Solutions and Notes for the book
Math Puzzles and Games
by Michael Holt

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To anyone who solves problems.

Introduction

Here you'll notes and solutions I wrote up as my daughter and I worked through this excellent book.

Number Problems

All the Fun of the Fair

For this problem we are simply looking for any number of items that sum to the target value of 50. Computing a few sums we find that $25 + 6 + 19 = 50$. Just for fun we automated this search procedure in the `python` file `all_the_fun_of_the_fair.py`. There we find the sum above is the only sum of such type.

Chessboard Problem

First note that we have $1 \times 1 = 1$ eight-by-eight square. We have $2 \times 2 = 4$ seven-by-seven square. In the same way we have

$$
\begin{aligned}
&3 \times 3 = 9 \text{ six-by-six squares} \\
&4 \times 4 = 16 \text{ five-by-five squares} \\
&5 \times 5 = 25 \text{ four-by-four squares} \\
&6 \times 6 = 36 \text{ three-by-three squares} \\
&7 \times 7 = 49 \text{ two-by-two squares} \\
&8 \times 8 = 64 \text{ one-by-one squares} \,.
\end{aligned}
$$

If we add up all of these squares we get a total of 204.

Cat and Mice

The cat starts by eating mice number one. Then counting five mice counterclockwise the cat next eats mice number two. Then the cat eats mice number four. Then the cats each mice number five. Finally the cat eats the third mouse. Thus the white mice is the third mice.

A Question of Ages

From the given statement if we let s be Sam's age and m be May's age we have that

$$
\begin{aligned}
s &= 3m \\
s + 2 &= 2(m + 2) \,.
\end{aligned}
$$

Putting the first equation into the second gives

$$3m + 2 = 2m + 4 \quad \text{or} \quad m = 2 \,.$$

Thus $s = 6$.

Another Question of Ages

From the given statement if we let u be the uncle's age and g be the girls age we have that

$$u = 3g$$
$$u + 20 = 2(g + 20)\,.$$

Putting the first equation into the second gives

$$3g + 20 = 2g + 40 \quad \text{or} \quad g = 20\,.$$

Thus $u = 60$.

Teen-Age Problem

From the given statement if we let s be Sam's age and j be Jo's age we have that

$$s + 4 = j$$
$$j + 5 = 2s\,.$$

Putting the first equation into the second gives

$$s + 9 = 2s \quad \text{so} \quad s = 9\,.$$

Thus $j = 13$.

15 Shuffle

Move the nine card from the last pile to the first pile to get ace, two, three, and nine there which sums to fifteen.

The Birthday Paradox

Our composer was born on February 29th in 1972 (i.e. a leap day of a leap year). Then he lived 76 years but only got a proper "birthday" every four years (when there was a leap year). The program `birthday_paradox.py` prints each year he had a birthday

```
Our composer is 76 years old but only had 18 birthdays in his life
[1792, 1796, 1804, 1808, 1812, 1816, 1820, 1824, 1828, 1832,
 1836, 1840, 1844, 1848, 1852, 1856, 1860, 1864, 1868]
```

The first year is his actual birthday.

Word Sums

We are told that $O = 4$. Thus adding the two O's together we get that E must be eight.

Lets try $W = 0$. Then when we add the two W's we would also get zero and thus $V = 0$. This can't be true since we assumed that $W = 0$.

Next lets try $W = 1$ then $V = 2$. Once we have picked a valid W we need to specify T. We need $T \geq 5$ since otherwise when we add the two T's they will sum to something less than 10 (and there would be no need for a F digit) For various possible choices for T we have

- If $T = 5$ then $I = 0$ and $F = 1$. This would give a contradiction to $W = 1$.
- If $T = 6$ then $I = 2$ and $F = 1$. This would give a contradiction to $V = 2$.
- If $T = 7$ then $I = 4$ and $F = 1$. This would give a contradiction to $O = 4$.
- If $T = 8$ then $I = 6$ and $F = 1$. This would give a contradiction to $E = 8$.
- If $T = 9$ then $I = 8$ and $F = 1$. This would give a contradiction to $E = 8$.

Lets try $W = 2$. Then when we add the two W we would get four and thus $V = 4$. This can't be true since we are told that O is four.

Next lets try $W = 3$ then $V = 6$. Once we have picked a valid W we need to specify T. For various possible choices of T we have

- If $T = 5$ then $I = 0$ and $F = 1$. This gives one solution.
- If $T = 6$ then $I = 2$ and $F = 1$. This would give a contradiction to $V = 6$.
- If $T = 7$ then $I = 4$ and $F = 1$. This would give a contradiction to $O = 4$.
- If $T = 8$ then $I = 6$ and $F = 1$. This would give a contradiction to $E = 8$ and $V = 6$.
- If $T = 9$ then $I = 8$ and $F = 1$. This would give a contradiction to $E = 8$.

Next lets try $W = 4$. Which would be a contradiction to the fact that $O = 4$.

Next lets try $W = 5$ then $V = 0$. Once we have picked a valid W we need to specify T (again $T \geq 5$). For various possible choices of T we have

- If $T = 5$ then $I = 1$ and $F = 1$. This would give a contradiction to $W = 5$.
- If $T = 6$ then $I = 3$ and $F = 1$. This would give one solution.

- If $T = 7$ then $I = 5$ and $F = 1$. This would give a contradiction to $W = 5$.
- If $T = 8$ then $I = 7$ and $F = 1$. This would give a contradiction to $E = 8$.
- If $T = 9$ then $I = 9$ and $F = 1$. This would give a contradiction to $T = 9$.

Next lets try $W = 6$ then $V = 2$. For various possible choices of T we have

- If $T = 5$ then $I = 1$ and $F = 1$. This would give a contradiction since we have two letters equal the digit one.
- If $T = 6$ then $I = 3$ and $F = 1$. This would give a contradiction since $W = 6$.
- If $T = 7$ then $I = 5$ and $F = 1$. This is one solution.
- If $T = 8$ then $I = 7$ and $F = 1$. This would give a contradiction to $E = 8$.
- If $T = 9$ then $I = 9$ and $F = 1$. This would give a contradiction to $T = 9$.

Next lets try $W = 7$. Which would mean that $V = 4$ and is a contradiction to the fact that $O = 4$.

Next lets try $W = 8$. Which is a contradiction to $E = 8$.

Finally lets try $W = 9$. Which would mean that $V = 8$ which is a contradiction to $E = 8$.

Thus the three possible solutions are

```
  534
+ 534
-----
 1068
```

and

```
  654
+ 654
-----
 1308
```

and

```
  764
+ 764
-----
 1528
```

Easy as ABC?

From the hint we have that $A + B + C = 19$. Thus when we add the first sum we get

```
  21
  AAA
  BBB
+ CCC
-----
 2109
```

From the given sum of the first three digit number ABC this equals the digits `FGHI` so we must have $F = 2$, $G = 1$, $H = 0$, and $I = 9$. Since the sum of the next two three digit numbers is also the digits `FGHI` we have that A, D, and E must satisfy the following sum

```
  AAA
  DDD
+ EEE
-----
 2109
```

Note that from the above we must then have that

$$A + D + E = 19\,. \tag{1}$$

The letters we don't know the values of : A, B, C, D, E must then come from the set S of digits not assigned thus far or $S = \{3, 4, 5, 6, 7, 8\}$. We will now try various choices for A and see if they can result in valid solutions.

If $A = 3$ then for $A + B + C = 19$ we must have $B + C = 16$ which is not possible using the remaining numbers in S.

If $A = 4$ then in the same way as above we must have $B + C = 15$ then $B = 7$ and $C = 8$ and no other choices of digits from S for B and C will sum to 15. From Equation 1 we must also have that $D + E = 15$ but with seven and eight removed from S this is not possible.

If $A = 5$ then $B + C = 14$ then $B = 6$ and $C = 8$ and no other choices from S for B and C will sum to 14. Then to have two digits D and E from S such that $D + E = 14$ also is not possible.

If $A = 6$ then $B + C = 13$ then $(B, C) = (6, 7)$ and $(B, C) = (5, 8)$ are two choices that work and no other choices from S for B and C will sum to 13. We can't use the pair with an element equal six for we started the assumption that $A = 6$. Thus this case is not possible.

If $A = 7$ then $B + C = 12$ and there are no two ways to sum two elements in S to get 12.

If $A = 8$ then $B + C = 11$ then $(B, C) = (5, 6)$ and $(B, C) = (4, 7)$ are two choices that work and no other choices from S for B and C will sum to 11. We can take one of the two assignments to (B, C) and then the other to (D, E) so that $D + E = 11$. Thus we have found the solution

```
  888                888
  555                444
+ 666              + 777
-----              -----
 2109               2109
```

You Can't Take It (All) with You

Lets say we can count for 24 hours. Then we could get

$$3600(24) = 86400\,,$$

dollars. This is not as much as one might think would be achievable.

Peasants' Multiplying

Here we will just do some simple problems using this method. First we compute 5×35 as

```
5  x   35
2       70 ... starts with an even number and is therefore dropped
1      140
```

Thus we need to sum $35 + 140 = 175$ which is correct.

Next lets do 13×10 as

```
13  x  10
 6     20 ... dropped
 3     40
 1     80
```

Thus we need to sum $10 + 40 + 80 = 130$.

Next an easier problem 13×1 as

```
13  x  1
 6     2 dropped
 3     4
 1     8
```

Thus we need to sum $1 + 4 + 8 = 13$.

Another larger problem of 56×25 is

```
56 x    25 ... dropped
28      50 ... dropped
14     100 ... dropped
 7     200
 3     400
 1     800
```

Thus we would need to sum $200 + 400 + 800 = 1400$.

Finally, lets compute 52×11 where we get

```
52  x   11 ... dropped
26      22 ... dropped
13      44
 6      88 ... dropped
 3     176
 1     352
```

Thus we would need to sum $44 + 176 + 352 = 572$.

Tear 'n' Stack

If we tear once and fold our stack is 2 sheets of paper high. Tear again our stack is $2^2 = 4$ sheets high. Tear a third time and our stack is $2^3 = 8$ sheets of paper high. If we do this 47 times our stack would be

$$2^{47} = 140737488355328\,,$$

sheets of paper high. If one sheet is 1/1000 of an inch then the above would be over 2.2 million miles high.

Grains of Wheat

If the sack was emptied before the 20th square it had to have less than

$$1 + 2 + 2^2 + 2^3 + \cdots + 2^{20-1} = 2^{20} - 1 = 1048575\,,$$

gains. The Grand vizier would end up obtaining

$$2^{64} - 1 = 18446744073709551615\,,$$

gains of wheat. This has 20 digits and is thus about 18 million million million grains.

Stock Taking

Let s be the number of sheep the farmer has. Let b be the number of sheep in the barn. Then we have

$$b = \frac{1}{3}s\,.$$

Let p be the number of sheep in the pasture. Then

$$p = \frac{1}{5}s\,.$$

Let n be the number of sheep that are newborn. Then

$$n = 3\left(\frac{s}{3} - \frac{s}{5}\right)\,.$$

The total number of sheep is the sum of b, p, and n plus one (for the daughter's pet). Thus

$$s = \frac{s}{3} + \frac{s}{5} + 3\left(\frac{s}{3} - \frac{s}{5}\right) + 1\,.$$

Solving this for s gives $s = 15$.

Slobodian Coin Puzzle

We can solve this problem by enumerating the ways we can get six slobs. As we can have 0, 1, 2, or 3 two-slob coins these determine how many one-slob coins we need. Table 1 shows how this is done.

Tug of War

Let B be the "strength" of a single boy, G the "strength" of a single girl and D the strength of a single dog. Then from the pictures we have

$$4B = 5G \tag{2}$$

$$2G + B = D\,. \tag{3}$$

Number of one-slob coins	Number of two-slob coins	Total slobs
6	0	6
4	1	6
2	2	6
0	3	6

Table 1: The number of ways to get a total of six slobs

To figure who will win the tug of war we need to determine which of $D + 3G$ or $4B$ is larger. Note that from Equation 2 we have that

$$4B = 5G = 2G + 3G\,.$$

Then from Equation 3 we have the above expression for $4B$ is equal to

$$D - B + 3G = D - \left(\frac{5}{4}G\right) + 3G = D + \frac{7}{4}G = D + 3G - \frac{5}{4}G\,.$$

Thus $4B < D + 3G$ and the side with the dog and three girls will win.

Check-out Check

Let g and b be the cost of a stick of gum and a bar of chocolate respectively. Then for 2.2 to be the total cost means that

$$2g + 3b = 2.2\,,$$

but if a stick of gum costs 0.1 then two sticks cost 0.2 so the above would become

$$3b = 2.0\,.$$

This can't be true as 2.0 is not divisible by three.

Puzzle Triangles

Following the given formula for how the circles are filled we have that the inner triangle would hold the numbers (starting at the top and going clockwise) 5, 3, and 1 while the outer triangle would hold the numbers (again from the top and clockwise) 2, 0, and 4.

Nice Work If You Can Get It

The payment on day d under the two methods would be

$$\left\{ \begin{array}{ll} 2^d & \text{cents} \\ 10 & \text{dollars} \end{array} \right. ,$$

The total payment received on day d (now both in dollars) then would be

$$\begin{cases} \frac{1}{100}\sum_{d'=1}^{d} 2^{d'} = \frac{2^{d+1}-2}{100} \\ d(10) = 10d \end{cases}$$

The first method (doubling every day) would result in more money on the day d where

$$\frac{2^{d+1}-2}{100} > 10d\,,$$

or

$$2^d > 500d + 1\,.$$

This inequality becomes true on (and after) day $d = 13$.

12 Days' Gifts

We would add the given numbers to get

$$1 + 2 + \cdots + 11 + 12 = \frac{1}{2}(12(13)) = 78\,.$$

Dividing-the-Line Code

When the message is received you know how many letters the message is. When constructing the "dividing-line-code" we need to make sure we have at least as many "divides" as characters in the message (thus we find the next power of two that is larger than the number of characters in the message). One does not need to write "divides" for characters we don't have in our message.

Holiday Message

The message is "GOOD HUNTING".

Bottle and Cork

Let b and c be the price of the bottle and cork respectively. Then we have

$$\begin{aligned} b &= c + 0.03 \\ b + c &= 0.5\,. \end{aligned}$$

Solving this we get $c = 0.01$ and $b = 0.04$.

Juggling and Balancing

If we let J, B, M, and P be the weights of the jug, bottle, mug, and plate respectively then from the shown weightings we can conclude that

$$\begin{aligned} J &= B \\ J &= M + P \\ 3P &= 2B\,. \end{aligned}$$

The second equation gives $M = J - P$. Putting in P from the third equation gives $M = J - \frac{2}{3}B$. Using the first equation this is the same as $M = J - \frac{2}{3}J = \frac{1}{3}J$. Thus $3M = J$.

Hidden Animal

This problem is made easier by having a quick integer to letter lookup table. One is provide in the `R` programming language in the `LETTERS` variable (perhaps in other programming languages also). Using that the answer to the first question is twelve and then using `R` we can quickly find out what letter that is using the command

```
> LETTERS[12]
[1] "L"
```

Thus to solve the full problem we can do

```
number_anwsers = c(12, 5, 15, 16, 1, 18, 4 )
paste0( LETTERS[number_anwsers], collapse="" )
[1] "LEOPARD"
```

Pictures by Numbers

From the given picture we can read various coordinate pairs that we need to shade. You can start at "one" and for each of the "out" arrows from that digit shade all of the boxes of the 8×8 grid. Then moving to "two" do the same thing. Doing this for each digit we eventually see a picture of a dog.

Number Tracks

I get that $\{6, 12, 18\}$ end up at A these numbers must be even (divisible by two) and divisible by three and thus divisible by 6. I get that $\{2, 4, 8, 10, 14, 16, 20\}$ end up at B, that $\{3, 9, 15\}$ end up at C and that $\{1, 5, 7, 11, 13, 17, 19\}$ end up at D.

Number Fourte!

Here is a small one

$$\text{six} \to \text{three} \to \text{five} \to \text{four}\,.$$

Here is a longer one (not counting the spaces and hyphen)

$$\text{one-hundred and fifteen} \to \text{twenty} \to \text{six} \to \text{four}\,,$$

using the transformations for six. Here is another just for fun

$$\text{one-hundred and nine} \to \text{seventeen} \to \text{nine} \to \text{four}\,.$$

In the above I did not count the spaces (or hyphen) in the textual representation of the number, but this does not seem to matter as we will still end with the word four if we did. As a comment, there exist automatic ways to derive the textual representation of an integer[1]. These could be used to automate a search to see if there are any textual representation of numbers that don't reduce to four.

Solve It in Your Head

Adding the two equations together gives

$$10(x+y) = 50 \quad \text{so} \quad x+y=5\,.$$

Subtracting the two equations gives

$$4(x-y) = 4 \quad \text{so} \quad x-y=1\,.$$

These last two equations are easily solved in ones head $x = 3$ and $y = 2$.

Time, Please

That would be one hour before two o'clock or one o'clock.

Picture and Frame

Let p and f be the price of the picture and frame respectively. Then we have

$$\begin{aligned} p &= f + 0.5 \\ p + f &= 2\,. \end{aligned}$$

Solving we get $f = 0.75$ and $p = 0.25$.

[1] http://stackoverflow.com/questions/309884/code-golf-number-to-words

Idle Ivan and the Devil

Let r be the amount of money that Ivan has in his pocket to start with. Then on the first crossing and payment he will have

$$2r - 8\,.$$

After the next crossing and payment we will have

$$4r - 16 - 8 = 4r - 24\,.$$

After his third crossing and payment he finds he has no money left or

$$2(4r - 24) - 8 = 0\,.$$

Solving this for r gives $r = 7$.

Happy Landing

If the flight takes seven hours and he leaves at 5 Alaskan time he will land at $5 + 7 = 12$ or noon Alaskan time.

Cracked-Clock Problem

We need to find the split that puts the larger valued numbers with the smaller valued numbers. This means that we would want to try splits that go "horizontally" across the clock face. The one that works splits the nine from the ten and the three from the four giving a sum of 39 on each side.

Stamp Strip Puzzle

Using the pictures of the four folded stamps given we see that the other three ways are *adcb*, *badc*, and *acdb*.

The Smallest Flock

One way to get a number that satisfies the given conditions (i.e. a a number that has one for the remainder when we divide by the numbers two, three, four, five, six, seven, eight, nine, and ten) would be

$$N = 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8 \cdot 9 \cdot 10 + 1\,.$$

Note that this number won't be the smallest such number. Writing the prime factors for each of the above the number N above can be written as

$$N = 2^8 3^4 5^2 7 + 1 \, .$$

The smallest such number can be obtained by setting each power of a primes to the smallest number such that each of given ten digits will still divide the number. For example in this problem this would be

$$2^3 3^2 5 \cdot 7 + 1 = 2520 + 1 = 2521 \, .$$

Letter Frame-up

Starting at C and walking clockwise skipping a letter each time we get the sequence

$$CHSPEEDAROLLINGSTONEGATHERSMU$$

Inserting vertical bars at what look like word boundaries we get

$$CH|SPEED|A|ROLLING|STONE|GATHERS|MU$$

This then reads "a rolling stone gather much speed".

The Farmer's Will

We can't divide 17 into halves but the closest we can do is to give Anne either 8 or 9 horses. We also can't divide 17 into thirds but the closest we can do is to give Bob 5 or 6 horses. We also can't divide 17 into ninths but the closest thing would be to give Charlie two horses. That means that Anne and Bob must have a total of 15 horses. This can happen if Anne has nine and Bob has six.

The Checkers Match

This is the number of pairs that we can form from five members and is given by $\binom{5}{2} = 10$.

The Spelling Bee

Notice that there is only one path that forms the "word" R. There are two paths that form the "word" RA (down the left and down the right). There are *four* paths that form the word RAT. These paths are: down the left diagonal all the way, down the right diagonal all the way, and down the left/right and then inwards to the last T. Thus for the three T characters in the third row there are 1, 2, and 1 ways to form the word RAT ending at each of the three T characters respectively. For the fourth row, there are 1, 3, 3, and 1 way to have a path end at the H in that row and spell the word RATH. In the same way there are 1, 4, 6, 4, 1 ways to form the word RATHS ending at each of the S characters in the fifth row respectively. This pattern can be diagrammed with the following figure

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

Where to form a new row we start and end with a one and then each number is the sum of the two numbers above it. This pattern of numbers is called Pascal's triangle[2] and has many mathematical properties.

Summing the numbers in each row of Pascal's triangle gives us the total number of paths that spell the given word. Thus we have one path that spells the word R, two paths that spell the word RA, four paths that spell RAT, eight paths that spell RATH, and sixteen paths that spell RATHS. Notice that the number of paths doubles as each row is added.

The sentence from Lewis Carroll's is translated as "And the grave land turtle squeaked out".

Number Oddity

Here we can show with `R` this pattern holds when we multiply the number 12345679 by each of the digits $1, 2, \ldots, 8, 9$.

```
n = 12345679
n * seq( 1, 9 ) * 9
[1] 111111111 222222222 333333333 444444444 555555555 666666666 777777777
[8] 888888888 999999999
```

The reason this works is that the order of multiplication does not matter and when we multiply our special number n by nine we get 111111111. Thus $9n$ times any number will give that number repeated.

Six 1s are 24?

We can divide six ones with three plus signs as

[2] https://en.wikipedia.org/wiki/Pascal%27s_triangle

```
1 1 + 1 1 + 1 + 1
```

meaning the sum $11 + 11 + 1 + 1 = 24$.

Pairing Puzzle

We would want to pair a large number with a small number to have the sums be close to each other. Thus we pair $(1, 8)$, $(2, 7)$, $(3, 6)$ and $(4, 5)$.

Diamonds of Marbles

Ned has two choices (white or black) for each of the four corners of the diamond. Thus he can create $2^4 = 16$ possible diamonds.

a	b	a-b
c	d	c-d
a-c	b-d	a-b-c+d

Table 2: The most general arraignment of a puzzle board where variables represent numbers and the assumed inequalities among the variables hold (see the text). Notice that the corner difference is $a - b - c + d$ no matter what direction we use to compute it.

7	3	4
4	2	2
3	1	**2**

Table 3: An valid puzzle board constructed as described in the text.

Number Patterns

Puzzle Boards

See Table 2 for a general labeling of a board like discussed in this problem. Here to have the differences be valid (i.e. positive) in the right-most column and bottom-most row we need to have $a \geq b$, $c \geq d$, $b \geq d$, and $a \geq c$ so that the subtractions on in the right and bottom column are of the correct sign. If we further assume that $a - c \geq b - d$ and $a - b \geq c - d$ then the two values in the lower right corner will both be equal and take the value of $a - c - b + d$. We can construct tables like the required ones by enforcing the given inequalities by trial an error. One example is given in Table 3.

Sum of the Whole Numbers

There are a lot of patterns that might work but one that seems to keep working is to append zeros to the two fives and concatenate the two numbers together at each step. Examples of this will make the process clearer. The sum of the numbers 1 to 100 is 5050. The sum of the numbers 1 to 1000 is 500500. The sum of the numbers 1 to 10000 is 50005000 etc.

Number Crystals

Part (A): Here we have

$$
\begin{aligned}
16 &= 4 \times 4 \\
1156 &= 34 \times 34 \\
111556 &= 334 \times 334 \\
11115556 &= 3334 \times 3334 \\
1111155556 &= 33334 \times 33334\,.
\end{aligned}
$$

Part (B): Here we have

$$
\begin{aligned}
09 &= 3 \times 3 \\
1098 &= 33 \times 33 \\
110889 &= 333 \times 333 \\
11108889 &= 3333 \times 3333 \\
1111088889 &= 33333 \times 33333\,.
\end{aligned}
$$

Part (C): Here we have

$$
\begin{aligned}
36 &= 6 \times 6 \\
4356 &= 66 \times 66 \\
443556 &= 666 \times 666 \\
44435556 &= 6666 \times 6666\,.
\end{aligned}
$$

Number Carousel

Starting with the given number we see that as we multiply by each of the numbers the digits "rotate" around. We can see this with the following R code

```
n = 142857
n * c( 1, 5, 6, 2, 3 )
```

which gives

```
[1] 142857 714285 857142 285714 428571
```

We can check if this pattern continues with the following R code

```
( n * c( 9, 10, 11 ) + 1 ) - 1000000
```

which gives

```
[1] 285714 428571 571428
```

from which we see that we do have the same digits and the pattern continues.

Take-away Number Squares

Trying this procedure on the numbering in the given square results in a final square with the number one at every vertex. A further iteration results in zeros at every vertex. Thus it looks like the limiting pattern is zeros at every vertex.

Take Any Three Digit Number

If we have a three digit number, say abc, we can get the number $abcabc$ by multiplying as

$$abc \times (1000 + 1) = abc \times 1001 = abcabc\,.$$

Note that the number 1001 factors as 7(11)(13) thus

$$abcabc = abc \times 1001 = abc \times 7 \times 11 \times 13\,,$$

which shows what we were tying to show.

Five 2s

There are many ways to do this. Here are the first ones that we thought of

$$\begin{aligned}
0 &= 2 - 2/2 - 2/2 \\
1 &= 2 - 2/2 + (2 - 2) \\
2 &= 2 \times 2 - 2 + (2 - 2) \\
3 &= 2 + 2/2 + (2 - 2) \\
4 &= 2 \times 2 + 2/2 \\
5 &= 2 + 2 + 2 - 2/2 \\
6 &= 2 \times 2 + 2 + (2 - 2) \\
7 &= 2 + 2 + 2 + 2/2 \\
8 &= 2 \times 2 \times 2 + (2 - 2) \\
9 &= 2 \times 2 \times 2 + 2/2 \\
10 &= 2 \times (2 + 2 + 2/2)\,.
\end{aligned}$$

Four 4s

Here are some ways that we thought of

$$
\begin{aligned}
1 &= (4 \times 4 - 4)/4 = (4 + 4 - 4)/4 \quad \text{producing the digit one seems relatively easy} \\
2 &= (4/4) + (4/4) \\
3 &= (4 + 4 + 4)/4 \\
4 &= ((4 - 4)/4) + 4 \\
5 &= (4 \times 4 + 4)/4 \\
6 &= ((4 + 4)/4) + 4 \\
7 &= (4 - (4/4)) + 4 \\
8 &= (4 - 4) + (4 + 4) \\
9 &= 4 + (4 + (4/4)) \\
10 &= (44 - 4)/4\,.
\end{aligned}
$$

In the expression to get 10 we join the first two fours together i.e. we don't use any operation between them.

Take-away Number Triangles

I get the triple of numbers two, two, and zero.

Missing Numbers

Part (a): Here we are adding three each time to get the sequence

$$2\,,5\,,8\,,11\,,14\,,17\,.$$

Part (b): Here we are adding two each time to get a ten between the eight and the twelve.

Part (c): Here we are adding five each time to get a 27 between the 22 and the 32.

Part (d): It looks like we are subtracting two and then adding three each time. Thus we would get a three in the missing space.

Another thing to note is that inside this sequence there seems to be two interlacing sequences one $1, 2, 3$ and another $3, 4, 5, 6$. This observation still gives the value of three for the missing number.

Part (e): These are the squares and the missing number is $6^2 = 36$.

Part (f): These are the powers of two and the missing number is then $2^5 = 32$.

Part (g): This pattern looks to be doubling the number and adding two to that. Thus the missing number would be $2(62) + 2 = 126$.

Part (h): The pattern here seems to be multiply the previous number by two and subtract that number by the index (minus one). This would give the missing number the value 103. For example, the numbers are generated according to

$$\begin{aligned} 9 &= 2(5) - 1 \\ 16 &= 2(9) - 2 \\ 29 &= 2(16) - 3 \\ 54 &= 2(29) - 4 \\ 103 &= 2(54) - 5 \\ 200 &= 2(103) - 6\,. \end{aligned}$$

Notice that the amount we subtract increases as we compute more terms.

Sums in the Head

Part (a): This would be $3000 - 3 = 2997$.

Part (b): We would pair these numbers in three sets of two groups where the sum in each group is 1000. The total sum is then 3000.

Part (c): The third row of numbers would be

```
604253
283012
112735
```

which would give the total of 1000000.

Part (d): The first ordering is easier as that is how addition is normally performed.

A Number Pattern

The next two patterns are

```
4 x 5 x 6 x 7 + 1 = 29 x 29
5 x 6 x 7 x 8 + 1 = 41 x 41
```

We get the number 29 by noting that going from 5 to 11 is a step of $11-5=6$. Then going from 11 to 19 is a step of size $19-11=8$ thus the step size increase by two each time so we should increase 19 by $8+2=10$ to get 29. We then increase 29 by $10+2=12$ to get 41.

Another Number Pattern

It looks like we are taking the number on the left (the one we multiply with) and then inserting an addition sign between two digits. If we do that to 147 we might break it into $14+7$. Then the right-hand-side are these two numbers (14 and 7) to the third power. Thus we have

$$147 \times (14+7) = 14^3 + 7^3$$
$$148 \times (14+8) = 14^3 + 8^3 \,.$$

Reverse Sums

These could be given by

$$24 + 3 = 27$$
$$24 \times 3 = 72 \,.$$

and

$$47 + 2 = 49$$
$$47 \times 2 = 94 \,.$$

Palindromes in Numbers

Following the procedure given we would have

```
   139
  +931
  1070
 +0701
  1771
```

which is a palindrome.

The Next Palindrome Year

This would be 1991.

Corridors of Numbers

Adding the numbers in a “corridor” gives n^3 where n is the corridor “number”.

Summing the numbers in the given section we find

$$4 + 7 + 10 + 5 + 9 + 13 + 6 + 11 + 16 = 81 = 92\,.$$

Multiplying Equals Adding?!

We want to numbers p and q such that $pq = p + q$. If we take $p = q$ we find that $p = 0$ or $p = 2$. I claim that there are an infinite number of such pairs of numbers. To see this, note that for a given value for p we can solve the above to find q given by

$$q = \frac{p}{p-1}\,.$$

If we take $p = 1.5$ (as suggested) then using the above we find $q = 3$.

Times-Table Triangle

Part (A): These are the squares of the positive integers.

Part (B): Four more rows would be (with some of the original rows for context)

```
                          26  27  28  29  30  31  32  33  34  35  36
                      37  38  39  40  41  42  43  44  45  46  47  48  49
                  50  51  52  53  54  55  56  57  58  59  60  61  62  63  64
              65  66  67  68  69  70  71  72  73  74  75  76  77  78  79  80  81
          82  83  84  85  86  87  88  89  90  91  92  93  94  95  96  97  98  99 100
     101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121
 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144
```

Using the above numbers and the pattern suggested in the book we have that $6 \times 12 = 72$ and $7 \times 13 = 91$.

Magic Square and Sliding Block Puzzles

Magic Squares

I find

```
618
753
294
```

Notice that in either direction the two numbers (other than the five) add to ten.

Cross Sums

Using the same logic as noted in the **Magic Squares** problem above we could solve this problem with

```
  9
  1
28573
  4
  6
```

Notice that five is "in the middle" and the number on either side sum to ten.

A Number Square

One such square is

```
978
654
231
```

A Triangle of Numbers

The triangle I came up with looked like

```
   2
  7 1
 6   9
5 3 4 8
```

Star of David

Consider this arrangement of numbers

```
    3
1 12 11 2
 6     8
5 10  7 4
    9
```

Number Rows

We can consider this arrangement of numbers

```
6   7
5 4 3
1   2
```

Multi-Magic Square

Consider

```
3   36  2
4    6  9
18   1 12
```