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(5.1) $K_1 = \sigma_1^2 I \qquad K_2 = \sigma_2^2 I$

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$$= \qquad\qquad Pr[A|\hat{x}] = \frac{p(\hat{x}|A)\,Pr(A)}{p(x)}$$

pick ω_1 iff

$$Pr[\omega_1|x] > Pr[\omega_2|x]$$

$$\rightarrow \frac{p(x|\omega_1)Pr(\omega_1)}{p(x)} > \frac{p(x|\omega_2)Pr(\omega_2)}{p(x)}$$

$$l = \frac{p(x|\omega_1)Pr(\omega_1)}{p(x|\omega_2)Pr(\omega_2)} > 1 \qquad \text{pick } \omega_1$$

$$< 1 \qquad \text{pick } \omega_2$$

$$\frac{p(x|\omega_1)}{p(x|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{Pr(\omega_2)}{Pr(\omega_1)} \qquad p(x|\omega_i) = \frac{1}{(2\pi)^{n/2}|K_i|^{1/2}}$$

$$\exp\left[-\tfrac{1}{2}(x-m)^T K^{-1}(x-m)\right]$$

$\Longrightarrow$

$$\frac{\frac{1}{\cancel{(2\pi)^{n/2}}|K_1|^{1/2}}\exp\left[-\frac{1}{2}(x-m_1)^T K_1^{-1}(x-m_1)\right]}{\frac{1}{\cancel{(2\pi)^{n/2}}|K_2|^{1/2}}\exp\left[-\frac{1}{2}(x-m_2)^T K_2^{-1}(x-m_2)\right]} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{Pr(\omega_2)}{Pr(\omega_1)}$$

$$|K_1| = (\sigma_1^2)^n \quad |K_2| = (\sigma_2^2)^n \qquad K_1^{-1} = \frac{1}{\sigma_1^2} I \ ; \quad K_2^{-1} = \frac{1}{\sigma_2^2} I$$

$$\frac{\left(\frac{1}{\sigma_1}\right)^n}{\left(\frac{1}{\sigma_2}\right)^n} \exp\left[-\frac{1}{2}x^T\left(\frac{1}{\sigma_1^2}\right)x\right] \exp\left[+\frac{1}{2}x^T\left(\frac{1}{\sigma_1^2}\right)x\right] \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{Pr(\omega_2)}{Pr(\omega_1)}$$

$$= \left(\frac{\sigma_2}{\sigma_1}\right)^n \exp\left[-\frac{1}{2\sigma_1^2}x^T x + \frac{1}{2\sigma_2^2}x^T x\right] \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{Pr(\omega_2)}{Pr(\omega_1)}$$

$\Rightarrow$

$$\left(\frac{\sigma_2}{\sigma_1}\right)^n \exp\left[\frac{1}{2}\left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}\right)x^T x\right] \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \frac{Pr(\omega_2)}{Pr(\omega_1)}$$

$$n\ln\left(\frac{\sigma_2}{\sigma_1}\right) + \frac{1}{2}\left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}\right)x^T x \gtrless \ln\left(\frac{Pr(\omega_2)}{Pr(\omega_1)}\right)$$ mult by 2

$$\left(\frac{1}{\sigma_2^2} - \frac{1}{\sigma_1^2}\right)x^T x + 2n\ln\left(\frac{\sigma_2}{\sigma_1}\right) \underset{\omega_2}{\overset{\omega_2}{\gtrless}} 2\ln\left(\frac{Pr(\omega_2)}{Pr(\omega_1)}\right) \equiv T$$

$$\left(\left(\frac{1}{\sigma_1}\right)^2 - \left(\frac{1}{\sigma_2}\right)^2\right) x^T x + n \ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right) \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} -2 \ln\left(\frac{Pr(\omega_2)}{Pr(\omega_1)}\right)$$

$$\left(\left(\frac{1}{\sigma_1}\right)^2 - \left(\frac{1}{\sigma_2}\right)^2\right) x^T x + n \ln\left(\frac{\sigma_1^2}{\sigma_2^2}\right) \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} 2 \ln\left(\frac{Pr(\omega_1)}{Pr(\omega_2)}\right)$$

5.2

$$R_i = \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$|R_i - \lambda I| = \begin{vmatrix} 1-\lambda & 1/2 & 0 \\ 1/2 & 1-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{vmatrix} = (1-\lambda)(1-\lambda)(2-\lambda) - \tfrac{1}{2}\left(\tfrac{1}{2}\right)(2-\lambda) = 0$$

$$\Rightarrow (2-\lambda)\left[(1-\lambda) - \tfrac{1}{4}\right] = 0$$

$$\lambda = 2; \quad (1-\lambda) = \pm\tfrac{1}{2}$$

$$1 \mp \tfrac{1}{2} = \lambda \quad \Rightarrow \quad \lambda = \tfrac{1}{2}, \tfrac{3}{2}$$

$$\lambda_1 = \tfrac{1}{2};\ \lambda_2 = \tfrac{3}{2};\ \lambda_3 = 2$$

ordering in this app is λ_1 = largest evalue

$\lambda_1 = 2;\ \lambda_2 = \tfrac{3}{2};\ \lambda_3 = \tfrac{1}{2}$

$e_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix};\ e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix};\ e_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 0 & 0 & 3/2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0 \qquad v_3 = 0, \quad v_1 = -v_2$$

$$e_1 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -1/2 & 1/2 & 0 \\ 1/2 & -1/2 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0 \qquad v_3 = 0, \quad v_1 = v_2 \qquad e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix} -1 & 1/2 & 0 \\ 1/2 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\varepsilon = \sum_{j=m+1}^{n} \lambda_j =$$

To make ε as small as possible pick $u_{m+1}, u_{m+2}, \ldots, u_n$ corresponding to the ~~smallest~~ ei vectors w/ the smallest e.values

(a) $\varepsilon = \sum_{j=2}^{3} \lambda_j = 4$

(b) $\varepsilon = \frac{1}{2}$

5.3

$$R = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$|R - \lambda I| = \begin{vmatrix} 2-\lambda & 1 & 0 \\ 1 & 2-\lambda & 0 \\ 0 & 0 & 7-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda)(2-\lambda)(7-\lambda) - 1(7-\lambda) = 0$$

$$\Rightarrow (7-\lambda)\left[(2-\lambda)^2 - 1\right] = 0$$

$$\lambda = 7 ; \quad (2-\lambda) = \pm 1 \Rightarrow 2 \mp 1 = \lambda \Rightarrow \lambda = 1, 3$$

$\lambda_1 = 7 ; \lambda_2 = 3 ; \lambda_3 = 1$ ordered w/ e_1 corresponding to the largest eivalue

$\lambda_1 = 7$:

$$\begin{bmatrix} -5 & 1 & 0 \\ 1 & -5 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \bar{0}$$

~~$v_3 = 0$~~ v_3 = anything

$-5v_1 + v_2 = 0 \Rightarrow v_2 = 5v_1$ put in 2nd eq

$\Rightarrow v_1 - 5(5)v_1 = 0 \Rightarrow v_1 = 0$

$\Rightarrow v_2 = 0$

$$e_1 = \begin{pmatrix} v_1 \\ 5v_1 \\ v_3 \end{pmatrix} = v_1 \cancel{\begin{pmatrix} 1 \\ 5 \\ 0 \end{pmatrix}} + v_3 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

0

$\lambda_2 = 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$v_3 = 0 \quad v_1 = v_2$

$$e_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$\underline{\lambda_3 = 1}$

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = 0$$

$v_3 = 0$

$v_1 = -v_2$

$e_3 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

$$e_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad e_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \quad e_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

If $x \cong \sum_{i=1}^{m} c_i e_i$ w/ $e_1 \Leftrightarrow$ normalized e.vector associated w/ the largest e.value

Then $\varepsilon = \sum_{j=m+1}^{n} \lambda_j$

So

$$\varepsilon = \sum_{j=2}^{3} \lambda_j = 4 \quad \text{using } \tilde{x} = c_1 e_1$$

Using $\tilde{x} = c_2 e_2 + c_3 e_3$ $\quad \varepsilon = 7$ so the error is $\underline{\underline{\text{smaller}}}$

<u>ΨS 91 Theme</u>

5.4

$$R_1 = \begin{bmatrix} 1 & 1/2 & 0 \\ 1/2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad R_2 = \begin{bmatrix} 1 & -1/2 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(a) From Section 5.4.1

let $Q = R_1 + R_2 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 2 \cdot I$

Find the transformation $S \ni S^T Q S = I$

$\Rightarrow S = \underline{\underline{\frac{1}{\sqrt{2}} I}}$ (check $S^T Q S = \frac{1}{\sqrt{2}} I \, Q \, \frac{1}{\sqrt{2}} I = I$ ✓)

Then the eigvalues + eigvectors of S are

$\lambda_{1,2,3} = \frac{1}{\sqrt{2}} \doteq .707$

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

let $R_i' \equiv S^T R_i S = \cancel{S}$

So $\cancel{R_1'} =$

(b) Given $R_1' \in \mathbb{R}^{3\times3}$ $\exists$ 3 e.vectors these are the same as the eigenvectors of R_2'. Compute the e.vectors of

$$R_1' = S^T R_1 S = \frac{1}{\sqrt{2}} I \cdot R_1 \cdot \frac{1}{\sqrt{2}} I = \frac{1}{2} R_1$$

$$R_1' = \begin{bmatrix} 1/2 & 1/4 & 0 \\ 1/4 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|R_1' - \lambda I| = 0 \Rightarrow \begin{vmatrix} 1/2-\lambda & 1/4 & 0 \\ 1/4 & 1/2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)\begin{vmatrix} \frac{1}{2}-\lambda & 1/4 \\ 1/4 & 1/2-\lambda \end{vmatrix} = (1-\lambda)\left[(\tfrac{1}{2}-\lambda)^2 - 1/16\right] = 0$$

$$\Rightarrow \lambda = 1 \qquad (\lambda - 1/2)^{\cancel{2}} = \pm\frac{1}{4}$$

$$\lambda = \frac{1}{2} \pm \frac{1}{4} = \left\{\frac{1}{4}, \frac{3}{4}\right\}$$

$$\lambda_1 = \frac{1}{4} \le \lambda_2 = \frac{3}{4} \le \lambda_3 = 1$$

Since the two e.vectors corresponding to the largest & the smallest e.value will be the best representation for the corresponding class, we are required to find them

$$R_1' - 1 \cdot I = \begin{bmatrix} -1/2 & 1/4 & 0 \\ 1/4 & -1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -1/2 & 0 \\ \cancel{4}1 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow$$

$$e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$R_1' - \frac{1}{4} \cdot I = \begin{bmatrix} 1/4 & 1/4 & 0 \\ 1/4 & 1/4 & 0 \\ 0 & 0 & \cancel{0}\,3/4 \end{bmatrix}$$

$$R_1' - \frac{1}{4} I = 0 \rightarrow v_1 = -v_2 \rightarrow e_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

Then the transformation

$$y = T x' = \begin{bmatrix} e_{j_1}^T \\ \vdots \\ e_{j_m}^T \end{bmatrix} S^T x$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{1}{\sqrt{2}} I x = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x$$

$$= \underbrace{\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T} x$$

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8.15 $$\vec{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_k \end{bmatrix} = \begin{bmatrix} e_1^{(1)T} \\ \vdots \\ e_1^{(N_c)T} \end{bmatrix} x = T x$$

$e_1^{(1)} \cdots e_1^{(N_c)}$

$$G_i = \frac{1}{N_c}\left[R_i + \sum_{\substack{k=1 \\ k \neq i}}^{N_c} (I - R_k) \right]$$

1st pick S $\ni$ $\sum_{k=1}^{N_c} S^T R_k S = I$

$\therefore$ If $z = Ax$ then in the z space

$$R_x = E[xx^T] = E[A^{-1}z \cdot (A^{-1}z)^T] = E[A^{-1} z z^T (A^{-1})^T]$$

$$= A^{-1} E[z z^T] (A^{-1})^T$$

$$= A^{-1} R_z (A^{-1})^T$$

?

Pg 92 Therrien

5,6 5,18 $H = -E[\ln p_Y(y)]$

4,57 $p(x) = \dfrac{1}{(2\pi)^{n/2}|K|^{1/2}} \exp\left\{-\frac{1}{2}(x-m)^T K^{-1}(x-m)\right\}$

eq 5,52 $H = -\frac{1}{2}\ln|K| + \frac{n}{2}(1+\ln(2\pi))$

eq (4,20) $\operatorname{tr}(AB) = \operatorname{tr}(BA)$

So

$$H = -E\left[\ln \frac{1}{(2\pi)^{n/2}|K|^{1/2}} \exp\left\{-\frac{1}{2}(x-m)^T K^{-1}(x-m)\right\}\right]$$

$$= -E\left[-\left(\frac{n}{2}\right)\ln(2\pi) - \frac{1}{2}\ln|K| + -\frac{1}{2}(x-m)^T K^{-1}(x-m)\right]$$

$$= \frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|K| + \frac{1}{2}E\left[(x-m)^T K^{-1}(x-m)\right]$$

$$= \frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|K| + \frac{1}{2}E\left[\operatorname{tr} K^{-1}(x-m)(x-m)^T\right]$$

$$= \frac{n}{2}\ln(2\pi) - \frac{1}{2}\ln|K| + \frac{1}{2}\cancel{E}\,\underbrace{\operatorname{tr}(K^{-1}K)}_{n}$$

$$\Rightarrow H = -\frac{1}{2}\ln|K| + \frac{n}{2}(\ln(2\pi)+1) \quad ✓$$

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(Sol) $C = (Da - q)^T H (Da - q) + \text{const}$

$$\frac{\partial C}{\partial \vec{a}} = D^T H (Da - q) + (Da - q)^T H D \quad *$$

V.S.

$C = (D_a^T - q^T) H (Da - q) + \text{const}$

$C = a^T D^T H D a + - a^T D^T H q - q^T H D a + q^T H q + \text{const}$

$$\frac{\partial C}{\partial \vec{a}} = \underbrace{D^T H D a + a^T D^T H D}_{2 D^T H D a} - \underbrace{D^T H q - q^T H D}_{-2 D^T H q} \overset{\text{set}}{=} 0$$

Not technicaly correct !!

$$\Rightarrow \quad a = (D^T H D)^{-1} D^T H q \quad ✓$$

From eq * we get

$$D^T \underline{\underline{H}} D a - D^T H q + \underline{(Da)^T H} D - q^T H D \overset{\text{set}}{=} 0$$

$$D^T H D a = D^T H q \Rightarrow a = \quad ✓$$

$y = A x$ column

$\frac{\partial y}{\partial x} = A$

$y = x^T A$ row

$\frac{\partial y}{\partial x} = A^T$

$$\mu_i = E[h(y) \mid \omega_i] \qquad h(y) = b^T y + c$$

$$\sigma_i^2 = E[(h(y) - \mu_i)^2 \mid \omega_i]$$

$$\mu_i = E[b^T y + c \mid \omega_i] = b^T m_i + c$$

$$\sigma_i^2 = E[(b^T y + \cancel{c} - b^T m_i - \cancel{c})^2 \mid \omega_i]$$

$$= E[(b^T y - b^T m_i)^2 \mid \omega_i] =$$

$$= E[(b^T (y - m_i))^2 \mid \omega_i]$$

How?

$$= E[(y - m_i)^T b \cdot b^T (y - m_i) \mid \omega_i] = b^T K_i b$$

$$\frac{\partial F}{\partial \vec{b}} = \cancel{\frac{\partial \mu_i}{\partial \vec{b}}} \frac{\partial F}{}$$

$$F = \frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}$$

$$\frac{\partial F}{\partial \mu_1} = \frac{2(\mu_1 - \mu_2)}{\sigma_1^2 + \sigma_2^2} \qquad \frac{\partial F}{\partial \mu_2} = \frac{-2(\mu_1 - \mu_2)}{\sigma_1^2 + \sigma_2^2}$$

$$\frac{\partial F}{\partial \sigma_1} = \cancel{-\frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)^2}} (2\sigma_1) \qquad \frac{\partial F}{\partial \sigma_2} = \cancel{-\frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)}} (2\sigma_2)$$

$$\frac{\partial F}{\partial \sigma_1^2} = -\frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)^2} \qquad \frac{\partial F}{\partial \sigma_1^2} = -\frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)^2}$$

$$\frac{\partial \mu_i}{\partial b} = m_i \qquad \frac{\partial \sigma_i^2}{\partial \vec{b}} = 2k_i b$$

$$\frac{\partial F}{\partial \vec{b}} = 0 \quad \Rightarrow$$

$$\frac{2(\mu_1 - \mu_2)}{(\sigma_1^2 + \sigma_2^2)} m_1 + \frac{2(\mu_1 - \mu_2)}{(\sigma_1^2 + \sigma_2^2)} (-m_2)$$

$$-\frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)^2} 2k_1 b - \frac{(\mu_1 - \mu_2)^2}{(\sigma_1^2 + \sigma_2^2)^2} 2k_2 b = 0 \qquad \text{eq } 6.24 \checkmark$$

(6.1)

$$k_1 = k_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1/4 \end{bmatrix} \qquad m_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad m_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

From $\ell(\hat{y}) = \dfrac{p(\hat{y}|\omega_1)}{p(\hat{y}|\omega_2)} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} \lambda$ as a decision fn

then w/ ~~eq~~ Assumptions that the class conditional pdfs are gaussian one gets eq 6.2 +

$$h(\hat{y}) = (\hat{y}-m_1)^T k_1^{-1}(\hat{y}-m_1) - (\hat{y}-m_2)^T k_2^{-1}(\hat{y}-m_2) + \ln\frac{|k_1|}{|k_2|} \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} T$$

In the case above

$$k_1^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} = k_2^{-1} \qquad |k_1| = |k_2|$$

So $h(\hat{y})$ become

$$= \left(\begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)^T \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \left(\begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right)$$

$$- \left(\begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right) \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \left(\begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right) + \ln(1)$$

$$(\hat{y}_1 \ \hat{y}_2) \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 \end{pmatrix} - (\hat{y}_1, \hat{y}_2 - 2) \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} \begin{pmatrix} \hat{y}_1 \\ \hat{y}_2 - 2 \end{pmatrix}$$

$h(\hat{y}) = \quad \ldots$

(6.2) ...

(6.3) ...

(6.4) $g_k(y) = p_{y|\omega_k}(y|\omega_k)\Pr[\omega_k]$

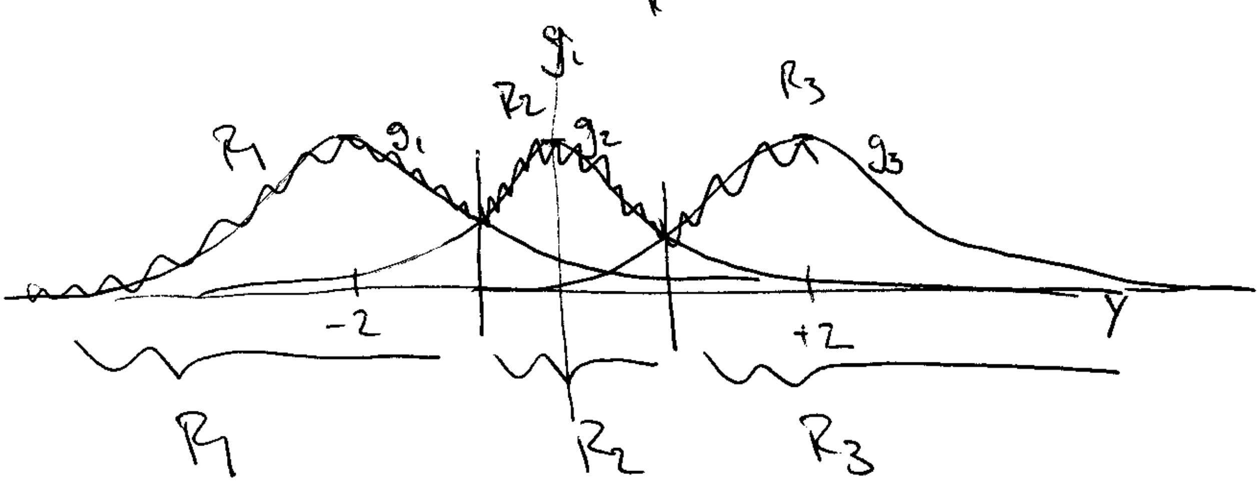
(a) $g_1(\hat{y}) = \frac{1}{3}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(\hat{y}+2)^2}{2}\right)$

$g_2(\hat{y}) = \frac{1}{3}\cdot\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(\hat{y})^2}{2}\right)$

$g_3(\hat{y}) = \frac{1}{3}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{(\hat{y}-2)^2}{2}\right)$

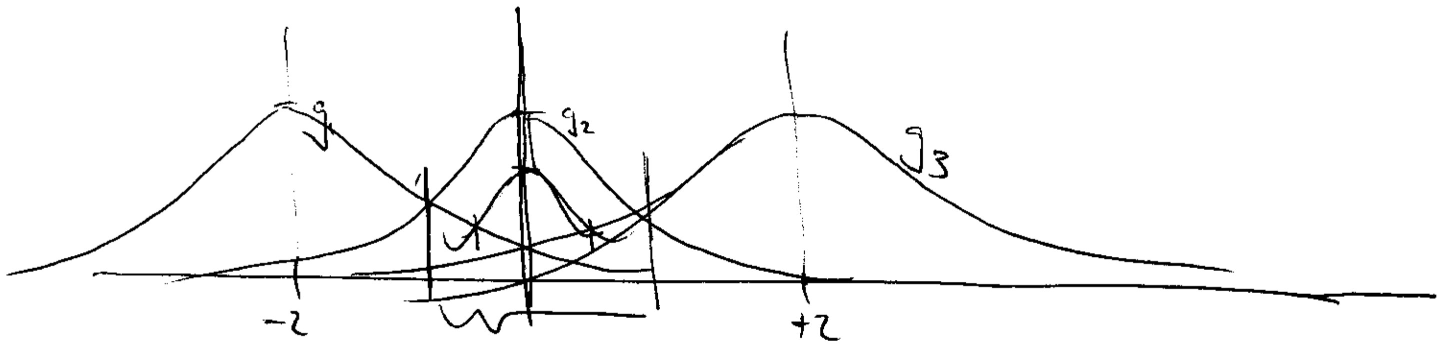
Discrimination is done when

Choose ω_i when $g_i(\hat{y}) = \max_k g_k(\hat{y})$



. .

(b) $Pr[\omega_2] \nearrow$ $\quad Pr[\omega_1] = Pr[\omega_3]$



Region R_2 increases in size

(c) $p(\hat{y}|\omega_2) = \frac{1}{\sqrt{2}} \tilde{p}(\hat{y}|\omega_2)$

Region R_2 decreases in size

6.5 $\quad g_k(y) = 2\ln\left[p(y|\omega_k) Pr[\omega_k] \right] + \frac{1}{2}\ln(2\pi)$

$= 2\ln[$

Note all of qualitative features ~~found~~ before still hold true, since we are simply operating on our original $g_k(y)$ w/ a monotone increasing fn.

(6.6) $$\cancel{g_1(y) = \frac{Pr\{\omega_1\}}{(2\pi)^{m/2}|k_1|^{1/2}} \exp\left[-\frac{1}{2}(y\right.}$$

$$g_1'(y) = \frac{Pr\{\omega_1\}}{(2\pi)^{2/2}(1/4)} \exp\left[-\frac{1}{2}\left[\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right]^T \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} \left[\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \end{pmatrix}\right]\right]$$

$|k_1| = \frac{1}{4}$ $\qquad k_1^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$

Alternatively

$g_1(y) = \quad \ldots \qquad$ eq 6.16

(6.7) $$b = \left[\tfrac{1}{2}(k_1 + k_2)\right]^{-1}(m_1 - m_2)$$

$k_1 = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ $\qquad k_2 = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$m_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\qquad m_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

plot decision boundary when $T = 1$

$$(m_1 - m_2)^T \left[\tfrac{1}{2}(k_1 + k_2)\right]^{-1} \hat{y} \underset{\omega_2}{\overset{\omega_1}{\lessgtr}} T = 1$$

5

6.8 $D_y^2(i) - D_y^2(0)$

8.65

$$2(y_{p_0} - y_{p_i})^T y = |y_{p_0}|^2 - |y_{p_i}|^2 + D_y^2(i) - D_y^2(0)$$

$$\underline{\underline{P}} = 2(y_{p_0} - y_{p_i})$$

$$q_{ith} = |y_{p_0}|^2 - |y_{p_i}|^2$$

feature z ith element

$$Py = q + z$$ eq 5.65 can be written as

linear classifier on z →

$$b^T z + c \underset{\omega_2}{\overset{\omega_1}{\gtrless}} T$$

$$b^T(Py - q) + c$$

$$b^T P y - b^T q + c$$

$$\cancel{\mathrm{E}}\ d^T y - \underbrace{b^T q + c}_{e} \rightarrow \quad d^T y - e \underset{\omega_2}{\overset{\omega_1}{\gtrless}} T$$

7.1 Maximum likelihood estimate

(a) $$p_T(\hat{T}) = \begin{cases} \alpha e^{-\alpha \hat{T}} & \hat{T} > 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{\alpha}_N = \frac{1}{\frac{1}{N}\sum_{i=1}^{N} \hat{T}^{(i)}}$$

(b) $E[\hat{\alpha}_N] = \alpha \Rightarrow$ estimate $\hat{\alpha}_N$ is unbiased.

How show?

(c) consistent $\Rightarrow$ $\lim_{N\to\infty} Pr[|\tilde{p}_N - p| < \epsilon] = 1$

$$\lim_{N\to\infty} Pr[|\tilde{\alpha}_N - \alpha| < \epsilon] \overset{?}{=} 1$$

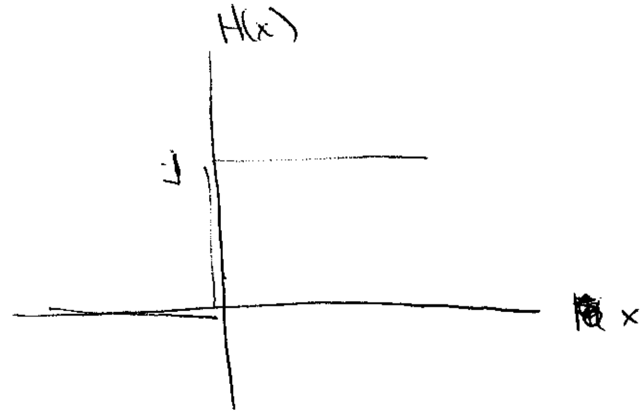
$$\Rightarrow \lim_{N\to\infty} Pr\left[\left|\frac{1}{\frac{1}{N}\sum_{i=1}^{N}\hat{T}^{(i)}} - \alpha\right| < \epsilon\right] = 1$$ How show?

(d) Minimum-variance estimate

$E[(\tilde{\alpha} - \alpha)^2] \overset{?}{\geq} j_{ii}^{-1}$ w/ $J = E[aa^T]$ $a = \frac{\partial \ln p_Y}{\partial \vec{p}}$

How show?

(7.2) $$P_Y(\hat{y}) = \begin{cases} \frac{1}{a} & 0 \le \hat{y} \le a \\ 0 & \text{otherwise} \end{cases}$$



(a) $P_{Y^{(1)};a}$ & $P_{Y^{(1)},Y^{(2)};a}$

$$P_{Y^{(1)};a}(y^{(1)};a) = \begin{cases} \frac{1}{a} & 0 \le y^{(1)} \le a \\ 0 & \text{otherwise} \end{cases} = \frac{1}{a}H(x) - \frac{1}{a}H(x+a)$$

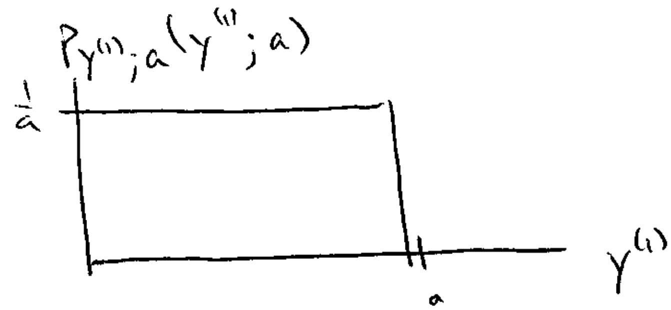
$$= \frac{1}{a}\left(H(x) - H(x-a)\right)$$

$$P_{Y^{(1)},Y^{(2)};a} = \begin{cases} \frac{1}{a} & 0 \le y^{(1)} \le a \\ 0 & \text{else} \end{cases} \cdot \begin{cases} \frac{1}{a} & 0 \le y^{(2)} \le a \\ 0 & \text{else} \end{cases}$$

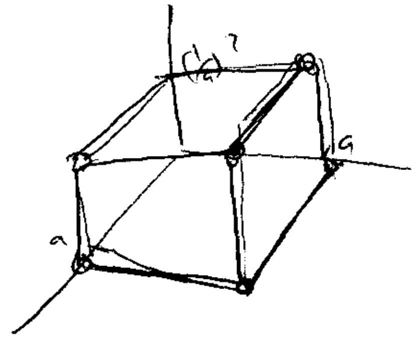
$$= \left(\frac{1}{a}\right)^2 \cancel{\left(H(x) - H(x-a)\right)}\,($$

$$= \left(\frac{1}{a}\right)^2 \left(H(y^{(1)}) - H(y^{(1)}-a)\right)\left(H(y^{(2)}) - H(y^{(2)}-a)\right)$$

(b)



$$P_{y^{(1)}, y^{(2)}; a}(y^{(1)}, y^{(2)}; a)$$



(b)
$$P_{y^{(1)} y^{(2)} \dots y^{(N)}; a} = \left(\frac{1}{a}\right)^N \prod_{k=1}^{N} \left(H(y^{(1)}) - H(y^{(k)} - a)\right)$$

$$\frac{\partial P_{y^{(1)} \dots y^{(N)}; a}}{\partial y^{(i)}} = \left(\frac{1}{a}\right)^N \prod_{\substack{k=1 \\ k \neq i}}^{N} \left(H(y^{(k)}) - H(y^{(k)} - a)\right)\left(\delta(0) - \delta(a)\right) \; ?$$

7.3
$$p(\alpha) = \begin{cases} \alpha \beta^2 e^{-\alpha\beta} & \alpha \geq 0 \\ 0 & \text{else} \end{cases}$$

(a) ?

(b) Meen Square

$$I(\tilde{\alpha}) = \int_{-\infty}^{\infty} |\tilde{\alpha} - \alpha|^2 p(\alpha)$$

$$\tilde{p} = \int_{-\infty}^{+\infty} p\, p(p \mid y^{(1)}, y^{(2)}, y^{(3)}, \cdots, y^{(N)})\, d\vec{p}$$

$$\therefore \quad \tilde{\alpha} = \int_{-\infty}^{+\infty} \alpha\, p(\alpha \mid \qquad ?$$

(c) $\rightarrow$

7.4 $\quad a + b\big|_{\tilde{p}} = 0$

$$a = \frac{\partial \ln p(y^{(1)}, y^{(2)}, \cdots, y^{(N)}}{\partial \vec{p}}$$

$$\vec{b} = \frac{\partial \ln p_{\vec{p}}}{\partial \vec{p}}$$

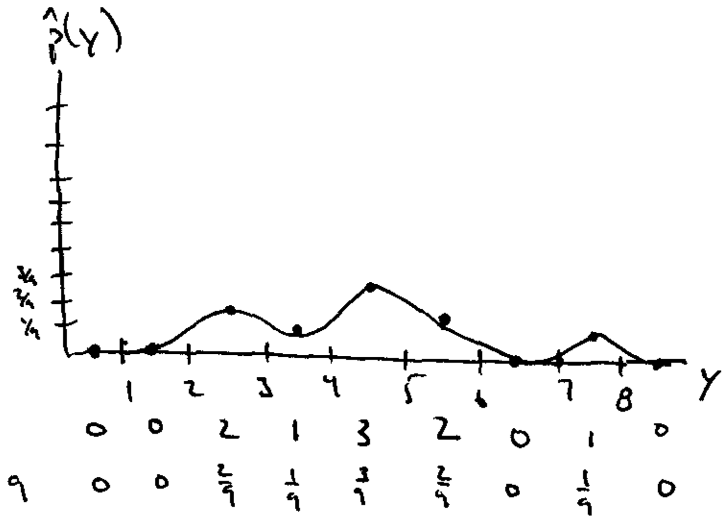
?

7.5 $\quad y^{(i)} \qquad ?$

(8.1) $Y^{(i)}$

$$\tilde{p}(\hat{y}) = \frac{k}{2hN}$$

$N = 9$



let $2h \cong 1$

Then $\hat{p}(\hat{y}) = \frac{k}{N}$

(8.2) $$\hat{p}(y) = \frac{1}{N}\sum_{i=1}^{N} \gamma(y - y^{(i)})$$

$$\rightarrow \quad \int_{-\infty}^{\infty} \gamma(z)\,dz = 1 \quad + \quad \int_{-\infty}^{+\infty} z\,\gamma(z)\,dz = 0$$

Then $$\tilde{m} \equiv \int_{-\infty}^{+\infty} y\,\tilde{p}(y)\,dy = \frac{1}{N}\sum_{i=1}^{N} \int_{-\infty}^{+\infty} y\,\gamma(y - y^{(i)})\,dy$$

$$= \frac{1}{N}\sum_{i=1}^{N} \int_{-\infty}^{+\infty} (y + y^{(i)})\,\gamma(y)\,dy = \frac{1}{N}\sum_{i=1}^{N} \Big[\underbrace{\int_{-\infty}^{+\infty} y\,\gamma(y)\,dy}_{0} + y^{(i)} \underbrace{\int_{-\infty}^{+\infty} \gamma(y)\,dy}_{1} \Big]$$

$$= \frac{1}{N}\sum_{i=1}^{N} y^{(i)}$$

(B.3) pooled 5-nearest neighbor.

$$\frac{k_1}{k_2} \underset{\omega_2}{\overset{\omega_1}{\gtrless}} 1 \qquad X = \text{Class } 1, \quad Y = \text{Class } 2$$

For A' $k_1 = 2$ $k_2 = 3$ ⇒ classify A as 2

For B' $k_1 = 3$ $k_2 = 2$ ⇒ " B as 1

For C $k_1 = 2$ $k_2 = 3$ ⇒ " C as 2

(B.4) $\min_i d(\hat{y}, \hat{y}^{(i)}) = \min_i d(\hat{y}, s^{(i)})$

(a)

$y^{(i)}$ $s^{(j)}$

⇒ $d(y, y^{(i)}) = d(y, s^{(j)})$

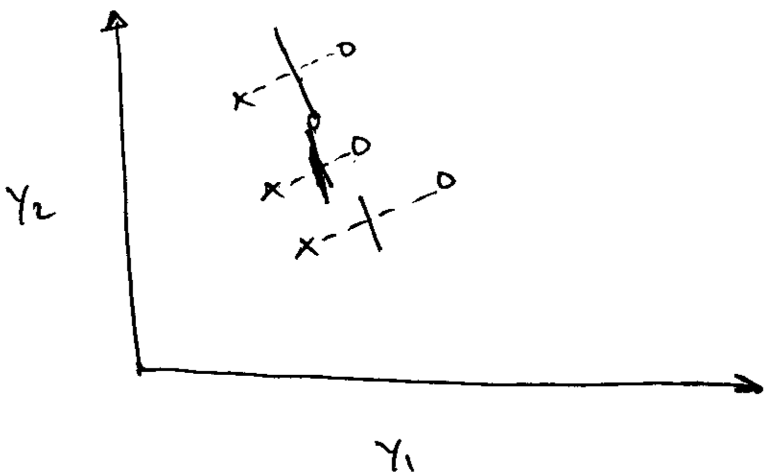
$y = (x, y)$

$$\sqrt{(x - x^{(i)})^2 + (y - y^{(i)})^2} = \sqrt{(x - s_x^{(j)})^2 + (y - s_y^{(j)})^2}$$

$$\Leftrightarrow \underbrace{(x - y_x^{(i)})^2 - (x - s_x^{(j)})^2}_{x^2 \text{ cancel}} + \underbrace{(y - y_y^{(i)})^2 - (y - s_y^{(j)})^2}_{y^2 \text{ cancel}} = 0$$

⇒ eq of a line.

(b)



~~each~~ many line segments between each data points.

(c) Removal of all pts to the left or right of the decision boundary.

8.5

(a) ?
(b) ?

9.1 $y' = Ay \Rightarrow y = A^{-1}y'$ so that $\frac{dy}{dy'} = A^{-1}$

$\Lambda(y) = \ln\left(\frac{p_1(y)}{p_2(y)}\right)$ then than divergence is given by

$$D = E[\Lambda(y)|\omega_1] - E[\Lambda(y)|\omega_2]$$

consider the PDF for y' w.r.t that of y.

$$p(y')\,dy' = p(y)\,dy$$

$$\Rightarrow \quad p(y') = p(y)\left|\frac{dy}{dy'}\right|$$

~~determinant~~ of ~~linear transformation~~ ~~determinant~~ or
absolute value of Jacobian determinant

$$\frac{\partial(y)}{\partial(y')} = \begin{vmatrix} \frac{\partial y_1}{\partial y_1'} & \frac{\partial y_1}{\partial y_2'} & \cdots & \frac{\partial y_1}{\partial y_n'} \\ \frac{\partial y_2}{\partial y_1'} & \frac{\partial y_2}{\partial y_2'} & \cdots & \frac{\partial y_2}{\partial y_n'} \\ \vdots & & & \\ \frac{\partial y_n}{\partial y_1'} & \cdots & & \frac{\partial y_n}{\partial y_n'} \end{vmatrix} = \begin{vmatrix} (A^{-1})_{11} & (A^{-1})_{12} & \cdots & (A^{-1})_{1n} \\ \vdots & & & \\ (A^{-1})_{n1} & (A^{-1})_{n2} & \cdots & (A^{-1})_{nn} \end{vmatrix}$$

$= |A^{-1}|$ determinant of A^{-1}

Thus

$$\Lambda(y') = \ln\left(\frac{\tilde{P}_1(y')}{\tilde{P}_2(y')}\right)$$

w/ $\tilde{P}_i(y') = P_i(y)\,\big|\,|A^{-1}|\,\big|$

So $$\Lambda(y') = \ln\left(\frac{\tilde{P}_1(y')}{\tilde{P}_2(y')}\right) = \ln\left(\frac{P_1(y)\,\cancel{|\,|A^{-1}|\,|}}{P_2(y)\,\cancel{|\,|A^{-1}|\,|}}\right) = \Lambda(y)$$

$\therefore$ $D(y') = D(y)$ ✓

Bhattacharyya distance:

$$B = -\ln\left[\int_{-\infty}^{+\infty}\sqrt{P_1(y)\,P_2(y)}\;dy\right]$$

$$B' = -\ln\left[\int_{-\infty}^{+\infty}\sqrt{\tilde{P}_1(y')\,\tilde{P}_2(y')}\;dy'\right] = -\ln\left[\int_{-\infty}^{+\infty}\sqrt{P_1(y)\,\big|\,|A^{-1}|\,\big|\cdot P_2(y)\,\big|\,|A^{-1}|\,\big|}\cdot\big|\,|A^{-1}|\,\big|\;dy\right]$$

$$dy' = \cancel{\big|\,|A^{-1}|\,\big|\,dy}\;\;\big|\,|A|\,\big|\,dy$$

$$\Rightarrow B' = -\ln\left[\int_{-\infty}^{+\infty} \sqrt{p_1(y)\,p_2(y)}\ \underbrace{\left| |A^{-1}| \right| \cdot \left| |A| \right|}_{\underbrace{\left| |A^{-1}|\,|A| \right|}_{=1}}\ dy \right]$$

$$|A^{-1}| = \frac{1}{|A|}$$

$$\Rightarrow B' = B \quad \checkmark$$

(b) $\quad y' = F(y) \qquad p(y') = p(y)\left|\frac{\partial y}{\partial y'}\right|$

$\quad y = F^{-1}(y')$

$\frac{\partial y}{\partial y'}$ is the Jacobian determinant of the transformation F^{-1}

So that

$$D = E[\Lambda(y)|\omega_1] - E[\Lambda(y)|\omega_2]$$

$$\Lambda(y) = \ln\left(\frac{p_1(y)}{p_2(y)}\right) \quad \text{so}$$

$$\Lambda(y') = \ln\left(\frac{\tilde{p}_1(y')}{\hat{p}_2(y')}\right) = \ln\left(\frac{p_1(y)\,\cancel{\left|\frac{\partial y}{\partial y'}\right|}}{p_2(y)\,\cancel{\left|\frac{\partial y}{\partial y'}\right|}}\right) = \Lambda(y) \quad \checkmark$$

$$B' = -\ln\left[\int_{-\infty}^{+\infty} \sqrt{\hat{p}_1(y')\,\hat{p}_2(y')}\, dy'\right]$$

$$= -\ln\left[\int_{-\infty}^{+\infty} \sqrt{p_1(y)\left|\frac{\partial y}{\partial y'}\right| p_2(y)\left|\frac{\partial y}{\partial y'}\right|} \cdot \left|\frac{\partial y'}{\partial y}\right| dy\right] = B \checkmark$$

$$dy' = \left|\frac{\partial y'}{\partial y}\right| dy = \frac{dy}{\left|\frac{\partial y}{\partial y'}\right|}$$

9.2 Eq 9.8 for divergence, Bhattacharyya

$J_M(\omega_1, \omega_2) > 0 \qquad \omega_1 \neq \omega_2$

$$D = E[\Lambda(y)|\omega_1] - E[\Lambda(y)|\omega_2] = H(1,2) + H(2,1) \quad \text{w/ each} > 0$$

$\omega_1 \neq \omega_2$

$$= \int_{-\infty}^{+\infty} \ln\left(\frac{p_1}{p_2}\right)$$

$\omega_1 \neq \omega_2$

$$B = -\ln\left[\int_{-\infty}^{+\infty} \sqrt{p_1(y)\,p_2(y)}\, dy\right] \qquad 0 \leq \sqrt{p_1(y)\,p_2(y)} < 1$$

$$\sqrt{p_1(y)p_2(y)} \le \begin{cases} p_1(y) & \text{if } p_2(y) < p_1(y) \\ p_2(y) & \text{if } p_1(y) < p_2(y) \end{cases}$$

$$\le \sqrt{p_1(y)p_2(y)} \le \operatorname{Max}(p_1(y), p_2(y))$$

If I can show $\displaystyle\int_{-\infty}^{+\infty} \sqrt{p_1 \cdot p_2}\, dy \le 1$ we are done.

the problem is w/ integrating this expression & proving that the integral is less than 1

$$J_m(\omega_1, \omega_1) = J_m(\omega_2, \omega_2) = 0$$

$$\Lambda(y) = 1 \quad \& \quad \ln(\Lambda) = 0 \qquad \text{yes for } D.$$

$$\int \sqrt{p_1 \cdot p_2}\, dy \equiv 1 \qquad \text{yes for } B$$

$$J_m(\omega_1, \omega_2) = J_m(\omega_2, \omega_1)$$

$$D = \int_{-\infty}^{+\infty} \ln\left(\frac{p_1}{p_2}\right) p_1\, dy - \int_{-\infty}^{+\infty} \ln\left(\frac{p_1}{p_2}\right) p_2\, dy$$

$$= - \int_{-\infty}^{+\infty} \ln\left(\frac{p_2}{p_1}\right) p_1\, dy + \int_{-\infty}^{+\infty} \ln\left(\frac{p_2}{p_1}\right) p_2\, dy$$

$$= E[\tilde{\lambda} \mid \omega_2] - E[\tilde{\lambda} \mid \omega_1] = J_m(\omega_2, \omega_1) \quad ✓.$$

$$B = -\ln\left[\int \right.$$ yes trivially

P_{VI}: $J_m(\omega_1, \omega_2) \le J_{m+1}(\omega_1, \omega_2)$

For divergence: $\int_{-\infty}^{+\infty} \ln$

$$\Lambda(y) = \ln\left(\frac{p_1}{p_2}\right)$$ Assuming <u>independent</u> components

$$= \ln\left(\frac{p_1(\hat{y})\,p_1(y_{m+1})}{p_2(\hat{y})\,p_2(y_{m+1})}\right) = \ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right) + \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right)$$

So

$$D = \int \ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right) p_1\,dy + \int \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right) p_1\,dy$$

$$- \int \ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right) p_2\,dy - \int \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right) p_2\,dy$$

Now $\int \ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right) p_1\,dy$ we integrate out y_{m+1} to get of which $\ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right)$ is a constant to get $\int \ln\left(\frac{p_1(\hat{y})}{p_2(\hat{y})}\right) p_1(\hat{y})\, d\hat{y}$

Sim for $\int \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right) p_1\, dy$ integrate out $y_1, y_2, \ldots, y_m$

to get $\int \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right) p_1(y_{m+1})\, dy_{m+1}$

$$\therefore D_{m+1} = \int \quad D_m + \int \ln\left(\frac{p_1(y_{m+1})}{p_2(y_{m+1})}\right) p_1(y_{m+1})\, dy_{m+1}$$

$$- \underbrace{\int \ln\left(\qquad \right) p_2(y_{m+1})\, dy_{m+1}}_{\text{Always positive}}$$

For $B_{m+1} = -\ln\left[\int_{-\infty}^{+\infty} \sqrt{p_1(\hat{y}) p_2(\hat{y})\, p_1(y_{m+1}) p_2(y_{m+1})}\, d\hat{y}\, dy_{m+1}\right]$

$$= - \ln\left[\int_{-\infty}^{+\infty} \sqrt{p_1 \qquad}\right.$$

$$\sqrt{p_1(\hat{y}) p_2(\hat{y})\, p_1(y_{m+1}) p_2(y_{m+1})} \leq \sqrt{p_1(\hat{y}) p_2(\hat{y})}$$

$\therefore \; -\ln\sqrt{P_1 \cdot P_2 \cdot P_1 \; P_2} \geq \ln\sqrt{P_1 P_2}$

$\therefore \; B_{m+1} \geq B_m$

(9.3) $C = \frac{1}{4}\left(m_1^T k_1^{-1} m_1 + m_2^T k_2^{-1} m_2 - 2 m_p^T k_p^{-1} m_p\right)$

w/ $m_p = \frac{1}{2} k_p (k_1^{-1} m_1 + k_2^{-1} m_2)$

& $k_p^{-1} = \frac{1}{2}(k_1^{-1} + k_2^{-1})$ put in the class

$\cancel{C = \frac{1}{4}\left(m_1^T k_1^{-1} m_1 + m_2^T k_2^{-1}\right.} m_2$

$- \cancel{\frac{4}{2^3}\left(k_1^{-1} m_1 + k_2^{-1} m_2\right)^T k_p^T} (k_1$

Pg 154 Therrien

(9.3) As the 1st part of this problem consider

$$\frac{|K_1|^{1/2}|K_2|^{1/2}}{|K_p|} = \frac{|K_1|}{|K_1|^{1/2}}\frac{|K_2|}{|K_2|^{1/2}}\left|\tfrac{1}{2}(K_1^{-1}+K_2^{-1})\right|$$

Since $\frac{1}{|K_p|} = |K_p^{-1}|$

$$= \frac{|K_1|\left|\tfrac{1}{2}(K_1^{-1}+K_2^{-1})\right||K_2|}{|K_1|^{1/2}|K_2|^{1/2}}$$

$$= \frac{\left|\tfrac{1}{2}(K_2+K_1)\right|}{(|K_1||K_2|)^{1/2}} = \text{eq } 9.21 \checkmark$$

Don't see?

$$k_A \equiv \frac{1}{2}(k_1 + k_2)$$

$$k_2^{-1} k_A k_1^{-1} = \frac{1}{2}(k_2^{-1} + k_1^{-1}) = k_p^{-1}$$

$$= k_1^{-1} k_A k_2^{-1}$$

Then $k_A^{-1} = \cancel{k_1}$?

$$k_1 k_A^{-1} k_2 = k_p \rightarrow k_A^{-1} = k_1^{-1} k_p k_2^{-1}$$

& $k_2 k_A^{-1} k_1 = k_p \rightarrow k_A^{-1} = k_2^{-1} k_p k_1^{-1}$ ✓

Then $k_A^{-1} - 2k_1^{-1} \underset{\substack{\uparrow \\ \text{using (1)}}}{=} k_1^{-1}(k_p k_2^{-1} - 2)$

$$\underset{\substack{\uparrow \\ \text{using (2)}}}{=} (k_2^{-1} k_p - 2) k_1^{-1}$$

Don't see?

(9.4) (9.20)

$\min\{a,b\} \le a^s b^{1-s} \qquad a,b,s \in (0,1] \qquad s=0$

$\Rightarrow \quad a=0 \quad b \in (0,1]$

$b=0$

~~a, b,~~

How?

(9.5) $\mu(s) = \ln \int_{-\infty}^{+\infty} p_1^s(y)\, p_2^{1-s}(y)\, dy$

w/ $p_1 = \frac{1}{(2\pi)^{m/2}|k_1|^{1/2}} \exp\left\{ -\frac{1}{2}(y-m_1)^T k_1^{-1}(y-m_1)\right\}$

$p_2 = \frac{1}{(2\pi)^{m/2}|k_2|^{1/2}} \exp\left\{ \quad \cdots \quad \right\}$

So

$$p_1^s(y)\, p_2^{1-s}(y) = \frac{1}{(2\pi)^{\frac{m}{2}s}(2\pi)^{\frac{m}{2}(1-s)}|k_1|^{s/2}|k_2|^{\frac{(1-s)}{2}}}$$

$$\exp\left\{ -\frac{s}{2}(y-m_1)^T k_1^{-1}(y-m_1) - \frac{(1-s)}{2}(y-m_2)^T k_2^{-1}(y-m_2)\right\}$$

So

$$p_1^{s}(y)\, p_2^{1-s}(y) = \frac{1}{(2\pi)^{m/2}} \frac{1}{|k_1|^{s/2} |k_2|^{\frac{1-s}{2}}} \exp\left\{ -\frac{1}{2}\Big(\right.$$

$$\left. s(y-m_1)^T k_1^{-1}(y-m_2) + (1-s)(y-m_2)^T k_2^{-1}(y-m_2)\Big) \right\}$$

expanding

$$s\left[y^T k_1^{-1} y - 2 m_1^T k_1^{-1} y + m_1^T k_1^{-1} m_1 \right]$$

$$+(1-s)\left[y^T k_2^{-1} y - 2 m_2^T k_2^{-1} y + m_2^T k_2^{-1} m_2 \right]$$

$$= y^T \left(s k_1^{-1} + (1-s) k_2^{-1} \right) y - 2 m_1^T s k_1^{-1} y - 2 m_2^T (1-s) k_2^{-1} y$$

$$+ s m_1^T k_1^{-1} m_1$$

(9.5) eq 9.30 is

$$\mu(s) = \ln \int_{-\infty}^{+\infty} p_1^s(y) p_2^{1-s}(y)\, dy$$

Now w/ $p_1(y) = \frac{1}{(2\pi)^{m/2} |K_1|^{1/2}} \exp\left\{ -\frac{1}{2} (y-m_1)^T K_1^{-1} (y-m_1) \right\}$

& $p_2(y) = \frac{1}{(2\pi)^{m/2} |K_2|^{1/2}} \exp\left\{ -\frac{1}{2} (y-m_2)^T K_2^{-1} (y-m_2) \right\}$

So that

$$p_1^s(y) p_2^{1-s}(y) = \frac{1}{(2\pi)^{\frac{m}{2} \cdot s} |K_1|^{\frac{s}{2}} (2\pi)^{\frac{m}{2}(1-s)} |K_2|^{\frac{(1-s)}{2}}} \cdot \exp\left\{ \frac{-1}{2} \cdot \left[s(y-m_1)^T K_1^{-1} (y-m_1) + (1-s)(y-m_2)^T K_2^{-1} (y-m_2) \right] \right\}$$

$$= \frac{1}{(2\pi)^{m/2} |K_1|^{\frac{s}{2}} |K_2|^{\frac{(1-s)}{2}}} \exp\left\{ \frac{-1}{2} \cdot \left[s y^T K_1^{-1} y - 2s m_1^T K_1^{-1} y + s m_1^T K_1^{-1} m_1 + (1-s) y^T K_2^{-1} y - 2(1-s) m_2^T K_2^{-1} y + (1-s) m_2^T K_2^{-1} m_2 \right] \right\}$$

$$= \frac{1}{(2\pi)^{m/2}|k_1|^{\frac{s}{2}}|k_2|^{\frac{1-s}{2}}} \exp\left\{\frac{-1}{2}\cdot\right.$$

$$\left.\left[y^T(sk_1^{-1}+(1-s)k_2^{-1})y - 2[sm_1^Tk_1^{-1}+(1-s)m_2^Tk_2^{-1}]y + sm_1^Tk_1^{-1}m_1+(1-s)m_2^Tk_2^{-1}m_2\right]\right\}$$

Define $k_p^{-1} = sk_1^{-1}+(1-s)k_2^{-1}$

& $k_A = sk_1+(1-s)k_2$

$\{y^2-2xy+x^2\}$

$(y-x)^2$

Then

$$= \frac{1}{(2\pi)^{m/2}|k_1|^{s/2}|k_2|^{\frac{1-s}{2}}} \exp\left\{\frac{-1}{2}\cdot\right.$$

$$\left[y^Tk_p^{-1}y - 2m_q^Tk_p^{-1}y + m_q^Tk_p^{-1}m_q \cancel{+} - m_q^Tk_p^{-1}m_q\right.$$

$$\left. - 2\cancel{[sm_1^Tk_1^{-1}+(1-s)m_2^Tk_2^{-1}]}y + sm_1^Tk_1^{-1}m_1+(1-s)m_2^Tk_2^{-1}m_2\right]$$

$$\therefore\ m_q^Tk_p^{-1} = sm_1^Tk_1^{-1}+(1-s)m_2^Tk_2^{-1}$$

$$m_q^T = sm_1^Tk_1^{-1}k_p+(1-s)m_2^Tk_2^{-1}k_p$$

$$m_q = k_p\left(s k_1^{-1} m_1 + (1-s) k_2^{-1} m_2\right) \qquad \text{Same as eq 9.186 ✓}$$

$$\therefore \quad p_1^{s}(y)\, p_2^{1-s}(y) = \frac{1}{(2\pi)^{m/2} |k_1|^{\frac{s}{2}} |k_2|^{\frac{1-s}{2}}} \exp\left\{ -\frac{1}{2}\Big[y^T k_p^{-1} y - 2 m_q^T k_p^{-1} y \right.$$

$$\left. + m_q^T k_p^{-1} m_q \underbrace{- m_q^T k_p^{-1} m_q + s m_1^T k_1^{-1} m_1 + (1-s) m_2^T k_2^{-1} m_2}_{q} \Big] \right\}$$

$$p_1^{s}(y)\, p_2^{1-s}(y) = \frac{1}{(2\pi)^{m/2} |k_1|^{s/2} |k_2|^{\frac{1-s}{2}}} \exp\left\{ -\frac{1}{2}\Big(y^T k_p^{-1} y - 2 m_q^T k_p^{-1} y \right.$$

$$\left. + m_q^T k_p^{-1} m_q \Big] \right\} \cdot \exp\left\{ \frac{1}{2} m_q^T k_p^{-1} m_q - \frac{1}{2} s m_1^T k_1^{-1} m_1 - \frac{1}{2}(1-s) m_2^T k_2^{-1} m_2 \right\}$$

Then

$$B = -\ln\left[\int_{-\infty}^{+\infty} \frac{|k_p|^{1/2}}{(2\pi)^{m/2} |k_1|^{s/2} |k_2|^{\frac{1-s}{2}}} \frac{e^{q}}{|k_p|^{1/2}} \exp\left\{ -\frac{1}{2}\left[(y-m_q)^T k_p^{-1} (y-m_q) \right] \right\} dy \right]$$

$$= -\ln\left[\frac{|k_p|^{1/2}}{|k_1|^{s/2} |k_2|^{\frac{1-s}{2}}}\, e^{q} \underbrace{\int_{-\infty}^{+\infty} \frac{1}{(2\pi)^{m/2} |k_p|^{1/2}} \exp\left\{ -\frac{1}{2} (y-m_q)^T k_p^{-1} (y-m_q) \right\} dy}_{\equiv 1} \right]$$

$$\rightarrow \quad B = -C_1 + \frac{1}{2}\ln\left(\frac{|k_1|^s |k_2|^{1-s}}{|k_p|}\right)$$

$$+ \quad C_1 = \frac{1}{2} m_q^T k_p^{-1} m_q - \frac{1}{2} s\, m_1^T k_1^{-1} m_1 - \frac{1}{2}(1-s) m_2^T k_2^{-1} m_2$$

Consider

$$\frac{1}{2}\ln\left(\frac{|k_1|^s |k_2|^{1-s}}{|k_p|}\right) = \frac{1}{2}\ln\left(|k_1|^s |k_2|^{1-s} \left|s k_1^{-1} + (1-s) k_2^{-1}\right|\right)$$

$$\frac{1}{|k_p|} = |k_p^{-1}| \qquad = \frac{1}{2}\ln\Big(|k_1|^s |k_2|^{1-s} |k_1|^{-1} |k_2|^{-1} \cdot \left|s k_2 + (1-s) k_1\right| \Big)$$

$$= \frac{1}{2}\ln\left(\frac{|s k_2 + (1-s) k_1|}{|k_1|^{1-s} |k_2|^s}\right) \quad \checkmark$$

Remains to simplify C_1.

$$C_1 = \frac{1}{2} m_q^T k_p^{-1} m_q - \frac{1}{2} s\, m_1^T k_1^{-1} m_1 - \frac{1}{2}(1-s) m_2^T k_2^{-1} m_2$$

w/ $m_q = k_p\left(s k_1^{-1} m_1 + (1-s) k_2^{-1} m_2\right)$

w/ $k_p^{-1} = s k_1^{-1} + (1-s) k_2^{-1}$

$$2G = m_q^T k_p^{-1} m_q - s m_1^T k_1^{-1} m_1 - (1-s) m_2^T k_2^{-1} m_2$$

$$k_p^{-1} = s k_1^{-1} + (1-s) k_2^{-1}$$

$$k_A = s k_1 + (1-s) k_2$$

Don't know how to turn G into

$$-\frac{1}{2} s(1-s)(m_1 - m_2)^T \left[(1-s) k_1 + s k_2\right](m_1 - m_2) \quad \dots$$

9.6 (3,5)

$$\varepsilon_1 = \Pr[\text{error} | \omega_1] = \int_{R_2} p(y|\omega_1)\, dy = \int$$

$$\varepsilon_2 = \Pr[\text{error} | \omega_2] = \int_{R_1} p(y|\omega_2)\, dy = \int_{\tau}^{\infty} p_{\Lambda|\omega_2}(\xi|\omega_2)\, d\xi$$

$$= \int_{\tau}^{\infty} e^{-s\xi + \mu(s)} p_{\xi}(\xi)\, d\xi$$

$$e^{-s\xi} < e^{-s\tau}$$

$$\therefore \varepsilon_2 \leq e^{-s\tau + \mu(s)} \int_{\tau}^{\infty} p_{\xi}(\xi)\, d\xi \leq e^{-s\tau + \mu(s)} \qquad \text{eq } 9.40$$

$$\cancel{\varepsilon_1 = \int_{-\infty}^{\tau} p_{\Lambda|\omega_1}(\xi|\omega_1)\, d\xi =}$$

$$\mu'(s') = \ln \int_{-\infty}^{+\infty} \left(\frac{p_2(y)}{p_1(y)}\right)^{s'} p_1(y)\, dy = \ln \int_{-\infty}^{+\infty} e^{s'\Lambda'(y)} p_1(y)\, dy$$

$$\text{w/ } \Lambda'(y) \equiv \ln\left(\frac{p_2(y)}{p_1(y)}\right) \qquad = \ln \int_{-\infty}^{+\infty} e^{s'\Lambda'(y)} p_{\Lambda'|\omega_1}(\Lambda'|\omega_1)\, d\Lambda'$$

$$\Rightarrow \quad e^{\mu'(s')} = \int_{-\infty}^{+\infty} e^{s'\Lambda'(y)} p_{\Lambda'|\omega_1}(\lambda'|\omega_1)\, d\lambda'$$

Define $p_\xi(\xi) = e^{s'\xi - \mu'(s')} p_{\Lambda'|\omega_1}(\lambda'|\omega_1)\, d$

Then $\varepsilon = \int_{R_2} \cdots = \int$

~~$\xi > -\lambda$ ... $-\xi < \tau$~~ $\quad \xi > -\tau \quad = \quad \xi$

$$\varepsilon = \int_{R_2} p(y|\omega_1)\, dy = \int_{-\tau}^{+\infty} p_{\Lambda|\omega_1}(\xi|\omega_1)\, d\xi$$

Via. * $\quad p_{\Lambda'|\omega_1}(\lambda'|\omega_1) = e^{-s'\xi + \mu'(s')} p_\xi(\xi)$

$$\therefore \quad \varepsilon = \int_{-\tau}^{+\infty} e^{-s'\xi + \mu'(s')} p_\xi(\xi)\, d\xi \lessgtr e^{\mu'(s') + s'\tau} \int_{-\tau}^{+\infty} p(\xi)\, d\xi$$

$$e^{-s'\xi} < e^{+s'\tau} \qquad < e^{\mu'(s') + s'\tau} \qquad ?$$

(b) eq 9.29 $\quad$ ~~$\varepsilon_B \le \Pr[\omega_1]^{1/2} \Pr[\omega_2]^{1/2} e^{-B}$~~

$$\varepsilon_B \le \Pr[\omega_1]^s \Pr[\omega_2]^{1-s} e^{\mu(s)}$$

$$\Pr[\text{error}] \le \Pr[w_1]e^{\mu(s)+(1-s)\tau} + \Pr[w_2]e^{\mu(s)-s\tau}$$

$$= \cancel{}$$

$$= e^{\mu(s)-s\tau}\left[\Pr[w_1]e^{\tau} + \Pr[w_2]\right]$$

$$\text{w/}\quad \tau = \ln\left[\frac{\Pr[w_2]}{\Pr[w_1]}\right]$$

$$= 2\Pr[w_2]e^{\mu(s)}e^{-s\tau} = 2\Pr[w_2]e^{\mu(s)}\left(\frac{\Pr[w_1]^s}{\Pr[w_2]^s}\right)$$

$$= 2\Pr[w_1]^s\Pr[w_2]^{1-s}e^{\mu(s)}$$

who?

Why 9.45?

M 174 Themi

(10.1)

(a) $x^{(1)} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 1 \\ 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $x^{(2)} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 2 \\ 0 \end{bmatrix}$

$$m = \frac{1}{4}\sum x^{(i)} = \cdots$$

$$k = E\left[(x^{i}-m)(x-m)^T\right] = \frac{1}{4}\sum_i (x^{(i)}-m)(x^{(i)}-m)^T$$

(b) time series mean (1) $= \frac{1}{7}(0+2+1+1+2+0+1+0) = \cdots$

tsm(2) $= \cdots$

..

tsm(4) $= \cdots$

$$\gamma(\ell) = E[(x_t-m_x)(x_{t-\ell}-m_x)] = E[x_t x_{t-\ell}] - m_x^2$$

$$= E[x_t x_{t-\ell} - x_t m_x - m_x x_{t-\ell} + m_x^2]$$

$$= E[x_t x_{t-\ell}] - m_x^2 .$$

For time series #1:

$$\gamma(1) = \frac{2(0) + 2(1) + (1)(1) + 1(2) + 2(0) + 1(0) + 1(0)}{7}$$

$$\gamma(2) = \frac{0(1) + 2(1) + (1)(2) + (1)(0) + \cdots}{5}$$

$$\gamma(3) = \frac{0(1) + \cdots}{}$$

10.2

$$\left[\begin{array}{c|c} K_k & r_{k+1} \\ \hline r_{k+1}^T & \sigma_{k+1}^2 \end{array}\right] \qquad \left[\begin{array}{c|c} I & 0 \\ \hline 0^T & 1 \end{array}\right]$$

Mult by 1rst set of rows by K_k^{-1}

$$\Rightarrow \left[\begin{array}{c|c} I & K_k^{-1} r_{k+1} \\ \hline r_{k+1}^T & \sigma_{k+1}^2 \end{array}\right] \qquad \begin{array}{c|c} K_k^{-1} & 0 \\ \hline 0^T & 1 \end{array}$$

Mult 1st set of rows by $-r_{k+1}^T$ & Add to last row

$$\left[\begin{array}{c|c} I & K_k^{-1} r_{k+1} \\ \hline 0^T & \sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1} \end{array}\right] \qquad \cancel{\begin{array}{c|c} K_k^{-1} & 0 \\ \hline 0^T & \end{array}}$$

$$\left[\begin{array}{c|c} K_k^{-1} & 0 \\ \hline -r_{k+1}^T K_k^{-1} & 1 \end{array}\right]$$

bottom row

$\div$ by $\sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1} \equiv e_{k+1}$

$$\Rightarrow \left[\begin{array}{c|c} I & K_k^{-1} r_{k+1} \\ \hline 0^T & 1 \end{array}\right] \qquad \left[\begin{array}{c|c} K_k^{-1} & 0 \\ \hline \dfrac{-r_{k+1}^T K_k^{-1}}{\sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1}} & \dfrac{1}{\sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1}} \end{array}\right]$$

Mult last row by $-K_k^{-1} r_{k+1}$ + Add to All rows above

$$\rightarrow \left[\begin{array}{c|c} I & 0 \\ \hline 0^T & 1 \end{array}\right] \qquad \left[\begin{array}{c|c} K_k^{-1} + e_{k+1}^{-1} g g^T & -e_{k+1}^{-1} g_{k+1} \\ \hline -e_{k+1}^{-1} g_{k+1}^T & e_{k+1}^{-1} \end{array}\right]$$

$\therefore$

$$K_{k+1}^{-1} \overset{?}{=} \left[\begin{array}{c|c} I & -g_{k+1} \\ \hline 0^T & 1 \end{array}\right] \left[\begin{array}{c|c} K_k^{-1} & 0 \\ \hline 0^T & 1/e_{k+1} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline -g_{k+1}^T & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} K_k^{-1} & -e_{k+1}^{-1} g_{k+1} \\ \hline 0^T & 1/e_{k+1} \end{array}\right] \left[\begin{array}{c|c} I & 0 \\ \hline -g_{k+1}^T & 1 \end{array}\right]$$

$$= \left[\begin{array}{c|c} K_k^{-1} + e_{k+1}^{-1} g_{k+1} g_{k+1}^T & -e_{k+1}^{-1} g_{k+1} \\ \hline -e_{k+1}^{-1} g_{k+1}^T & e_{k+1}^{-1} \end{array}\right]$$

Yes ✓

$$K_{k+1}^{-1} = \left[\begin{array}{c|c} K_k^{-1} & 0 \\ \hline 0^T & 0 \end{array}\right] + \left[\begin{array}{c|c} g_{k+1} e_{k+1}^{-1} g_{k+1}^T & -g_{k+1} e_{k+1}^{-1} \\ \hline -e_{k+1}^{-1} g_{k+1}^T & e_{k+1}^{-1} \end{array}\right]$$

$$+ \frac{1}{e_{k+1}} \cancel{\left[\begin{array}{c|c} g_{k+1} & g_{k+1} \\ \hline & 1 \end{array}\right]} \cancel{\left[\begin{array}{c|c} g_{k+1} & \\ \hline g_{k+1}^T & \end{array}\right]}$$

$$- \frac{1}{e_{k+1}} \cancel{\left[\begin{array}{c|c} 1 & \\ \hline g_{k+1}^T & \end{array}\right]} \cancel{\left[\begin{array}{c|c} 1 & g_{k+1} \\ \hline & \end{array}\right]}$$

$$\frac{1}{e_{k+1}} \left[\begin{array}{c} -g_{k+1} \\ \hline 1 \end{array}\right] \left[\begin{array}{c|c} -g_{k+1}^T & 1 \end{array}\right]$$ ✓

(103) 10.3c

$$\underbrace{\begin{bmatrix} I & -g_{k+1} \\ 0^T & 1 \end{bmatrix}\begin{bmatrix} K_k^{-1} & 0 \\ 0^T & e_{k+1}^{-1} \end{bmatrix}\begin{bmatrix} I & 0 \\ -g_{k+1}^T & 1 \end{bmatrix}}\begin{bmatrix} K_k & r_{k+1} \\ r_{k+1}^T & \sigma_{k+1}^2 \end{bmatrix}$$

$$\begin{bmatrix} K_k^{-1} + e_{k+1}^{-1} g_{k+1} g_{k+1}^T & -g_{k+1} e_{k+1}^{-1} \\ -e_{k+1}^{-1} g_{k+1}^T & e_{k+1}^{-1} \end{bmatrix}\begin{bmatrix} K_k & r_{k+1} \\ r_{k+1}^T & \sigma_{k+1}^2 \end{bmatrix}$$

$$= \begin{array}{c|c} I + e_{k+1}^{-1} g_{k+1} g_{k+1}^T K_k + \cancel{0} - e_{k+1}^{-1} g_{k+1} r_{k+1}^T & \cancel{-e_{k+1}^{-1} g_{k+1}^T r_{k+1}} \\ \hline -e_{k+1}^{-1} g_{k+1}^T K_k + e_{k+1}^{-1} r_{k+1}^T & \end{array}$$

$$\begin{array}{|c} K_k^{-1} r_{k+1} + \underline{e_{k+1}^{-1} g_{k+1} g_{k+1}^T r_{k+1}} - \underline{g_{k+1} e_{k+1}^{-1} \sigma_{k+1}^2} \\ \hline -e_{k+1}^{-1} g_{k+1}^T r_{k+1} + e_{k+1}^{-1} \sigma_{k+1}^2 \end{array}$$

w/ $g_{k+1} \equiv K_k^{-1} r_{k+1}$

$$e_{k+1} \equiv \sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1}$$

$$\Rightarrow \left[\begin{array}{c|c} I + e_{k+1}^{-1}(K_k^{-1} \cancel{r_{k+1}})(r_{k+1}^T K_k^{-1}) K_k - e_{k+1}^{-1}(\cancel{K_k^{-1}} r_{k+1} r_{k+1}^T) & K_k^{-1} r_{k+1} + \underline{e_{k+1}^{-1}(K_k^{-1} r_{k+1})(r_{k+1}^T K_k^{-1}) r_{k+1}} - K_k^{-1} r_{k+1} e_{k+1}^{-1} \sigma_{k+1}^2 \\ \hline -e_{k+1}^{-1} r_{k+1}^T \cancel{K_k^{-1}} K_k + \cancel{} & \underbrace{-e_{k+1}^{-1} r_{k+1}^T K_k^{-1} r_{k+1} + e_{k+1}^{-1} \sigma_{k+1}^2}_{\equiv 1} \end{array}\right]$$

look at elt (1,2)

$$K_k^{-1} r_{k+1} + e_{k+1}^{-1}(K_k^{-1} r_{k+1})\Big[\underbrace{r_{k+1}^T K_k^{-1} r_{k+1} - \sigma_{k+1}^2}_{-e_{k+1}^{-1}}\Big] = 0 \checkmark$$

(10.4) Assume problem should be

(1) ε_n orthogonal to x_{n-1}

then $y = A x_{n-1}$

$$E[\varepsilon_n y] = A E[\varepsilon_n x_{n-1}] = 0 \quad \checkmark$$

(2) $\varepsilon_n \equiv x_n - \tilde{x}_n = x_n - b^T x_{n-1}$

& $\varepsilon_n' = x_n - c^T x_{n-1}$

$\therefore \varepsilon_n - \varepsilon_n' = -(b-c)^T x_{n-1}$

$\Rightarrow \varepsilon_n' = \varepsilon_n + (b-c)^T x_{n-1} \quad \checkmark$

(3) $$E[(\varepsilon_n')^2] = E[(\varepsilon_n + (b-c)^T x_{n-1})^2]$$

$$= E[\varepsilon_n^2] + \overset{0}{\cancel{2E[\varepsilon_n (b-c)^T x_{n-1}]}} + E[((b-c)^T x_{n-1})^2]$$

$$= E[\varepsilon_n^2] \quad \checkmark$$

(4) $\varepsilon_n = x_n - b^T x_{n-1}$

$$\varepsilon_{min} = E[\varepsilon_n^2] = E[\varepsilon_n (x_n - b^T x_{n-1})]$$

$$= E[\varepsilon_n x_n] - b^T \cancelto{0}{E[\varepsilon_n x_{n-1}]} =$$

10.5 $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 1/3 & 4/5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5/3 & 0 \\ 0 & 0 & 8/5 \end{bmatrix} \begin{bmatrix} 1 & 2/3 & 1/3 \\ 0 & 1 & 4/5 \\ 0 & 0 & 1 \end{bmatrix}$$

Normal eqs: 10.43 are

$k_k g_{k+1} = r_{k+1}$ w/ $k_{k+1} = \left[\begin{array}{c|c} k_k & r_{k+1} \\ \hline r_{k+1}^T & \sigma_{k+1}^2 \end{array} \right]$

So w/ $k=1$ one gets

$k_1 = 3$; $k_2 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$; $k_3 = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$r_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

So $k_1 g_2 = r_2$

So $3 g_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \rightarrow g_2 = \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$

& $k_2 g_3 = r_3 \Rightarrow \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$

$$k_2 = \left[\begin{array}{c|c} k_1 & r_2 \\ \hline r_2^T & \sigma_2^2 \end{array}\right]$$

So $k_1 = 3$ ✓ ; $r_2 = 2$;

$\therefore$ Normal eq $\quad k_1 g_2 = r_2 \Rightarrow 3 g_2 = 2 \Rightarrow g_2 = 2/3$

$$k_3 = \left[\begin{array}{c|c} k_2 & r_3 \\ \hline r_3^T & \sigma_3^2 \end{array}\right]$$

w/ $k_2 = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$ $\quad r_3 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Normal eqs $\quad k_2 g_3 = r_3 \Rightarrow \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} g_3^{(1)} \\ g_3^{(2)} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} g_3^{(1)} \\ g_3^{(2)} \end{bmatrix} = \frac{1}{9-4}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{5}\begin{bmatrix} 3-4 \\ -2+6 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 \\ 4 \end{bmatrix} ** $$

consider $(1, g_2^T) = (1, 2/3)$ ✓

$(1, g_3^T) = (1, -1/5, 4/5) \sim X$

Eq 10.6 b

$$g_{k+1} = K_k^{-1} r_{k+1}$$

$$e_{k+1} = \sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1}$$

so $e_2 = \sigma_2^2 - 2(1/3)2 = 3 - \frac{4}{3} = \frac{5}{3}$ ✓

+ $e_3 = \sigma_3^2 - r_3^T K_2^{-1} r_3 = 3 - (1,2)\ \frac{1}{5}\begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= 3 - \frac{1}{5}(1,2)\begin{bmatrix} -1 \\ 4 \end{bmatrix} = 3 - \frac{1}{5}(-1+8)$

$= \frac{15}{5} - \frac{7}{5} = \frac{8}{5}$ ✓

(b) Eq 10.4

$$K_{k+1}^{-1} = \left[\begin{array}{c|c} I & -g_{k+1} \\ \hline 0^T & 1 \end{array}\right]\left[\begin{array}{c|c} K_k^{-1} & 0 \\ \hline 0^T & 1/e_{k+1} \end{array}\right]\left[\begin{array}{c|c} I & 0 \\ \hline -g_{k+1}^T & 1 \end{array}\right]$$

so $\cancel{K_1 = 3 \quad e_2 = \frac{5}{3}}$

$g_{k+1} = K_k^{-1} r_{k+1}$; $e_{k+1} = \sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1}$

(c) 10.47 − 10.49 + 10.51

10.51

$$e_{k+1} = e_k(1-\rho_k^2)$$

$$g_{k+1} = \begin{bmatrix} 0 \\ \hline g_k \end{bmatrix} + \begin{bmatrix} 1 \\ \hline -\tilde{g}_k \end{bmatrix}\rho_k$$

$$\rho_k = \left[x(k) - g_k^T \tilde{r}_k\right](1/e_k) \qquad e_k = x(0) - g_k^T r_k$$

So $k=1$ give ~~$e_1 = x(0) - g_1^T r_1 = 3 -$~~

$$e_2 = x(0) - g_2^T r_2 = 3 - (2/3)(2) = \frac{9-4}{3} = \frac{5}{3}$$

$$\rho_2 = \left[x(2) - g_2^T \tilde{r}_1\right](1/e_2)$$

$$= \left[1 - (2/3)(2)\right](3/5) = \left(-\tfrac{1}{3}\right)(3/5) = -1/5$$

Then

$$g_3 \stackrel{?}{=} \begin{bmatrix} 0 \\ 2/3 \end{bmatrix} + \begin{bmatrix} 1 \\ -2/3 \end{bmatrix}(-1/5) = \begin{bmatrix} -1/5 \\ \frac{2}{3}\left(1+\frac{1}{5}\right) \end{bmatrix} = \begin{bmatrix} -1/5 \\ \frac{2}{3}\left(\frac{6}{5}\right) \end{bmatrix}$$

$$= \begin{bmatrix} -1/5 \\ \frac{4}{5} \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -1 \\ 4 \end{bmatrix} \checkmark \text{ Yes}$$

(10.6) $\gamma_x(\ell) = \dfrac{\sigma_w^2}{1-a^2} a^{|\ell|}$

Show

$$g_k = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ a \end{bmatrix} \qquad e_k = \sigma_w^2 \qquad \forall k > 1$$

$$K_k = \frac{\sigma_w^2}{1-a^2} \begin{bmatrix} 1 & a & a^2 & \cdots & a^{k-1} \\ a & 1 & a & \cdots & a^{k-2} \\ \vdots & & & & a \\ a^{k-1} & & & & 1 \end{bmatrix}$$

So Normal eqs for g_{k+1} are $\quad g_{k+1} = K_k^{-1} r_{k+1}$

& $$r_{k+1} = \begin{bmatrix} a^k \\ a^{k-1} \\ \vdots \\ a \end{bmatrix} = a \begin{bmatrix} a^{k-1} \\ a^{k-2} \\ \vdots \\ 1 \end{bmatrix} \cdot \frac{\sigma_w^2}{1-a^2}$$

So $$g_{k+1} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ a \end{bmatrix}$$ ✓

Then

$$e_{k+1} = \sigma_{k+1}^2 - r_{k+1}^T K_k^{-1} r_{k+1} = \sigma_{k+1}^2 - r_{k+1}^T g_{k+1}$$

$$= \sigma_{k+1}^2 - \frac{a\sigma_w^2}{1-a^2} a = \frac{\sigma_w^2}{1-a^2}(1-a^2) = \sigma_w^2$$ ✓

(10.7) $x_k = a x_{k-1} + w_k$

$$R_x(l) = \frac{\sigma_w^2}{1-a^2} a^{|l|} \qquad R_k = \frac{\sigma_w^2}{1-a^2}\begin{bmatrix} 1 & a & a^2 & & a^{k-1} \\ a & 1 & a & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}$$

so Assuming ($x_0 = 0$)

$x_1 = w_1 = .226$

$x_2 = a x_1 + w_2 = (.95)(.226) + .403 = \cdots$

$\vdots$

$m^{(1)} = m^{(2)} = 0 \qquad e_1^{(i)}$

$\varepsilon^1_{k+1} = x^1_{k+1} - g^{(i)T}_{k+1} x^{(i)}$? Not sure ...

Pg 212 Thermic

(11.1) eq 11.3

$$P_i^{(r+1)}(k) = \underset{l \neq k}{\mathrm{Avg}} \left[\frac{\lambda_i^{(l)}(k;l)\, P_i^{(r)}(k)}{\sum_{j=1}^{N_c} \lambda_j^{(r)}(k;l)\, P_j^{(r)}(k)} \right] \qquad P_i^{(0)}(k)$$

?

(11.2)

$$P_i^{(r+1)}(k) = \frac{P_i^{(r)}(k) \prod_{l=1}^{N} \lambda_i^{(r)}(k;l)}{\sum_{j=1}^{N_c} P_j^{(r)}(k) \prod_{l=1}^{N} \lambda_j^{(r)}(k;l)}$$

v.s.

$$P_i^{(r+1)}(k) = \frac{\lambda_i^{(r)}(k)\, P_i^{(r)}(k)}{\sum_{j=1}^{N_c} \lambda_j^{(r)}(k)\, P_j^{(r)}(k)}$$

?

(11.3) ?

(11.4)

$$p_i(y) = \frac{\sum_{k=1}^{N} r(y - y(k))\, P_{i|F}(k)}{\sum_{k=1}^{N_c} P_{i|F}(k)}$$

?

11.5 ?

11.6

(a) ?

(b)

11.7 11.53 $$\sum_{\underline{\ell}_t} C(\underline{\ell}, \underline{\ell}_t) \Pr[\underline{\ell}_t | P] \prod_{k=1}^{N} p_Y(y_k, \omega_{t_k})$$

?

11.8 ?

(12.1) Eq $J = \sum_{k=1}^{N_c} \sum_{y^{(i)} \sim \omega_k} |y^{(i)} - \mu_k|^2$ 12.4

Let $\Upsilon = \{y^{(i)}\}$ + $\Omega = \{\omega_{s_i}\}$ be fixed then minimize

J. ~~w~~ w.r.t. μ_k

$$\frac{\partial J}{\partial \mu_k} = \sum_{y^{(i)} \sim \omega_k} \frac{\partial}{\partial \mu_k} |y^{(i)} - \mu_k|^2 = 0$$

Now $\frac{\partial}{\partial \vec{x}} |y - x|^2 = \frac{\partial}{\partial x_i} \sum_{l=1}^{n} (y_l - x_l)^2 = 2(y_i - x_i)(-1)$

$$\therefore \quad \frac{\partial J}{\partial \mu_k} = \sum_{y^{(i)} \sim \omega_k} -2(y^{(i)} - \mu_k) \underset{\substack{\uparrow \\ \text{set}}}{=} 0$$

Here $y^{(i)}$ + μ_k are vectors

Thus $\Rightarrow$ $\sum_{y^{(i)} \sim \omega_k} y^{(i)} = \sum_{y^{(i)} \sim \omega_k} \mu_k = N_k \mu_k$ since μ_k is constant w.r.t. $y^{(i)}$

Thus $\mu_k = \frac{1}{N_k} \sum_{y^{(i)} \sim \omega_k} y^{(i)}$ ✓

Assume μ_k are fixed then classify $y^{(i)}$ to class ω_{s_i} that minimizes

$|\Delta J_i|$ is

$$\Delta J_i = \sum_{k=1}^{N_c} \sum_{\substack{y^{(i)} \sim \omega_k \\ y^{(i)} \in \omega_{k_1}}} |y^{(i)} - \mu_k|^2 - \sum_{k=1}^{N_c} \sum_{\substack{y^{(i)} \sim \omega_k \\ y^{(i)} \in \omega_{k_2}}} |y^{(i)} - \mu_k|$$

$$= \underbrace{|y^{(i)} - \mu_{k_1}|^2}_{y^{(i)} \text{ is classified as } k_1} - \underbrace{|y^{(i)} - \mu_{k_2}|^2}_{y^{(i)} \text{ is classified as } k_2}$$

So to minimize ΔJ_i [redacted] [redacted] w.r.t. k_1 + k_2, for k_1 fixed

+ All k_2 + sim for k_2 fixed + $\forall$ k_1

Then $\min_{(k_1,k_2)} |\Delta J_i| = \ldots$

(12.2) $p(\hat{T}) = \begin{cases} 30 e^{-30\hat{T}} \\ 0 \end{cases}$

(a) Decide if Grumble Little has closed

Q: Are all 4 measurements independent?

$p(T) = \prod_{i=1}^{4} p(\hat{T})$

?

(12.3) $p(x|H_1) \sim N(m_x, \sigma_x^2)$ $p(x|H_2) \sim N(0, \sigma_x^2)$

$p(y|H_1) \sim N(m_y, \sigma_y^2)$ $p(y|H_2) \sim N(0, \sigma_y^2)$

$P_{\Lambda_x|\omega_i} =$? $P_{\Lambda_y|\omega_i} =$?

?

(12.4) ?