Some Notes on the Book:
How Would You Move Mt. Fuji?
by William Poundstone

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Introduction

As I was reading this book I worked a few of the problems. The answers presented are sometimes somewhat different than the ones presented in the book. In many of my answers I phrase the statement in algebra and proceed two work the problem from there. This is often results in a different solution (from the one presented in the book) to the given problem.

How Many Times a Day do a Clocks Hands Overlap?

Lets denote the angle of the hour hand from twelve (due North) as θ_{hour} and the angle of the minute hand (again from twelve) as θ_{minute}. Now in twelve hours the hour hand goes from twelve and back to twelve again, while in one hour the minute hand goes from twelve back to twelve again. If measure time t in hours and take $t = 0$ to be when the minute hand and the hour hand point towards the twelve then as a function of time the location of the hour hand is given by

$$\theta_{\text{hour}} = \left(\frac{2\pi}{12}\right)t \,.$$

As a function of time t the minute hand is given by

$$\theta_{\text{minute}} = 2\pi t \,.$$

For this problem we are looking for the number of times when

$$\theta_{\text{minute}} - \theta_{\text{hour}} = 0 \,(\text{mod}\, 2\pi) \,.$$

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The above is equal to

$$\left(2\pi - \frac{2\pi}{12}\right)t = 2\pi\left(1 - \frac{1}{12}\right) = 2\pi\left(\frac{11}{12}\right)t.$$

We want this to be equal to 0, 2π, 4π, 6π, 8π, i.e. $2\pi n$ for $n = 0, 1, 2, \ldots$. This will happen when

$$2\pi\left(\frac{11}{12}\right)t = 2\pi n,$$

or solving for t we get

$$t = \frac{12}{11}n.$$

Now we need to count the number of n such that the t above belongs to the same day i.e. $t \leq 24$. Using the above and solving for n we get $0 \leq n \leq 22$. Note that $n = 0$ is midnight i.e. both hands pointing straight up, $n = 11$ is noon both hands again pointing straight up, and $n = 22$ is the following midnight. If we count this second midnight as part of the same day then we have $22 - 0 + 1 = 23$ times in 24 hours when the two hands overlap. If this second midnight is not to be part of the same day then the number is 22.

Three Intersecting Ants

For the ants to not collide means that all ants must pick the counter clockwise or the clockwise direction to travel around the triangle. If we label the three vertices of the triangle as A, B, and C then the ants must move in the direction $A \to B \to C \to A$ or in the direction $A \to C \to B \to A$. The first notation means that the ant at A goes to B, the ant at B goes to C etc. All of the ants will move in one of the two above directions if each ant happens to pick the same direction independently. Since three things must happen this event will happen with a probability of $\frac{1}{2^3}$. As there are two directions the ants can move and not intersect the total probability of non-intersection is twice the previous number or $\frac{1}{2^2} = \frac{1}{4}$.

Counting in Base Negative -2

Recall that a number $n_m n_{m-1} \cdots n_2 n_1 n_0$ in base b is given by the expression

$$n_0 + n_1 b + n_2 b^2 + \cdots + n_{m-1}b^{m-1} + n_m b^m.$$

For this problem we want to find the coefficients $n_m n_{m-1} \cdots n_2 n_1 n_0$ of the powers of -2 that sum to the given natural numbers $1, 2, \cdots, 6, 7$. For example, note that we can write

$$
\begin{aligned}
1 &= (-2)^0 \\
2 &= (-2)^2 + (-2)^1 \\
3 &= (-2)^2 + (-2)^1 + 1(-2)^0 \\
4 &= (-2)^2 \\
5 &= (-2)^2 + 0(-2)^1 + (-2)^0 \\
6 &= 2(-2)^2 + (-2)^1 + 0(-2)^0 \\
7 &= 2(-2)^2 + (-2)^1 + (-2)^0 \,.
\end{aligned}
$$

We can continue in this way as long as needed. Based on the above counting in -2 the first seven natural numbers are

$$
001 \,, 110 \,, 111 \,, 100 \, 101 \,, 210 \,, 211 \ldots
$$

Maximizing the Probability we Draw a Red Marble

The probability of a red marble is drawn is given by

$$
\begin{aligned}
P(\text{Red}) &= \frac{1}{2} P(\text{Red}|\text{Jar1}) + \frac{1}{2} P(\text{Red}|\text{Jar2}) \\
&= \frac{1}{2} \left(\frac{N_r^1}{N_r^1 + N_b^1} \right) + \frac{1}{2} \left(\frac{N_r^2}{N_r^2 + N_b^2} \right) \,.
\end{aligned}
$$

Here N_r^1 and N_b^1 are the number of red and blue marbles put into the first jar (the notation for the second jar is N_r^2 and N_b^2). Since we have 100 red and 100 blue marbles we must have $N_r^1 + N_r^2 = 50$ and $N_b^1 + N_b^2 = 50$ so that the above is given by

$$
\begin{aligned}
P(\text{Red}) &= \frac{1}{2} \left(\frac{N_r^1}{N_r^1 + N_b^1} \right) + \frac{1}{2} \left(\frac{50 - N_r^1}{50 - N_r^1 + 50 - N_b^1} \right) \\
&= \frac{1}{2} \left(\frac{N_r^1}{N_r^1 + N_b^1} \right) + \frac{1}{2} \left(\frac{50 - N_r^1}{100 - N_r^1 - N_b^1} \right) \,.
\end{aligned}
$$

We would then want to maximize the above expression as a function of N_r^1 and N_b^1 where each are restricted to be $0 \le N_r^1 \le 50$ and $0 \le N_b^1 \le 50$. The book presents a better argument about how to find the maximum value of $P(\text{Red})$ but we could (not really in an interview) numerically. Using the following R code where we sequentially evaluate $P(\text{Red})$ for each of the possible values for N_r^1 and N_b^1

```
max_value = -10000.00
for( ri in 0:50 ){
  for( bi in 0:50 ){
    P_red = 0.5 * ( ri / ( ri + bi ) ) + 0.5 * ( ( 50 - ri ) / ( 100 - ri - bi ) )
    if( is.finite(P_red) & (P_red > max_value) ){
```

```
    max_value = P_red
    max_r = ri
    max_b = bi
   }
  }
}
```

We compute that $N_r^1 = 1$ and $N_b^1 = 0$ with $P(\text{max}) = 0.7474747$ meaning that we put only one red marble and no blue marbles in one jar. The other 49 red marbles and 50 blue marbles will go in the other jar. One could perhaps use calculus to take the derivatives of the expression for $P(\text{red})$ with respect to N_r^1 and N_b^1 set the results equal to zero and solve for N_r^1 and N_b^1.

Measuring Four Quarts of Water

We can measure four quarts of water as follows. First fill the five liter bucket fully. Poor what liquid you can into the three liter bucket. This leaves two quarts of water in the five liter bucket. Empty out the three liter bucket. Poor the the two litters of water in the five liter bucket into the three liter bucket which (when finished) will leave space in that bucket for one more liter of water. Next fill the five liter bucket with water. Poor what water you can (one liter) from the five liter bucket into the three liter bucket. Once done this will leave four quarts of water (the desired amount) in the five liter bucket.

The Correct Labeling of Fruit

Go to the box labeled "apples and oranges" since it must be labeled wrong it must be on a "pure" box of fruit either apples or oranges. We can then smell (or feel) the given fruit to determine what type of fruit that basket really contains. Say you determine it contains oranges. Then you would label that box correctly as oranges. Because of this the basket that is incorrectly labeled apples (which is actually oranges or "apples and oranges") must now be "apples and oranges" since it can't be just oranges (we found that box already). Thus having labeled two of the boxes correctly (oranges and "apples and oranges") the final box which was labeled oranges must actually be apples and we have correctly ordered the fruit.

The Adulterous Village Problem

On page 85 the book presents a problem where there are 50 couples of men and women in which all of the husbands have been unfaithful. None of the wife's know when their own husbands have been unfaithful but each can tell when another woman's husband has been unfaithful. One should read the full description of the problem there. The book presents a

solution to this problem but I found it somewhat hard to follow. I thought a bit about this problem and these notes are the result.

As a first step notice what happens if there is only *one* unfaithful husband say H_1. Then his wife W_1 looks around and sees no unfaithful husbands. All the other wives W_i for $i = 2, 3, \cdots, 49, 50$ look around and see only a single unfaithful husband. When the queen says that there is at least one unfaithful husband all of these wives W_i for $i = 2, 3, \cdots, 49, 50$ don't learn anything new since they already see one unfaithful husband. Wife W_1 however sees 49 faithful husbands and thus has to conclude that the cheating husband is her own. She would then kill him right then. Thus the *other* wives W_i for $i = 2, 3, \cdots, 49, 50$ will see husband H_1 die and know that their husbands have been faithful.

Now consider the case when there are two unfaithful husbands. Then the two wives W_1 and W_2 only see one unfaithful husband (just as the 49 wives in the previous case did) while the other wives W_i for $i = 3, 4, \cdots, 49, 50$ see two unfaithful husbands. When the queen makes her announcement to all of the wives this is not new information. Since the wives W_1 and W_2 each see only one unfaithful husband they can determine that their respective husband has been unfaithful in the following way. The wife W_1 would reason as follows: if my husband has been faithful then the only unfaithful husband is the one I see (i.e. H_2) and as in the previous case above I should see him killed as in the case above. When that husband is *not* killed the reason must be that the other wife sees that *my* husband has been unfaithful. Thus I should kill my husband H_1. Notice that all wives that see two unfaithful husbands will see two deaths and can conclude that their own husbands have been faithful.

If there are three unfaithful husband H_1, H_2, and H_3 then the wives W_1, W_2, and W_3 see two unfaithful husbands while the other wives W_i for $i = 4, 5, \cdots 49, 50$ see three unfaithful husbands. Each of W_1, W_2, and W_3 can now reason as follows, if my husband has been faithful then I am like one of the women with a faithful husband in the previous case and I should quickly see the two husbands I'm looking at die. When I don't see that happen it must be because my own husband has been unfaithful and should be killed. Thus the wives that see three unfaithful husbands will quickly see all three husbands killed.

This pattern continues in that with 4 unfaithful husbands most women see all 4 husbands while the others see 3 unfaithful husbands. The ones that see only three reason that if there are only three then as in the previous case we should see them killed. When there are not killed this means that there must be four unfaithful husbands and then all four is killed. The women who originally saw 4 now see them all killed.

This pattern continues until we get to the case where all husbands have been unfaithful. Now all wives see 49 unfaithful husbands but don't know about their own. Each wife reasons that if my husband was faithful the other wives would see 48 unfaithful husbands and I should see these husbands killed. When I don't see that happen it must be because my husband has been unfaithful and he should be killed. This results in all husband being killed.

Which Switch Works the Light at the End of the Hallway?

Flip any two switches and wait a long time (like five minutes). After that time turn one of the switches that was on into the off position. Walk to the end of the hall and inspect the light. Then if

- The light is on then the switch that is in the on position controls it.

- The light is off but the bulb is warm means that of the two switches initially turned into the on position the one that is now in the off position controls the light.

- The light is off and the bulb is cold means that the third switch (the one never activated) controls the light.

Notes on Burning Two Fuses

The book provides one solution to this problem but I think the following solution is different and presents a different way to solve the problem. If we light one of the fuses at both ends these two ends will meet in 30 minutes. It then remains to find a way to measure an additional 15 minutes. We can do this by taking the second fuse, bending it into two equal haves (to find the middle) and then lighting each end and the (just found) *midpoint* of the fuse. This will result in *four* flames progressing into the fuse; the two on each end and then the two that result from lighting the middle of the fuse. This means that all flames will finish burning in $\frac{60}{4} = 15$ minutes. Thus we can measure 45 minutes by lighting the first fuse at both ends, wait until the flames meet (using up 30 minutes), and then lighting the second fuse at both ends and at the middle, waiting until the fuse is burnt up (using up 15 minutes), to give a total of 45 minutes.