Pg 13 Strohbehn 04-20-03 1

$$n-1 = 77.6\left(1+\frac{7.52\cdot 10^{-3}}{\lambda^2}\right)\left(\frac{P}{T}\right)10^{-6}$$

$$dn = 77.6\left(1+\frac{7.52\cdot 10^{-3}}{\lambda^2}\right)\left[\frac{dP}{T} - \frac{P\,dT}{T^2}\right]\cdot 10^{-6}$$

$\lambda \approx 0.6\cdot 10^{-6}\,m$

$$dn = 77.6\left(1+\frac{7.52\cdot 10^{-3}}{\lambda^2}\right)\frac{P}{T}\left[\frac{dP}{P} - \frac{dT}{T}\right]\cdot 10^{-6}$$

$\lambda = .6\cdot 10^{-6}\,m$

$\lambda = .6\,\mu m$ formula corresponds to $\mu m$

$$= \frac{79.22\,P}{T}\left(\frac{dP}{P} - \frac{dT}{T}\right)\cdot 10^{-6} \quad \text{eq } 2.8 \checkmark$$

$\Theta \equiv T - \gamma_a z$ potential temperature $d\Theta = dT$ assuming $dz \approx 0$

$$\delta n = -\frac{79\,P}{T}\,\frac{d\Theta}{T}\,10^{-6} = -\frac{79\,P}{T^2}\,d\Theta\cdot 10^{-6} \quad \text{eq } 29 \checkmark$$

$$C_n^2 = \left(\frac{79\,P}{T^2}\cdot 10^{-6}\right)^2 C_\Theta^2$$

$$n_1(r) = \int dN(k)\, e^{i\vec{k}\circ\vec{r}}$$

$$B_n(\vec{r}_1 + r, \vec{r}_1) = \langle n_1(\vec{r}+\vec{r}_1)\, n_1(\vec{r}_1)\rangle$$

$$= \langle \iint dN(\vec{k})\, e^{i\vec{k}\circ(\vec{r}+\vec{r}_1)}\, dN(\vec{k}')\, e^{-i\vec{k}'\circ\vec{r}_1} \rangle$$

$$= \iint e^{i\vec{k}\circ(\vec{r}+\vec{r}_1) - i\vec{k}'\circ\vec{r}_1} \langle dN(\vec{k})\, dN^*(\vec{k}')\rangle = B_n(\vec{r})$$

By assumption of statistical homogeneity

$\therefore$

$$\langle dN(\vec{k})\, dN^*(\vec{k}')\rangle = \delta(\vec{k}-\vec{k}')\, \Phi_n(k)\, d^3k\, d^3k'$$

$\Phi_n(\vec{k})$ is the 3D spectral density of the ~~ere~~ refractive index

$\overline{\tau}$

$$B_n(\vec{r}) = \iint e^{i\vec{k}\circ(\vec{r}+\vec{r}_1) - i\vec{k}'\circ\vec{r}_1}\, \delta(\vec{k}-\vec{k}')\, \Phi_n(k)\, d^3k\, d^3k' \quad \text{do } k' \text{ integral}$$

$$= \int e^{i\vec{k}\circ(\vec{r}+\vec{r}_1) - i\vec{k}\circ\vec{r}_1}\, \Phi_n(k)\, d^3k = \int e^{i\vec{k}\circ\vec{r}}\, \Phi_n(k)\, d^3k$$

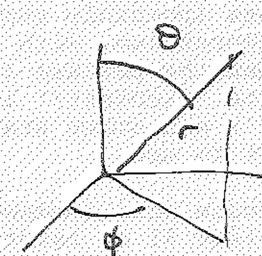
b

$$\Phi_n(\bar{k}) = \frac{1}{(2\pi)^3} \int B_n(\bar{r}) e^{-i\bar{k}\cdot\bar{r}} d^3r$$

$B_n(\bar{r}) = B_n(r)$ isotropic assumption.

$\Phi_n(\bar{k}) = \Phi_n(k)$ i.e. 3D ~~spatial~~ spectral density fn depends on magnitude of k only

Then




$$B_n(r) = \int \cancel{(\sin\theta\, r\, d\phi)(r d\theta)}\, dr$$

$$= \int d^3k\, e^{i\bar{k}\cdot\bar{r}} \Phi_n(\bar{k}) = \int (k^2 \sin\theta\, d\theta\, d\phi\, dk)\, e^{ikr}\, \Phi_n(k)$$

$$= 4\pi \int_0^\infty k^2 e^{ikr} \Phi_n(k)\, dk$$

How get

$$= \frac{4\pi}{r} \int_0^\infty dk \cdot k\, \Phi_n(k) \sin(kr)$$

?

$$\text{Sim } \Phi_n(k) = \frac{1}{2\pi^2} \int_0^\infty dr\, r\, B_n(r) \sin(kr)$$

$$D_n(r) = \langle [n_1(\bar{r}_1+\bar{r}) - n_1(\bar{r}_1)]^2 \rangle$$

$$= \langle n_1^2(\bar{r}_1+\bar{r}) - 2n_1(\bar{r}_1+\bar{r})n_1(r_1) + n_1^2(\bar{r}_1) \rangle$$

$$= 2B_n(0) - 2\langle n_1(\bar{r}_1+\bar{r})n_1(r_1)\rangle$$

$$= 2B_n(0) - 2B_n(r)$$

$$\Rightarrow B_n(0) - B_n(r) = \frac{1}{2}D_n(r)$$

eq 2.28 is

$$B_n(r) = \frac{4\pi}{r}\int_0^\infty dk\cdot k\,\Phi_n(k)\sin(kr)$$

$$B_n(0) = \lim_{r\to 0} \frac{4\pi}{r}\int_0^\infty dk\,k\,\Phi_n(k)\sin(kr) \qquad \frac{\text{"0"}}{\text{"0"}}$$

$$= \lim_{r\to 0} \frac{4\pi \int_0^\infty dk\,k^2\Phi_n(k)\cos(kr)}{1} = 4\pi\int_0^\infty dk\,k^2\,\Phi_n(k)\cancel{\cos(kr)}$$

$$\therefore \cancel{D_n(r)} = 2\left[\cancel{4\pi\int_0^\infty dk\,k^2\,\Phi(k)\cos(kr) - \frac{4\pi}{r}\int_0^\infty dk\,k\,\Phi(k)\sin(kr)}\right]$$

$$D_n(r) = 2\left[4\pi\int_0^\infty dk\, k^2 \Phi_n(k) - \frac{4\pi}{r}\int_0^\infty dk\, k \sin(kr)\right]$$

$$= 8\pi \int_0^\infty dk\, k^2 \Phi_n(k)\left[1 - \frac{\sin(kr)}{kr}\right] \qquad \text{eq 2.32} \checkmark$$

Invert the above relationship w.r.t. $D_n(r)$

$$\Phi_n(k) = \frac{1}{4\pi k^2}\int_0^\infty \frac{\sin(kr)}{kr} \frac{d}{dr}\left[r^2 \frac{d}{dr} D_n(r)\right] dr$$

$$D_n(r) = C_n^2 r^{2/3} \qquad l_0 \ll r \ll L_0$$

$$\frac{d}{dr} D_n = \frac{2}{3} C_n^2 r^{-1/3}$$

$$r^2 \frac{d}{dr} D_n = \frac{2}{3} C_n^2 r^{5/3}$$

$$\frac{d}{dr}\left[r^2 \frac{d}{dr} D_n(r)\right] = \frac{5}{3}\cdot\frac{2}{3} C_n^2 r^{2/3}$$

$$\therefore$$

$$\Phi_n(k) = \frac{1}{4\pi k^2}\int_0^\infty \frac{\sin(kr)}{kr}\left(\frac{10}{9}\right) C_n^2 r^{2/3}\, dr$$

$$= \frac{1}{4\pi k^3}\cdot\frac{10}{9}\, C_n^2 k^{-3}\int_0^\infty \sin(kr)\, r^{-1/3}\, dr$$

$$\cong \frac{5}{18\pi}\, C_n^2 k^{-3}\int_{l_0}^{L_0} dr\, \sin(kr)\, r^{-1/3}$$ eq 2.35 ✓

integrating from 0 to ∞

$$\int_0^\infty dr\, \sin(kr)\, r^{-1/3} = \dots$$ find integral ...

$v = kr$ $\quad dv = k\,dr$

$$= \int_0^\infty \sin(v)\left(\frac{v}{k}\right)^{-1/3}\left(\frac{dv}{k}\right) = \left(\frac{1}{k}\right)^{-1/3}\left(\frac{1}{k}\right)\int \dots$$

$$= \left(\frac{1}{k}\right)^{2/3}$$

$$-3-\frac{2}{3} = -\frac{9}{3}-\frac{2}{3} = -\frac{11}{3}$$ ~ eq 2.36 ✓

$$2\pi L_0^{-1} \ll k \ll 2\pi l_0^{-1}$$

$$\nabla \times \vec{E} = ik\vec{H}$$

$\nabla \times \Rightarrow$

$$\nabla \times (\nabla \times \vec{E}) = ik \nabla \times \vec{H}$$

$$\nabla(\nabla \circ \vec{E}) - \nabla^2 \vec{E} = ik \nabla \times \vec{H}$$

$$\Rightarrow -\nabla^2 \vec{E} + \nabla(\nabla \circ \vec{E}) = ik(-ikn^2 E) = k^2 n^2 E \quad \text{eq } 2.47$$

eq 2.46

$$\nabla \circ (n^2 E) = 0$$

$$\Rightarrow \nabla(n^2) \circ \vec{E} + n^2 \nabla \circ \vec{E} = 0$$

$$\Rightarrow \nabla \circ \vec{E} = \frac{-\nabla(n^2) \circ \vec{E}}{n^2} = \frac{-2n \nabla(n) \circ \vec{E}}{n^2} =$$

$$= \frac{-2\nabla(n) \circ \vec{E}}{n} \quad \text{put into eq } 2.47 \text{ + get}$$

$$-\nabla^2 \vec{E} + \nabla\left(\frac{-2\nabla(n) \circ \vec{E}}{n}\right) = k^2 n^2 E$$

$$\nabla^2 \vec{E} + k^2 n^2 E + 2\nabla\left(\frac{\nabla n}{n} \circ \vec{E}\right) = 0$$

$$\Rightarrow \nabla^2 \vec{E} + k^2 n^2 E + 2\nabla\left(\vec{E}\cdot\nabla \ln(n)\right) = 0$$ eq 2.48 ✓

- - - - -

$$\nabla^2 E + k^2 n^2 E = 0$$

$\cancel{\nabla \times}$ $\nabla \times \rightarrow$

$$\nabla^2(\nabla\times E) + \nabla\times(k^2 n^2 E) = 0$$

$$\nabla^2(\nabla\times E) + \underbrace{k^2\, \nabla\times(n^2 E)} = 0$$

$$k^2\left[\nabla(n^2)\times\vec{E} + n^2\nabla\times\vec{E}\right] =$$

$$\nabla^2 E + k^2 n^2 E = 0$$

let $E = E_0 + E_1 + \cdots$ just take 1st 2 terms

$n = 1 + n_1$

$$\nabla^2 E_0 + \nabla^2 E_1 + k^2(1 + 2n_1 + \cancel{n_1^2}^{\approx 0})(E_0 + E_1) = 0$$

$$\underline{\nabla^2 E_0} + \underline{\underline{\nabla^2 E_1}} + \underline{k^2 E_0} + \underline{\underline{k^2 E_1}} + 2k^2 \underline{\underline{n_1}} E_0 + 2k^2 \cancel{n_1 E_1}^{0} = 0$$

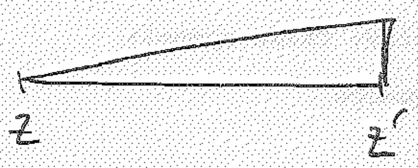
$$\Rightarrow \quad \nabla^2 E_0 + k^2 E_0 = 0 \qquad \text{eq 2.52}$$

$$\& \quad \nabla^2 E_1 + k^2 E_1 + 2k^2 n_1 E_0 = 0 \qquad \text{eq 2.53}$$

$E_0 = e^{ikz}$ th

$$\nabla^2 E_1 + k^2 E_1 = -2k^2 n_1 e^{ikz}$$

$$E_1 = \frac{1}{4\pi} \int_V d^3r' \frac{e^{ik|\bar{r}-\bar{r}'|}}{|\bar{r}-\bar{r}'|} \left[ 2k^2 n_1(\bar{r}') e^{ikz'} \right]$$




$|\rho-\rho'| \ll |z-z'|$

$$|\vec{r}-\vec{r}'| = \sqrt{(x-x')^2+(y-y')^2+(z-z')^2}$$

$$= \sqrt{(z-z')^2\left[1+\frac{(x-x')^2+(y-y')^2}{(z-z')^2}\right]}$$

$$= |z-z'|\sqrt{\left[1+\frac{(x-x')^2+(y-y')^2}{(z-z')^2}\right]}$$

$$= |z-z'|\Big[\,1$$

$$\sqrt{1+x} = \cancel{\sum_{k\geq 0}\binom{1/2}{k}x^k} \Leftarrow \sum_{k\geq 0}\binom{1/2}{k}x^k = 1+\binom{1/2}{1}x^1+\binom{1/2}{2}x^2+\cdots$$

$$= 1+(1/2)x+\frac{1/2(1/2-1)}{2}x^2+O(x^3)$$

$$\binom{p}{n} = \frac{p(p-1)(p-2)\ldots(p-n+1)}{n!}$$

$$= 1+\frac{1}{2}x-\frac{1}{8}x^2+O(x^3)$$

$$|\vec{r}-\vec{r}'| = |z-z'|\left[1+\frac{|\rho-\rho'|^2}{|z-z'|^2}\right]^{1/2}$$

$$\approx |z-z'|\left[1+\frac{1}{2}\frac{|\rho-\rho'|^2}{|z-z'|^2}-\frac{1}{8}\frac{|\rho-\rho'|^4}{|z-z'|^4}+\dots\right] \quad \text{eq 2.56 ✓}$$

$$\frac{|\rho-\rho'|^4}{|z-z'|^4} \sim \frac{(\lambda L/l_0)^4}{L^4} = \left(\frac{\lambda}{l_0}\right)^4$$

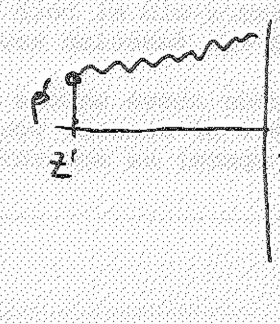
How are they estimating the magnitude of ~~the~~ each term?

$$E_1(r) = \frac{1}{4\pi}(2k^2)\int_V d^3r' \, \frac{e^{ik|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\, n_1(\vec{r}')\, e^{ikz'}$$

$$\approx \frac{k^2}{2\pi}\int_V d^3r' \exp\left\{ik(z-z')\left[1+\frac{(\rho-\rho')^2}{2(z-z')^2}\right]\right\}\frac{n_1(\vec{r}')}{(z-z')}\, e^{ikz'}$$

$$= \frac{k^2}{2\pi}e^{ikz}\int_V d^3r' \exp\left\{\frac{ik(\rho-\rho')^2}{2(z-z')}\right\}\frac{n_1(\vec{r}')}{(z-z')} \quad \text{eq 2.57 ✓}$$

Fresnel diffraction formula



$$E = E_0 + E_1$$

$$\frac{E}{E_0} = 1 + \frac{E_1}{E_0} = \frac{A}{A_0}\exp[i(S - S_0)]$$

here written $E = Ae^{iS}$ $E_0 = A_0 e^{iS_0}$

$$\ln\left(1 + \frac{E_1}{E_0}\right) = \ln\left(\frac{A}{A_0}\right) + i(S - S_0)$$

let $A = A_0 + A_1$

$$= \ln\left(1 + \frac{A_1}{A_0}\right) + i(S - S_0) \quad \text{eq } 2.59 \checkmark$$

$$\frac{E_1}{E_0} \approx \frac{A_1}{A_0} + i(S - S_0)$$

$$\frac{A_1}{A_0} = \mathrm{Re}\left[\frac{E_1}{E_0}\right] = \mathrm{Re}\left[\frac{k^2 \cancel{e^{ikz}}}{2\pi}\int_V d^3r' \exp\left\{\frac{ik(\rho - \rho')^2}{2(z - z')}\right\}\frac{n_1(\vec{r}')}{(z - z')}\right]$$

Assuming $\cancel{E_0 \approx 1}$ $E_0 = 1e^{ikz}$

$$= \cancel{\mathrm{Re}\left[\frac{k^2 e^{ikz}}{2\pi}\right]} \quad \cancel{\mathrm{Re}\left[\frac{k^2}{2\pi}\int_V d^3r'\right]}$$

$\Rightarrow$

$$\frac{A_1}{A_0} = \frac{k^2}{2\pi}\int_V d^3r \cos\left[\frac{k(\rho - \rho')^2}{2(z - z')}\right]\frac{n_1(\vec{r}')}{(z - z')}$$

$$S - S_0 = \cancel{\text{Re}} \ \text{Im}\left[\frac{k^2}{2\pi}\int_V d^3r' \exp\left[\frac{ik(\rho-\rho')^2}{2(z-z')}\right]\frac{n_1(\vec{r}')}{(z-z')}\right]$$

$$= \frac{k^2}{2\pi}\int_V d^3r' \sin\left(\frac{k(\rho-\rho')^2}{2(z-z')}\right)\frac{n_1(\vec{r}')}{(z-z')} \qquad \text{eq } 2.62 \checkmark$$

$\chi \quad n_1(\vec{r}')$

$n_1(\vec{r}') = \int d\nu(\vec{k}, z') e^{i\vec{k}\cdot\rho'}$ expanding in

$$\begin{bmatrix}\chi(r)\\ S_1(r)\end{bmatrix} = \frac{k^2}{2\pi}\int_V\int_{\vec{k}} d^3r' \left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho-\rho')^2}{2(z-z')}\right]\right\} \frac{1}{(z-z')} d\nu(\vec{k}, z') e^{i\vec{k}\cdot\vec{\rho}'}$$

$$= \frac{k^2}{2\pi}\int\int_0^z dz' \frac{d\nu(\vec{k}, z')}{(z-z')} \int d^2\rho' e^{i\vec{k}\cdot\vec{\rho}'} \left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho-\rho')^2}{2(z-z')}\right]\right\}$$

$$\int d^2\rho' e^{ik\cdot\rho'} \left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho-\rho')^2}{2(z-z')}\right]\right\}$$

let $\rho'' = \rho' - \rho \qquad d^2\rho' = d^2\rho''$

$$= \int d^2\rho'' e^{ik\cdot(\rho''+\rho)} \left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho'')^2}{2(z-z')}\right]\right\}$$

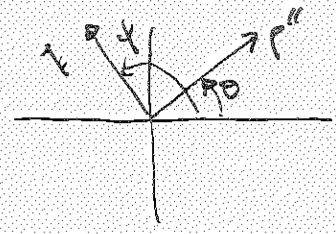
$$= e^{ik_0\rho}\int d^2\rho''\, e^{ik_0\rho''}\left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho'')^2}{2(z-z')}\right]\right\}$$

$\rho'' = (\rho'', \theta)$

$$= e^{ik_0\rho}\int_0^\infty\int_0^{2\pi}\rho''\, d\theta\, d\rho''\, e^{ik_0\rho''}\left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho'')^2}{2(z-z')}\right]\right\}$$

$$= e^{ik_0\rho}\int_0^\infty d\rho''\,\rho''\left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k(\rho'')^2}{2(z-z')}\right]\right\}\underbrace{\int_0^{2\pi} d\theta\, e^{ik\rho''\cos(\theta-\psi)}}_{2\pi J_0(k\rho'')}$$

eq 2.65




$\psi - \theta$

$$I_{1,2} = 2\pi e^{ik_0\rho}\underbrace{\int_0^\infty d\rho''\,\rho''\, J_0(k\rho'')\left\{\begin{matrix}\cos\\ \sin\end{matrix}\left[\frac{k\rho''^2}{2(z-z')}\right]\right\}}_{\text{Henkel transform}}$$

$$= \frac{2\pi(z-z')}{k}\, e^{ik_0\rho}\left\{\begin{matrix}\sin\\ \cos\end{matrix}\left[\frac{k^2(z-z')}{2k}\right]\right\}$$

$$\begin{bmatrix} \mathcal{T}(r) \\ S_1(r) \end{bmatrix} = \Big\{ \frac{k^2}{4\pi} \int \int_0^z dz' \frac{d\nu(k, z')}{(z-z')} \frac{\cancel{2\pi}}{\cancel{k}} e^{i\underline{k}\circ\underline{\rho}} \left\{ \cancel{\infty} \begin{matrix} \cos \\ \sin \end{matrix} \left[ \frac{\underline{k}^2 (z-z')}{2k} \right] \right\} \Big\}$$

$$= k \int \int_0^z dz' \frac{d\nu(\underline{k}, z')}{(z-z')} e^{i\underline{k}\circ\underline{\rho}}$$

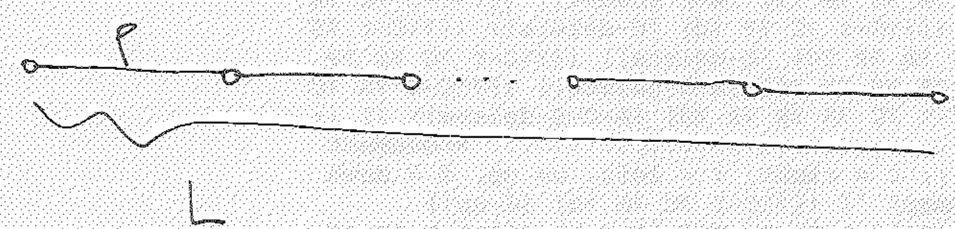
$$= \int e^{i\underline{k}\circ\underline{\rho}} \left( k \int_0^z dz' \, d\nu(\underline{k}, z') \left\{ \begin{matrix} \cos \\ \sin \end{matrix} \left[ \frac{\underline{k}^2 (z-z')}{2k} \right] \right\} \right)$$

$\Delta S_1 \sim k\rho\,\Delta n(\rho)$

$\langle \Delta S_1^2(\rho)\rangle \sim k^2\rho^2 \langle \Delta n^2(\rho)\rangle$

w/ $\langle \Delta n^2(\rho)\rangle \cong C_n^2 \rho^{2/3}$

$\Rightarrow \langle \Delta S_1^2(\rho)\rangle \sim k^2\rho^2 C_n^2 \rho^{2/3} = k^2 C_n^2 \rho^{8/3}$



$N = \frac{L}{\rho}$

$\langle \Delta S^2(\rho)\rangle \sim N \langle \Delta S_1^2(\rho)\rangle \sim N\cdot k^2 C_n^2 \rho^{8/3} = L k^2 C_n^2 \rho^{5/3}$ q 3.80 ✓

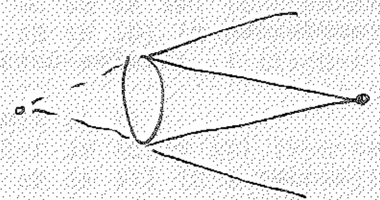
$\langle \Delta S^2(\rho)\rangle \sim N \langle \Delta S_1^2(\rho)\rangle \sim N k^2 \rho^2 \langle \Delta n^2(\rho)\rangle$

$\sim \frac{L}{l_0} k^2 \rho^2 C_n^2 l_0^{2/3} (\rho/l_0)^2 \qquad \rho \ll l_0$

$= L k^2 C_n^2 (l_0^{-1} l_0^{2/3} l_0^{-2})\,\rho^4 = L k^2 C_n^2 l_0^{7/3} \rho^4 \qquad \rho \ll l_0$

$-3 + \frac{2}{3} = \frac{7}{3}$

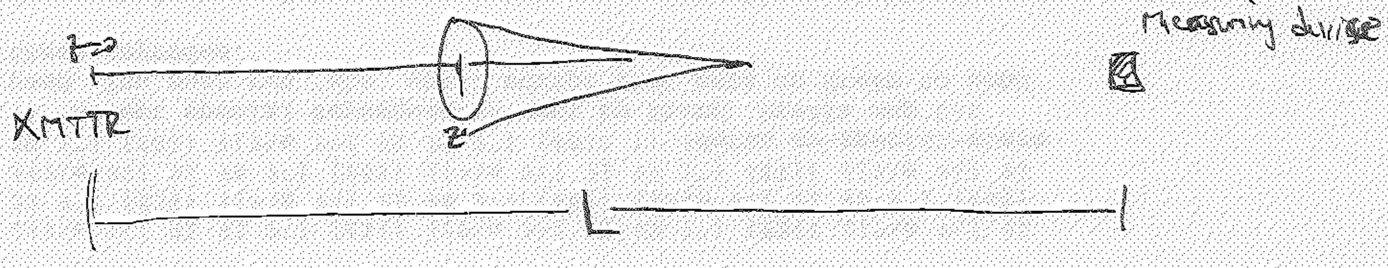
$$F \approx \frac{l}{\Delta n}$$



$$l \sim l_0 - L_0 \sim 10^{-3} m - 1 m$$

$$\Delta n \sim 10^{-6} - 10^{-8}$$

$$F \sim 10^3 m - 10^8 m.$$




$$A_0^2 (l/2)^2 = A_r^2 z^2$$

$$\delta A = \left( \frac{l}{2r} - 1 \right) A_0 =$$

$$\frac{l}{2r} = \frac{F}{F-L+z}$$

$l_c = \frac{l^2}{\lambda}$ distance geometrical optics is valid.

$\theta = \frac{\lambda}{l} =$ diffraction angle

$$\frac{\delta A}{A} = \frac{L-z}{F-L+z} A_0$$

$$\frac{\delta A}{A_0} = \frac{L-z}{\quad}$$

$$\langle \Delta n^2(\ell) \rangle \sim C_n^2 \ell_1^{2/3}$$

Assume $\Delta n \sim C_n \ell_1^{1/3}$

$$F = \frac{\ell}{(C_n \ell_1^{1/3})}$$

From $$\frac{\delta A}{A_0} = \frac{C_n \ell_1^{1/3}}{\ell}(L-z)$$

$$\frac{\delta A}{A_0} \sim C_n \ell_0^{-2/3}(L-z)$$

$$\frac{\delta A}{A_0} \sim$$

$$\overline{(L-z)^2} = \frac{1}{L}\int_0^L (L-z)^2 dz = \frac{1}{L}\int_0^L (z-L)^2 dz = \frac{1}{L}\frac{(z-L)^3}{3}\Big|_0^L$$

$v = L - z$

$dv = -dz$

$$= \frac{1}{L}\left(+\frac{L^3}{3}\right) = \frac{L^2}{3}.$$

$$\left\langle \left(\frac{\delta A}{A_0}\right)^2 \right\rangle \sim C_n^2 L^3 \ell_0^{-7/3}$$ eq 3.89.

$$\ell_c \sim \sqrt{\lambda L}$$

$$\left\langle \left(\frac{\delta A}{A_0}\right)^2 \right\rangle \sim C_n^2 L^3 \cdot (\lambda L)^{-7/6}$$

$$= C_n^2 L^3 \cdot L^{-7/6} k^{7/6}$$

$$= C_n^2 L^{11/6} k^{7/6} \qquad l_0 < \sqrt{\lambda L} \qquad \text{eq } 3.91$$

$$\left|\frac{\delta A}{A_0}\right|_l \sim \frac{(L-z)^2}{F^2} \quad l^2$$

---

Fig 78

$$\langle \Delta S^2(\rho) \rangle \sim k^2 C_n^2 L \rho_0^{5/3} \qquad l_0 \ll \rho \ll L_0$$

$$\parallel$$

$$\pi^2$$

$$\Rightarrow \rho_0 = \left(\frac{\pi^2}{k^2 C_n^2 L}\right)^{3/5} \qquad \text{eq } 3.98 \quad \checkmark$$

3.96b

$$\left|\frac{\delta A}{A_0}\right|_l^2 \sim C_n^2 k^2 L l^{5/3}$$

$$\left.\begin{matrix} W_\chi(\omega) \\ W_S(\omega) \end{matrix}\right\} = \frac{8\pi^2 k^2}{V}(.033 C_n^2)\int_0^L d\eta \, \mathrm{Re}(G_1 \mp G_2)$$

$$G_1 = \frac{\sqrt{\pi}}{2} C^{-8/3} \Gamma\left(\frac{1}{2},\right.$$

$$W_\chi \to .8506 \frac{C_n^2}{V} k^{2/3} L^{7/3}$$

V = cross wind velocity.

$$\frac{W_b^2}{W_0^2} = \exp\left(-2\langle \chi_t \rangle - 2\langle(\chi_t - \langle \chi_t \rangle)^2\rangle\right)\Big|_{\rho=0}$$

$$\frac{W_\chi(k_2)}{W_\chi(k_1)} = \left(\frac{k_2}{k_1}\right)^{2/3} \qquad f \to 0$$

$$\frac{W_\chi(k_2)}{W_\chi(k_1)} = \left(\frac{k_2}{k_1}\right)^{2} \qquad f \to \infty$$

---

$$W_\chi(\omega, k_1, k_2) = \left(\frac{8\pi^2 k_1 k_2}{V}\right)(0.033 C_n^2) k_1^{-4/3} L^{7/3} I_\chi$$

$$\Phi_n(k,\eta) = 0.033\, C_n^2(\eta)\, k^{-11/3}$$

$$B_\chi(\rho,\tau) = 8\pi^2 k^2 \int_0^L d\eta \int_0^\infty k\, dk\, J_0\left(k\left|\frac{\rho\eta}{L} - v\tau\right|\right) \sin^2\left[\frac{\eta(L-\eta)}{2kL} k^2\right] \Phi_n(k,\eta)$$

$$l_0 \ll \sqrt{\lambda L} \ll L$$

$$\left.\frac{\partial B_\chi}{\partial \tau}\right|_{\tau=0} =$$

$$B_\chi(\rho,\tau) = \int_0^L d\eta\, k(\rho,\tau,\eta)\, C_n^2(\eta)$$

$$k(\rho,\tau,\eta) =$$

$$B_\chi(\rho_i,\tau_i) = \sum_{j=1}^{N} a(\rho_i,\tau_i,\eta_j)\, C_n^2(\eta_j)$$

$$a(\rho_i,\tau_i,\eta_j) = W_j\, k(\rho_i,\tau_i,\eta_j)$$

$$g = Af \qquad g_i = B_\chi(\rho_i,\tau_i) \qquad a_{ij} = W_j\, k(\rho_i,\tau_i,\eta_j)$$

$$|g - Af|^2 = \min.$$

$$f = (A^+ A)^{-1} A^+ g.$$

$\delta f = (A^tA)^{-1}$

$f = f + \delta f$ , $f = (A^tA)^{-1}A^tg$

$g = g + \delta g$

$\rightarrow \delta f = (A^tA)^{-1}A^t\delta g$

$\Delta_f = \frac{\|\delta f\|}{\|f\|}$ $\|f\| = \max_i |f_i|$

$$\Delta_f = \frac{\|\delta f\|}{\|f\|} \le \frac{\|(A^tA)^{-1}\|\cdot\|A^t\|\|\delta g\|}{\|f\|} \overset{?}{=} \|(A^tA)^{-1}\|\|A^t\|\frac{\|\delta g\|}{\|g\|}$$

how get?

$\|f\| \le \|(A^tA)^{-1}\|\cdot\|A^t\|\|g\|$

$\Delta_{f_{max}} = \|(A^tA)^{-1}\|\|A^t\|\Delta_g$ $\|A\|_\infty = \max_i \sum_j |a_{ij}|$

---

$g = Af$

$g_d = g + n$ $g_d$ measured value + known $M \times 1$ matrix

$g = g_d - n$

$g_d - n = Af$

$g_d = Af + n$

$\langle g_d \rangle = 0$ $\langle f \rangle = 0$ $\langle n \rangle = 0$

$$\langle |(f - Bg_d)^{\dagger} a_1| \rangle^2 = \min$$

$$B = (R_{ff} A^{\dagger} + R_{fn})(A R_{ff} A^{\dagger} + R_{fn}^{\dagger} A^{\dagger} + A R_{fn} + R_{nn})^{-1}$$

$$R_{ff} = \langle f f^{\dagger} \rangle \qquad R_{fn} = 0$$

$$\Rightarrow \quad B = (R_{ff} A^{\dagger})(A R_{ff} A^{\dagger} + R_{nn})^{-1}$$

$$f_d = B g_d \qquad \Delta_{f\max} = \|A\| \cdot \|B\|$$

$0 \le x \le L$

$$\cancel{f(x) = \bar{f} + \sum_{k=0}^{k} c \cos\left(\frac{k\pi x}{L}\right)}$$

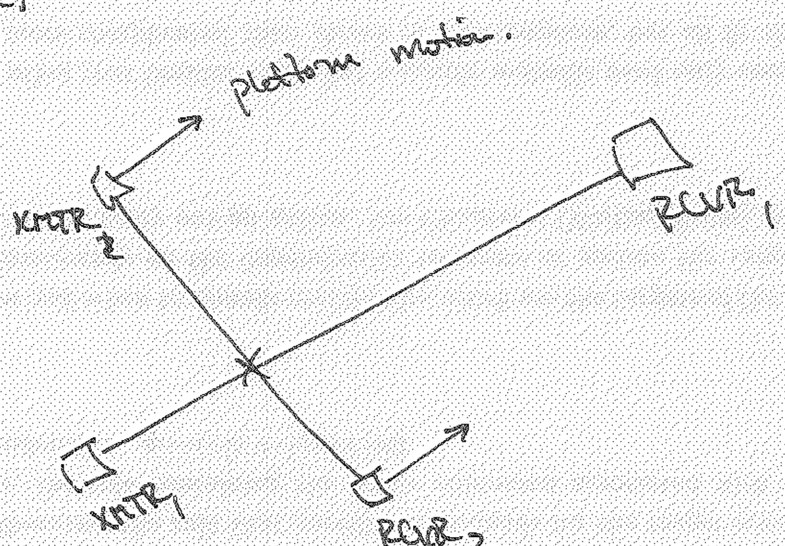
$\cos(nx) \qquad 0 \le x \le 2\pi$

$$\cancel{R_{ff}(r_{ij}) \quad r_{ij} =}$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi}{L} x\right) + b_n \sin\left(\frac{n\pi}{L} x\right)\right) \qquad g(x) = f\left(\frac{P}{\pi} x\right)$$

2L periodic fn $f(x)$

$g' = A'$




measuring the cross correlation of RCVR$_1$ + RCVR$_2$ would tell us about the atmosphere at X.

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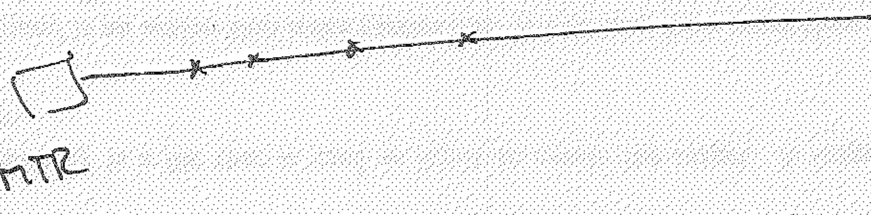
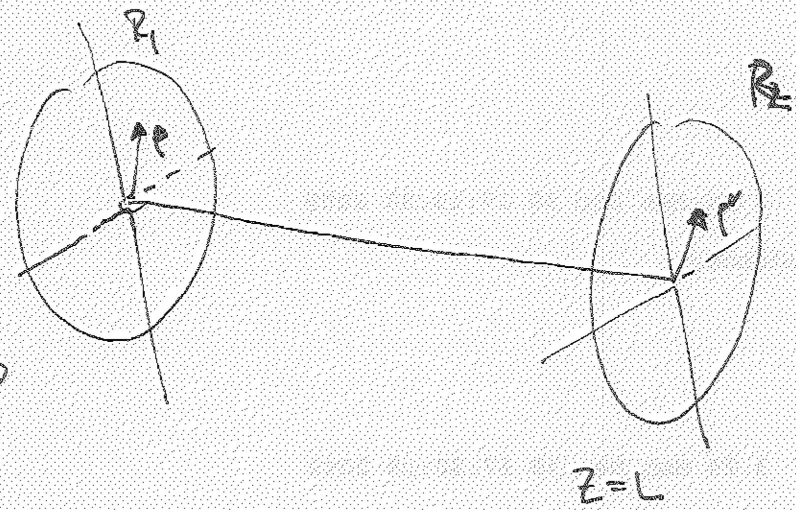
Spacially filtering technique very interesting, see at end of this chart




By changing the spacial filtering placed on the transmitter one can "probe" all locations along the beam line.

$h_{21}(\rho', \rho, t)$




$$h_{21}(\rho', \rho, t) = \int d\tau \, G(\rho', \rho, t; \tau) \exp(2\pi i f_0 \tau)$$

$V_i(\rho', t)$

$$V_0(\rho, t) = \int_{R_2} d\rho' \, V_i(\rho', t - L/c) \, h_{12}(\rho, \rho', t)$$

Coherence time for atmos turbulence $\sim 10^{-3}$ s. $= 1$ ms

$\varepsilon \rightarrow h_{21} \qquad \varepsilon(\bar{r}, t, f_0) \qquad h_{21}(\rho', \rho, t)$

$$\varepsilon(\bar{r}) = \varepsilon_0 (1 + n_1(\bar{r}))^2$$

$h_{21}(\rho', \rho)$

$$\nabla^2 U(\bar{r}) + k^2 (1 + n_1(\bar{r}))^2 U(\bar{r}) = 0 \qquad k = \frac{2\pi f_0}{c}$$

$$\langle n_1(\bar{r}_1) n_1(\bar{r}_2) \rangle = C_n^2[(z_1 + z_2)/2] \cdot \int d\upsilon \, S_n(|\upsilon|) \exp[i\upsilon \cdot (\bar{r}_1 - \bar{r}_2)]$$

$$h_2(\rho',\rho) = \{\exp[ik(L + |\rho'-\rho|^2/2L)]/i\lambda L\}$$
$$\cdot \exp[\chi(\rho,\rho') + i\phi(\rho,\rho')]$$

$$m_\chi = \langle \chi(\rho,\rho') \rangle$$
$$m_\phi = \langle \phi(\rho,\rho') \rangle$$

$m_\chi = -C_\chi(0,0)$ $m_\phi = 0$ Assume.

$$C_\chi(\rho',\rho) = 4\pi^2 k^2 \int_0^L dz \int_0^\infty du\, u\, C_n^2(z)\, \Phi_n(u)\, J_0(du) \sin^2\left[\frac{u^2 z(L-z)}{2kL}\right]$$

$$C_\phi(\rho',\rho) = 4\pi^2 k^2 \int_0^L dz \int_0^\infty$$

$$C_{\chi,\phi}(\rho',\rho) = \int_0^L dz \int_0^\infty du\, u\, C_n^2(z)\, \Phi_n(u)\, J_0(du) \ldots$$

$$D_\chi(\rho',\rho) = 2[C_\chi(0,0) - C_\chi(\rho',\rho)]$$

$$D_\phi(\rho',\rho) = 2[C_\phi(0,0) - C_\phi(\rho',\rho)]$$

$$D_{\chi,\phi}(\rho',\rho) =$$

$\rho = 0$

$\langle v_0(\rho') \rangle \approx 0$

$\langle v_0(\rho_1') v_0^*(\rho_2') \rangle$

w/ $v_0(\rho', t) = \int_{R_1} d\rho \, v_i(\rho, t - \frac{L}{c}) \, h_{21}(\rho', \rho, t)$

$v_0(\rho_1') v_0^*(\rho_2') = \int_{R_1} \int_{R_1} d\rho_1 \, d\rho_2 \, v_i(\rho_1) v_i^*(\rho_2) \langle h_{21}(\rho_1$

$$n-1 = kp$$
$$n_0 - 1 = kp_0$$

$$\frac{\delta(n-1)}{n_0-1} = \frac{\delta(kp)}{kp_0} = \frac{\delta p}{p_0} \equiv s \qquad \text{eq 7.4.}$$

$$\rightarrow \quad \delta(n-1) = s(n_0-1)$$
$$\delta n = s(n_0-1) \qquad \text{eq 7.5.}$$

$$pV = \mu RT \qquad \beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\Big|_p\right)$$

$$V = \frac{\mu RT}{p}$$

$$\frac{\partial V}{\partial T}\Big|_p = \frac{\mu R}{p} \qquad \beta = \frac{1}{V}\left(\frac{\mu R}{p}\right) = \frac{1}{T} \quad \text{eq 7.7} \ \checkmark$$

$$c_p - c_v = R \quad \rightarrow \quad c_v = c_p - R$$

$$c_p = c_v + R$$

$$\gamma \equiv \frac{c_p}{c_v} = 1 + \frac{R}{c_v}$$

$$\gamma - 1 = \frac{R}{c_p - R} = \frac{1}{\frac{c_p}{R} - 1} \quad \rightarrow \quad \frac{c_p}{R} - 1 = \frac{1}{\gamma - 1}$$

$$c_p = R\left[1 + \frac{1}{\gamma-1}\right] = \frac{\gamma}{\gamma-1}R \qquad \text{eq 7.8}$$

$$\frac{\delta\rho}{\rho} = \frac{-v^{-2}\delta v}{v^{-1}} \qquad \rho \equiv v^{-1}$$

$$\delta\rho = -v^{-2}\delta v$$

$$= -\left.\frac{\delta v}{v}\right|_p = -\left.\frac{\delta v}{\delta T}\right|_p \cdot \frac{\delta T}{v} = -v\cdot\beta\,\frac{\delta T}{v} = -\beta\,\frac{dQ}{c_p}$$

$$\{dQ = c_p\,dT\} \qquad = \cancel{\frac{\beta}{c_p}} = -\frac{1}{T}\cdot\left(\frac{\gamma-1}{\gamma R}\right) dQ \qquad \text{eq } 7.9 \checkmark$$

$$\frac{\delta\rho}{\rho} \equiv S \quad \text{\&} \quad dQ = \frac{RT}{p_0}\, q_v$$

$$q_v = \alpha I t_0$$

$$S = -\frac{(\gamma-1)}{\gamma R T}\cdot\frac{RT}{p_0}\cdot\alpha I t_0 = -\frac{(\gamma-1)}{\gamma p_0}\,\alpha I(\bar r)\,t_0 \qquad \text{eq } 7.12$$

$$\delta N = \cancel{\phantom{xx}}\ \frac{L}{\lambda}\,\delta n = \frac{L}{\lambda}(n_0-1)S(r)$$

$$= -\frac{(n_0-1)L}{\lambda}\frac{(\gamma-1)}{\gamma p_0}\,\alpha I(\bar r)\,t_0 = -(n_0-1)\frac{L}{\lambda}\frac{(\gamma-1)}{\gamma p_0}\,\alpha I_0\,\frac{a}{v} \qquad \text{eq } 7.14 \checkmark$$

$$L \simeq \frac{4a^2}{\lambda} \qquad P \simeq \pi a^2 I_0$$

$$-1 = -(n_0 - 1)\,\frac{4\cancel{a^2}}{\lambda^2}\,\frac{(r-1)}{r p_0}\,\alpha\,\frac{P}{\pi \cancel{a^2}}\,\frac{a}{v}$$

$$1 = (n_0-1)\,\frac{4}{\lambda^2}\,\frac{(r-1)}{r p_0}\,\alpha\,\frac{P}{\pi}\,\frac{a}{v}$$

$$P \cong \frac{\lambda^2 r p_0 \pi v}{4(n_0-1)(r-1)\alpha a} = \frac{\pi}{4}\left(\frac{r p_0}{r-1}\right)\frac{1}{\alpha(n_0-1)}\,\frac{\lambda^2 v}{a} \qquad \text{eq } 7.17 \checkmark$$

$a = 10\,\mu m \qquad p_0 = 1\,atm = 10^5\ N/m^2 \qquad n_0 - 1 = 3\cdot10^{-4}$

$r = 1.4 \qquad \lambda = ? \qquad$ How get $\lambda$?

$$P_t = \frac{\pi}{4}\,\frac{(1.4)(10^5\ N/m)}{(.4)\,\alpha\,(3\cdot10^{-4})}\,\frac{(10^{-10}\ m^2)}{a}\,v = .09\,\frac{v}{\alpha a} \cong .1\,\frac{v}{\alpha a}$$

$\alpha = 3\cdot10^{-3}\ m^{-1}$

$$P_t = .1 \frac{v}{\alpha a}$$

$$= \frac{.1\ (.3\ m/s)}{(3\cdot 10^{-3}\ 1/m)(10^{-2}\ m)} = \frac{.03\ m/s}{3\cdot 10^{-5}} = \frac{3\cdot 10^{-2}}{3\cdot 10^{-5}}\ [\ ]$$

$$= 10^{3}\ W$$

$$\frac{\delta T}{T} = -\frac{\delta \rho}{\rho} = -s$$

$$PV = \mu RT$$

$$P = \mu \rho RT$$

$$\rho = \frac{P}{\mu RT}$$

$$\cancel{\delta\rho\Big|_p = -\frac{P}{\mu RT^2}\,\delta T\Big|_p}$$

$$\cancel{\frac{\delta T}{T}\Big|_p =}$$

$$T = \frac{P}{\mu R \rho}$$

$$\delta T\Big|_p = \frac{-P}{\mu R \rho^2}\,\delta\rho$$

$$\therefore \frac{\delta T}{T}\Big|_p = \frac{-(P/\mu R\rho^2)\,\delta\rho}{(P/\mu R\rho)} = -\frac{\delta\rho}{\rho} \equiv -s \qquad \text{eq 7.20} \checkmark$$

$$\delta = -\frac{(\gamma-1)}{\gamma p_0}\alpha I_0 \frac{a}{v}$$

$$= -\frac{(.4)}{1.4(10^5\,N/m^2)}\cdot(3\cdot10^{-3}\,m^{-1})\cdot\left(\frac{P_0}{\pi a^2}\right)\frac{10^{-2}\,m}{(1/3\,m/s)}$$

$$= \frac{-(.4)(3\cdot10^{-3})(10^3)(10^{-2})}{1.4(10^5)\pi(10^{-4})(1/3)}\left(\frac{\cancelto{1}{W}}{\cancel{N\cdot m/s}}\right)$$

$$= -8.1\cdot10^{-4}$$

$$\delta T = -T\delta \cong (273\,k)(8.1\cdot10^{-4}) = .2235\,k \quad \sim\checkmark$$

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$$\nabla \times E = -\frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$$

$$\bar{D} = \cancel{\varepsilon H}\ \varepsilon \bar{E}\ , \quad \bar{B} = \mu \bar{H}$$

$$\nabla \times (\nabla \times E) = -\frac{\partial}{\partial t}(\nabla \times B)$$

$$\Rightarrow \nabla(\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}(\nabla \times \mu H) = -\mu \frac{\partial}{\partial t}(\nabla \times H)$$

$$= -\mu \frac{\partial^2}{\partial t^2} \bar{D}$$

$$= \cancel{-\mu\varepsilon \frac{\partial^2}{\partial t^2} E} = \cancel{\mu\varepsilon \frac{\partial^2}{\partial t}}$$

$$= -\mu \frac{\partial^2}{\partial t^2}(\varepsilon \bar{E})$$

$$\Rightarrow \nabla(\nabla \cdot E) = \nabla^2 E - \mu \frac{\partial^2}{\partial t^2}(\varepsilon E) \qquad \text{eq } 7.24$$

$\nabla \cdot D = 0$ in absence of free charges:

$$\nabla \cdot (\varepsilon E) = \nabla \varepsilon \cdot E + \varepsilon \nabla \cdot E = 0 \rightarrow \nabla \cdot E = -\frac{1}{\varepsilon} \nabla \varepsilon \cdot E$$

$$\Rightarrow \nabla^2 E - \mu \frac{\partial^2}{\partial t^2}(\varepsilon E) = \nabla\left(\frac{1}{\varepsilon} \nabla \varepsilon \cdot E\right)$$

$$= \nabla(E \cdot \nabla(\log \varepsilon)) \qquad \text{eq } 7.26$$

$$\varepsilon(\bar{r},t) = \varepsilon_0 n^2(\bar{r},t)$$

$$c^2 = \frac{1}{\mu \varepsilon_0}$$

eq 7.26 becomes:

$$\nabla^2 E - \mu \frac{\partial^2}{\partial t^2}(\varepsilon_0 n^2 E) = -\nabla\left[E \circ \nabla(\log(\varepsilon_0 n^2))\right]$$

$$\Rightarrow \nabla^2 E - \mu \varepsilon_0 \frac{\partial^2}{\partial t^2}(n^2 E) = -\nabla\left[E \circ \nabla(\log(\varepsilon_0 n^2))\right]$$

$$\Rightarrow \nabla^2 E - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(n^2 E) = \underbrace{-\nabla\left[E \circ \nabla(\log(\varepsilon_0 n^2))\right]}$$

$$= -\nabla\left[E \circ \nabla\left(\cancelto{0}{\log \varepsilon_0} + 2\log n\right)\right]$$

$$= -\nabla\left[E \circ \nabla(2\log n)\right]$$

$$= -2\nabla\left[E \circ \nabla(\log n)\right] \qquad \text{eq 7.29}$$

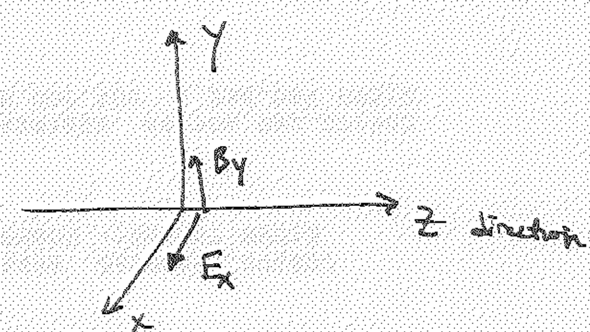
variations in $n$ cannot occur faster than an acoustic transit time for the beam profile. For a 1cm diameter

$$t \sim 3 \cdot 10^{-5} s$$

$t \sim$ electric field variations $\sim 10^{-14} - 10^{-16}$ s

$n \sim$ changes $O$(acoustic time scales) $\sim 10^{-5}$ s

$$\therefore \quad t_n \gg t_E$$




$$-2\nabla\left[E \cdot \nabla(\log n)\right]$$

$$\nabla^2 E - (n^2/c^2)\frac{\partial^2 \bar{E}}{\partial t^2} = 0$$

$E(x,y,z,t)$

$$E_x(x,y,z,t) = \psi(x,y,z)\exp(ikz - i\omega t) \qquad k \sim n$$

$$\nabla^2 E_x - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}E_x = 0$$

$$\nabla^2 = \nabla_\perp^2 + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 E = \left(\nabla_\perp^2 + \frac{\partial}{\partial z^2}\right)\psi(x,y,z)\exp(ikz - i\omega t)$$

$$= \exp(ikz - i\omega t)\,\nabla_\perp^2 \psi + \frac{\partial^2}{\partial z^2}\left(\psi(x,y,z)\,e^{ikz - i\omega t}\right)$$

$$= e^{ikz - i\omega t}\,\nabla_\perp^2 \psi + \frac{\partial}{\partial z}\left(\psi_z e^{ikz - i\omega t} + \psi(ik)e^{ikz - i\omega t}\right)$$

$$= e^{ikz - i\omega t}\,\nabla_\perp^2 \psi + \psi_{zz} e^{ikz - i\omega t} +$$

$$+ \psi_z(ik)e^{ikz - i\omega t} + \psi_z(ik)e^{ikz - i\omega t}$$

$$+ \psi(ik)^2 e^{ikz - i\omega t}$$

$$= e^{ikz - i\omega t}\left[\nabla_\perp^2 \psi + \psi_{zz} + 2ik\psi_z + \psi(ik)^2\right]$$

$$=$$

$$\frac{\partial^2 E}{\partial t^2} = \frac{\partial^2}{\partial t^2}\left[\psi e^{ikz - i\omega t}\right] = e^{ikz}\frac{\partial^2}{\partial t^2}\left[\psi e^{-i\omega t}\right]$$

$$= e^{ikz}\frac{\partial}{\partial t}\left[\psi_t e^{-i\omega t} - i\omega\psi e^{-i\omega t}\right]$$

$$\Rightarrow \quad \frac{\partial^2 E}{\partial t^2} = e^{ikz}\Big[ \psi_{tt} e^{-i\omega t} - i\omega \psi_t e^{-i\omega t}$$

$$- i\omega \psi_t e^{-i\omega t} + (-i\omega)^2 \psi e^{-i\omega t} \Big]$$

$$= e^{ikz - i\omega t}\Big[ \psi_{tt} - 2i\omega \psi_t + (-i\omega)^2 \psi \Big]$$

$\therefore$

$$\nabla^2 E - \left(\frac{n^2}{c^2}\right)\frac{\partial^2}{\partial t^2} E = 0$$

$$\Rightarrow e^{ikz - i\omega t}\Big[ \nabla_\perp^2 \psi + \psi_{zz} + 2ik\psi_z + \psi (ik)^2 \Big]$$

$$- \left(\frac{n}{c}\right)^2 e^{ikz - i\omega t}\Big[ \cancelto{0}{\psi_{tt}} - 2i\omega \cancelto{0}{\psi_t} + (-i\omega)^2 \psi \Big] = 0$$

$$\Rightarrow \nabla_\perp^2 \psi + \psi_{zz} + 2ik\psi_z - k^2 \psi - \left(\frac{n}{c}\right)^2 (-i\omega)^2 \psi = 0$$

$$\Rightarrow \nabla_\perp^2 \psi + \cancelto{0}{\psi_{zz}} + 2ik\psi_z - k^2 \psi + \left(\frac{n\omega}{c}\right)^2 \psi = 0$$

$$\nabla_T^2 \psi + 2ik\frac{\partial\psi}{\partial z} - k^2\psi + \frac{n^2\omega^2}{c^2}\psi = 0$$

$$\omega = \frac{kc}{n_0} \qquad k = \underline{2\pi n_0}$$

$$\left\{ v \equiv \frac{c}{n_0} \right\} \qquad \cancel{\omega k = v = \frac{c}{n_0}} \qquad \frac{\omega}{k} = v = \frac{c}{n_0}$$

$$\omega = \frac{kc}{n_0}$$

$$k = \frac{2\pi}{\lambda_{vac}} \cdot n_0$$

$\Rightarrow$

$$\nabla_T^2 \psi + 2ik\frac{\partial\psi}{\partial z} - k^2\psi + \frac{n^2}{\cancel{c^2}} \cdot \frac{k^2 \cancel{c^2}}{n_0^2}\psi = 0$$

$$\Rightarrow \nabla_T^2 \psi + 2ik\frac{\partial\psi}{\partial z} - k^2\psi + \frac{n^2 k^2}{n_0^2}\psi = 0$$

$$\nabla_T^2 \psi + 2ik\frac{\partial\psi}{\partial z} + \underbrace{k^2\left[\frac{n^2}{n_0^2} - 1\right]\psi}$$ $= 0$

$$k^2\left[\frac{n^2(x,y,z,t)}{n_0^2} - 1\right]\psi = 0$$

7.34 ✓

eq ~~7.33~~ ✓

Set $n(x,y,z) \equiv n_0$

$$\Rightarrow \underbrace{\nabla_T^2}\psi + 2ik\frac{\partial\psi}{\partial z} = 0 \qquad \psi = \psi(r,z)$$

$$\Rightarrow \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + 2ik\frac{\partial\psi}{\partial z} = 0 \qquad \text{eq } 7.35$$

$r J_0(\lambda r) \Rightarrow$ multiply & integrate from $r = 0,\infty$

$$= -\lambda^2 F(\lambda) + 2ik\underbrace{\int_0^\infty r\cdot\frac{\partial\psi}{\partial z}\cdot J_0(\lambda r)\,dr}_{2ik\frac{\partial}{\partial z}F(\lambda)} = 0$$

$$= -\lambda^2\bar{\psi}(\lambda,z) + 2ik\frac{\partial\bar{\psi}}{\partial z}(\lambda,z) = 0 \qquad \text{eq } 7.36.$$

$$\Rightarrow \frac{d\psi}{dz} = \frac{\lambda^2}{2ik}\psi = -\frac{\lambda^2 i}{2k}\psi$$

$$\psi(\lambda,z) = \psi(\lambda)e^{-\frac{i\lambda^2}{2k}z} \qquad \text{eq } 7.37.$$

$\psi(\lambda)$ det. by I.C.

At $z=0$ pick

$$\psi(r,0) = \psi_0\exp\left(-\frac{r^2}{r_0^2} - \frac{ikr^2}{2f}\right) \qquad \text{How get?}$$

w/ $f_0$ take there

$$f_0 \int_0^\infty r \exp\left(\frac{-r^2}{r_0^2} - \frac{ikr^2}{2f}\right) J_0(sr)\, dr$$

How?

$$= f_0 \frac{r_0^2 f}{(2f + ikr_0^2)} \exp\left[\frac{-s^2 f r_0^2}{2(2f + ikr_0^2)}\right]$$

$\therefore$

$$f(s,z) = \frac{f_0 r_0^2 f}{(2f + ikr_0^2)} \exp\left[\frac{-s^2 f r_0^2}{2(2f + ikr_0^2)} - \cancel{\frac{is^2}{2f}}\ \frac{is^2 z}{2k}\right]$$

$$= \frac{f_0 r_0^2 f}{(2f + ikr_0^2)} \exp\left[-\left(\frac{f r_0^2}{2(2f + ikr_0^2)} + \frac{iz}{2k}\right) s^2\right]$$

inverse Henkel transform How?

$$f(r,z) = \frac{f_0}{\left((1 - z/f) + i\frac{2z}{kr_0^2}\right)} \exp\left[\frac{-r^2}{w^2(z)} - \frac{ikr^2}{2R(z)}\right]$$

$$w^2(z) = r_0^2 (1 - z/f)^2 + 4z^2/(k^2 r_0^2)$$

$$\frac{dw^2}{dz} = r_0^2 (2)(1 - z/f)(-1/f) + \frac{8z}{k^2 r_0^2} \underset{set}{=} 0$$

$$= \frac{4z}{k^2 r_0^2} - \frac{r_0^2}{f}(1 - z/f) = 0$$

$$\left(\frac{4}{k^2 r_0^2} + \frac{r_0^2}{f^2}\right) z = \frac{r_0^2}{f}$$

$$z_w = \frac{r_0^2/f}{\left(\frac{4}{k^2 r_0^2} + \frac{r_0^2}{f^2}\right)} \cdot \frac{k^2 r_0^2 f^2}{k^2 r_0^2 f^2} = \frac{r_0^4 k^2 f}{4f^2 + k^2 r_0^4}$$ eq 7.42 ✓

$$R(z) = \frac{f - 2z + z^2/z_w}{1 - z/z_w}$$

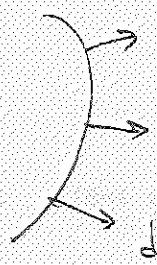
$$I(r,z) = |t(r,z)|^2$$

$$|t(r,z)|^2 = \frac{|t_0|^2}{\left[(1 - z/f)^2 + 4z^2/k^2 r_0^4\right]} \exp\left[-\frac{2r^2}{w^2(r)}\right]$$

$$= \frac{|t_0|^2 r_0^2}{w^2(z)} \exp\left[-\frac{2r^2}{w^2(r)}\right]$$ eq 7.45 ✓

$$I(r,z) = I_0 \exp\left(-\frac{r^2}{a^2}\right) \qquad a \equiv \frac{w(r)}{\sqrt{2}}$$

geometrical optics




$$\vec{E}(\vec{r},t) = e(r)\exp\left[ik_0 S(\vec{r}) - i\omega t\right]$$ put into Maxwells eqs

$$\left[\nabla S(r)\right]^2 = n^2(r)$$

$S(r)$ = constant on surfaces of constant phase.

$$\hat{t} = \frac{d\vec{r}}{ds}$$

so see $n\hat{t} = \nabla S$

$$\frac{d}{ds}\left(n\frac{dr}{ds}\right) = \nabla n(r) \quad w/\ \hat{t} = \frac{dr}{ds}$$

$$n\frac{dt}{ds} + \frac{dn}{ds}\cdot t = \nabla n$$

$$k = \frac{dt}{ds} \quad \Rightarrow \quad n\vec{k} = \nabla n - \frac{dn}{ds}\hat{t}$$

$I(r)$

$$\int_A I t\cdot d\sigma = -\int_V \alpha I\, dV$$

$d\sigma =$

$dV =$

$$\nabla \circ (I t) + \alpha I = 0$$

kirchoff diffraction theory:

$$\psi(R) = \frac{-1}{4\pi} \int_{S_1} \frac{\exp(ikR)}{R} n \circ \left[ \nabla \psi + ik\left(1 + \frac{i}{kR}\right) \frac{\bar{R}}{R} \psi \right] ds'$$

perpendicular beam: $z = 0$

$$\left.\frac{\partial \psi}{\partial z}\right|_{z=0} =$$

$$n \circ (\nabla' \psi) = ik\psi$$

$$kR \gg 1 \quad \hat{n} \circ \left(\frac{\bar{R}}{R}\right) = 1$$

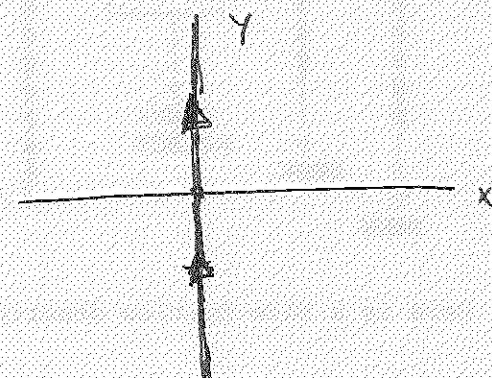
$$\psi(r) = -\frac{ik}{4\pi} \int_{S_1} \psi(S_1) \frac{\exp(ikR)}{R}$$

$$f(x,y,z) = \frac{-ik}{2\pi}\frac{\exp(ikR_0)}{R_0}\int_{S_1} u(\xi,\eta)\exp(ikf(\xi,\eta))\,d\xi\,d\eta$$

$$f(\xi,\eta) = -(l\xi + m\eta) + \frac{(\xi^2+\eta^2)}{2R_0} \qquad l = \frac{x}{R_0},\quad m = \frac{y}{R_0}$$

w/ $y \equiv 0$

$$f(\xi,\eta) = -l\xi + \frac{(\xi^2+\eta^2)}{2R_0} \quad + \quad u(\xi,\eta) = \ ?$$




$$\int_{-\infty}^{+\infty}\exp(ik\eta^2/2R_0)\,d\eta = \sqrt{\frac{2R_0}{k}}\int_{-\infty}^{+\infty}\exp(iv^2)\,dv$$

let $v = \sqrt{\frac{k}{2R_0}}\,\eta$

$$= 2\sqrt{\frac{2R_0}{k}}\int_0^{\infty}\exp(iv^2)\,dv$$

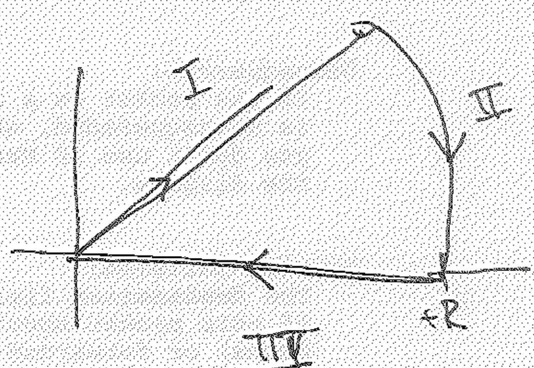
$dv = \sqrt{\frac{k}{2R_0}}\,d\eta$

let $u = \sqrt{i}\,v$     w/ $\sqrt{i} = (e^{i\pi/2})^{1/2} = e^{i\pi/4}$

$du = \sqrt{i}\,dv = e^{i\pi/4}\,dv$

$$\therefore\ I = 2\sqrt{\frac{2R_0}{k}}\int_0^{+\infty\cdot e^{i\pi/4}}\exp(u^2)\cdot e^{-i\pi/4}\cdot du$$

$$= 2e^{-i\pi/4}\sqrt{\frac{2R_0}{k}}\int_0^{+\infty e^{i\pi/4}}\exp(u^2)\,du$$



$\Rightarrow \cancel{2e^{i\pi/4}\sqrt{\frac{2R_0}{K}}\left[\lim_{R\to\infty}\int_{\mathrm{I}} + \int\right.}$

Now By C-I. Th.

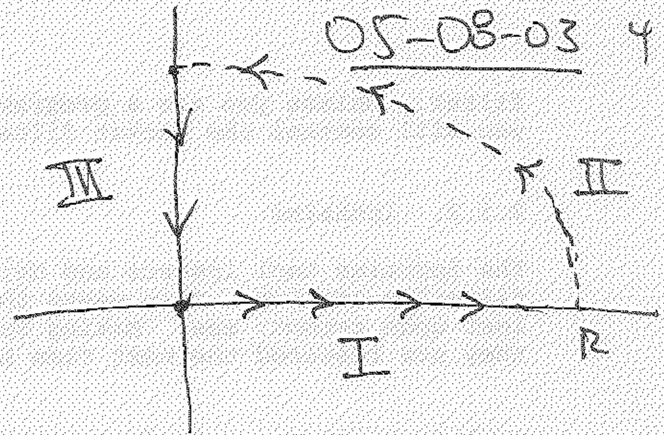
$$\int_{\mathrm{I}} + \int_{\mathrm{II}} + \int_{\mathrm{III}} = 0 \qquad \text{For } \forall \text{ fixed } R$$

$$\Rightarrow \int_{\mathrm{I}} = -\int_{\mathrm{II}} - \int_{\mathrm{III}}$$

$$= \lim_{R\to\infty} \int_{\pi/4}^{0} \exp(R^2 e^{2i\theta}) R \qquad u = Re^{i\theta}$$

diverges !! What's wrong?

$$\int_{-\infty}^{+\infty} \exp\left(\frac{ik\eta^2}{2R_0}\right) d\eta = 2\int_{0}^{\infty} \exp\left(\frac{ik\eta^2}{2R_0}\right) d\eta$$




Require a path to close the loop ∋

if $z = \eta = Re^{i\theta}$

$\cancel{i\eta^2 \leftarrow 0}$ $i\eta^2$ has negative real part !!

$\Rightarrow \cancel{iR^2 e^{i2\theta}} \Leftarrow 0 \Rightarrow$

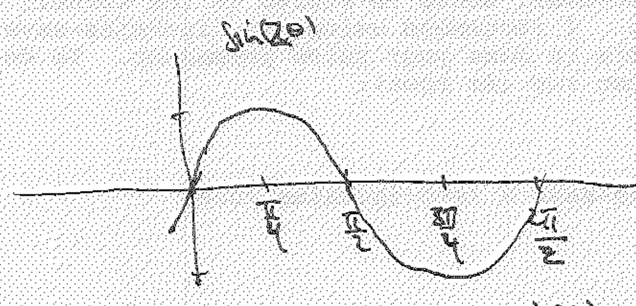
$$i\eta^2 = iR^2 e^{i2\theta} = iR^2\left[\cos(2\theta) + i\sin(2\theta)\right]$$

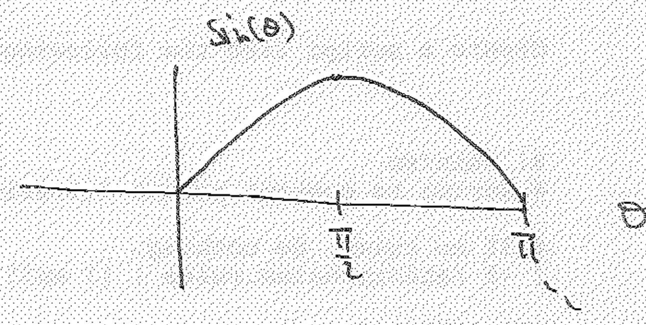
$$= iR^2\cos(2\theta) - R^2\sin(2\theta)$$ has negative real part if

$\sin(2\theta) > 0$

$\Rightarrow 0 \le 2\theta \le \pi$

$0 \le \theta \le \frac{\pi}{2}$




Then $\displaystyle\int_{I \cup II \cup III} = 0$

$$\Rightarrow \int_{I} = -\int_{II} + -\int_{III}$$

$$\rightarrow \int_I = -\left\{ \begin{array}{l} \eta = Re^{i\theta} \\ 0 \le \theta \le \pi/2 \end{array} \right\} - \left\{ \begin{array}{l} \eta = i\xi \\ \xi \text{ from } R \text{ to } 0 \end{array} \right\}$$

$$\therefore \int_0^\infty \exp\left(\frac{ik\eta^2}{2R_0}\right) d\eta = -\int_0^{\pi/2} \exp\left(\frac{ikR^2 e^{2i\theta}}{2R_0}\right) iRe^{i\theta}\, d\theta$$

$$- \int_R^0 \exp\left(\frac{ik(-1)\xi^2}{2R_0}\right) i\, d\xi$$

$$= -iR \int_0^{\pi/2} \exp\left(\frac{ik}{2R_0} R^2 \left[\cos(2\theta) + i\sin(2\theta)\right]\right) e^{i\theta}\, d\theta$$

$$- i \int_R^0 \exp\left(\frac{-ik\xi^2}{2R_0}\right) d\xi$$

$$\therefore \int_0^\infty \exp\left(\frac{ik\eta^2}{2R_0}\right) d\eta = \cancel{-iR \int_0^{\pi/2}} \lim_{R\to\infty} \underbrace{-iR \int_0^{\pi/2} \exp\left(\frac{ikR^2 \cos 2\theta}{2R_0}\right) \exp\left(-\frac{kR^2}{2R_0}\sin(2\theta)\right) \cdot e^{i\theta}\, d\theta}_{\to 0}$$

$$- i \int_\infty^0 \exp\left(-\frac{ik\xi^2}{2R_0}\right) d\xi$$

$$\Rightarrow \int_0^\infty \exp\left(\frac{ik\eta^2}{2R_0}\right) d\eta = i \int_0^\infty \exp\left(\frac{-ik\eta^2}{2R_0}\right) d\eta$$

$$I = i\bar{I}$$

$$\Rightarrow \int_0^\infty \left(\exp\left(\frac{ik\eta^2}{2R_0}\right) - i\exp\left(\frac{-ik\eta^2}{2R_0}\right)\right) d\eta = 0$$

$$\cancel{\int_0^\infty e^{-u^2}} \qquad \int_0^{+\infty} e^{-\alpha v^2} dv = \cdots \text{ known } \checkmark.$$

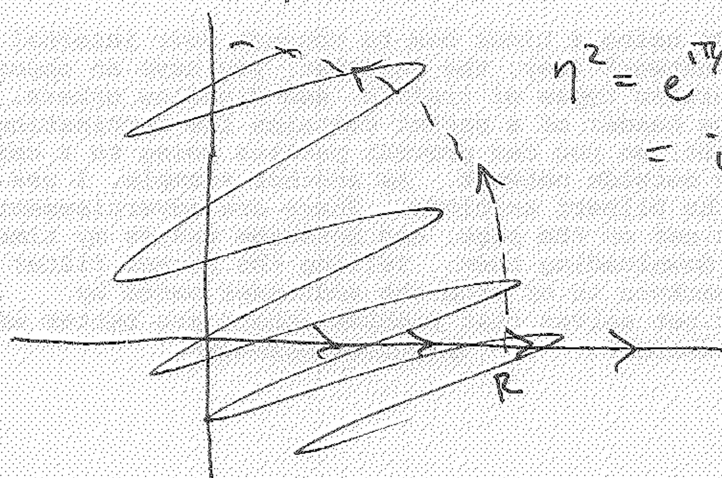
What path in $\eta$ gives

$$i\eta^2 = -x^2$$

if $\eta^2 = i\xi^2 \Rightarrow \eta = \ldots$

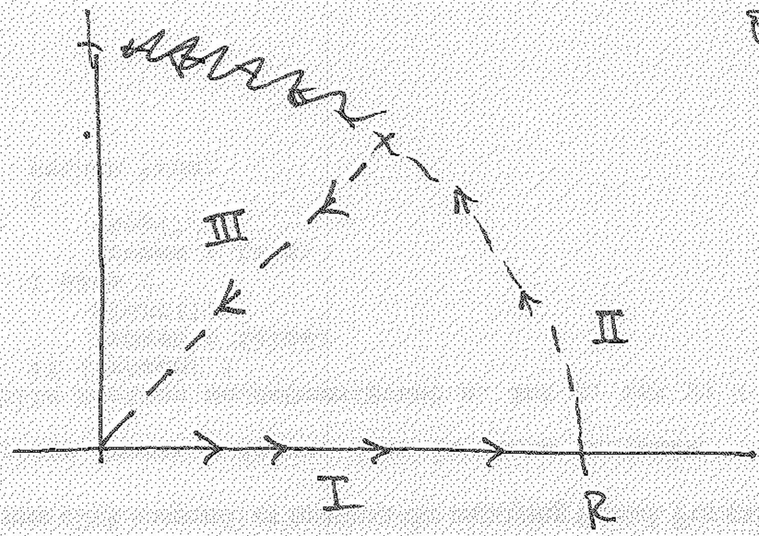
I know this is $\eta = e^{i\pi/4}\cdot\xi$. becaus

<u>Third attempt</u>:




$$\eta^2 = e^{i\pi/2}\xi^2 = i\xi^2.$$

"$\eta$" plane




"$\eta$" plane

$$\int_{\mathrm{I}} = -\int_{\mathrm{II}} - \int_{\mathrm{III}}$$

$\downarrow$ $\eta = Re^{i\theta}$, $0 \le \theta \le \pi/4$

$\downarrow$ $\eta = \xi e^{i\pi/4}$, $\xi$ from $R$ to $0$.

Since + $\theta$ $\ni$ $0 \le \theta \le \frac{\pi}{2}$ has a negative real part.

part.

As $R \to \infty$ This integral vanishes.

$$\therefore \int_0^\infty \exp\left(\frac{ik\eta^2}{2B_0}\right) d\eta = -\int_R^0 \exp\left(\frac{ik}{2B_0}\cdot \xi^2 \cdot i\right) e^{i\pi/4} \cdot d\xi \qquad \text{As } R \to \infty$$

$$= + e^{i\pi/4} \int_0^R \exp\left(\frac{-k\xi^2}{2B_0}\right) d\xi \qquad \text{As } R \to \infty.$$

$\therefore$ ~~$\alpha$~~ $e^{i\pi/4}$ let $v = \sqrt{\frac{k}{2R_0}}\,\xi$

$$dv = \sqrt{\frac{k}{2R_0}}\,d\xi \Rightarrow d\xi = \sqrt{\frac{2R_0}{k}}\cdot dv$$

$$\int_0^\infty \exp\left(\frac{ik\eta^2}{2R_0}\right)d\eta = e^{i\pi/4}\cdot\int_0^\infty \exp(-v^2)\cdot\sqrt{\frac{2R_0}{k}}\,dv$$

$$= \sqrt{\frac{2R_0}{k}}\,e^{i\pi/4}\underbrace{\int_0^\infty \exp(-v^2)\,dv}_{\frac{\sqrt{\pi}}{2}\ \checkmark}$$

check $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\exp(-x^2-y^2)\,dx\,dy = I^2$

$$= 2\pi\int_0^\infty r e^{-r^2}dr$$

$$= 2\pi\cdot\left.\frac{e^{-r^2}}{2(-1)}\right|_0^\infty$$

$$= \pi$$

$$\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi}$$

$\therefore$

$$\int_{-\infty}^{+\infty}\exp\left(\frac{ik\eta^2}{2R_0}\right)d\eta = \cancel{2}\frac{\sqrt{\pi}}{\cancel{2}}\cdot\sqrt{\frac{2R_0}{k}}\cdot e^{i\pi/4}$$

$$= \sqrt{2}\sqrt{\frac{\pi R_0}{k}}\,e^{i\pi/4}$$

Now $e^{i\pi/4} = \cos(\pi/4) + i\sin(\pi/4)$

$$= \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$\therefore$ $\int_{-\infty}^{+\infty}\exp\left(\frac{ik\eta^2}{2R_0}\right)d\eta = (1+i)\sqrt{\frac{\pi R_0}{k}}$ eq 2.64 $\checkmark$

$$f(x,z) = \frac{\cancel{f(\xi)}}{\cancel{z}} \quad \frac{-ik}{2\pi} \frac{\exp(ikR_0)}{R_0} \cdot \frac{(1+i)}{1} \sqrt{\frac{\pi R_0}{k}} \cdot ?$$

$$= \frac{(-i+1)}{2\pi} \cdot \frac{\sqrt{\pi}}{\sqrt{R_0}} \frac{k}{\sqrt{k}} \exp(ikR_0) \cdot ?$$

$$= \frac{(1-i)}{2} \sqrt{\frac{k}{\pi R_0}} \exp(ikR_0) \cdot \int_{S_1} U(\xi) \exp\left(ik\left(-l\xi + \frac{\xi^2}{2R_0}\right)\right) d\xi$$

let $U(\xi) = U_0 (1 - \xi^2/a^2)^{1/2} \exp\left(-\frac{ik\xi^2}{2f}\right)$ $\qquad |\xi| < a$

$$\Rightarrow f(x,z) = \frac{(1-i)}{2} \sqrt{\frac{k}{\pi R_0}} \exp(ikR_0) U_0 \int_{-a}^{+a} \left(1 - \left(\frac{\xi}{a}\right)^2\right)^{1/2} \exp\left(\cancel{\frac{-ik\xi^2}{2f}} - ikl\xi + \cancel{\frac{ik\xi^2}{2R_0}}\right) d\xi$$

w/ $R_0 = f$ why?

$$= \frac{(1-i)}{2} \sqrt{\frac{k}{\underset{\underset{f}{\parallel}}{\pi R_0}}} \exp(ik\underset{\underset{f}{\parallel}}{R_0}) U_0 \int_{-a}^{+a} \left(1 - \left(\frac{\xi}{a}\right)^2\right)^{1/2} \underbrace{\exp(-ikl\xi)}_{\cos(kl\xi) + i\sin(kl\xi)} d\xi$$

$$\int_{-a}^{+a} \left(1 - \left(\frac{\xi}{a}\right)^2\right)^{1/2} \cos(kl\xi)\, d\xi$$

How get?

$$= \pi \frac{J_1(kla)}{kl} \qquad \text{eq } 7.68$$

$$I(x,z) = |\,t(x,z)\,|^2$$

$$I(x,z) = \left|\frac{1-i}{2}\right|^2 \frac{k}{\pi z} v_0^2 \pi^2 \frac{J_1^2(kla)}{k^2 l^2}$$

$$= \frac{(1-i)}{2}\frac{(1+i)}{2}\frac{k}{\pi z} v_0^2 \pi^2 \frac{J_1^2(kla)}{k^2 l^2}$$

$$= \frac{v_0^2}{2} \frac{\pi z}{k x^2} J_1^2(kax/z)$$

Don't see

$$I(0,z) = \;?$$

$$\lim_{x\to\infty} \frac{J_1^2(\frac{ka}{z}x)}{x^2} \qquad \frac{0}{0} \qquad J_1(x)$$

Check:

$$= \frac{\cancel{2} J_1(\frac{ka}{z}x) J_1'(\frac{ka}{z}x)\cdot\frac{ka}{z}}{\cancel{2}x}$$

$$\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}$$

$$\frac{d}{dx}\left[x^{-p} J_p(x)\right] = -x^{-p} J_{p+1}(x)$$

correct ✓.

DO

$$J_p(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(k+p+1)}\left(\frac{x}{2}\right)^{2k+p}$$

$$J_1(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,\Gamma(k+2)}\left(\frac{x}{2}\right)^{2k+1} = \sum_{k=0}^{\infty}\frac{(-1)^k}{k!\,(k+1)!} \sim 1\cdot\left(\frac{x}{2}\right)$$

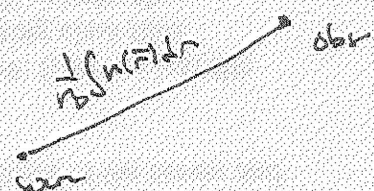
$$\lim_{x\to\infty} \frac{J_1^2\left(\frac{ka}{f}x\right)}{x^2} = \lim_{x\to\infty}\frac{\left(\frac12\frac{ka}{f}x\right)^2}{x^2} = \lim_{x\to\infty}\frac14\frac{k^2a^2}{f^2}$$

$\therefore$

$$I(0,f) = \frac{U_0^2}{2}\cdot\frac{\pi f}{k}\cdot\frac14\frac{k^2a^2}{f^2} = \frac{U_0^2}{8}\pi\frac{k}{f}\cdot a^2 \qquad \text{eq 7.71} \checkmark$$

Eq 7.58

$$U(R) = \frac{-ik}{2\pi}\int_{S_1} U(S_1)\,\frac{\exp(ikR)}{R}\,da' \qquad e^{ikR}$$




$$\nabla^2 U - \frac{n^2}{c^2}\frac{\partial^2 U}{\partial t^2} = 0$$

$U(r,\theta,\phi) \qquad n(r,\theta,\phi)$

$e^{-i\omega t} \qquad U = U(\bar r)e^{-i\omega t}$

$$\frac{\partial U}{\partial t} = -i\omega\, U(r)\, e^{-i\omega t}$$

$$\frac{\partial^2 U}{\partial t^2} = (-i\omega)^2 U(r)\, e^{-i\omega t} = -\omega^2 U(r)\, e^{-i\omega t}$$

$$\Rightarrow \frac1r\frac{\partial^2}{\partial r^2}(rU) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial\theta^2} + \frac{n^2\omega^2}{c^2}U = 0$$

$F(r,\theta,\phi) = r\Psi(r,\theta,\phi)$

$\omega = k_v c$

$k_v = \frac{2\pi}{\lambda_v}$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(F) + \frac{n^2\omega^2}{c^2}\cdot\frac{F}{r} = 0$$

$$\frac{\partial^2 F}{\partial r^2} + \frac{n^2\omega^2}{c^2}F = 0 \Rightarrow \frac{d^2F}{dr^2} + n^2 k_v^2 F = 0$$

n constant $\Rightarrow$ $F(r) \propto e^{\pm n k_v r i}$

Wentzel-Kramers-Brillouin WKB appr

$F(r) = \exp(g(r))$

$F' = e^{g(r)} g'(r)$ $\quad F'' = e^{g(r)}(g'(r))^2 + e^{g(r)} g''(r)$

$$\Rightarrow (g'(r))^2 + g''(r) + n^2 k_v^2 g = 0$$

$$\cancelto{0}{g''(r)} + (g'(r))^2 + n^2 k_v^2 = 0 \qquad \text{eq} \quad 7.78.$$

$$g'(r) = \pm i n k_v$$

$$g'(r) = \pm i k_v n(r)$$

$$g(r) = \pm i k_v \int n(r)\, dr$$

If $g(r) = \pm i k_r \int n(r) dr$

$$\frac{dg}{dr} = \pm i k_r n(r)$$

$$\frac{d^2g}{dr^2} = \pm i k_r \frac{dn(r)}{dr}$$ put in $g''(r) + g'(r)^2 + k_r^2 n^2(r) = 0$ for $g''(r)$ leaving $g'(r)$ as the unknown

$$g''_{n-1}(r) + (g'_n(r))^2 + k_r^2 n^2(r) = 0 \qquad g_0 = 0.$$

$$\Rightarrow \pm i k_r \frac{dn}{dr} + k_r^2 n^2(r) + (g'(r))^2 = 0$$

$$(g'(r))^2 = \mp i k_r \frac{dn}{dr} - k_r^2 n^2(r)$$

$$= -n^2(r) k_r^2 \left[ 1 \pm \frac{i}{n^2 k_r} \frac{dn}{dr} \right] \qquad \text{eq 7.80}$$

$$g'(r) = \pm i n(r) k_r \left(1 \pm \frac{i}{n^2 k_r} \frac{dn}{dr}\right)^{1/2} \cong \pm i n(r) k_r \left(1 \pm \frac{i}{2n^2 k_r} \frac{dn}{dr}\right)$$

eq 7.81

$$g(r) = \pm i k_r \int n(r) dr - \int \frac{1}{2n(r)} \cdot \frac{dn}{dr} \cdot dr$$

$$= \pm i k_r \int n(r) dr - \frac{1}{2} \int \frac{d}{dr} (\ln(n(r))) \, dr$$

$$g(r) = \pm i k_v \int n(r)\,dr - \tfrac{1}{2}\ln(n(r))$$

So $\quad F(r) = \exp(g(r))$

$$= e^{\pm i k_v \int n(r)\,dr} \cdot e^{\ln(n(r))^{-1/2}}$$

$$= \frac{e^{\pm i k_v \int n(r)\,dr}}{\sqrt{n(r)}}$$

So

$$\psi(r,\theta,\phi) = \frac{F(r,\theta,\phi)}{r} = \frac{\exp\left(\pm i k_v \int n(r)\,dr\right)}{r\sqrt{n(r)}}$$

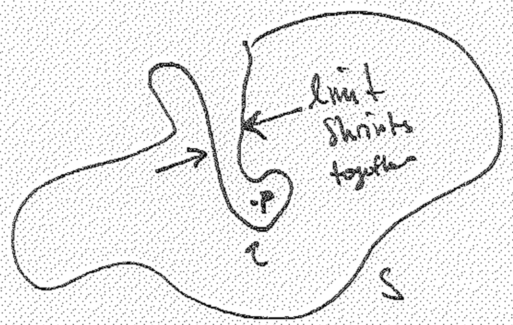
Kirchoff Diffraction

$$\int_V (U\nabla^2 V - V\nabla^2 U)\,dv = -\int_S \left(U\frac{\partial V}{\partial t} - V\frac{\partial U}{\partial t}\right)ds$$

U + V satisfy

$$\nabla^2\psi + n^2 k_v^2 \psi = 0$$

Then $\displaystyle\int_V (U\nabla^2 V - V\nabla^2 U)\,dv = \int_V \left(U(-n^2k_v^2 V) - V(-n^2 k_v^2 U)\right) dv \equiv 0.$




$$\int_S + \int_P \left( u \frac{\partial v}{\partial t} - v \frac{\partial u}{\partial t} \right) ds = 0$$

$$\frac{\partial u_A}{\partial t} = i l u_A$$

$$n-1 = 3\cdot 10^{-4}$$

$$\rho = \rho_0(1+\delta(r,t))$$

$$\left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right)\delta(r,t) = 0$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right)\delta(r,t) = 0$$

$$h_v(r,t) = \frac{\partial q_v}{\partial t} \qquad \cancel{\delta Q = \frac{RT}{\rho_0} q_v}$$

$$\delta(r) = -\frac{(\gamma-1)}{\gamma p_0}\alpha I(r)\, t_0 \qquad \text{eq } 7.12$$

$$\frac{\partial \delta}{\partial t} = -\frac{(\gamma-1)}{\gamma p_0} h_v(r,t)$$

$$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right)\delta(r,t) \cong \nabla^2\frac{\partial \delta}{\partial t} = \nabla^2\left(-\frac{(\gamma-1)}{\gamma p_0}h_v(r,t)\right)$$

$$= -\frac{(\gamma-1)}{\gamma p_0}\nabla^2 h_v(r,t) \qquad \text{eq } 7.105.$$

$$h_v(r,t) = \alpha I(r,t)$$

$$h_r(r,t) = \alpha I(r,t)$$

$$\nabla^2 I \qquad t = 0^+$$

(2.105) $$\frac{\partial}{\partial t}\left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right)s(r,t) = -\frac{(r-1)}{r_{p0}}\nabla^2 h_r(r,t)$$

$$s(r,t) = \frac{t^3}{3!}\frac{(r-1)}{r_{p0}}c_s^2\,\alpha\,\nabla^2 I$$

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05-11-03

$$p = C\rho^{\gamma}$$

$$\delta p = \gamma C \frac{\rho^{\gamma}}{\rho} \delta\rho = \gamma \frac{p}{\rho} \delta\rho = \gamma p \frac{\delta\rho}{\rho} = \gamma p \cdot s \qquad \text{eq } 7.245 ✓$$

$$f_v - \gamma p_0 \nabla s = 0$$

w/ $f_v = \frac{n-1}{2c} \nabla I$

$$\Rightarrow \frac{(n-1)}{2c} \nabla I - \gamma p_0 \nabla s = 0$$

$$s = \frac{(n-1)}{2c\gamma p_0} I(r) \qquad \text{eq } 7.247 ✓$$

---

$$\left(\nabla^2 - \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\right) s(r,t) = \frac{n-1}{2\gamma p_0 c} \nabla^2 I(r,t)$$

$$F(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) \exp(i\alpha x)\, dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(\alpha) \exp(-i\alpha x)\, d\alpha$$

$$F^{(r)}(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left[\frac{d^r}{dx^r} f(x)\right] e^{ix\alpha}\, dx$$

$u = e^{ix\alpha} \longleftrightarrow v = \frac{d^{r-1} f}{dx^{r-1}}$

$du = i\alpha e^{i\alpha x} \quad dv = \frac{d^r}{dx^r} f$

$d(uv) = v\,du + u\,dv$

$\int u\,dv = uv - \int v\,du$

$$= \frac{1}{\sqrt{2\pi}} \left[ \left(\frac{d^{r-1}}{dx^{r-1}} f(x)\right) e^{i\alpha x} \Big|_{-\infty}^{+\infty} - \int_{-\infty}^{+\infty} i\alpha \left(\frac{d^{r-1}}{dx^{r-1}} f\right) e^{i\alpha x}\, dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[ e^{i\alpha x} \left(\frac{d^{r-1}}{dx^{r-1}} f(x)\right) \Big|_{-\infty}^{+\infty} - i\alpha \int_{-\infty}^{+\infty} \frac{d^{r-1} f}{dx^{r-1}} e^{i\alpha x}\, dx \right]$$

eq A.4 ✓.

$$F^{(r)}(\alpha) = (-i\alpha)^r F(\alpha)$$

$F_n(\lambda)$

$$F_n(\lambda) \equiv \int_0^\infty r f(r) J_n(\lambda r)\, dr$$

$$f(r) = \int_0^\infty \lambda F_n(\lambda) J_n(\lambda r)\, d\lambda$$

$$\int_0^\infty r\left(\frac{d^2 f}{dr^2} + \frac{1}{r}\frac{df}{dr}\right) J_0(\lambda r)\, dr =$$

$$= \int_0^\infty r \frac{d^2 f}{dr^2} J_0(\lambda r)\, dr + \int_0^\infty r \cdot \frac{1}{r}\frac{df}{dr} \cdot J_0(\lambda r)\, dr$$

$u = r J_0(\lambda r)$ $\quad v = \frac{df}{dr}$ | $u = J_0(\lambda r)$ $\quad v = f$

$du = \frac{d}{dr}(r J_0(\lambda r))$ $\quad dv = \frac{d^2 f}{dr^2}$ | $du = \frac{d}{dr}(J_0(\lambda r))$ $\quad dv = \frac{df}{dr}$

$$= r J_0(\lambda r) \cdot \frac{df}{dr}\Big|_0^\infty - \int_0^\infty \frac{df}{dr} \cdot \frac{d}{dr}(r J_0(\lambda r))\, dr$$

$$+ J_0(\lambda r) f\Big|_0^\infty - \int_0^\infty f \cdot \frac{d}{dr}(J_0(\lambda r))\, dr$$

WRONG

$= \ \ldots ?$ $\quad \left\{ \dfrac{dJ_0(\lambda r)}{dr} = -\lambda J_1(\lambda r) \right.$ ✓ Yes!!

Think :

$=$

$$\frac{d}{dx}\left[ x^p J_p(x) \right] = x^p J_{p-1}(x)$$

$$\frac{d}{dx}\left[ x^{-p} J_p(x) \right] = -x^{-p} J_{p+1}(x)$$

$$dv = r\frac{d^2 f}{dr^2}$$

$$\int r\frac{d^2 f}{dr^2} = r\frac{df}{dr} - \int 1\,\frac{df}{dr} = r\frac{df}{dr} - f$$

$\dfrac{d}{dr} \Rightarrow$

$$r\frac{d^2 f}{dr^2} = \frac{df}{dr} + r\frac{d^2 f}{dr^2} - \cancel{\frac{df}{dr}} =$$

$$= \int_0^\infty r \frac{d^2 f}{dr^2} J_0(\lambda r)\, dr + \int_0^\infty r \frac{1}{r} \frac{df}{dr} \cdot J_0(\lambda r)\, dr$$

let

$u = J_0(\lambda r)$ $\quad$ $v = \int r \frac{d^2 f}{dr^2} \cdot dr$

$du = -\lambda J_1(\lambda r) dr$ $\quad$ $dv = r \frac{d^2 f}{dr^2}$

let

$u = J_0(\lambda r)$ $\quad$ $v = f$

$du = -\lambda J_1(\lambda r)\, dr$ $\quad$ $dv = \frac{df}{dr}$

Now

$$v = \int r \frac{d^2 f}{dr^2} dr = r \frac{df}{dr} - \int 1 \cdot \frac{df}{dr} \cdot dr$$

$$= r \frac{df}{dr} - f \quad \checkmark$$

$\therefore$ Above becomes:

$$= J_0(\lambda r) \left[ r \frac{df}{dr} - \cancel{f} \right] \Big|_0^\infty + \lambda \int_0^\infty J_1(\lambda r) \left( r \frac{df}{dr} - \cancel{f} \right) dr$$

$$+ \cancel{J_0(\lambda r) f} \Big|_0^\infty + \cancel{\lambda \int_0^\infty J_1(\lambda r) f \, dr}$$

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$$= \cancel{J_0(\lambda r)\, r \frac{dZ}{dr}}\Big|_0^\infty + \lambda \int_0^\infty J_1(\lambda r)\, r \frac{dZ}{dr}\, dr$$

By hypothesis.

let: $u = r J_1(\lambda r)$ $\qquad v = Z$

$$= \cancel{\&}.$$

$du = \frac{d}{dr}(r J_1(\lambda r))$ $\qquad dv = \frac{dZ}{dr}$

$$= \frac{1}{\lambda}\frac{d}{dr}(\lambda r J_1(\lambda r))$$

$$= \frac{1}{\lambda}\frac{d}{d\xi}(\xi J_1(\xi)) \cdot \frac{d\xi}{dr} \qquad \xi = \lambda r$$

$$\frac{d\xi}{dr} = \lambda$$

$$= \frac{d}{d\xi}(\xi J_1(\xi))$$

$$= \xi J_0(\xi) = \lambda r J_0(\lambda r)$$

$$= \lambda \cdot \left[ \cancel{r J_1(\lambda r) Z}\Big|_0^\infty - \int_0^\infty Z \cdot \lambda r J_0(\lambda r)\, dr \right]$$

By Hypothesis

$$= -\lambda^2 \int_0^\infty r Z(r) J_0(\lambda r)\, dr = -\lambda^2 F(\lambda) \quad \text{eq A.8} \checkmark.$$