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$$h_a = 0$$

$$\xi_t = \sum_{s=a+1}^{t} Y_t Y_s^{-1} w_s$$

$$\xi_t$$

$$\xi_t = \sum_{s=-\infty}^{t} Y_t Y_s^{-1} w_s$$

Show ξ_t satisfies $z_t = A(t) z_{t-1} + w_t$

$$\xi_t - A(t)\xi_{t-1} = \sum_{s=-\infty}^{t} Y_t Y_s^{-1} w_s - A(t) \sum_{s=-\infty}^{t-1} Y_{t-1} Y_s^{-1} w_s$$

$$= Y_t Y_t^{-1} w_t + \sum_{s=-\infty}^{t-1} [Y_t - A(t) Y_{t-1}] Y_s^{-1} w_s$$

$$\xi_t = \sum_{r=0}^{\infty} Y_t Y_{t-r}^{-1} w_{t-r}$$

$$|A \circ B| \le |A|\,|B|$$

$$\le \left|\sum_i a_i b_i\right| \le \Big(\sum_i |a_i|^2\Big)^{1/2} \Big(\sum_i |b_i|^2\Big)^{1/2}$$

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$$\alpha_i^*(t) = \alpha_{q-i}(t+q-i) \qquad \text{eq } 3.7$$

$$\alpha_i^{**}(t) = \alpha_{q-i}^*(t+q-i) = \alpha_{q-(q-i)}(t+q-(q-i))$$

$$= \alpha_i(t+i)$$

---

eq 3.13 $\quad H(s,r) = -H^*(r-q,s)$

$$\Rightarrow \quad H^*(r-q,s) = -H(s,r)$$

$$\Rightarrow \quad H^*(r,s) = -H(s,r+q)$$

$$\Rightarrow \quad H^{**}(r,s) = -H^*(s,r+q) = -\left[-H(r+q,s+q)\right]$$

$$= H(r+q,s+q)$$

$$\Rightarrow \quad H(r,s) = H^{**}(r-q,s-q) \qquad \text{eq } 3.16 \ \checkmark$$

$$t^{(m)} = t(t-1)(t-2)\cdots(t-m+1) \qquad t^{(1)} = t$$

$$P(t) = \sum_{m=0}^{n} b_m t^{(m)} \qquad t^{(0)} = 1 \qquad t^{(2)} = t(t-1)$$

$$t^{(3)} = t(t-1)(t-2)$$

$$\vdots$$

$$b_m \equiv \frac{\Delta^m P(0)}{m!}$$

eq 22 is $\eta_t = \int_C z^{t-1} v(z)\,dz$

$$t^{(1)}\eta_{t-k} = \int_C t z^{t-k-1} v(z)\,dz$$

$$= \int_C t\cdot z^{t-1}\cdot z^{-k} v(z)\,dz$$

$$= \int_C D[z^t]\cdot z^{-k} v(z)\,dz$$

& $t^{(2)}\eta_{t-k} = \int_C t(t-1) z^{t-k-1} v(z)\,dz$

$$= \int_C t(t-1) z^{t-2}\cdot z^{-k} v(z)\,dz$$

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$$t^{(2)}\eta_{t-k} = \int_C D^2[z^t]\, z^{-k+1}\, v(z)\, dz$$

$$\cancel{t^{(3)}\eta_{t-k} = \int_C D^3[z^t]\, z^{-k-2}\, v(z)\, dz}$$

$$\vdots$$

$$\cancel{t^{(j)}\eta_{t-k} = \int_C z}$$

$$t^{(3)}\eta_{t-k} = \int_C t(t-1)(t-2)\, z^{t-k-1}\, v(z)\, dz$$

$$= \int_C t(t-1)(t-2)\, z^{t-3} \cdot z^{-k+2}\, v(z)\, dz$$

$$= \int_C D^3[z^t]\, z^{-k+2}\, v(z)\, dz$$

Thus:

$$t^{(j)}\eta_{t-k} = \int_C D^j[z^t]\, z^{-k+j-1}\, v(z)\, dz$$

$$\int_C D^j[z^t]\, z^{-k+j-1}\, v(z)\, dz$$

$= \; [illegible] \; -$

$$= z^{-k+j-1}\, D^{j-1}[z^t]\, v(z)\Big|_C - \int_C D^{j-1}[z^t]\, D^1[z^{+j-k+1}\, v(z)]\, dz$$

$$= \cancel{z^{-k+j-1}\, D^{j-1}[z^t]\, v(z)\Big|_C} -$$

$$= D^{j-1}[z^t] \cdot z^{-k+j-1}\, v(z)\Big|_C - \int_C D^{j-1}[z^t] \cdot D^1[z^{j-k+1}\, v(z)]\, dz$$

$$= D^{j-1}[z^t]\, z^{-k+j+1}\, v(z)\Big|_C - D^{j-2}[z^t]\, D^1[z^{+j-k+1}\, v(z)]\Big|_C$$

$$+ \int_C D^{j-2}[z^t]\, D^2[z^{j-k+1}\, v(z)]\, dz$$

$$= D^{j-1}[z^t]\, z^{j-k+1} v(z) \Big|_C - D^{j-2}[z^t]\, D^1[z^{j-k+1} v(z)] \Big|_C$$

$$+ D^{j-3}[z^t]\, D^2[z^{j-k+1} v(z)]\, dz \Big|_C - \int_C D^{j-3}[z^t]\, D^3[z^{j-k+1} v(z)]\, dz$$

$$= \sum_{i=1}^{j} D^{j-i}[z^t]\, D^{i-1}[z^{j-k+1} v(z)]\, dz \Big|_C (-1)^{i+1}$$

$$(-1)^j \int_C z^t D^j [z^{j-k+1} v(z)]\, dz$$

$$D^1[z^t] = t^{(1)} z^{t-1}$$

$$D^2[z^t] = t(t-1) z^{t-2}$$

$$\vdots$$

$$D^k[z^t] = t^{(k)} z^{t-k}$$

$\therefore$ Above gives:

$$= \sum_{i=1}^{j} t^{(j-i)} z^{t-j+i}\, D^{i-1}[z^{j-k+1} v(z)]\, dz\, (-1)^{i+1}$$

$$R(t)y_t = \alpha_0(t)y_t + \alpha_1(t)y_{t-1} + \cdots + \alpha_q(t)y_{t-q}$$

$$\alpha_k(t) = \sum_{j=0}^{n_k} a_{kj} t^{(j)}$$

$$\alpha_k(t)\eta_{t-k} = \sum_{j=0}^{n_k} a_{kj} t^{(j)} \eta_{t-k}$$

$$= \sum_{j=0}^{n_k} a_{kj}\pi_{jk} + \sum_{j=0}^{n_k} a_{kj}(-1)^j \int_C z^t \Delta^j \left[z^{j-k-1} v(z)\right] dz$$

Then

$$R(t)\eta_t = \sum_{j=0}^{n_0} a_{0j}\pi_{j0} + \sum_{j=0}^{n_0} a_{0j}(-1)^j \int_C z^t \Delta^j \left[z^{j-1} v(z)\right] dz$$

$$+ \sum_{j=0}^{n_1} a_{1j}\pi_{j1} + \sum_{j=0}^{n_1} a_{1j}(-1)^j \int_C z^t \Delta^j \left[z^{j-1-1} v(z)\right] dz$$

$$+ \cdots$$

$$+ \sum_{j=0}^{n_q} a_{qj}\pi_{jq} + \sum_{j=0}^{n_q} a_{qj}(-1)^j \int_C z^t \Delta^j \left[z^{j-q-1} v(z)\right] dz$$

$$P(t)\eta_t = \underbrace{\sum_{k=0}^{q}\sum_{j=0}^{n_k} a_{kj}\pi_{jk}}_{\equiv \Pi_c} + \underbrace{\sum_{k=0}^{q}\sum_{j=0}^{n_k} a_{kj}(-1)^j \int_C z^t D^j\left[z^{j-k-1} v(z)\right] dz}_{\checkmark}$$

Require $W(z) \equiv 0$. then $P(t)\eta_t = \Pi_c$

$n_k \le 1 \quad \forall \; k$

$$\sum_{k=0}^{q}\sum_{j=0}^{1} (-1)^j a_{kj} z^t D^j\left[z^{j-k-1} v(z)\right] = 0$$

$$\Rightarrow \sum_{k=0}^{q} a_{k0} z^t D^0\left[z^{-k-1} v(z)\right] - a_{k1} z^t D^1\left[z^{-k} v(z)\right] = 0$$

$$\Rightarrow \sum_{k=0}^{q} a_{k0} z^{t-k-1} v(z) - a_{k1} z^t\left(-kz^{-k-1} v(z) + z^{-k} v'(z)\right) = 0$$

$$\Rightarrow v'(z)\sum_{k=0}^{q} a_{k1} z^{t-k} - v(z)\left[\sum_{k=0}^{q}\left(a_{k0} z^{t-k-1} + k z^{t-k-1} a_{k1}\right)\right] = 0$$

$$\eta_t = A_\xi \int_C z^{t-1} \exp\left[-\tfrac{1}{2}\xi(z - z^{-1})\right] dz$$

$$z = e^{i\phi}$$

$$dz = i e^{i\phi} d\phi$$

$$\eta_t = A_\xi \int_0^{2\pi} e^{i\phi(t-1)} \quad \dots$$

$$\vdots$$

$$A_\xi (z^2 - 2\xi z + 1)^{1/2} z^{t-1} \Big|_C$$

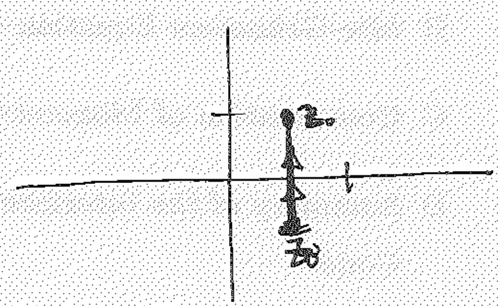
$|\xi| < 1$

$$z_0 = \frac{+2\xi \pm \sqrt{4\xi^2 - 4}}{2} = \frac{\xi \pm \sqrt{\xi^2 - 1}}{1}$$

$$= \xi \pm \sqrt{\xi^2 - 1}$$

$|\xi| < 1 \quad \rightarrow \quad z_0 \equiv \xi + i\sqrt{1 - \xi^2}$

$$\bar{z}_0 = \xi - i\sqrt{1 - \xi^2}$$



$$\eta_t = \int_{\bar{z}_0}^{z_0} z^{t-1} \cdot A_\xi z (z^2 - 2\xi z + 1)^{-1/2} dz = A_\xi \int_{\bar{z}_0}^{z_0} z^{t} (z^2 - 2\xi z + 1)^{-1/2} dz$$

$$\frac{z^2 - \xi z + 1}{z} = \frac{v'(z)}{v(z)}$$

$$z - \xi + \frac{1}{z} = \frac{v'}{v}$$

$$\frac{z^2}{2} - \xi z + \ln z + C = \ln v$$

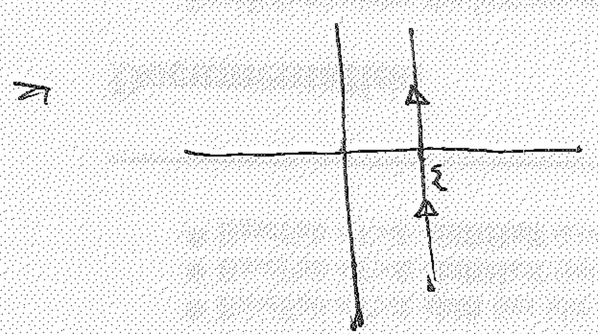
$$v(z) = C' z \exp\left(\frac{z^2}{2} - \xi z\right)$$

$$= C' z \cancel{\exp\left(\tfrac{1}{2}(z^2 - 2\xi z + \xi^2)\right)}$$

$$= C' z \exp\left(\tfrac{1}{2}(z^2 - 2\xi z + \xi^2 - \xi^2)\right)$$

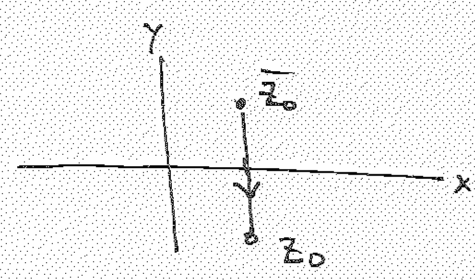
$$= C'_\xi z \exp\left(\tfrac{1}{2}(z-\xi)^2\right) \quad ✓$$

$\Gamma_c = 0$ eq 2.13



$$\eta_t = A_\xi = \int_{\xi - i\infty}^{\xi + i\infty} z^{t-1} \cdot z \exp\left(\tfrac{1}{2}(z-\xi)^2\right) dz \qquad \text{eq } 2.25 \ ✓$$

$$\eta_t = A_\xi \int_{\bar{z}_0}^{z_0} z^t (z^2 - 2\xi z + 1)^{-1/2}\, dz$$



let $z = \xi + i\sqrt{1-\xi^2}\cos\phi \qquad 0 \le \phi \le \pi$

$dz = -i\sqrt{1-\xi^2}\sin\phi$

$$\eta_t = \int_0^\pi (\xi + i\sqrt{1-\xi^2}\cos\phi)^t \left((\xi + i\sqrt{1-\xi^2}\cos\phi)^2 - 2\xi(\xi + i\sqrt{1-\xi^2}\cos\phi) + 1 \right)^{-1/2} dz$$

Consider $\left. z^2 - 2\xi z + 1 \right|_{z = \xi + i\sqrt{1-\xi^2}\cos\phi}$

$\Leftarrow \cancel{\xi^2 - (1-\xi^2)\cos^2\phi = -2\xi}$

$= \xi^2 + 2\xi i\sqrt{1-\xi^2}\cos\phi - (1-\xi^2)\cos^2\phi - 2\xi(\xi + i\sqrt{1-\xi^2}\cos\phi) + 1$

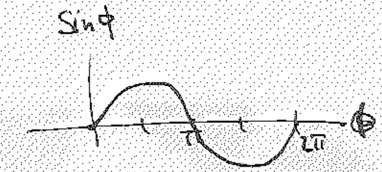
$= \underline{\xi^2} + \cancel{2i\xi\sqrt{1-\xi^2}\cos\phi} - \cos^2\phi + \xi^2\underline{\cos^2\phi}$

$\quad - \underline{2\xi^2} - \cancel{2i\xi\sqrt{1-\xi^2}\cos\phi} + 1$

$= (\cos^2\phi - 1)\xi^2 + 1 - \cos^2\phi \qquad 1 - \cos^2\phi = \sin^2\phi$

$= (1-\cos^2\phi)(1-\xi^2) = \sin^2\phi\,(1-\xi^2)$

Then $dz = i\sqrt{1-\xi^2}\,(-\sin\phi)\,d\phi$



So

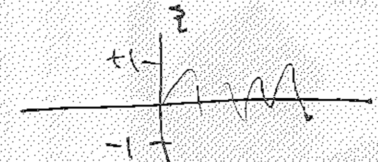
$$\eta_t = A_\xi \int_0^\pi \left(\xi + i\sqrt{1-\xi^2}\cos\phi\right)^t \cdot \left(\cancel{\sin^2\phi}\,\cancel{(1-\xi^2)}\right)^{-1/2} \cdot i\,\cancel{\sqrt{1-\xi^2}}\,(-\cancel{\sin\phi})\,d\phi$$

$$= -iA_\xi \int_0^\pi \left(\xi + i\sqrt{1-\xi^2}\cos\phi\right)^t d\phi$$

didn't get neg sign!?

let $\xi = \cos\theta$ $\quad \sqrt{1-\xi^2} = |\sin\theta|$

$|\xi| < 1$



$$\eta_t = A_\theta \int_0^\pi \left(\cos\theta + i|\sin\theta|\cos\phi\right)^t d\phi$$

How know positive?

2.23 let $i\phi = z - \xi \quad \Rightarrow$

~~to~~ when $z = \xi - i\infty$

$i\phi = -i\infty \quad \Rightarrow \quad \phi = -\infty$

when $z = \xi + i\infty$

$i\phi = \xi + i\infty - \xi = i\infty \quad \Rightarrow \quad \phi = \infty$

$$\eta_t = A_\xi \int_{-\infty}^{+\infty} (\xi + i\phi)^t \exp\left[\tfrac{1}{2}(i\phi)^2\right] i\,d\phi$$

$$= i A_\xi \int_{-\infty}^{+\infty} (\xi + i\phi)^t e^{-\frac{1}{2}\phi^2} \, d\phi \qquad \text{eq } 2.26 \checkmark$$

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$$\xi_t = \sum_{s=-\infty}^{t} \gamma_t \gamma_s^{-1} u_s$$

$$\xi_s = \sum_{r=-\infty}^{s} \gamma_s \gamma_r^{-1} u_r$$

so $$\xi_t \cdot \xi_s' = \left(\sum_{r=-\infty}^{t} \gamma_t \gamma_r^{-1} u_r\right)\left(\sum_{p=-\infty}^{s} \gamma_s \gamma_p^{-1} u_p\right)'$$

$$= \left(\sum_{r=-\infty}^{t} \gamma_t \gamma_r^{-1} u_r\right)\left(\sum_{p=-\infty}^{s} u_p' \gamma_p'^{-1} \gamma_s'\right)$$

$$= \sum_{r=-\infty}^{t} \sum_{p=-\infty}^{s} \gamma_t \gamma_r^{-1} u_r u_p' \gamma_p'^{-1} \gamma_s' \qquad \text{eq 1.8}$$

so $$E\left[\xi_t \cdot \xi_s'\right] =$$