Jeans Pg 64

9-4-01

By dimensional Analysis given the tension in the string + the length determine the period + the frequency of vibration of a stretched string.

It is ~~known~~ that (experimentally) the frequency or period depends on

1) tension T
2) mass of string (mass per unit length) μ
3) length of string between two clamped ends l

?? What about the gravitational constant?
1) ~~I would imagine that this constant is in the~~
How does the frequency + period depend on this #?
1) I would imagine ~~that~~ that this constant is in the constant C_f.

Now

$$[T] = F = \frac{ML}{T^2}$$

$$[\mu] = \frac{M}{L} \quad + \quad [l] = L$$

$$\therefore \quad \left[\frac{T}{\mu}\right] = \frac{L^2}{T^2}$$

So

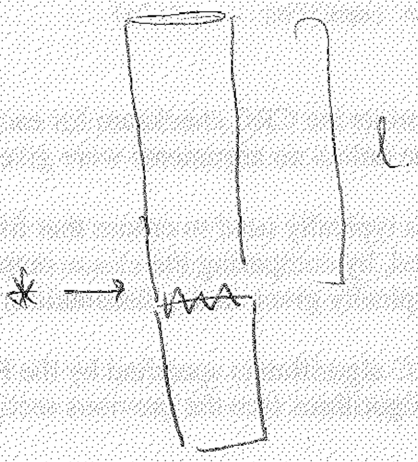
$$\left[\frac{T}{\mu l^2}\right] = \frac{1}{T^2} = [f^2] \Rightarrow f \propto \sqrt{\frac{T}{\mu l^2}} = \frac{\sqrt{T}}{l\sqrt{\mu}}$$

$$\Rightarrow f = \frac{C_f\sqrt{T}}{l\sqrt{\mu}} \quad + \quad T = \frac{l\sqrt{\mu}}{C_f\sqrt{T}}$$

Jeans pg 110

Dimensional Analysis of vibrations of a free column of Air

Spring force ~ pressure.



Experimentally known freq depends on

~~Base~~ height of column of air l

think pressure of Air P

$$[P] = \frac{[F]}{[A]} = \frac{ML/T^2}{L^2} = \frac{M}{LT^2}$$

$$[l] = L$$

$$[f] = \frac{1}{T}$$

Note: it would actually be greater ~~at~~ At the bottom of the tube due to the weight of the column of Air above

Use density of air Assumed constant $[\rho_{Air}] = \frac{M}{L^3}$

$$f \underset{\sim}{\propto} \sqrt{\frac{P}{(\rho_{Air}) l^2}} = \frac{1}{l} \frac{\sqrt{P}}{\sqrt{\rho_{Air}}}$$

$$\left[\frac{P}{\rho_{Air}}\right] = \frac{L^2}{T^2}$$

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10-13-01 1

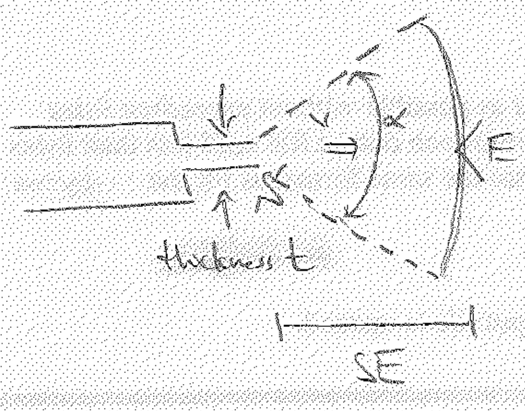
$$40 \text{ miles/hr} = 40 \cdot \frac{\text{miles}}{\text{hr}} \cdot \frac{1 \text{ hr}}{3600 \text{ s}} \cdot \frac{5 \cdots \text{ ft}}{1 \text{ mile}} \cdot \frac{12 \text{ inch}}{1 \text{ ft}} =$$

diameter = .5 inch

$$\text{Thus period of whirlwinds produce} = \frac{(5\,2/5)(.5 \text{ inch})}{\text{speed of free stream}}$$

$$\text{It equals } 704 \text{ inchs/s} = 1408$$

$$\begin{array}{r} 704 \\ 2 \\ \hline 1408 \end{array}$$



Claim:

Assuming conservation of Mass we obtain a 1/r dependence on velocity from a directional source point such as the one above. Note: this is only true in the 2Dimensional case.

The flux of mass at source S is

$$\underbrace{\rho_{Air} \underbrace{\overset{\uparrow}{t} \; \overset{\uparrow}{V_S}}_{\text{Volume flux}}}_{\text{Mass flux}} = \text{Flux of mass at point E.}$$

thickness of outlet

velocity at S

Assuming an equal amount of mass goes in the wedge shown above. The area of this 2Dimensional semi-circle is

$$\rho_{Air} (\alpha r) V_E$$

$$\Rightarrow \quad t V_S = \alpha r V_E \Rightarrow V_E = \left(\frac{t}{\alpha}\right) \frac{V_S}{r}$$

Note: Since the air exiting the flue has been traveling at some speed before it exists it will have a built up momentum that should need time to defuse before the ∢ profile shown above is correct. I.E. the true flow will prob. look ~~take~~ something like the following

Region of
Basically straight
Flow Not influenced
by opening yet.

The momentum carried by the flow does not let the velocity drop (by spreading) so quickly, thus generally in a case like this the $\overset{}{\underset{\text{linear}}{\wedge}}$ decay occurs after point P

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$1\frac{2}{3}$ diameters

8 feet in length

$(1\frac{2}{3})(6 \text{ inch}) = 6 + 4 = 10$ inches

⇒ effective length of l = 8 feet 10 inches

⇒ Since pipe is open $\lambda = 2l$ = 16 feet + 20 inch

= 16 feet + 1 feet + 8 inch

= 17 feet + 8 inch.

For a wooden 4-~~feet~~ pipe

$2(6 \text{ inch}) = 12$ inch

⇒ effective length = 5 feet

+ λ of stopped wooden pipe $= 4l =$ 20 feet.

10-18-01

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$C\,E\,G\,C' \xrightarrow[\text{How?}]{} 4:5:6:8$

$$\begin{array}{r} 4 \\ 15 \\ 8 \\ \hline 120 \end{array}$$

$4 > 2^2$
$5 = 5$
$6 = 2 \cdot 3$
$8 = 2^3$

$L.C.M = 2^3 \cdot 3 \cdot 5 = 8 \cdot 15 = 120$

$C\,E\,G\,B \xrightarrow[\text{How}]{} 8:10:12:15$

$8 = \underline{2}^3$
$10 = \underline{2} \cdot 5$
$12 = \underline{2}^2 \cdot 3$
$15 = 3 \cdot 5$

$L.C.M. = 2^3 \cdot 3 \cdot 5 = 120$

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10-19-01 1

Wood block floor: reflects 97% c'

78% c''

After 10 reflections how much sound is left.

After 1 reflection $I_1 = (.97) I_0$ c'

$I_1 = (.78) I_0$ c''

So after 10 reflections $I_{10} = (.97)^{10} I_0$ c'

$I_{10} = (.78)^{10} I_0$ c''

$\Rightarrow \frac{I_{10}}{I_0} = (.97)^{10}$ = percentage still in room or reflected of c' note = .73

$\frac{I_{10}}{I_0} =$ [illegible] $(.78)^{10}$ = percentage still in room or reflected of c'' note.

= .08

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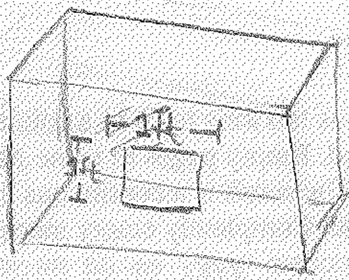
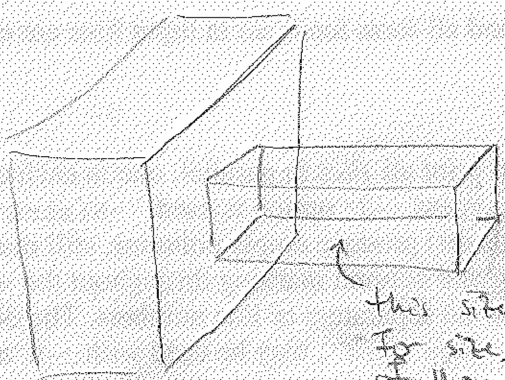
10-19-01





this sized volume (see below for size) is emptied every second of the energy in the room.

Flux of sound energy leaves at 1100 ft/s thus volume that flows off is

$$1100\ \text{ft/s}\,(1\ \text{ft}^2) = 1100\ \text{ft}^3/\text{s}$$

If Vol of room $= x(1100\ \text{ft}^3/\text{s})$ time to empty room $= x$

1) only half of reflected sound is traveling towards the window side

2) Remaining 1/2 has velocity of $\frac{1100\ \text{ft/s}}{2}$

Thus rate of which volume of sound energy is emptied is $\frac{1100\ \text{ft}^3/\text{s}}{4}$

$$= 275\ \text{ft}^3/\text{s}$$

Additionally we have not taken into account that (every second) the energy in the room decreases by $(275\ \text{ft}^3/\text{s})\rho$ w/ ρ the current density of sound energy (of the room)

Assume the initial energy in the room has density ρ_0

Let the volume of the room be written as

$$V = 4(275)x$$

Then in the 1st second the room looses energy of the amount

$$E_1 = E_0 - (275\ \mathrm{ft^3/s})(1\mathrm{s})\rho_0$$

$$= 4x(275\ \mathrm{ft^3})\rho_0 - (275)\rho_0$$

$$= (4x-1)(275)\rho_0$$

Then the new energy density in the room is

$$\rho_1 = \frac{E_1}{V} = \frac{E_1}{4x(275)} = \frac{(4x-1)}{4x}\rho_0$$

$$= \left(1-\frac{1}{4x}\right)\rho_0$$

So that in the next second

$$E_2 = E_1 - (275)\rho_1 = (275)(4x-1)\rho_0 - (275)\left(1-\frac{1}{4x}\right)\rho_0$$

$$=$$

10-19-01

w/ all these constants its hard to see whats going on.

let initial energy density be ρ_0 & velocity that energy flows at of the box be V^*, let the initial volume of the room be V_R & the Area of the window A_w

Then $E_0 = V_R \rho_0$

In 1 second the new energy is $E_1 = E_0 - (A_\omega) V^* \rho_0$

$$= (V_R - A_\omega V^*) \rho_0$$

$$= \left(1 - \frac{A_\omega V^*}{V_R}\right) V_R \rho_0 = \left(1 - \frac{A_\omega V^*}{V_R}\right) E_0$$

At which point the density is then

$$\rho_1 = \frac{E_1}{V_R} = \left(1 - \frac{A_\omega V^*}{V_R}\right) \rho_0$$

Then in another second the energy in the room is

$$E_2 = E_1 - A_\omega V^* \rho_1$$

$$= \left(1 - \frac{A_\omega V^*}{V_R}\right) E_0 - (A_\omega V^*)\left(1 - \frac{A_\omega V^*}{V_R}\right) \rho_0$$

$$= \left(1 - \frac{A_\omega V^*}{V_R}\right)\left[E_0 - \frac{A_\omega V^*}{V_R} E_0\right] = \left(1 - \frac{A_\omega V^*}{V_R}\right)^2 E_0$$

At which point the density is then

$$\rho_2 = \frac{E_2}{V_R} = \left(1-\frac{A_w v^*}{V_R}\right)^2 \rho_0$$

∴ by induction

$$E_n = \left(1-\frac{A_w v^*}{V_R}\right)^n E_0 \quad \& \quad \rho_n = \left(1-\frac{A_w v^*}{V_R}\right)^n \rho_0$$

Using the #'s given in the text $V_R = 4(275)x\ ft^3$

$A_w = 1\ ft^2$

$v^* = 275\ ft/s$

$$\frac{A_w v^*}{V_R} = \frac{(1\ ft^2)(\cancel{275}\ ft/s)}{4(\cancel{275})x\ ft^3/s} = \frac{1}{4x}$$

desire to know when will

$$\frac{E_n}{E_0} = 10^{-6}$$

$$\rightarrow \left(1-\frac{1}{4x}\right)^n = 10^{-6}$$ take $\log_{10}$ of both sides

$$n \log\left(1-\frac{1}{4x}\right) = -6$$

$$n = \frac{-6}{\log_{10}\left(1-\frac{1}{4x}\right)}$$

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$\log_b x = \frac{\log_c x}{\log_c b}$ $\therefore$ $\log_{10}\left(1-\frac{1}{4x}\right) = \frac{\log\left(1-\frac{1}{4x}\right)}{\log 10}$

$\therefore$ $n = \frac{-6\log 10}{\log\left(1-\frac{1}{4x}\right)}$

Assuming $x >> \underline{1}$ $\log(1-x) = -\sum_{k=0}^{+\infty} \frac{x^{k+1}}{(k+1)} \doteq -\left[\frac{x}{1}+\frac{x^2}{2}+\cdots\right]$

$\frac{1}{1-x} = \sum_{k=0}^{+\infty} x^k$

$-\log(1-x) = \sum_{k=0}^{+\infty} \frac{x^{k+1}}{k+1}$

Thus $\log\left(1-\frac{1}{4x}\right) \cong -\left[\frac{1}{1}\left(\frac{1}{4x}\right)+\cdots\right]$

$= -\frac{1}{4x}$

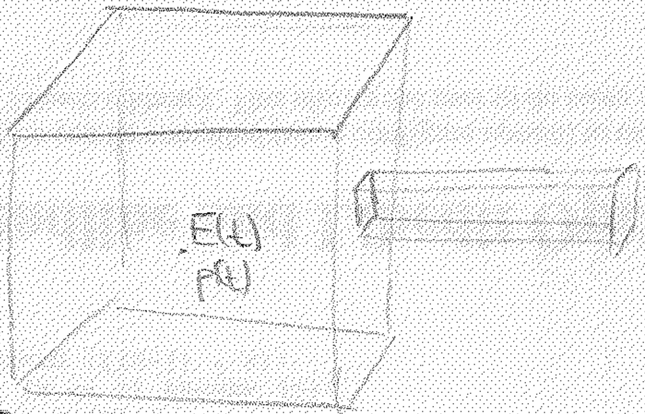
$\therefore$ $n \cong (6\log 10)\,4x = 24\log 10\; x$

$\log 10 \approx ?$ $3 \le \log 10 \le 4$

so $18 \le 6\log 10 \le 24$ too large!!

The problem of this approach is that it is discrete while the actual process is <u>continuous</u>!!

10-19-01



$$\boxed{\rho(t) \equiv \frac{E(t)}{V_R}} \qquad dE = -(A_0 v^*)\, dt\, (\rho(t)) \overset{-}{=}_{\wedge} (A_0 v^*)\, dt\, \frac{E(t)}{V_R}$$

$$\Rightarrow \frac{dE}{dt} = -\left(\frac{A_0 v^*}{V_R}\right) E(t)$$

Thus $$\frac{dE}{E(t)} = -\left(\frac{A_0 v^*}{V_R}\right) dt$$

$$\ln E(t) = -\left(\frac{A_0 v^*}{V_R}\right) t + C_0$$

$t=0 \quad E(t) = E_0$

$$\Rightarrow \ln E_0 = C_0 \quad \Rightarrow \quad E(t) = E_0 e^{-\left(\frac{A_0 v^*}{V_R}\right) t}$$

Again to mean $$\frac{E(t)}{E_0} \cong 10^{-6}$$

$$10^{-6} = e^{-\left(\frac{A_0 v^*}{V_R}\right) t} \quad \rightarrow \quad -\left(\frac{A_0 v^*}{V_R}\right) t = (-6) \log 10$$

$$t = \left(\frac{V_R}{A_0 v^*}\right)(6 \log 10)$$

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Now $t = \left(\frac{4x(\cancel{x})}{\cancel{x}}\right) 6\log 10$

$= 4x(6\log 10)$

Approximate $\log_e 10 = ? = p = e^p = 10$ $e \hat{=} 2.7$

$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ $\therefore$ $e^1 \hat{=} \frac{1}{1!} + \frac{1}{2} + \frac{1}{6} + \cdots$

find a zero of $f(x) = e^x - 10 = 0$

$f'(x) = e^x$

Newtons method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{(e^{x_n} - 10)}{e^{x_n}}$$

$$= x_n - 1 + 10e^{-x_n}$$

Take a guess $x_0 \hat{=} 3$

$x_1 \hat{=} 3 - 1 + 10e^{-3} \hat{=} 2 + \frac{10}{(2.7)^3}$

$= 2.50805$

$$\begin{array}{r} {}^{4} \\ 2.7 \\ 2.7 \\ \hline 189 \\ 540 \\ \hline 7.29 \end{array} \qquad \begin{array}{r} {}^{2\,4} \\ 7.29 \\ 2.7 \\ \hline 5103 \\ 14580 \\ \hline 19.683 \end{array} \checkmark$$

$$19.683 \overline{\big)\, 10.000}^{\;.9}$$

$= .50805$

$x_2 = 2.50805 - 1 + 10e^{-(2.50805)}$

e

10-19-06

let $V_R = 4_x V^*$ ⇒ $4_x = \frac{V_R}{V^*}$

$$\Rightarrow t = V_R \frac{(6\log 10)}{V^*} = V_R\left(\frac{6\log 10}{275}\right)$$

Pg 204 Jeans

10-19-01

$A_{room} = 6\ell^2$

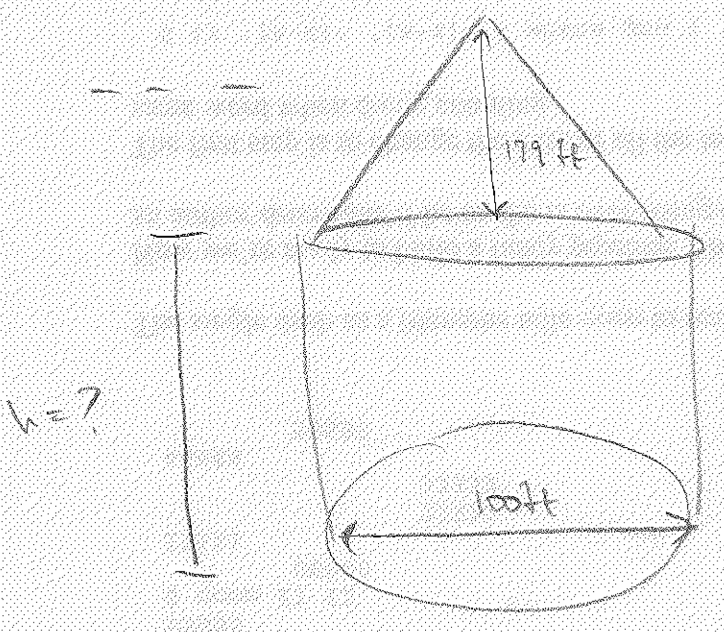
Adsorbing power total $= \frac{1}{40} 6\ell^2 = \quad n$

$$\frac{3}{20}\ell^2 = n$$

$$> \frac{\ell^2}{7}$$

$V = \ell^3$

So that reverberation time $= \frac{V}{20n} = \frac{\ell^3}{20\left(\frac{3}{20}\ell^2\right)} = \frac{\ell}{3}$



Baptistry at Pisa.

Since h is not given I'll assume the building looks more like a cone.

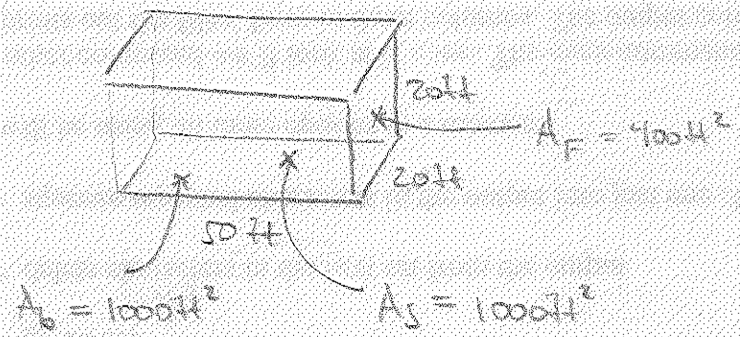
Area Room = 50,000 ft^2

$V_R = 1\ 10^6$ ft^3

$n = A_R\left(\frac{1}{100}\right) = \frac{5\ 10^4}{10^2} = 5 \cdot 10^2$ ✓

$t_{Rev} = \frac{V}{20n} = \frac{10^6}{20 \cdot 5 \cdot 10^2} = \frac{10^6}{10000}$

$= 10^2 s > 1.5$ mi



$A_{walls} = 2(1000ft^2) + 2(400 ft^2) = 2800ft^2$

Absorbtion of C $(.09)(2800) = 252$

$$\begin{array}{r} \overset{7}{2}8 \\ 9 \\ \hline 252 \end{array}$$

$V_{Room} = 50(20)^2 = 20,000$

$$\frac{V_{Room}}{20n} = \frac{20,000}{20n} = \frac{1000}{n}$$

— — ——

What does table mean of seated audience minus 3/4 inch blocks?

I would think that one would just replace the floor of wood blocks w/ one of an audience thus keeping All the calculation charge but correcting 500 ft² of miss recorded data.

Seated Audience minus wood blocks = .72 − .05 = .67 ⇐

$(.67) 500 = 5(67) = 350-15 = 335$

10-19-01

By subtracting the absorbtion coefficient of the wood blocks from that of the seated audience we have added a negative wood block section. When is is added blindly to the results from the table above we cancel the effect of the initial inclosion of the wood blocks.

Contribution from the floor should be (w/ audience now)

$$(500\,ft^2)(a_{seated\ audience}) + (1000\,ft^2)(a_{wood\ blocks})$$

$$= \underbrace{(500\,ft^2)(a_{seated\ audience} - a_{wood\ blocks}) + \underbrace{(500\,ft^2)(a_{wood\ blocks}) + 500\,ft^2\,(a_{wood\ blocks})}_{+\,1000\,ft^2\,(a_{wood\ blocks})}}_{\text{Computed in table}}$$

Computed in table

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10-19-01

Measure size of my room + then add additional pieces of material on the wells + floors to cultivate the optimum period of reverberation.

$$V = 12 \cdot 10^3 \qquad \tau = \frac{V}{20n} \Rightarrow \frac{(1.03)(20)}{12 \cdot 10^3} = \frac{1}{n}$$

$$\Rightarrow n = \frac{12 \cdot 10^3}{(1.03)(20)} = \frac{6 \cdot 10^2}{1.03}$$

$$< 600$$

Thus I can measure my room + compute the units of absorbtion + increase or decrease these to make my room match the table given on this page

Also plot Vol of room in ft^3 vs reverberation period + then interpolate to fit my room in.

Pg 214 Ideas

10-19-01

Material absorbs 1/3 of sound impeding on it.

$$\tau = \frac{V}{20n} = \frac{V}{20(S(1/3))} = \frac{3}{20}\left(\frac{V}{S}\right)$$

$n = S(1/3)$

So to become inaudable

$$I_n = (2/3)^n I_0 \Rightarrow (2/3)^n = 10^{-6}$$

$$n\log(2/3) = -6\log 10$$

$$n = \frac{-6\log 10}{-\log(2/3)} = \frac{6\log 10}{\log(3/2)}$$

$$= 6\log_{3/2}10 = ?$$

$\frac{1}{3} +$