}D\,.$$

The time he needs to drive the distance $\frac{1}{5}D$ at the above rate is then

$$\frac{\frac{1}{5}D}{\frac{4}{5}D} = \frac{1}{4}\,,$$

or an hour or 15 minutes. Thus he must have left his home 15 minutes before 4:05 or at 3:50. To reach the dock he must travel $\frac{4}{5}D$ from the point he is at at 4:05. This takes him

$$\frac{\frac{4}{5}D}{\frac{4}{5}D} = 1\,,$$

hour. Thus he reaches the doc at 5:05.

Meeting the Stone Cutter

The cart driver meets the stone cutter at some point on his way to the ferry landing. This means that the cart driver does not have to travel the distance from where he meets the stone cutter to the ferry landing. This distance normally would have to be traveled *twice* once going to the ferry landing and then once he has picked up the stone cutter and again from the ferry landing to home. Since the stone cutter arrived 20 minutes early, traveling these *two* segments must normally take 20 normally to travel. Thus traveling the distance from where the stone cutter and the cart driver meet to the ferry landing must take half of this time or 10 minutes. As the cart driver normally gets to the ferry landing at 6:00 subtracting this 10 minutes of travel time he would normally have to travel means that the two meet at 5:50.

How Many Handshakes?

This would be $\binom{15}{2} = \frac{15(14)}{2} = 105$.

Mugs Clinking

This would be the number of segments (or gaps) between the knights. In this case this is 15.

The Shopping Trip

Let R, C, M, and B be the costs of the robe, the chest, the mantle, and the bracelet respectively. Then the statements of the problem state that

$$\begin{aligned} R + C + M &= 80 \\ B &= M \\ R + B &= 5C - 10 \\ R &= C + M \,. \end{aligned}$$

We can write everything in terms of R, C, and M to get

$$\begin{aligned} R + C + M &= 80 \\ R - 5C + M &= -10 \\ R - C - M &= 0 \,. \end{aligned}$$

When we solve this system of equations we get $R = 40$, $C = 15$, and $M = 25$.

The Jewel Chest

Let E be the number of pairs of earrings, R the number of rings, and P the number of pings. Then the statements from the problem give

$$R + 2E + P = 26$$
$$R = \frac{5}{2}P$$
$$E = R - 4\,.$$

We can solve this system of equations for P, R, and E and we find $P = 4$, $R = 10$, and $E = 6$. Amarina will have to draw items until she gets two earrings of the same type. Given that she cannot see what she is selecting then to guarantee that she gets two earrings of the same type in the worse case she will have to draw all of the pins out (of which there are four of them) and then all of the rings (of which there are 10 of them). Finally she must draw three earrings out to make sure that she gets a duplicate of one of the two types. This gives a total of 17 draws needed.

How Many Schlockels?

Let N be the number of schlockels Altus has. If we assume that N is a multiple of 5 then we must have $N = 5n$ and the given bounds mean that $1 < 5n < 19$ or $1 \le n \le 3$ so that $N \in \{5, 10, 15\}$. In this case N is not a multiple of 8 but none of these numbers above are between 20 and 29 required by the second clue. If we next assume that N is a multiple of 8 then we would have $N = 8n$. Then by the third clue since N is not a multiple of 10 (assuming that $n \neq 10n'$) the number N must be between 30 and 39. In this case $30 < 8n < 39$ so that $4 \le n < 5$. Thus $n = 4$ and we see that $N = 32$. One can then check that each of the three clues for this problem are true.

How Many Eggs?

This problem can be solved by just considering every possible number of eggs from 5 until 39 and doing the requested divisions. Thus given a hypothetical number of eggs we need to check if the remainder when divided by two is one, the remainder when divided by 3 is zero, and the remainder when divided by 5 is three. While this is not a very clever method of solution it is important to be able to phrase a problem as a procedure or algorithm that a computer could solve. Solving this problem using this method in the `R` script `code.R` I first consider various possible numbers of eggs, compute the remainders obtained when we divide by 2, 3, and 5 and then subtract the required remainder values as specified by the problem. When I do this and tabulate these values the correct number of eggs will sit atop a column of zeros. Some of the output from this table looks like

```
eggs        5    6    7    8 ...  31    32    33    34    35   ...
```

```
mod 2 - 1     0    -1     0    -1 ...    0     -1      0     -1      0    ...
mod 3         2     0     1     2 ...    1      2      0      1      2    ...
mod 5 - 3    -3    -2    -1     0 ...   -2     -1      0      1     -3    ...
```

From the above table we see that 33 eggs (due to all the zeros in the column headed by that number) are the number that satisfy the required conditions.

Perforce Arrives Too Late

Let T_0 be the original amount of time king Perforce originally planned for his march (in days). Then the delays will contribute to the new time for the march in the form of

$$T_0 + \frac{1}{2} + \frac{1}{10}T_0 + 3 = \left(1 + \frac{1}{3}\right) T_0 \,.$$

If we solve for T_0 we get $T_0 = 15$ days for the original time to perform for the march and then the total number of days for the march (including delays) is then given by $\left(1 + \frac{1}{3}\right) T_0 = 20$.

Switching Allegiance

Let L and M be the number of troops that Lavar and Malcavar have initially respectively. To start, we are told that

$$10 < L + M < 30 \,. \tag{31}$$

Then when Colin decides to leave Malcavar's troop and join Lavar's we are told that the number of solders is equal or that

$$L + 1 = M - 1 \,. \tag{32}$$

Colin then switches back to his original allegiance. When Draal decides to switch allegiance from Laval to Malcavar the number of troops assigned to Malcavar and Laval are now given by

$$M + 1 \quad \text{and} \quad L - 1 \,.$$

We are told that both these numbers are prime numbers. From Equation 32 we have that $M = L + 2$ this means that using Equation 31 written in terms of L only is

$$10 < 2L + 2 < 30 \quad \text{or} \quad 4 < L < 14 \,.$$

In terms of M only this is

$$6 < M < 16 \,.$$

Thus to solve this problem lets enumerate both L and M over the ranges above. For each value of L and M we will then compute $M + 1$ and $L - 1$. When we do this we get the following table

L	5	6	7	8	9	10	11	12	13
M	7	8	9	10	11	12	13	14	15
M+1	8	9	10	11	12	13	14	15	16
L-1	4	5	6	7	8	9	10	11	12

From the above table that the pairs $L = 8$ and $M = 10$ have $M + 1 = 11$ and $L - 1 = 7$ are prime numbers. This is the number of troops before any change allegiance. When Draal switches from Laval to Malcavar we then have $L - 1 = 7$ troops for Laval and $M + 1 = 11$ troops for Malcavar.

Able And Gallant Soldiers

If we let A_f the the number of foot soldiers for Sir Able, A_m be the number of mounted troops of Sir Able with G_f, and G_m having the same meaning for Sir Gallant. Then the statements given for this problem indicate that

$$\begin{aligned} A_f + A_m + G_f + G_m &= 50 \\ G_m &= A_f \\ A_f + A_m + 2 &= G_f + G_m \\ A_m + 4 &= G_m \,. \end{aligned}$$

This is a system of four equations and four unknowns which possibly has a unique solution. When we solve this for the four unknowns we get $A_f = 14$, $A_m = 10$, $G_f = 12$, $G_m = 14$.

More Jousters

Let N be the number of known knights that were to compete originally. Then $\binom{N}{2}$ are the number of competitions. If M more unknown knights show up then there are $\binom{N+M}{2}$ pairs to compete against one another. We are told that

$$\binom{N+M}{2} - \binom{N}{2} = 26 \,.$$

We can expand the binomial coefficients to get

$$\frac{1}{2}(N + M)(N + M - 1) - \frac{1}{2}N(N - 1) = 26 \,.$$

When we simplify the above we get

$$M(2N + M - 1) = 52 \,.$$

Given the above lets look at what the factors of 52 are. We can write 52 as

$$2 \cdot 26 \quad \text{and} \quad 4 \cdot 13 \,.$$

A_3 Value	Factors	Sum $A_1 + A_2 + A_3$
9	$(1, 9)$, $(3, 3)$	19, 15
8	$(1, 8)$, $(2, 4)$	17, 14
7	$(1, 7)$	15
6	$(1, 6)$, $(2, 3)$	13, 11
5	$(1, 5)$	11
4	$(1, 4)$, $(2, 2)$	9, 8
3	$(1, 3)$	7
2	$(1, 2)$	5
1	$(1, 1)$	3

Table 3: The Sons of Blythe

Since when M and N are integers we have that $M < 2N + M - 1$ for $N \geq 1$. Then in the first case we would have $M = 2$ so that

$$2N + M - 1 = 2N + 1 = 26\,,$$

which means that N would have to be a non integer and this cannot be a solution. If $M = 4$ then we would have

$$2N + M - 1 = 2N + 3 = 13\,,$$

so that $N = 5$. Thus we have shown that $N = 5$ original knights and $M = 4$ new knights.

The Sons of Blythe

Let the ages of the three sons be given by A_1, A_2, and A_3. Then the problem tells us that their ages are ordered as

$$A_1 \leq A_2 \leq A_3 < 10\,,$$

the product of the two youngest sons equals the age of the oldest or

$$A_1 A_2 = A_3\,,$$

and that $A_1 + A_2 + A_3$ is a prime number. Since all ages are integers we will use the product fact to determine the ages by picking A_3 and then considering the factors of A_3. Once we have the factors of A_3 we can look at the sums of the three numbers and check if it is prime number. In Table 3 we enumerate the different possible values for A_1, A_2, and A_3. If we assume that two children can be of the same age then we get several different solutions satisfying the given conditions. If we assume that all children must have a different age then we see that the ages that satisfy the conditions and meet this property are $A_1 = 2$, $A_2 = 3$, and $A_3 = 6$.

Who Rode Faster?

Let P_{rode}, P_{rested}, C_{rode}, and C_{rested} be the amount of time that Pure rode, rested and Chaste rode and rested respectively. We are told in the problem that

$$P_{\text{rode}} = 3C_{\text{rested}}$$
$$C_{\text{rode}} = 4P_{\text{rested}}$$
$$P_{\text{rode}} + P_{\text{rested}} = C_{\text{rode}} + C_{\text{rested}}\,.$$

Putting the first two equations into the third equation (all in terms of P_{rode} and C_{rode}) we get

$$P_{\text{rode}} + \frac{1}{4}C_{\text{rode}} = C_{\text{rode}} + \frac{1}{3}P_{\text{rode}}\,.$$

This means that

$$P_{\text{rode}} = \frac{9}{8}C_{\text{rode}}\,.$$

Thus since we have to multiply C_{rode} by a number larger than one to get P_{rode} we must have that

$$C_{\text{rode}} < P_{\text{rode}}\,.$$

Because the time Chaste rode was less than the time that Pure rode, Chaste must have rode faster.

How Far From Castleton to Devil's Peak?

The first horseman left Castleton going to Devil's Peak and then meet the second horseman traveling the other direction after four miles. If v_1 is the velocity of the first horseman, then the time the first horseman traveled before he met the second horseman is $\frac{4}{v_1}$. If the total distance between the two towns is D then the second horseman has traveled $D - 4$. If v_2 is the velocity of the second horseman the time he traveled before meeting the first horseman is $\frac{D-4}{v_2}$. These two times are equal and we have

$$\frac{4}{v_1} = \frac{D-4}{v_2}\,.$$

Both horsemen travel to their destinations and then turn around to head home where they meet two miles from Devil's Peak. This means that the first horseman has traveled $D + 2$ and the second horseman has traveled $D + D - 2 = 2D - 2$. The times to travel these two distances must be the same and we have

$$\frac{D+2}{v_1} = \frac{2D-2}{v_2}\,.$$

If we divide these two equations we get

$$\frac{4}{D+2} = \frac{D-4}{2D-2}\,.$$

Solving this for D we get $D = 10$. Putting this value of D into the first equation gives

$$\frac{4}{v_1} = \frac{6}{v_2} \quad \text{or} \quad \frac{v_2}{v_1} = \frac{3}{2}\,.$$

How Did the Archers Cross the River?

Start with the state where we have two children on the side of the bank with the archers and no one on the far side of the river. Have two children cross the river and one comes back with the boat. A first archer takes the boat over and the child that was left on the far side of the bank brings the boat back. This produces the situation where we have two children on one side of the bank and a single archer on the far side of the bank. Thus we have transported one archer across. This procedure can be repeated as many times as needed to get all the archers across the river.

How Can Everyone Cross the River?

We first ferry the human over to the other side of the river. This leaves the elf and the dwarf together which is fine since they get along. Next the half-elf ferries the dwarf to the other side of the river. The half-elf cannot leave the dwarf with the human since they do not get along, thus he must bring the human back across the river leaving the dwarf on the far side of the river. The half-elf returns to the near side of the river with the human drops him off in exchange for the elf which he then proceeds to ferry across the river. When the elf is carried across the river he will be deposited with the dwarf which is an acceptable configuration. Finally the half-elf goes back across the river to pick up the human and bring him to the far side completing the task of carrying everyone across the river.

How Fast Does Lucien's Horse Need to Run?

Let M and W be the rates at which Mnemo's and Wister's horse can run. The first half of the race (run by Mnemo's horse) will be finished in $\frac{1}{M}$ time and the second half of the race will be finished in $\frac{1}{W}$ time. Since we are told that $M = 4W$ we have that the total time for Mnemo's and Wister's horses will be running is given by

$$\frac{1}{M} + \frac{4}{M} = \frac{5}{M}\,.$$

For Lucien's horse to finish at the same time as Wister's means that it runs the two miles in the same time as the above or with a rate given by

$$\frac{2}{\frac{5}{M}} = \frac{2}{5}M\,.$$

Thus Lucien's horse must run at $\frac{2}{5}$ the speed of Mnemo's horse.

The Price of Candy

Let B, S, and C be the price per piece of candy in shukels. She then bought $B+S+C$ candies. The statement that she paided an average price per piece is then the statement

$$\frac{B^2+S^2+C^2}{B+S+C} = 7\,. \tag{33}$$

The fact that each kind of candy means that $B \neq S \neq C$. The ordering of the candy and the fact that each one is more than 4 shukels means that

$$B > S > C > 3\,. \tag{34}$$

The fact that the total price paided for all candy is less than 150 means that

$$B^2+S^2+C^2 < 150\,.$$

Finally the fact that compared to bonbons one of the other two kinds of candy costs three shukels less per piece means that

$$S = B-3 \quad \text{or} \quad C = B-3\,.$$

Since the average cost per shukel is 7 and bonbons are the most expensive candy then $B > 7$ or else the average cost of the candy would be less than 7. Thus we know that $B \geq 8$. If we let B take on values from eight onward then either S or C is $B-3$ and we can determine the values of all but one term in Equation 33. Thus we can solve the quadratic equation for this last unknown value and then we would have all three values of B, S, and C. For example, if we let $B = 8, 9, 10$ then we would get the values of $5, 6, 7$ for either S or C. It does not matter which one since the other variable is then explicitly determined by Equation 33 which is a quadratic. In terms of the "other" variable (the one that is three less than B) and the "unknown" variable (the one that we would want to solve for Equation 33 as

$$B^2 + \text{other}^2 + \text{unknown}^2 = 7(B + \text{other} + \text{unknown})\,,$$

or to solve for unknown we would solve

$$\text{unknown}^2 - 7\text{unknown} + B^2 + \text{other}^2 - 7B - 7\text{other} = 0\,.$$

I'll call the expression

$$B^2 + \text{other}^2 - 7B - 7\text{other}\,,$$

the "c" term since we will assign values for a, b, and c in the quadratic template $ax^2+bx+c=0$. As this is a quadratic we can solve it with the quadratic equation. This requires us to compute the square root of the discriminant of b^2-4ac. We will evaluate the discriminant for the three various values of B selected above. We get

```
Bs         8    9   10
other      5    6    7
C_term    -2   12   30
descrim   57    1  -71
```

Thus we can only take the square root of the first two values. For the value of $B = 8$ this would give solutions

$$\text{unknown} = \frac{7 \pm \sqrt{57}}{2} \in \{-0.2749172, 7.2749172\} .$$

If we are to assume that all valid values for B, S, and C must be integers than cannot be the correct solution. For the second value of $B = 9$ this would give

$$\text{unknown} = \frac{7 \pm \sqrt{1}}{2} \in \{3, 4\} .$$

As all prices must be greater than 3 the only value that works is for 4. Given this we have that $B = 9$, $S = 6$, and $C = 4$.

Feeding the Horses

Let h be the amount of food per day per horse needed to feed a single horse. The total amount of food that we have at the start (denoted by T) is then

$$30 \cdot 6 \cdot h = T .$$

The six days of feeding the six horses will use $6 \cdot 6 \cdot h$ amount of food and so the amount of food remaining is given by

$$T - 36h = 360h - 36h = 324h .$$

After this six days of time Distal sells the horses. The amount of food we need for the remaining time for the remaining horses is given by

$$2(75 - 6)h = 138h .$$

Since this is less than the amount of food we have $324h$ Distal should have enough food.

The Island of Odds

If we let T, L, and N represent the truth tellers, the liars, and the normal people respectively then meeting any ordering of two of them has a probability of $\frac{1}{9}$ of happening. The cases where we have at least one normal person are

$$(T, N) , (L, N) , (N, T) , (N, L) , (N, N) ,$$

or five such cases. We can see this by creating a two dimensional grid with the type of the first person meet corresponds to the rows and the type of the second person meet corresponds to the columns. This gives a probability of $\frac{5}{9}$.

A Tiger or a Treasure

This is a variant of the "Monte Hall Problem" and one should switch selections once the new information (as to which door/cave is not desirable) is revealed.

Drawing Stones

Let B and W be the number of black and white stones initially in the jar. Then after having observed that the first two draws are black stones the probability of the next stone being black is given by

$$\frac{B-2}{B+W-2}.$$

The probability that the first draw is black is given by

$$\frac{B}{B+W}.$$

We are told in the problem that

$$\frac{B-2}{B+W-2} = \frac{9}{10}\left(\frac{B}{B+W}\right), \tag{35}$$

and that $B+W \in \{7, 8, 12, 13\}$. Since we know that we can draw at least three black stones we must have $B \geq 3$. Once we are given a value for $B+W$ then certainly $B \leq B+W$. Thus to solve this problem we will specify a value of $B+W$ from the above choices and then let B range $3 \leq B \leq B+W$ and see for what value of B (if any) Equation 35 is satisfied. For example, if we assume that $B+W = 12$, then Equation 35 then becomes

$$\frac{B-2}{10} = \frac{9}{10}\left(\frac{B}{10}\right).$$

Another perhaps simpler method would be just to solve the above equation for B and see if B is an integer. When we do this we find that $B+W = 12$ and $B = 8$ (so that $W = 4$) gives equality in Equation 35.