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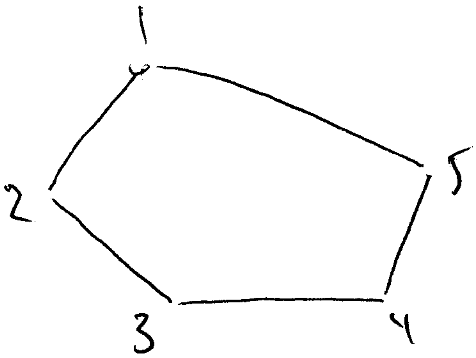
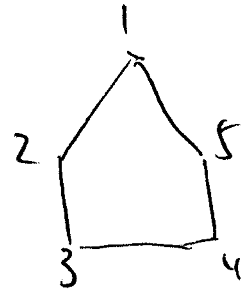
05-26-02



C_5



C_5

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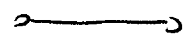
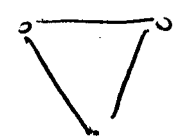
K_1

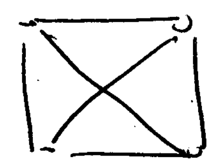
K_2



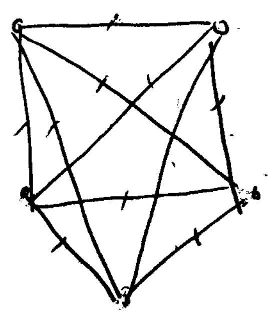
K_3



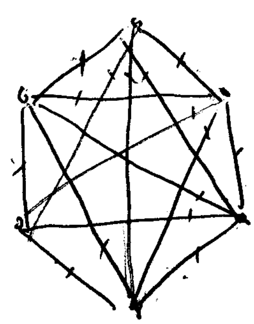
K_4



K_5



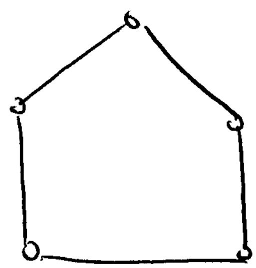
K_6



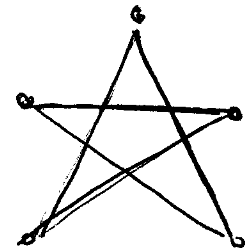
K_7

Drawn in text.

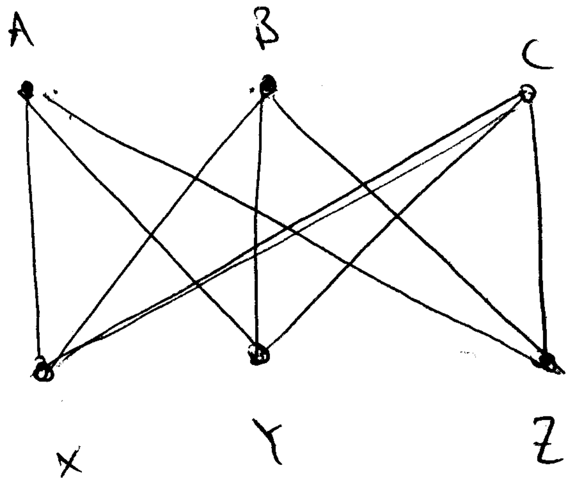
v	e
1	0
2	1
3	3
4	6
5	10
6	15
7	21
8	28



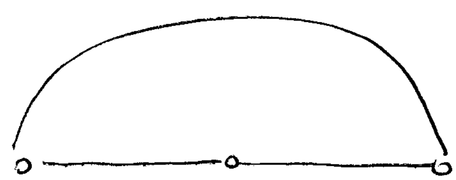
C_5

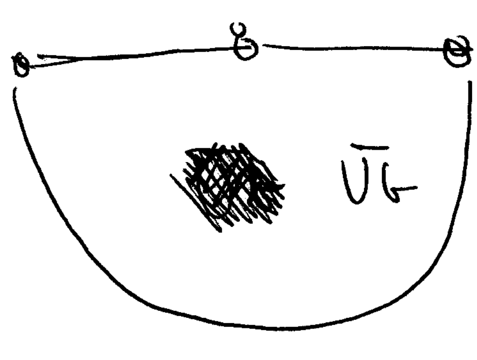


$\overline{C}_5$



UG





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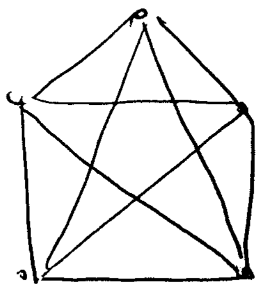
K_5



K_5

(1) ϕ, $\{1,2,3\}$
$\{1\}$ $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$

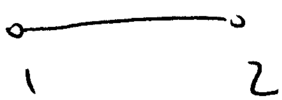
(2) Pr: J($=\phi$) is a subset of A, cannot be ~~~~ false?

(3) If the barber shaves himself then there is a contradiction in that the barber only shaves men who do not shave themselves. If the barber does not shave himself, then there is a contradiction in that the barber is a man who does not shave himself & lives in the village. Thus by his definition he should be shaved by himself but this again is a contradiction

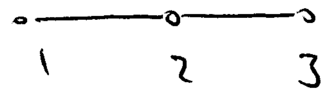
This is equivalent to Russell's Paradox in that the ~~barber~~ rule that determines the barbers membership does not allow a consistent (w/ the rules of logic) membership for him.

(4) ~~~~ No, S is not a set since the discription of S is less than 25 words. thus S would contain itself. As having S contain itself is forbidden by the definition of a set S cannot be a set

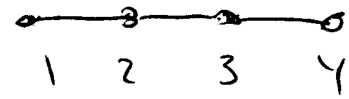
(5) P_2



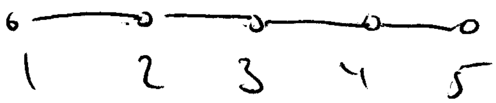
P_3



P_4



P_5



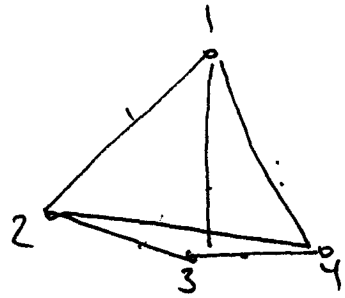
v	e
2	1
3	2
4	3
5	4

conjecture $v = e + 1$ or $e = v - 1$

(6)

W_4 vertex set $\{1,2,3,4\}$

edge set $\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{4,2\}\}$

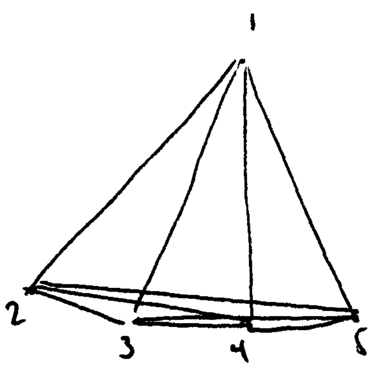


$e_4 = 6$

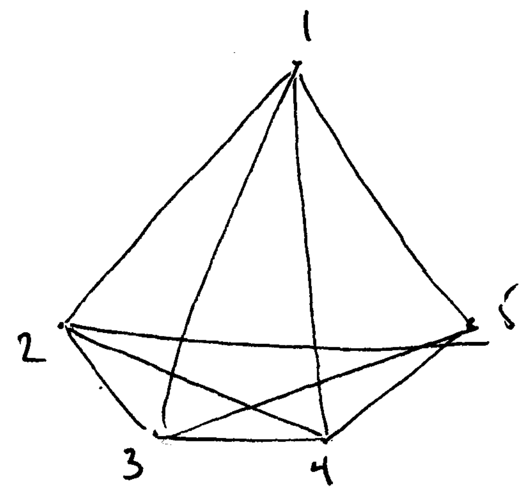
$= 3+2+1$

W_5 v. set $\{1,2,3,4,5\}$

edge set $\{\{1,2\},\{1,3\},\{1,4\},\{1,5\},\{2,3\},\{2,4\},\{2,5\}$
$\{3,4\},\{3,5\},\{4,5\},\{5,2\}\}$

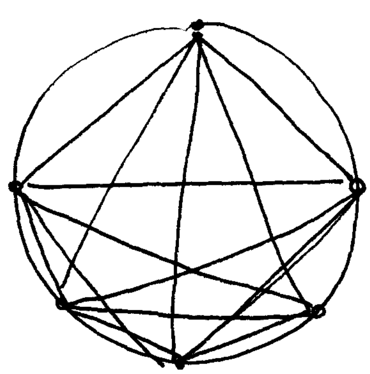


or



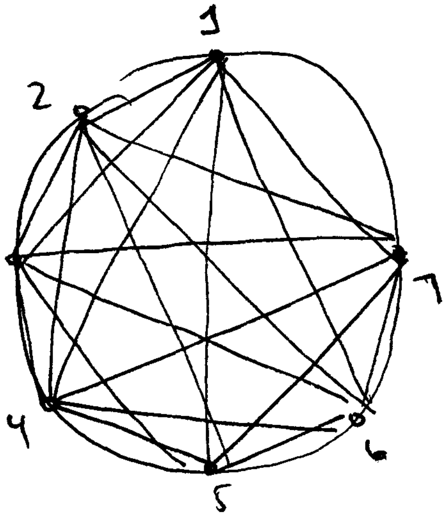
$e_5 = 4+3+2+1$

W_6

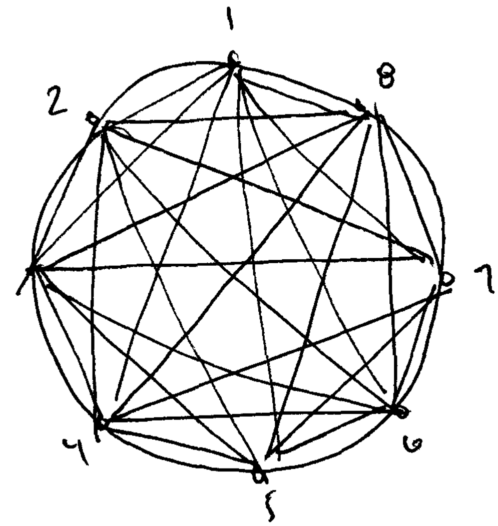


$$e_6 = \sum_{k=1}^{5} k$$

W_7



W_8



This look alot like $\underline{\underline{K_8}}$

edges in wheel graph is given by

$$e = \binom{t}{2}$$

$$e_n = \sum_{k=1}^{n-1} k = \frac{n(n-1)}{2}.$$

How do I prove this like the book would like me too?

(7) ~~B~~. From Thm #2

$$e = \frac{1}{2}v(v-1) = (v-1) + (v-2) + \cdots + 2 + 1$$

edges from vertex 1 to all others

of edges from vertex # 2 to all oth

(8) Max # of edges given v vertices is $\binom{v}{2}$

# of edges in the graph K_v

So the # of edges $\bar{G}$ must have would be $\binom{v}{2} - e$

$$\Rightarrow \frac{v!}{2!(v-2)!} - e = \frac{v(v-1)}{2} - e$$

(9) G has $v = 6$ then K_6 has $\frac{1}{2}6(5) = 15$ edges odd.

Any given graph that has v vertices is a subgraph of K_v.

+ thus can have at most $\frac{1}{2}v(v-1)$ edges

$\therefore$ $e_G + e_{\bar{G}} = 15$

$\Rightarrow$ that either G or $\bar{G}$ has ~~an~~ more than 6.5 edges i.e. 7 edges

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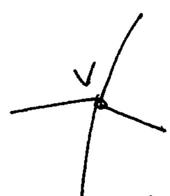
(10) w/ 6 people, considering each person as a vertex, Problem #9 prs that either G or $\bar{G}$ contains a subgraph isomorphic to K_3. Considering vertices to be people & edges to be the relationship friend ship. the theorem in problem #9 claims that at least 3 people are mutual friends (~~a edge~~ G contains a subgraph of K_3) or 3 people are mutually unaquainted. ($\bar{G}$ contains a subgraph of K_3)

(11) consider a vertex. v



As the degree of this vertex (d) represents a count on "d" edges. For each of the d edges the opposing vertex would also contain the edge. Thus

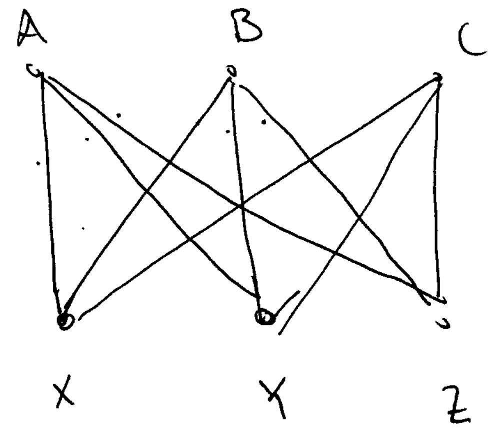
$$\sum_{v \in V} \deg(v) = 2e$$

(12) (a) $\sum_{v \in V} \deg(v) = 2e$

$= 4(3) + 5(2) + 6(2) + 8 =$

$= 12 + 10 + 12 + 8 = 30 + 12 = 42. \Rightarrow \#\text{ edges} = 21$

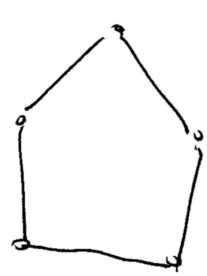
(b)



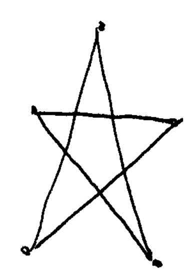
For utility graph: $\sum_{v \in V} \deg(v) = 6(3) = 18 = 2e \Rightarrow e = 9$ yes ✓.

For hypothetical graph: $\sum_{v \in V} \deg(v) = 7(3) = 21$ is odd $\rightarrow\!\!\!\!\times$.

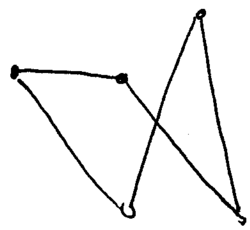
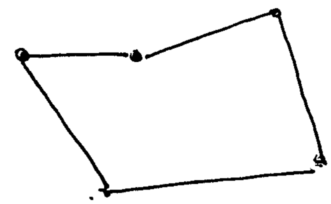
(13) $\underline{C_5}$:



$\overline{C_5}$:

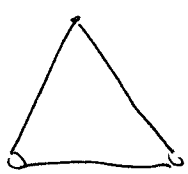


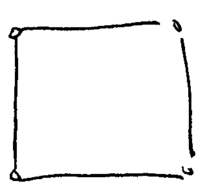
Pr $C_5 \cong \bar{C}_5$.

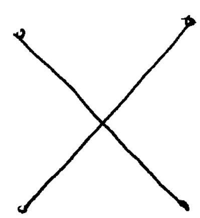
$\bar{C}_5 \cong$  $\cong$  $\cong C_5$ ✓.

Pr: No othe cyclic graph is isomorphic to its complement

C_3 ⇒ would be.



C_4: 

$\bar{C}_4$ 

$C_4 \not\cong \bar{C}_4$

The # of edges of the complementary graph is
$\bar{C}_v$ has $|K_v| - v$.

$$= \frac{1}{2}v(v-1) - v = v$$

$$= v^2 - v - 2v = 2v \qquad \div v$$

$$= v - 1 - 2 = 2 \quad \Rightarrow \quad v = 5.$$ the only one

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(14) $G \cong \bar{G}$

let $e = \#$ of edges in Graph G

& $v = \#$ of vertices in Graph G.

Then $\#$ edges in graph $\bar{G}$ is $\frac{1}{2}v(v-1) - e = e$

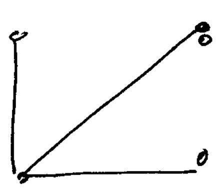
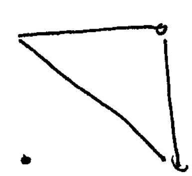
$\Rightarrow \frac{1}{2}v(v-1) = 2e$

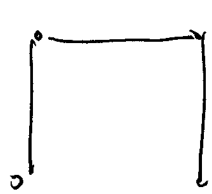
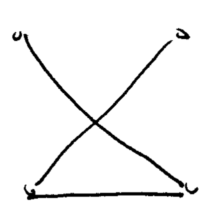
$\Rightarrow \frac{1}{4}v(v-1) = e$ since e is an integer either v or

$v-1$ must be $\div$ by 4.

(15) let $v-1 = 4 \Rightarrow v = 5$. $e = \frac{1}{4}(5)(4) = 5$.

let $v = 4$. $\Rightarrow e = \frac{1}{4}(4)(3) = 3$

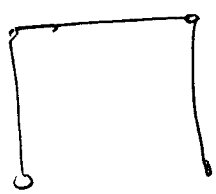
G:  $\bar{G}$:  No. $G \not\cong \bar{G}$

G:  $\bar{G}$:  Yes isomorphic let ~~[illegible]~~ $v = 8$ # $e = \frac{1}{4}(8)(7) = 14$.

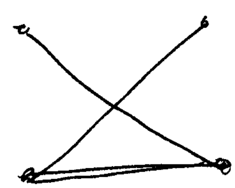
. . .

(16) # graphs w/ $v=8$ = ~~[illegible]~~ 12,346
unique (ie: equivalent up to isomorphism)

(17) The self complementary graph is



w/ complement



(18)

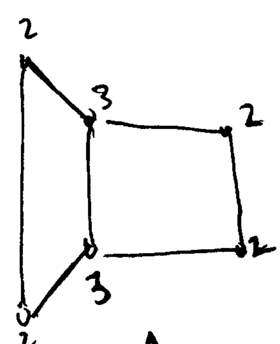
n choices for 1st mapping $\quad n-1$ choices for 2nd $\quad \ldots \quad 2 \quad = n!$

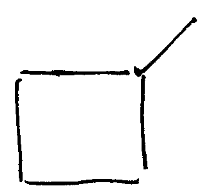
(19)

$$\begin{Bmatrix} 1 \\ 2 \\ 3 \\ \vdots \end{Bmatrix} \xrightarrow{\cdot 2} \begin{Bmatrix} 2 \\ 4 \\ 6 \\ \vdots \end{Bmatrix}$$

$\therefore x_2 = 2x_1$ is a 1-1 correspond

(20) consider the subgraph A C E G this is a 4 sided 4 vertex object.





Since ↑ has a vertex of order 3 I have to take a subgraph of the right graph that I take 2, 4 or 5 + 7

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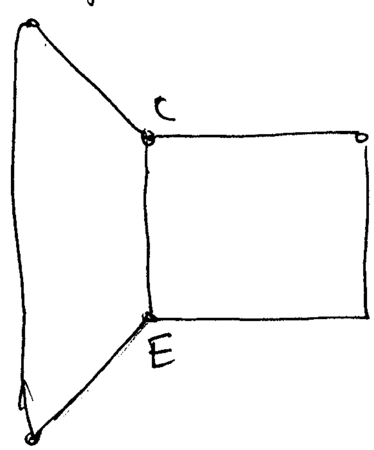
(20) ~~B~~ One way to say two graphs are not isomorphic is to find a subgraph of one graph that is not a member of the other graph

Pick as a subgraph:



The $\exists$ two vertices of degree 3. Since $\Rightarrow$ we would have to pick vertex 7, 5, 4, or 2 to correspond to C or E

Since C + E are connected by an edge. we must select 7, 5 or 4, 2 as the matches for C + E (irrespective) of order

If I map C $\leftrightarrow$ 4 + E $\leftrightarrow$ 2

— — — —

It suffices to say that C_6 is a subgraph of the 2nd graph while C_6 is not a subgraph of the 1st graph

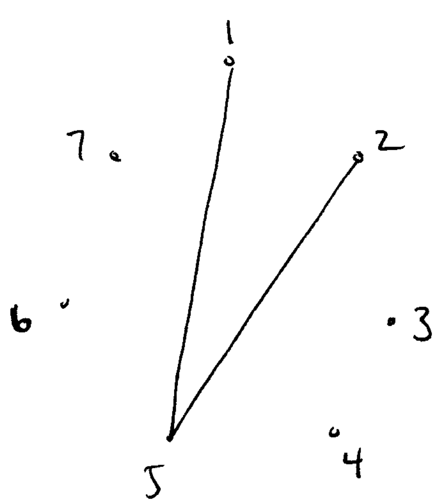
(21) ?

(22) ?

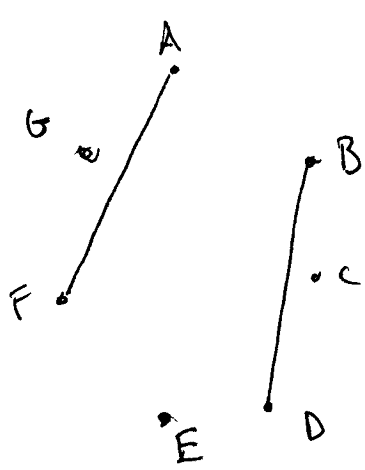
(23) The graph on the left has a subgraph isomorphic to C_3. While the graph on the right has no subgraphs isomorphic to C_3

(24) When a graph has many edges it can help to look at the complements of both graphs, if they are not isomorphic then the original graphs are not either

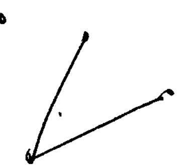
The left graph in Fig 31 becomes:



The right graph in Fig 31 becomes:

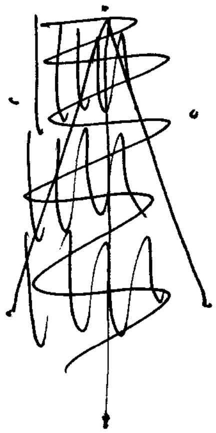


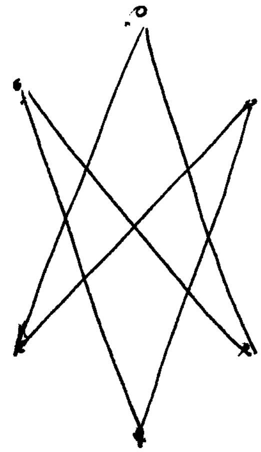
Since the left graph's complement has a graph isomorphic to



& the right graph does not, they cannot be isomorphic.

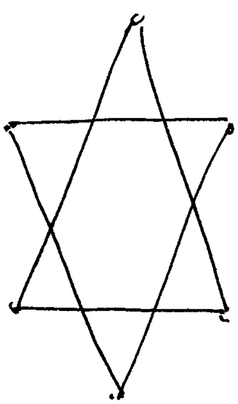
(25) Consider the complement of the graph on the left in Fig 46.





This graph is isomorphic to C_6 & has 1 component.

Consider the complement of the graph on the right in Fig 46.



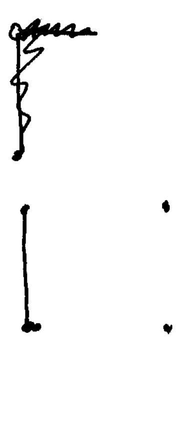
This graph has 2 components.

& ∴ The original two graphs cannot be isomorphic.

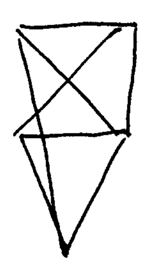
(26)

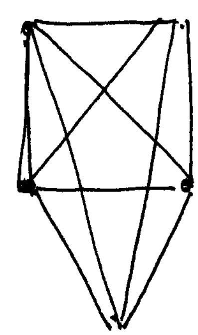
N_5

0 edges



1 edges

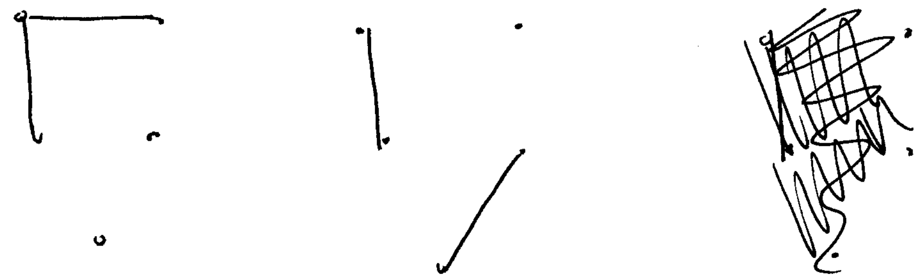




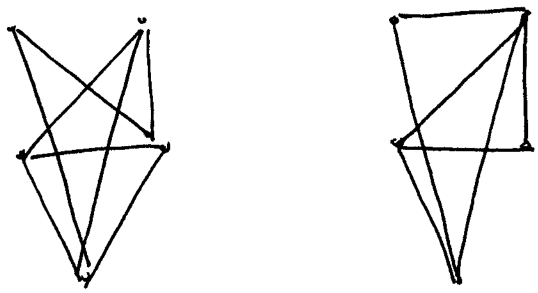
K_5

$\frac{1}{2}(5)(4) = 10$ edges

2 edges



w/ complements:



$$\begin{array}{r} 34 \\ -8 \\ \hline 26. \end{array}$$

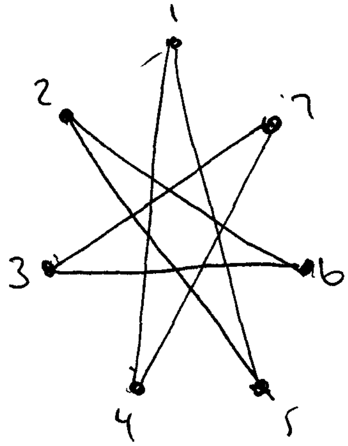
...

(27) w/ $v=4$ one graph has the property that $\forall$ $\bar{G} = G$.

& specifically this happens when $\# \text{ edges} = \dfrac{\# \text{edges max. } \frac{1}{2}v(v-1)}{2}$

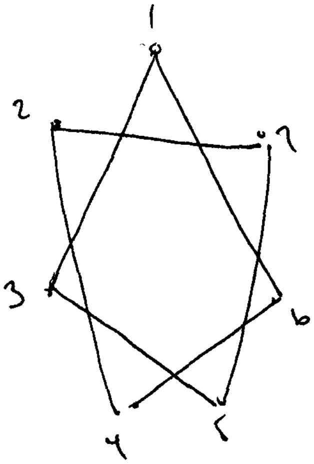
$\Rightarrow \# \text{edges} = \frac{1}{4}v(v-1)$ ... ?

(28) ~~Bt~~ ~~on~~ when there are a great # of edges working w/ the complement graph is often easier.



This is clearly isomorphic to C_7.

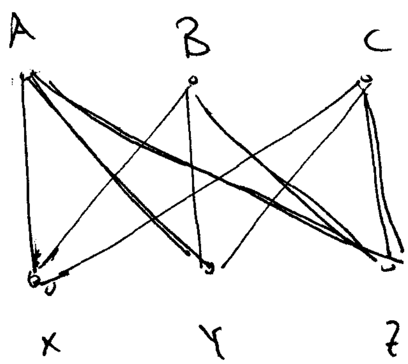
Now for the 2nd graph in Fig 47.



This has 2 components each isomorphic to C_4.

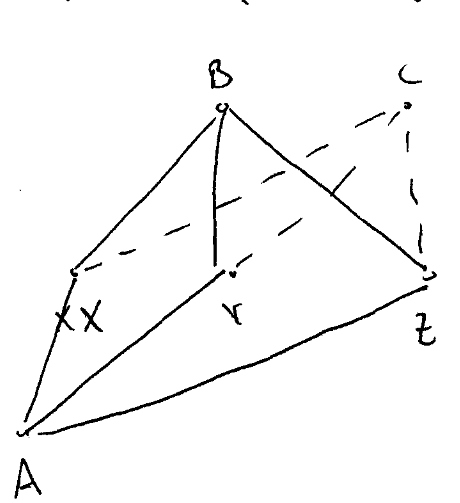
I think that the graphs are not isomorphic

(29)

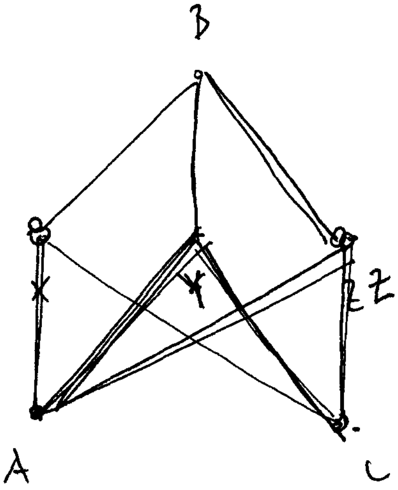


Utility graph

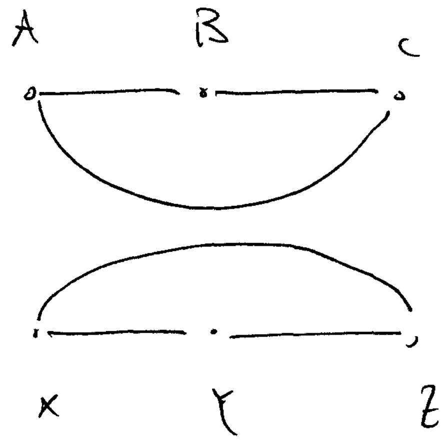
$\cong$



$\cong$



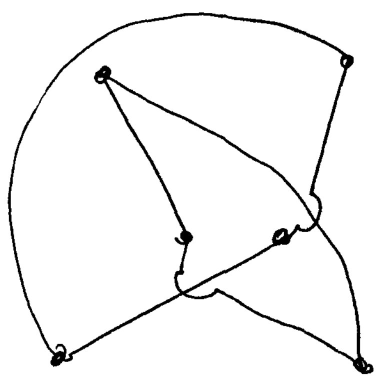
The complement of the utility graph should be particularly simple since (A utility graph should be close to being connected



It is isomorphic to 2 C_3 graphs

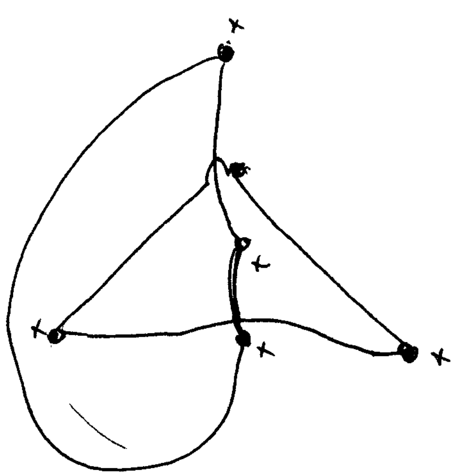
Now for each of the graphs in Fig 48, the complements are...

(a)



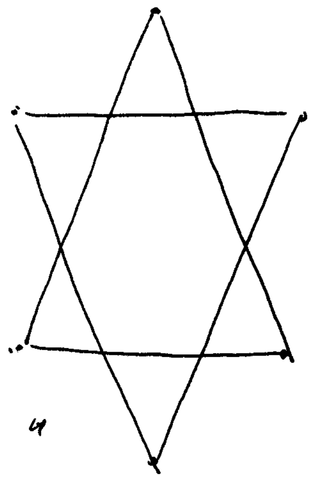
As this graph is isomorphic to two graphs isomorphic to C_3 the result follows

(b)



Again isomorphic to 2 C_3 graphs.

(c) ..



Isomorphic to 2 C_3 graphs. ✓

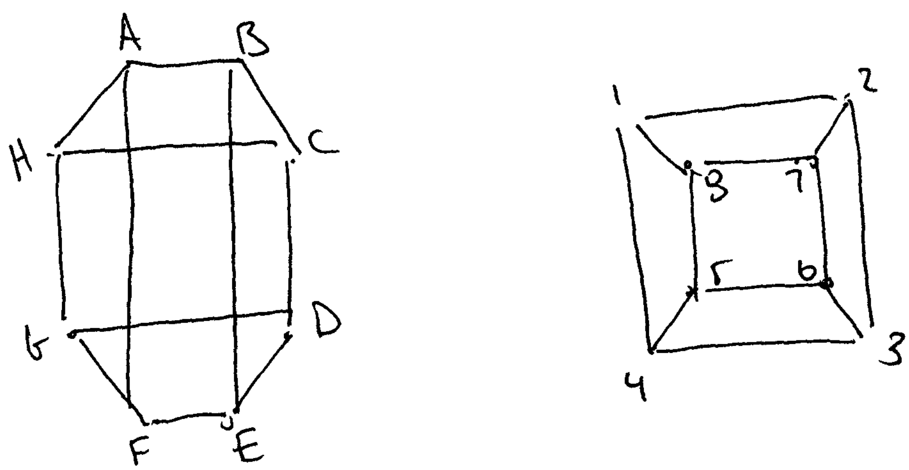
30-37

Figure 49

Not isomorphic, the graph on the right has a vertex w/ degree 4 & the graph on the left does not.

Fig 50:

These graphs are isomorphic. consider the labelling:



let $A \leftrightarrow 8$ $\quad H \leftrightarrow 1$ be the reordering.

$7 \leftrightarrow B$ $\quad C \leftrightarrow 2$

$F \leftrightarrow 5$ $\quad G \leftrightarrow 4$

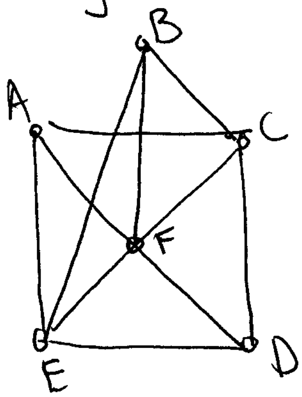
$E \leftrightarrow 6$ $\quad D \leftrightarrow 3$

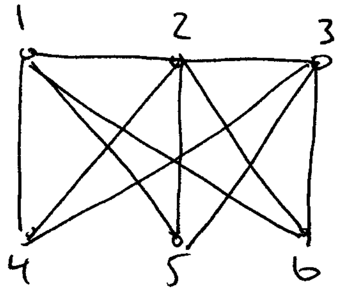
$A \to B \to C \approx 8 \to 7 \to 2$ ✓

Fig 51:

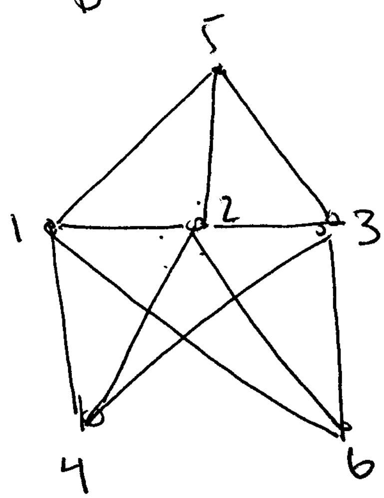
Not isomorphic. The left graph has a subgraph isomorphic to C_3 but the right graph does not.

Fig 52:

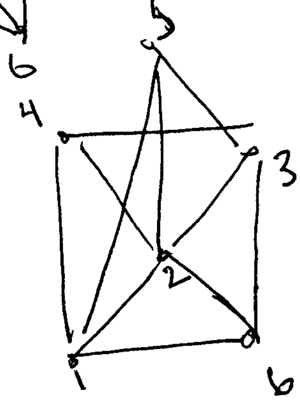




$\cong$



$\cong$



Yes they are isomorphic

Thus the correspondence is

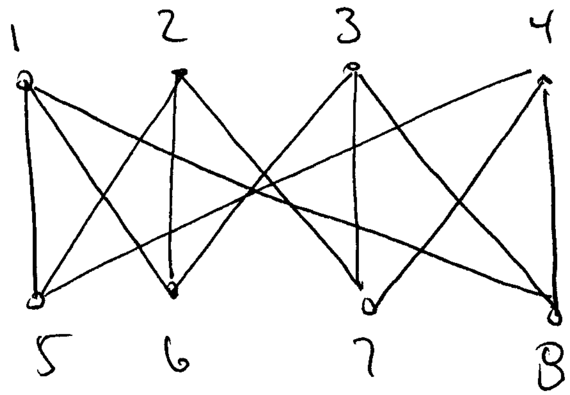
$E \leftrightarrow 1$ $\quad B \leftrightarrow 5$

$D \leftrightarrow 6$ $\quad A \leftrightarrow 4$

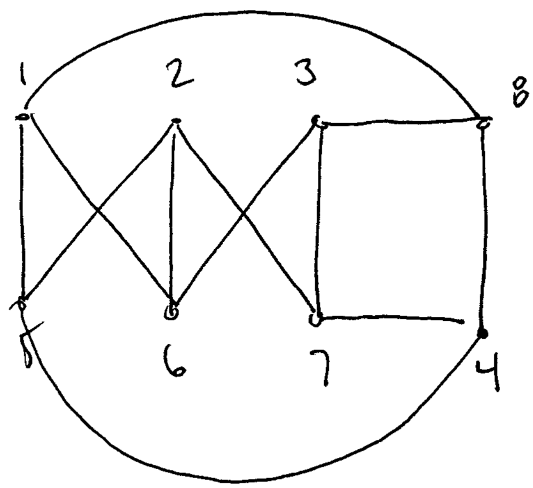
$F \leftrightarrow 2$

$C \leftrightarrow 3$

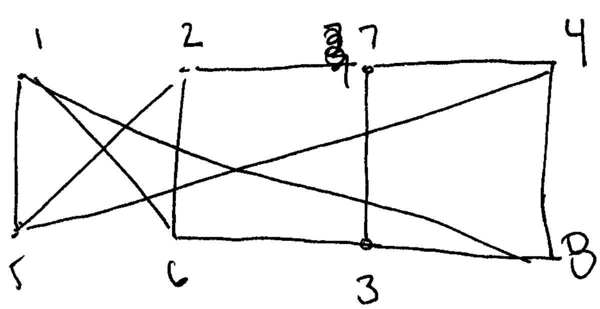
Fig 03:



$\cong$ flip 4-8

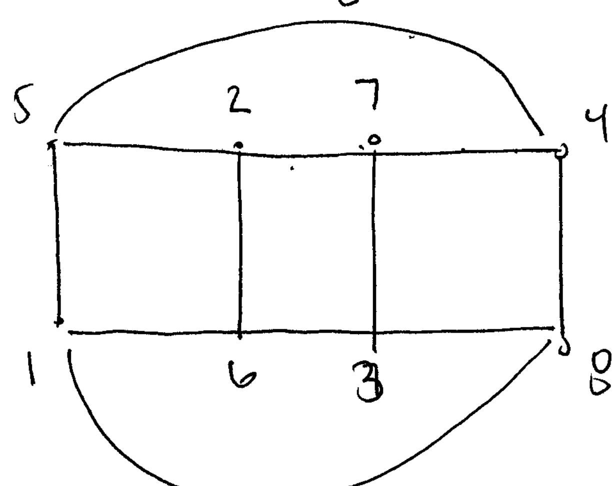


$\cong$ flip squen 3,8,4,7

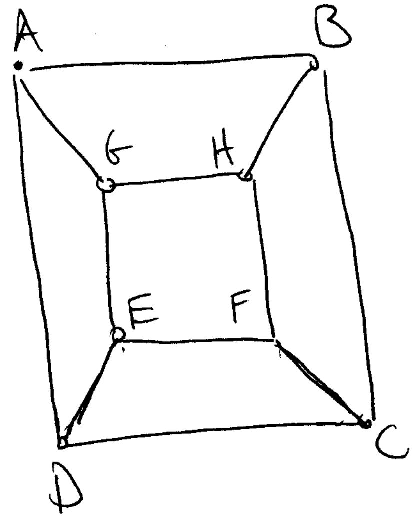


$\cong$ flip 1 + 5

$\Rightarrow$



This we can see is isomorphic to the graph in the left w/ Relabeling



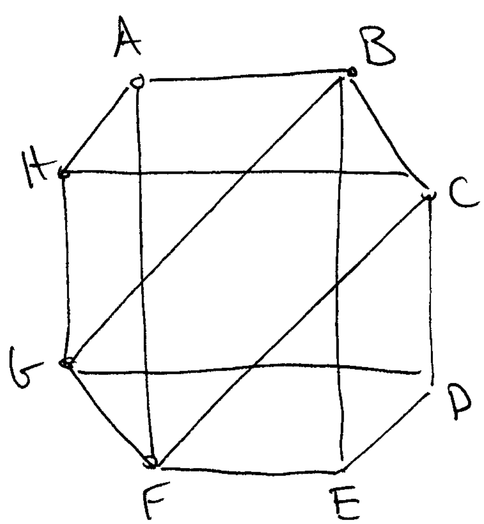
A ↔ 5 1 ↔ D

2 ↔ G 6 ↔ E

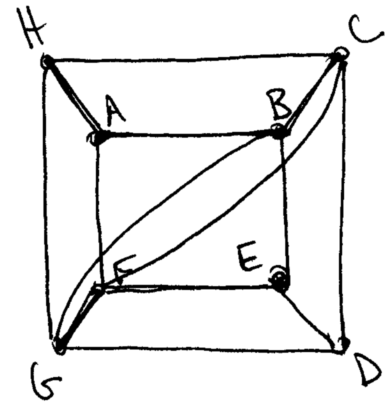
7 ↔ H 3 ↔ F

4 ↔ B 8 ↔ C.

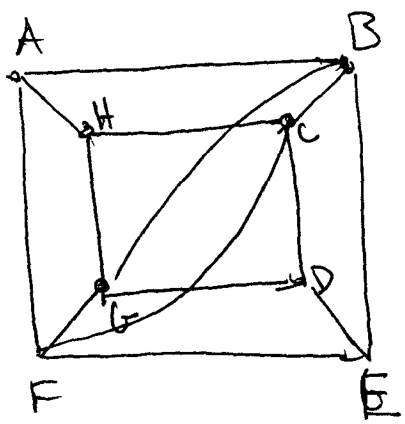
Figure 54



$\cong$

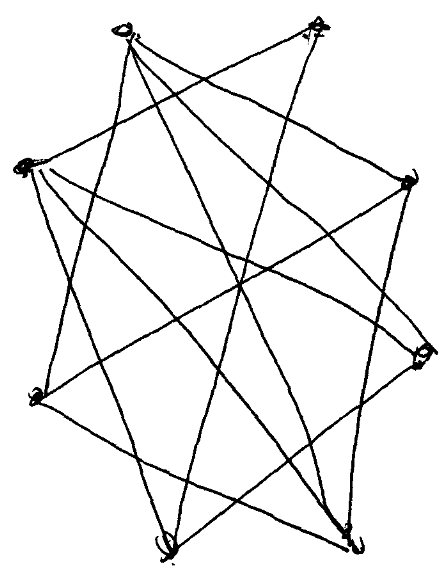


$\cong$

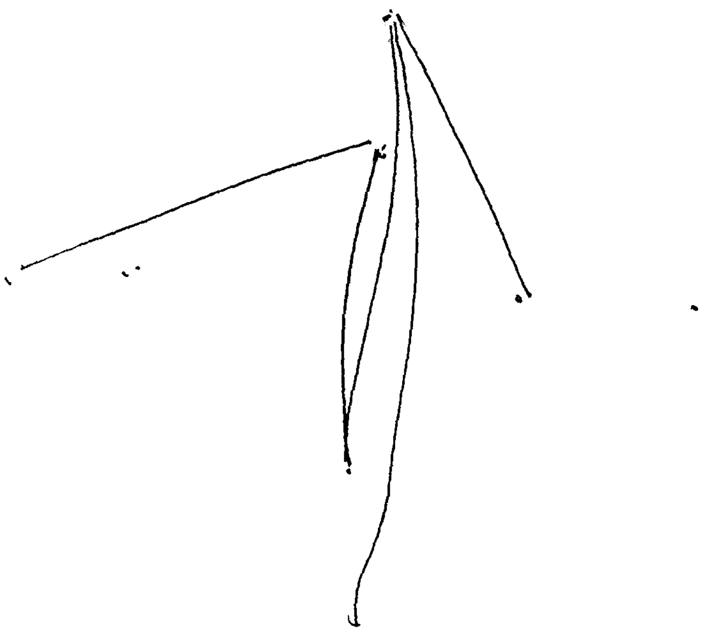


As this gets harder I am starting to think that it is not possible

Consider the complement of the graph on the left

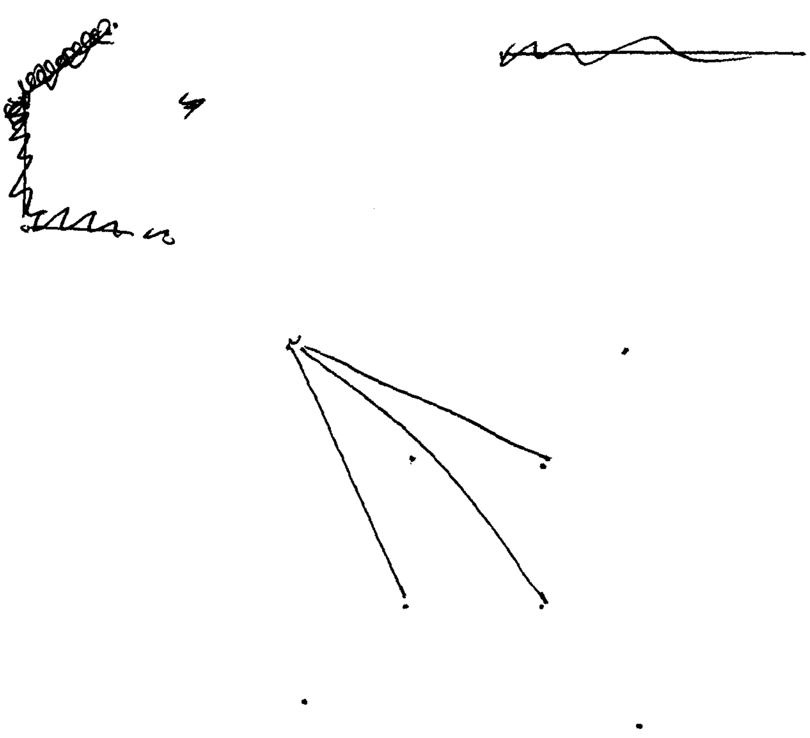


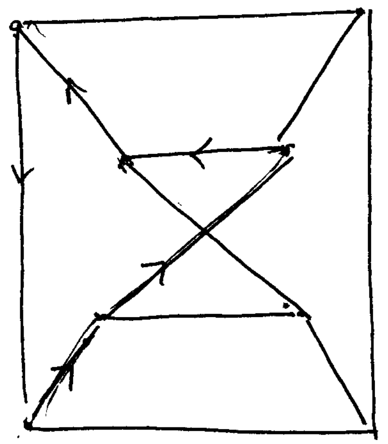
The complement of the graph on the Right is.



... No they are not isomorphic The graph on the right has several graphs isomorphic to C_3 while none exist in the left graph

Fig 55:



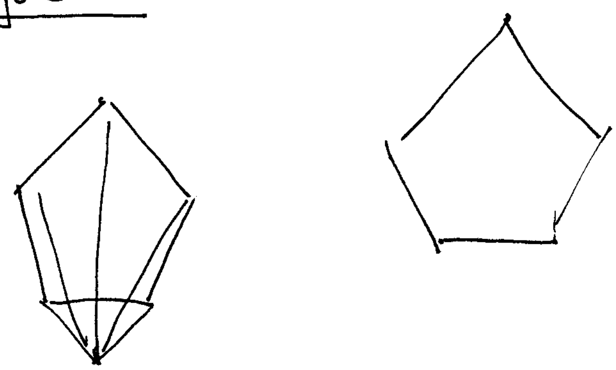


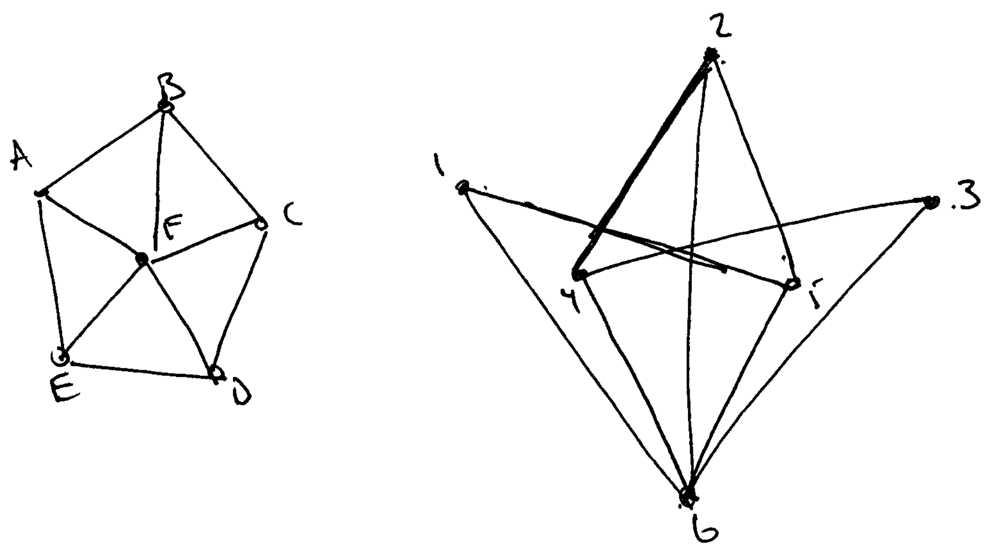
Note: path shown is isomorphic to C_5

Z_5.

Since the graph on the right has no such path the two graphs cannot be isomorphic

Figure 8b:





An isomorphism is

F ⇔ 6 B ⇔ 2 6 − 4 − 3 ⇒ F − E − A

E ⇔ 4

D ⇔ 5

C ⇔ 1

A ⇔ 3

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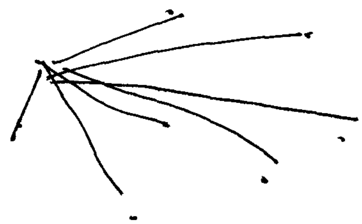
(38)



Think of each player being a node & the # of edges being a match. This is the # of edges in the complete graph

$k_{20} = \frac{1}{2}(20)(20-1)$

$k_{25} = \frac{1}{2}(25)(25-1)$

(39) Given a graph G w/ v vertices

We can lable this graph differently by placing different labels at each vertex, I would think that there would be v! different labeling.

RJ62 Trudea

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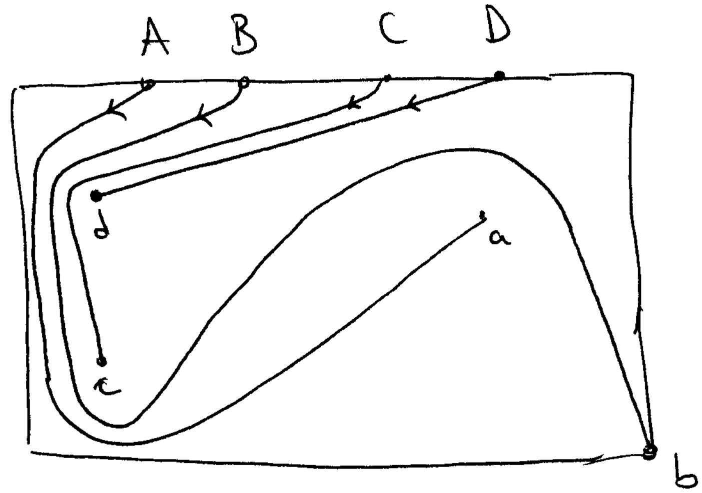
40



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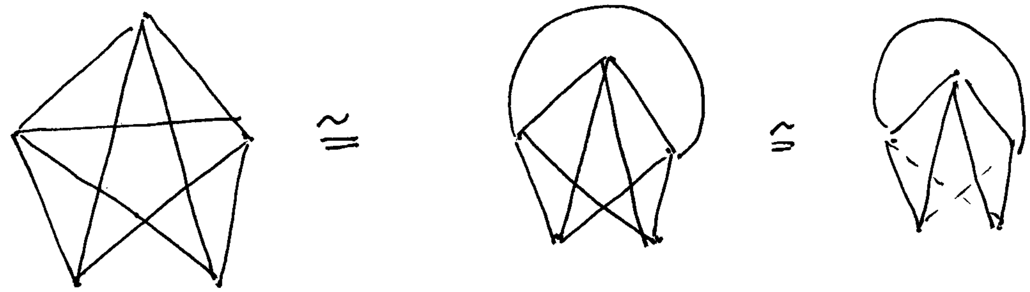
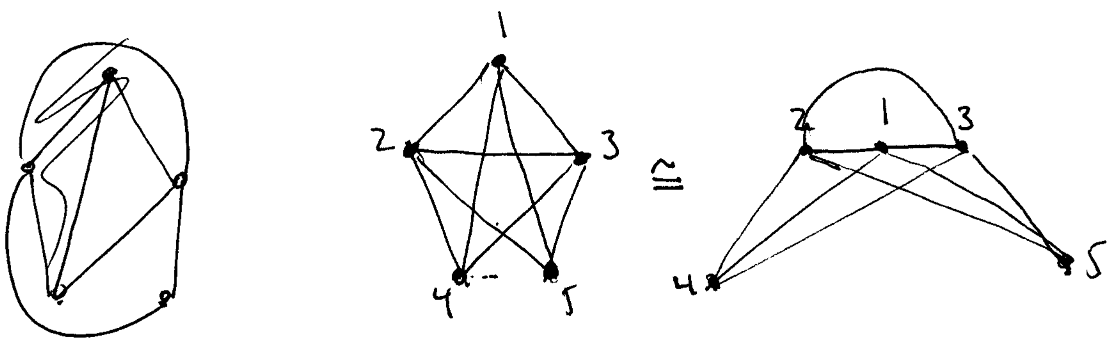
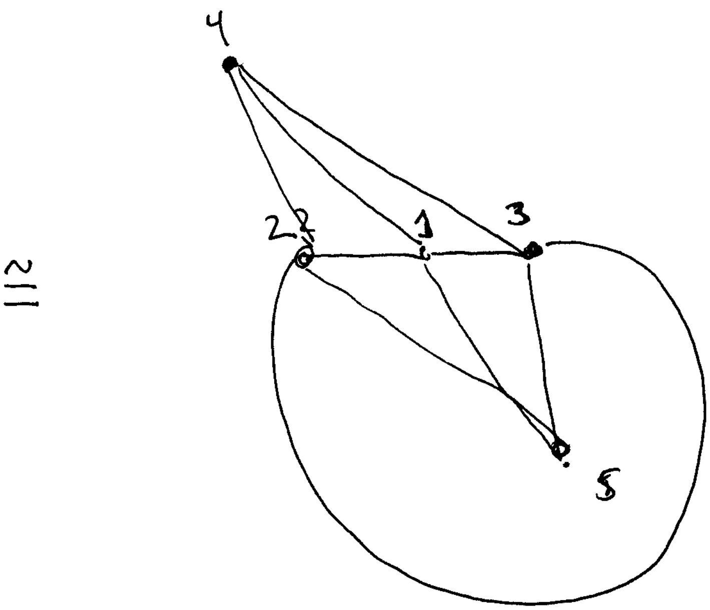


Fig 58 (a)





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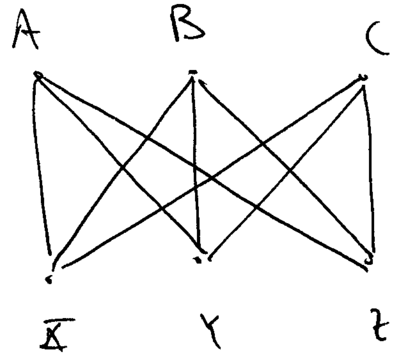
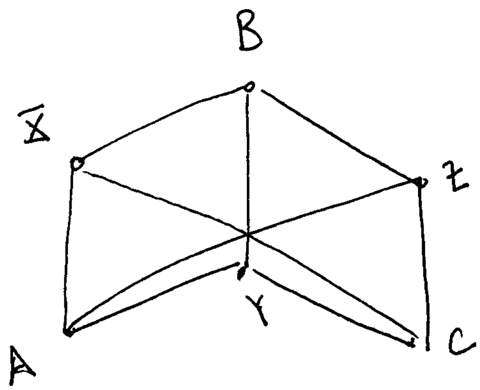
Uf.



≅



≅

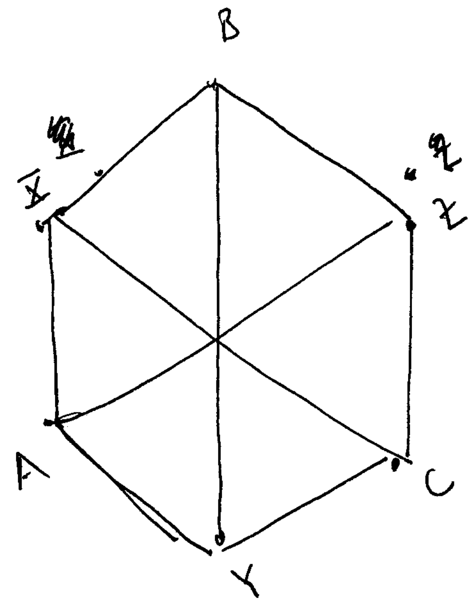
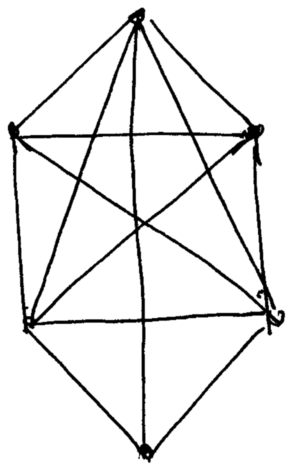
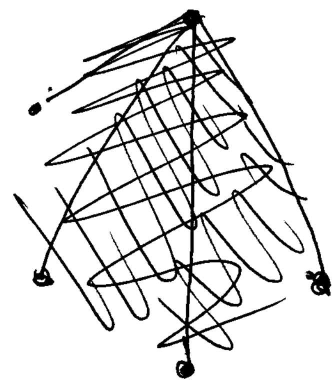


Fig 61 (a) ✓.

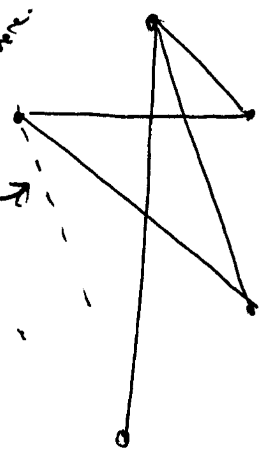
Fig 65 c:

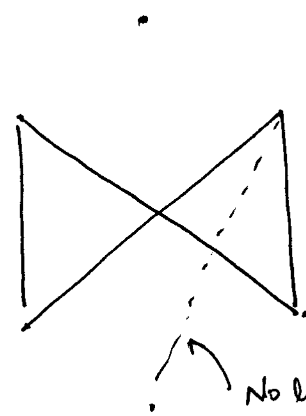


⇒ ≥ subgraph



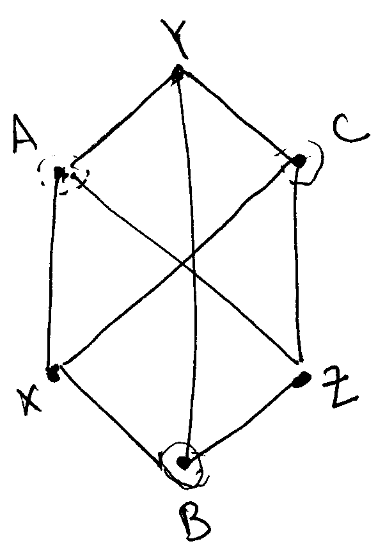
no line here.





No line.

∴ can't get the ~~utility gr~~ utility graph from this arrangement.



This is the subgraph that is isomorphic to the utility graph

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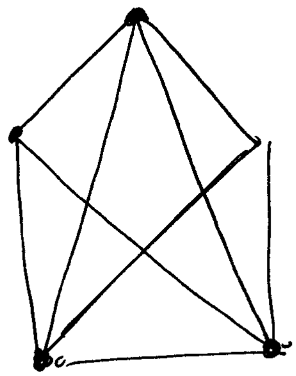
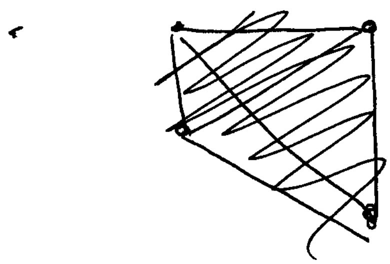
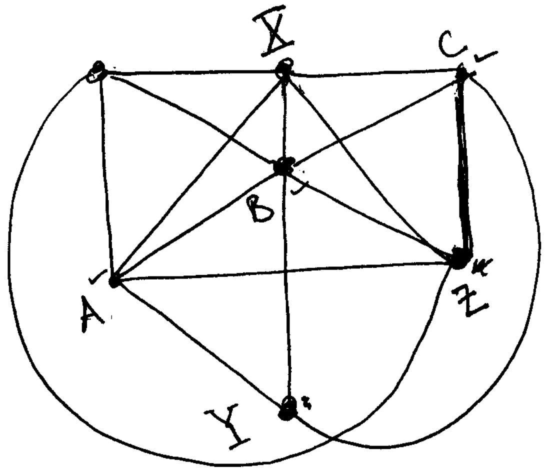
look for the graph that is isomorphic to K_5.



is K_5 ✓.

Fig 65 d:

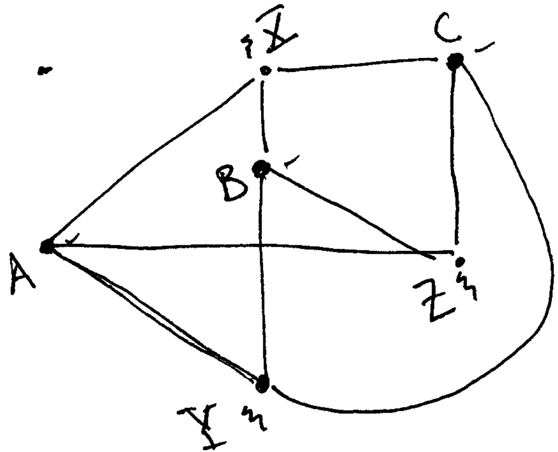




Find the subgraph isomorphic to UG.

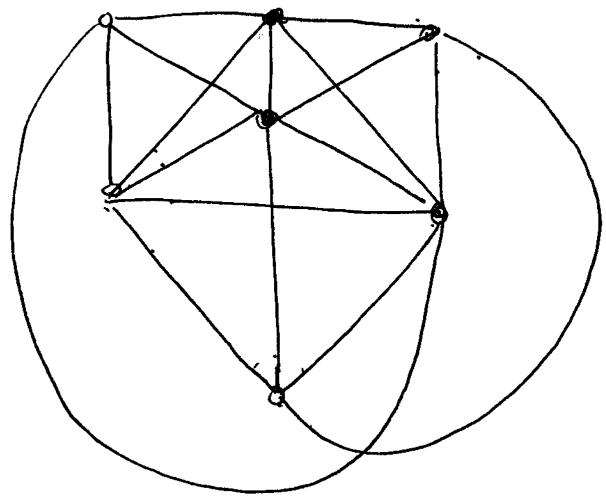
✓ denote team A.

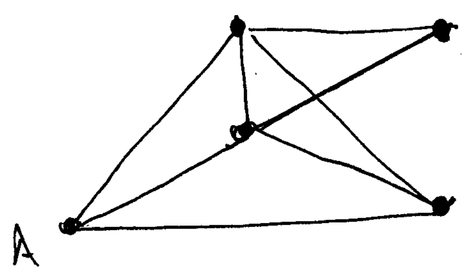
ξ " " B.



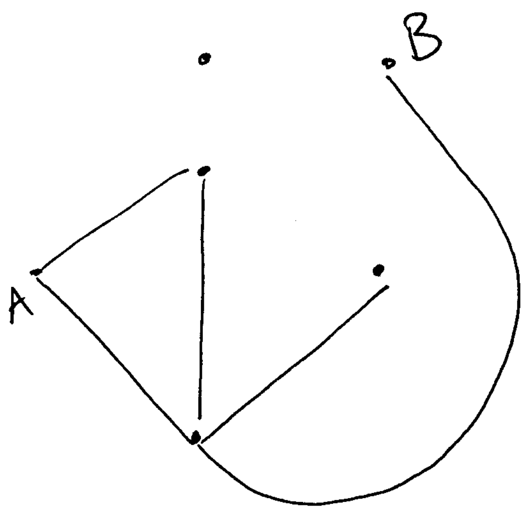
this graph is isomorphic to the utility graph UG.

Find the subgraph of Fig 65 d is omorphic to ~~the~~ K_5

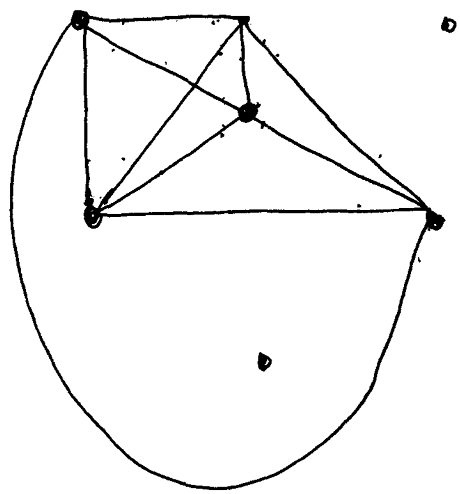




won't work since A has degree 3 only. Not true reflect from original drawing...



won't work since A does not connect to B.



this w/o two lone pts, is isomorphic to K_5.

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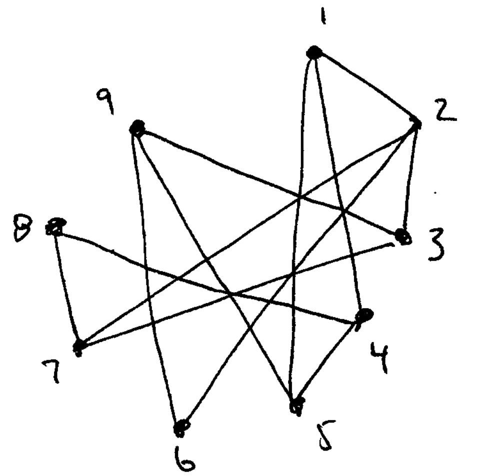
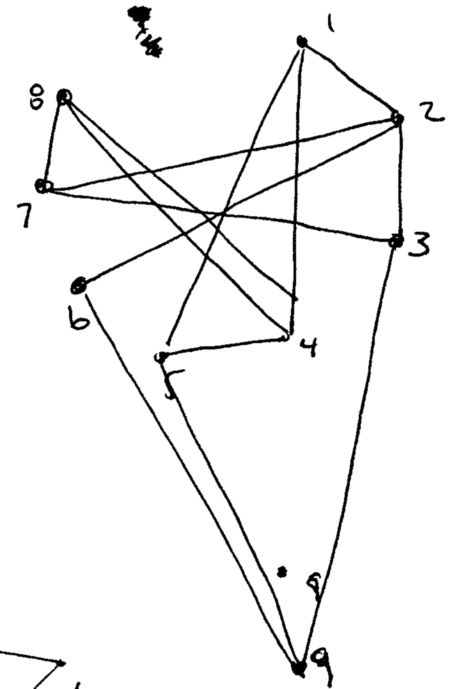
Fig 77:

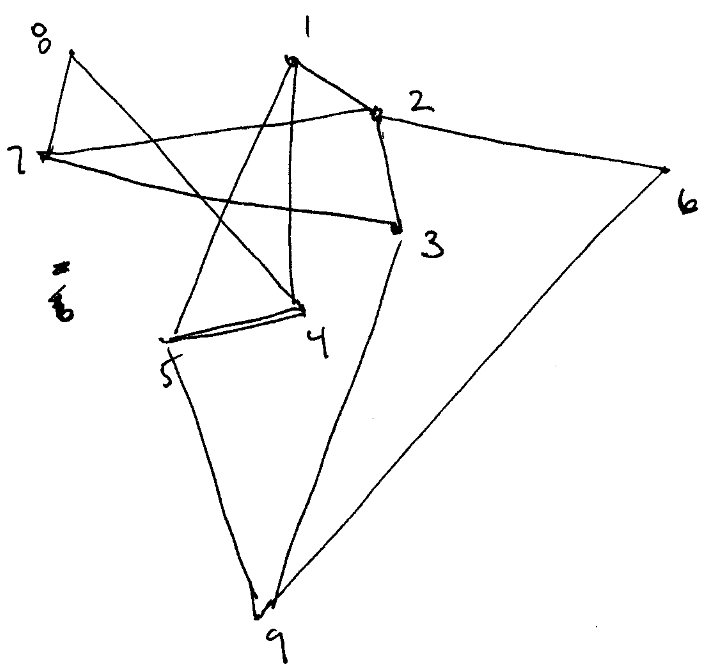
92 - 87 = 5



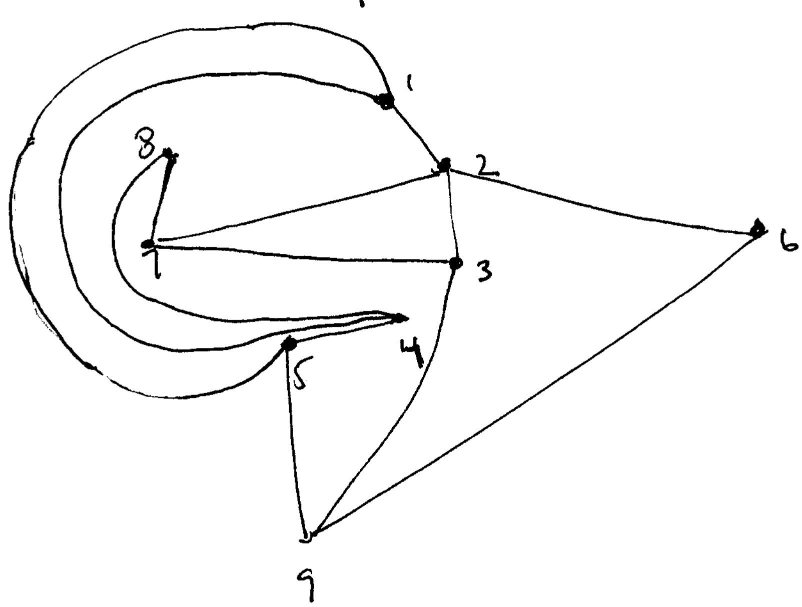
≅



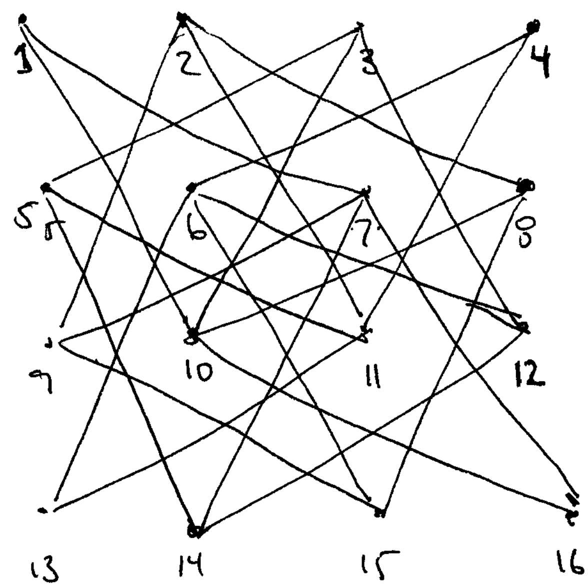
≅



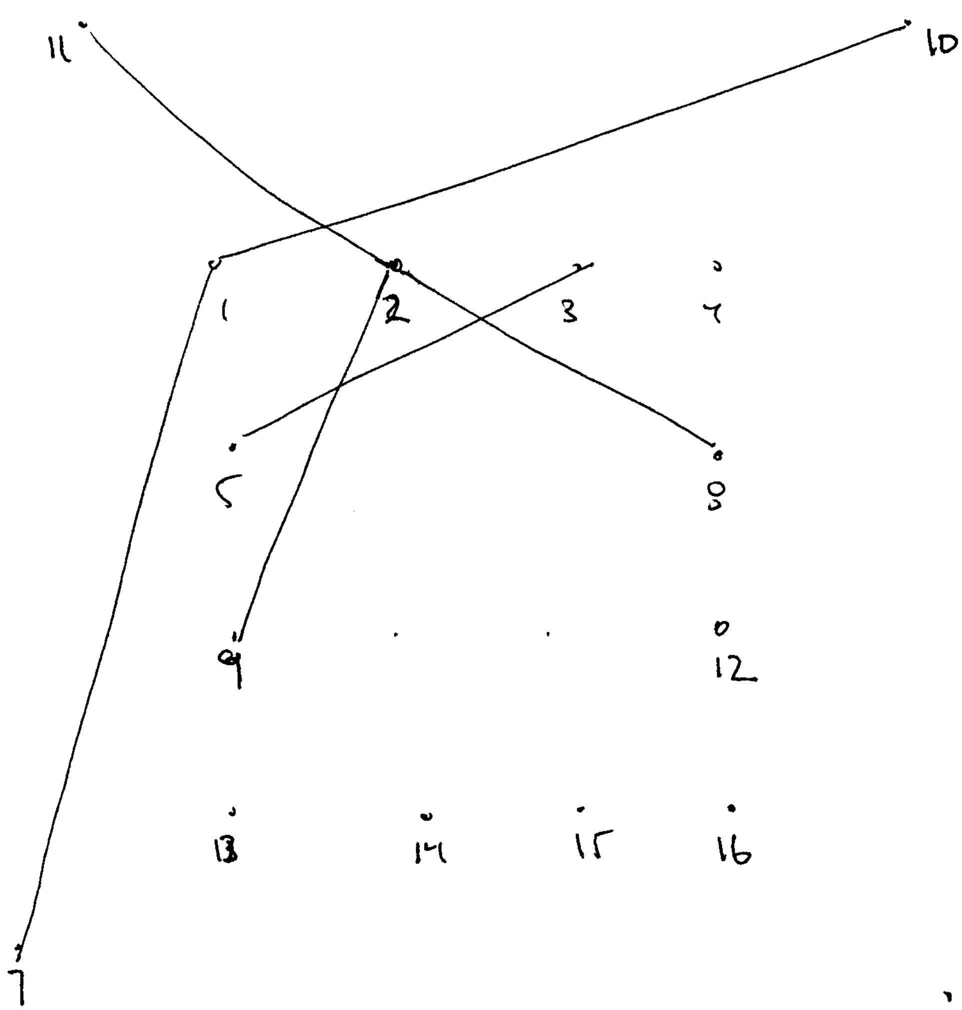
≅



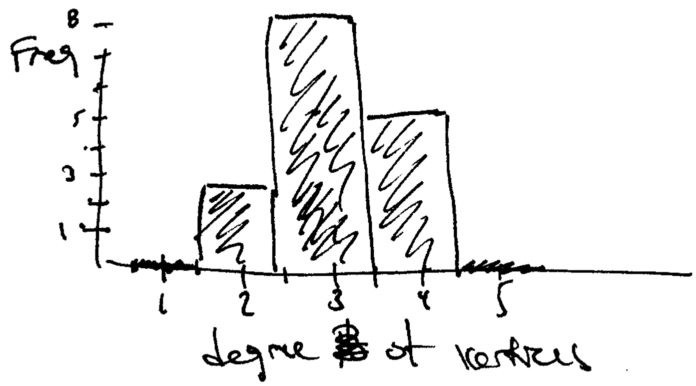
This graph is planar. ✓



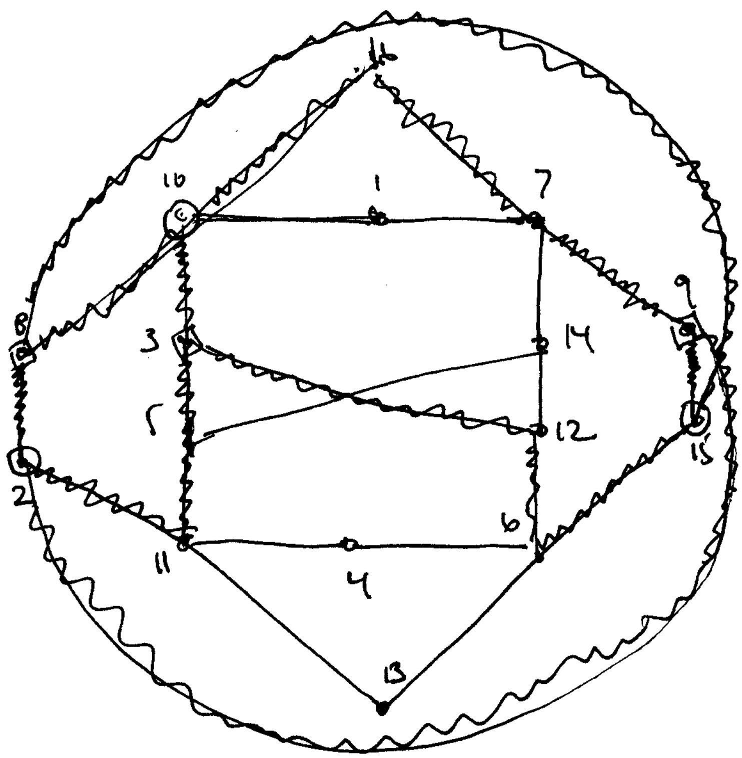
move #6 out.
now #7 out
now #10 + #11 out.

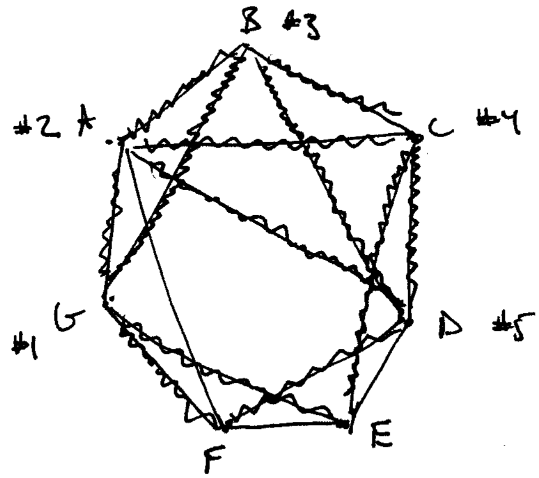


Histogram of # of



degree of vertex	Freq
1	
2	III
3	~~IIII~~ III
4	~~IIII~~
5	

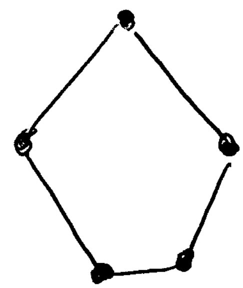




Need paths from #1 → #4 → Expansion path ~~GF~~ GEC
" " " #1 → #5 → Expansion path GFD
#4 → #1 ✓
#5 → #1 ✓

① Fig 63 (c)

(a)



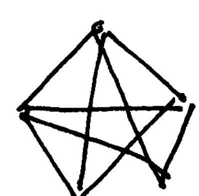
3 edges would have to connect to 6 different vertices to have none of them adjacent. Since our graph has only 5 vertices, this is not possible. ∴ 2 edges must be adjacent

(b) The same logic used in part (a) holds here too.

(c) If v odd $\Rightarrow$ $v+1$ is even

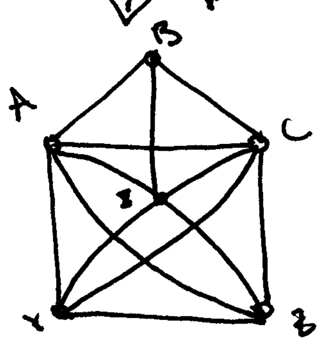
& a graph w/ $\frac{1}{2}(v+1)$ edges will have $v+1$ end points. If no 2 end points coincide then one would require a graph w/ $v+1$ vertices, since our graph has v vertices, 2 edges must be adjacent.

② Fig 80(a)



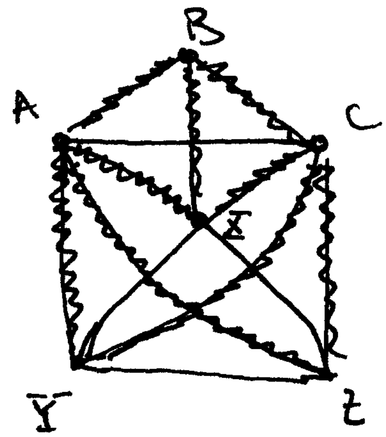
please see notes on pg 65.

(b)

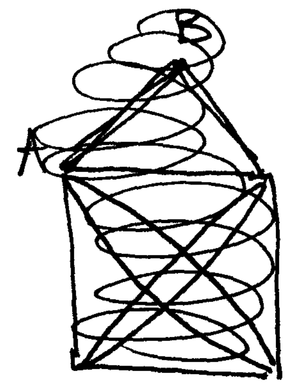


Look for a super graph of an extension to UG.

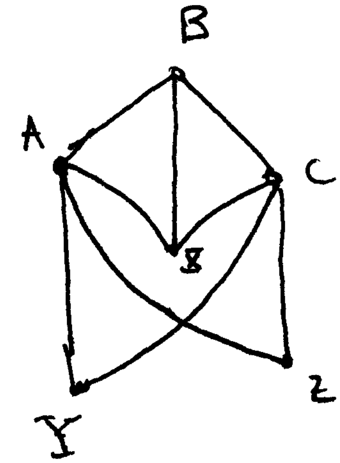
try relabelling shown 02-30-02



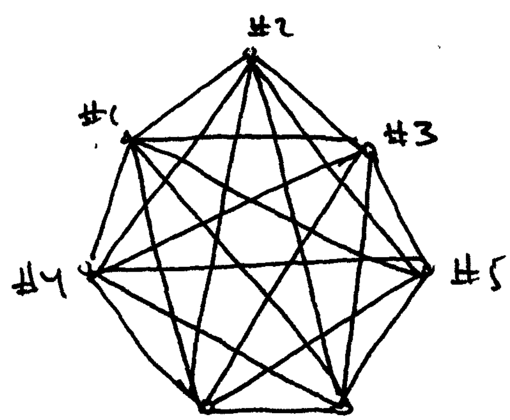
~~crossing~~
crossing
unneeded edges



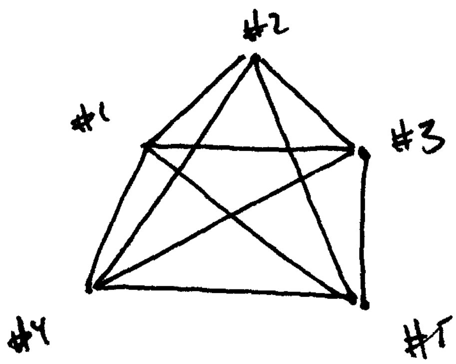
This is isomorphic to UG. & ∴ the original graph is not planar by kuratowski's theorm.



(c) Fig 58 (c)

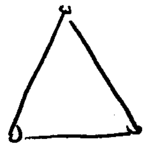


Since this is K_7 it prob contains K_5

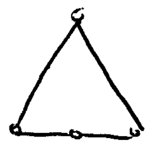


Thus Fig 58 (c) is a super graph of K_5 & ∴ it is non planar by kuratowski's thm.

3 K_3

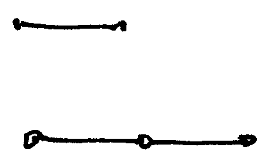


To show something is a supergraph of ~~something~~ another graph, one must be able to erase vertices + edges + obtain the desired target graph



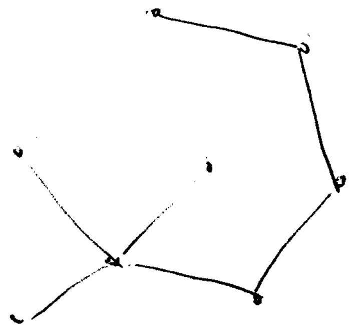
Since for any of $C_3, C_4, \ldots$ we cannot delete nodes + obtain cyclic graphs, none of the expansions of K_3 can be supergraphs of K_3 except itself.

To find a graph that has every expansion of G to be a supergraph consider "line graphs"

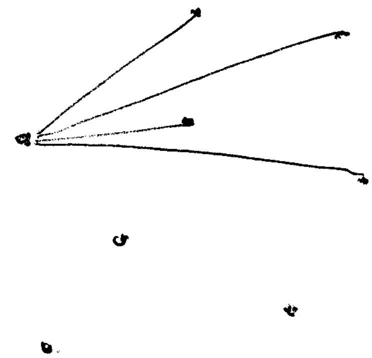


etc Then every ~~the~~ expansion is a supergraph of the above graphs.

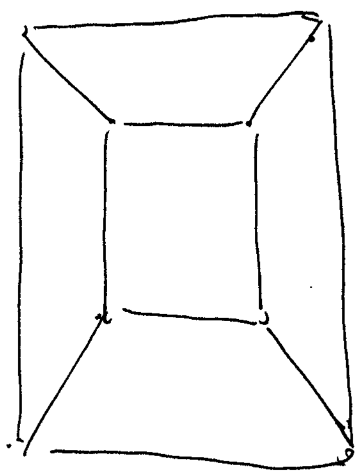
4



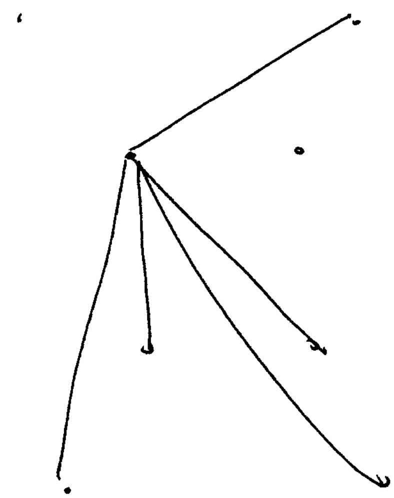
G



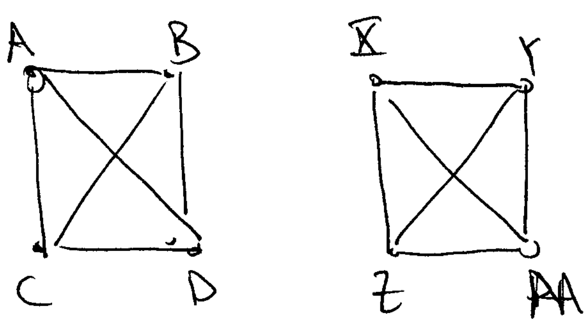
$\bar{G}$



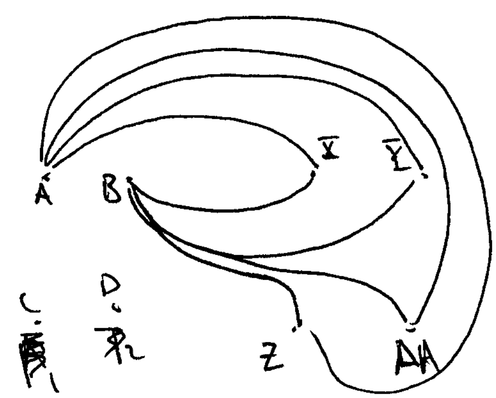
G



$\bar{G}$

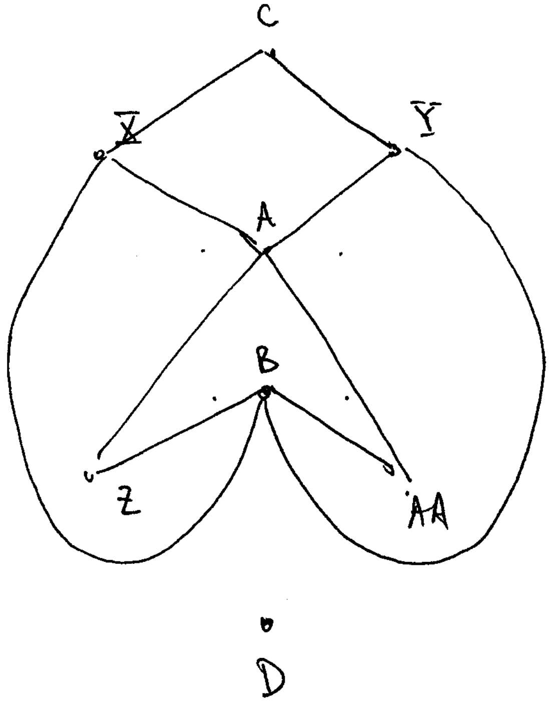


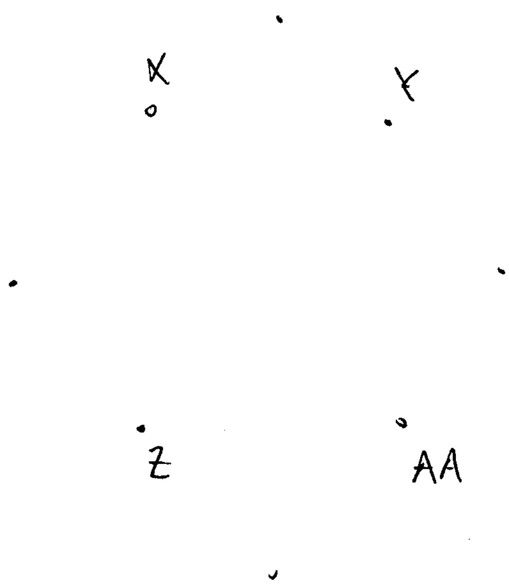
G

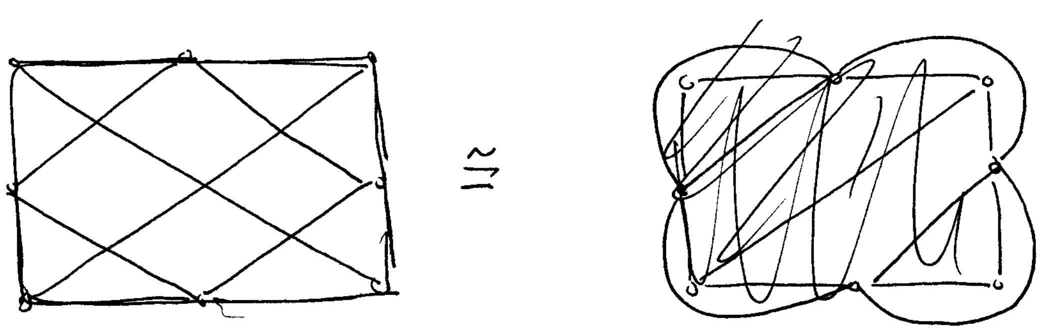


$\bar{G}$

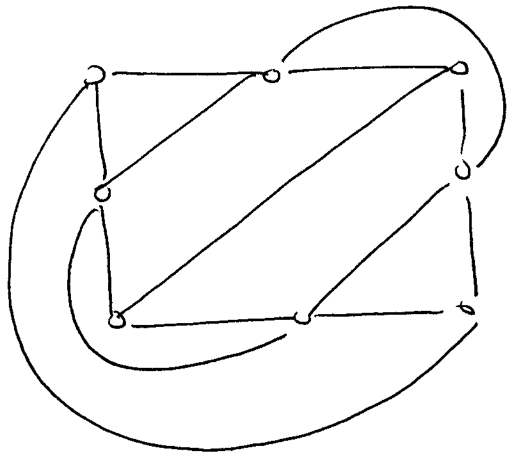
Not fixed yet, need to draw edge from $\overset{C}{P_1}$ + $\overset{D}{P_2}$ still


C
X
Y
A
B
Z
AA
D

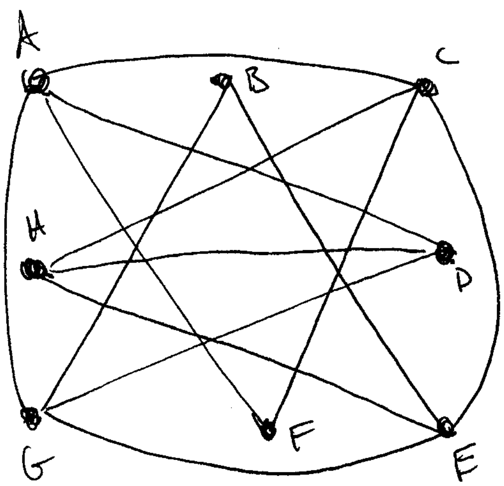

X
Y
Z
AA


≅

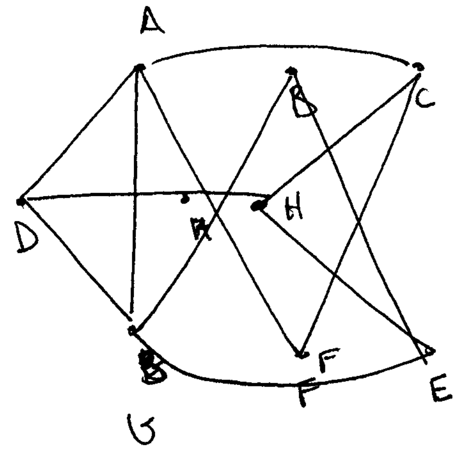
$\cong$



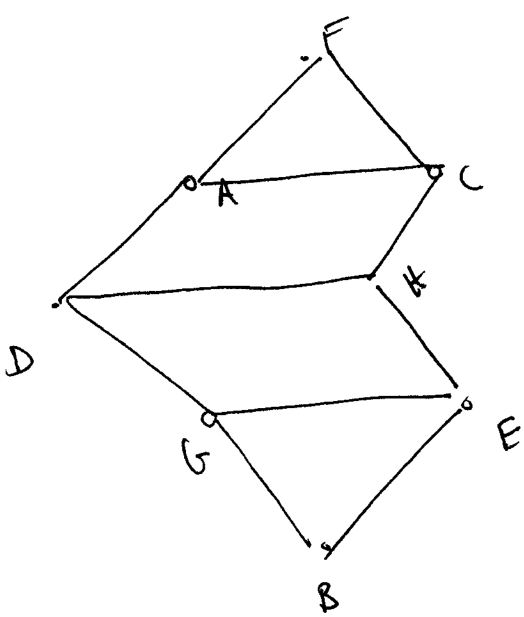
is the planar representation of this graph its complement is



$\cong$

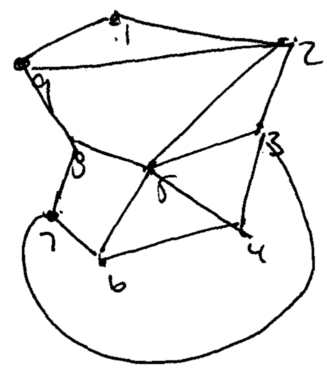


$\cong$

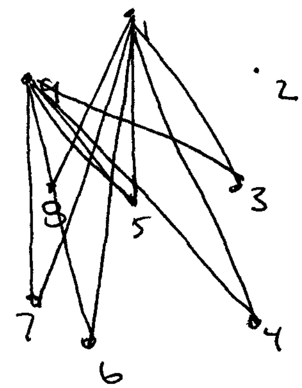


this is planar

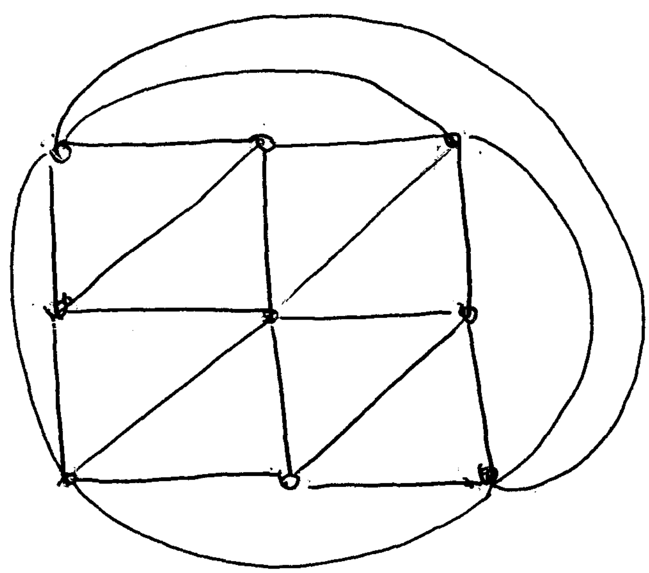
5



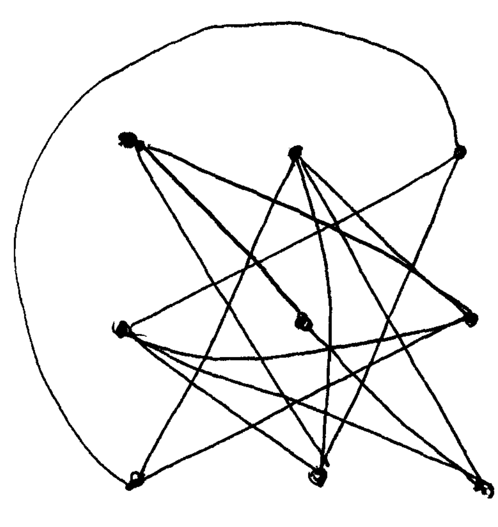
G is planar



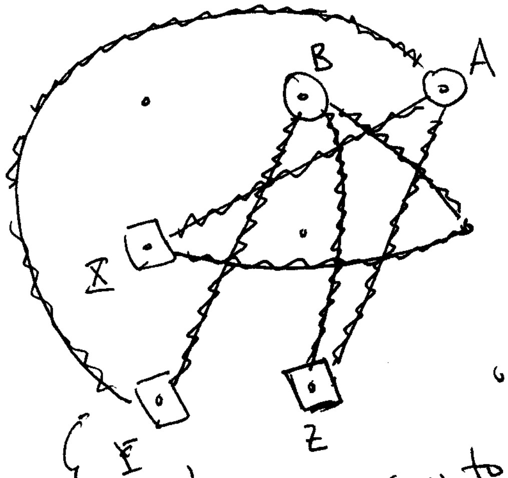
$\bar{G}$ ... begins to get difficult ...

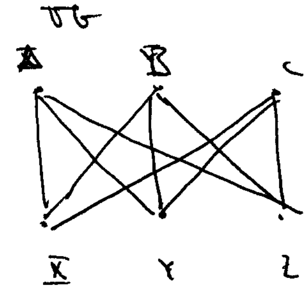


G is planar



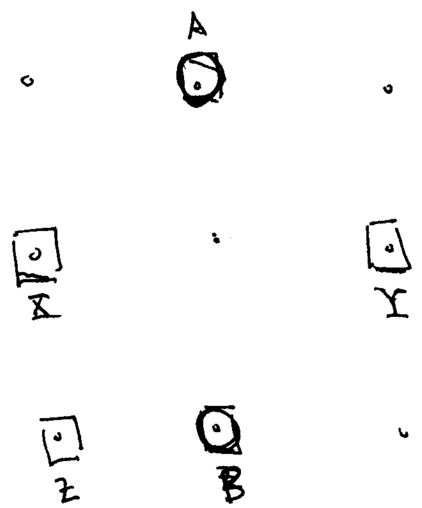
$\bar{G}$ Claim $\bar{G}$ is not planar.

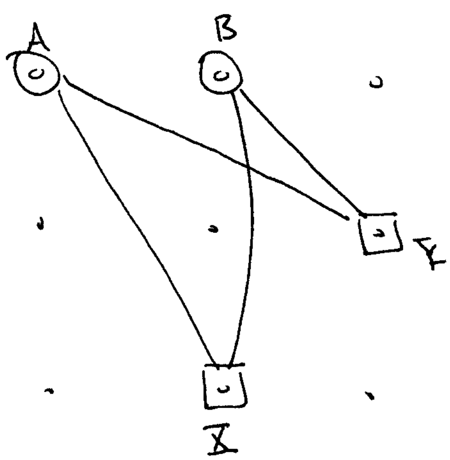




This company seems to be difficult to get it.

look for a ~~supp~~ super graph of an extension of TG

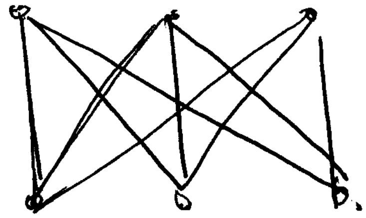




... Seems more complicated than originally thought

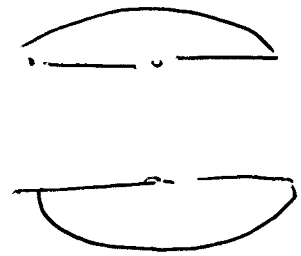
(6)

UG



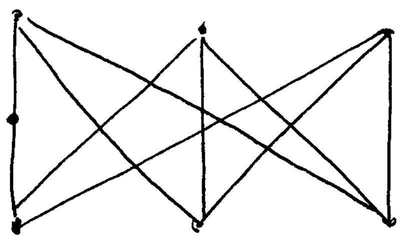
K_5

$\overline{UG}$

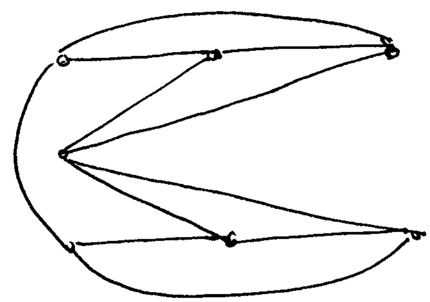


$\overline{K_5} = N_5$.

1 extension of UG.

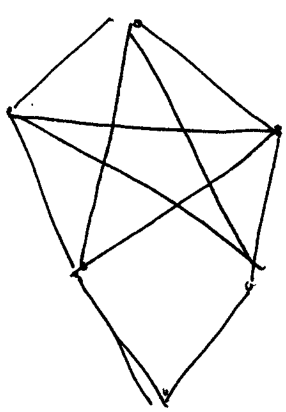


1 extension of $\overline{UG}$

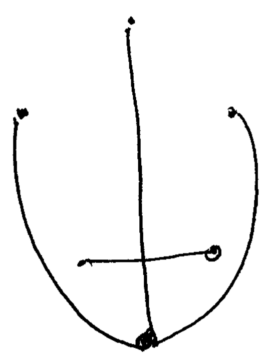


is planar.

1 extension of K_5



1 extension of $\overline{K_5}$



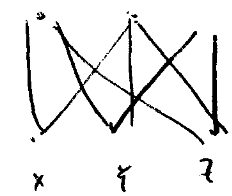
is planar. . . .

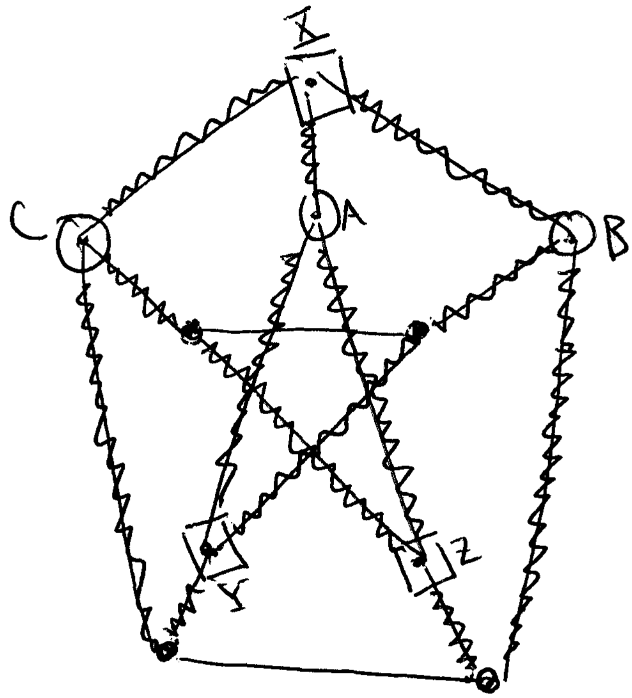
pg 93 Trudeau

08 - 01 - 02

1

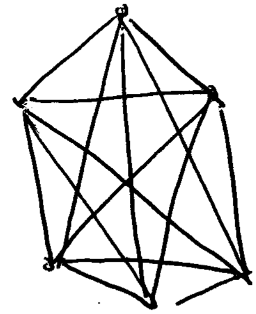
(7) Consider this super graph of ~~Km~~ an extension of UG





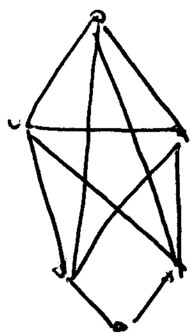
(8)

(8) K_6 will certainly be nonplanar



K_6

Extensions of K_5 will certainly be non-planar



How many unique extensions are there (think only 1)

I would think that I would remove edges from K_6 & show that each graph that results is non planar . . .

This would be my direction of solving this problem

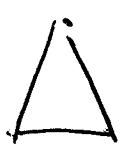
(9) If H is an expansion of G, then we have inserted vertices into edges of G ~~For every vertex we~~ ~~insert we delete~~ Say we insert n vertices into the graph G. Then we have added $\underline{\underline{n}}$ additional edges.

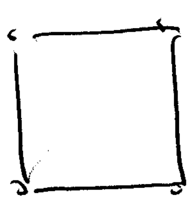
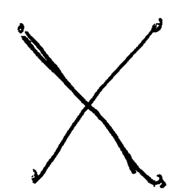
Thus $v_H - e_H = v_G - e_G =$

$\Rightarrow v_H + e_G = e_H + v_G$.

(10) In both cases (this problem & the choice)

$v_H - e_H$ is incorrect.

(11) C_3  $\bar{C}_3 = C_3$

C_4  $\bar{C}_4$ 

C_v has v vertices & v edges

K_v has ~~$\frac{1}{2}v(v+1)$~~ v vertices & $\frac{1}{2}v(v+1)$ edges

$\therefore$ $\bar{C}_v$ has $\frac{1}{2}v(v+1) - v$ edges w/ v vertices

$\overline{G}$ has 

$3 \cdot 3 = 6$ edges

K_6 has $\frac{5}{2}(6) = 15$ edges

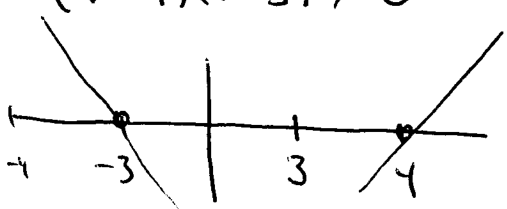
$\bar{G}$ has $\frac{1}{2}v^2 + \frac{v}{2} - v = \frac{1}{2}v - \frac{v}{2} = \frac{1}{2}v(v-1)$ edges.

~~Since every expression~~ ~~$\neq$~~ ~~$\overline{G}$~~ Thus I know at least

$\frac{1}{2}v(v-1) \geq 6$

$v^2 - v - 12 \geq 0$ $\qquad v = \frac{+1 \pm \sqrt{1 + 4(12)}}{2} = \frac{+1 \pm \sqrt{49}}{2}$

$(v-4)(v+3) \geq 0$

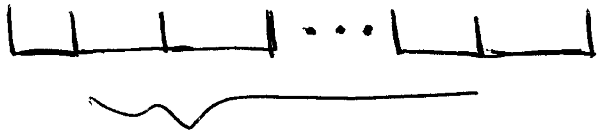


$= \frac{1 \pm 7}{2} = \left\{ 4 \right.$

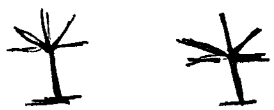
$\Rightarrow \boxed{v \geq 4}$

But otherwise how does one guess?

(12)



n-boxes m boxes.



n = # of trees m = # of leaves

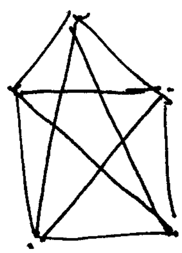
n < m. at. ... Don't see how to do this.? Seems rather obvious but...

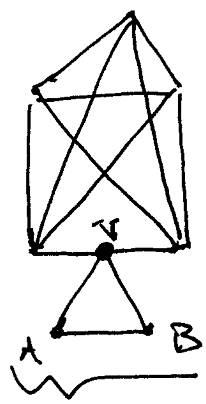
(13) ~~fat~~

Given any graph G any non trivial expansion of that graph is not a subgraph of G. This is because ~~deleting~~ Using an eraser to ~~delete~~ one would never be able to get back the original graph

(14) Find an supergraph of an expansion of K_5, that is not an expansion of a supergraph of K_5.

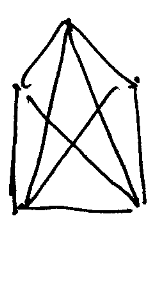
K_5:





This is not an expansion of a super graph of K_5 because in forming a super graph one is not allowed to insert vertices in arbitrary locations & the vertex labelled V would not be allowed to exist. (or at least & not connect vertices A & B)

i.e.

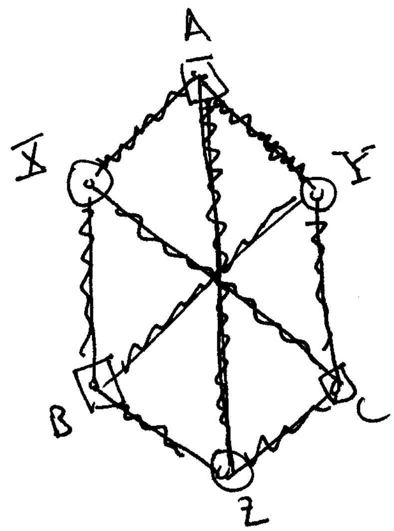


is a supergraph of K_5, but no expansion of this graph can give rise to the graph shown above

(15) Figs 46, 51, 54 & 55

~~En~~ In Fig 46 the left graph is planar.

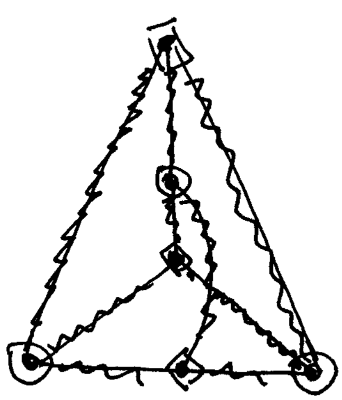
& I claim the right graph is non-planar



this graph contains UG & thus cannot be planar by ~~Kuraw~~ Kuratowski's Thm

In Fig 51 The left graph is planar

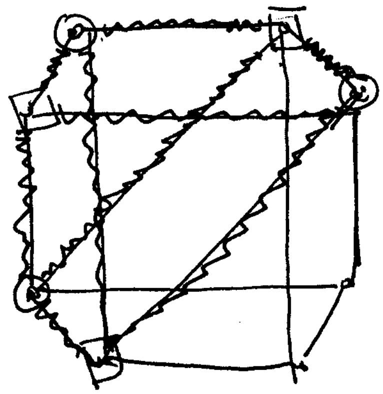
&



As again this graph is a super graph contains UG it is non-planar

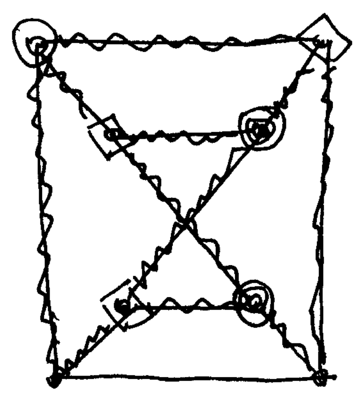
In Fig 54 The right graph is planar

If I can show the left graph is non planar then the two graphs are non-isomorphic



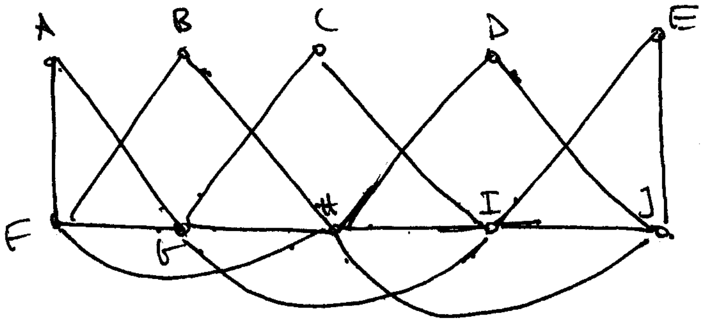
This is thus a super graph of UG + by Kuratowski's thm is non-planar.

Fig 55: The left graph is planar, ∴ if I can show the right graph is non-planar I will have shown that the two graphs cannot be isomorphic.

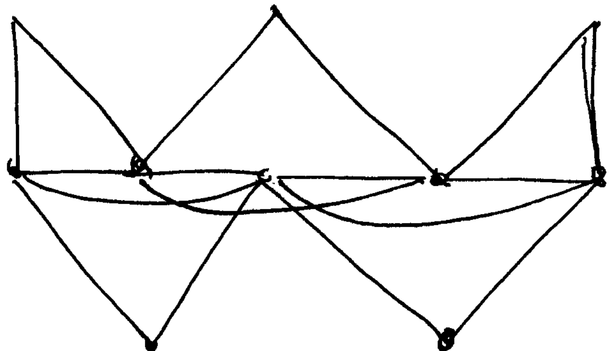


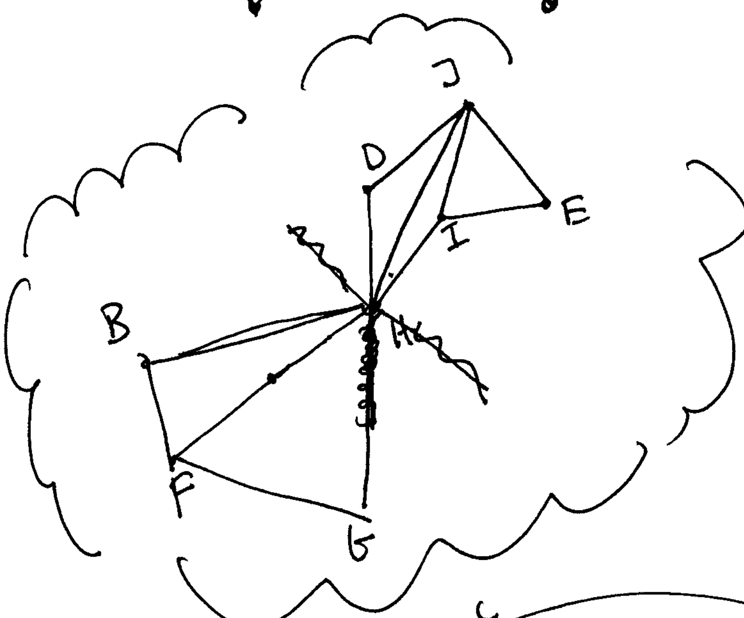
As this is isomorphic to a supergraph of UG. this graph cannot Be planar.

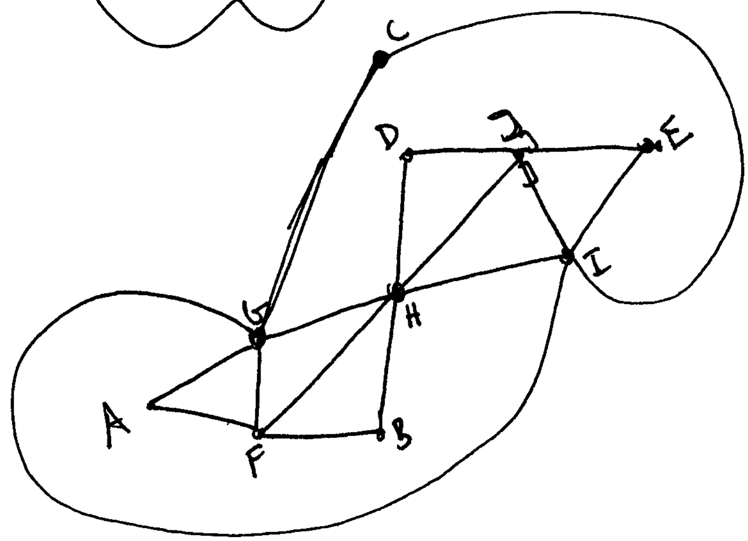
(16)



$\cong$

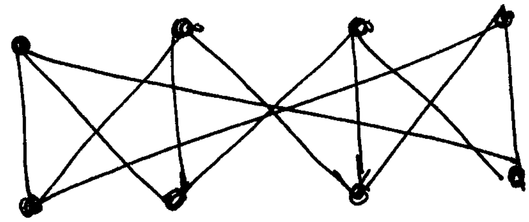




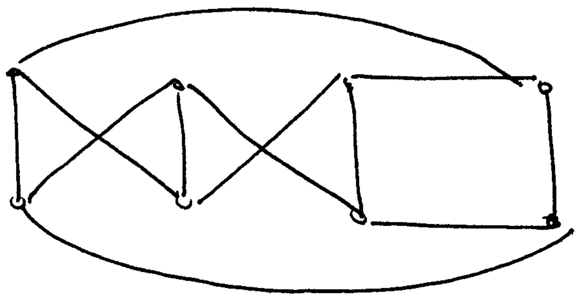


This is a planar graph isomorphic to the original & thus we are finished.

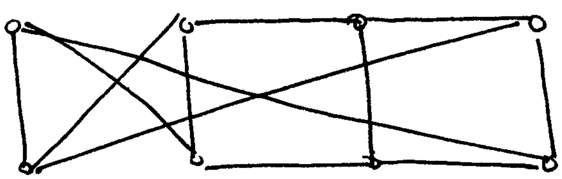
Fig 39 (b)



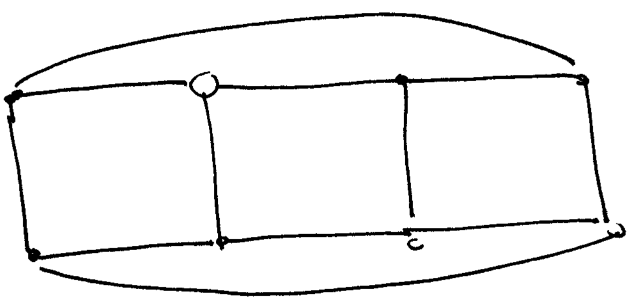
$\cong$



$\cong$

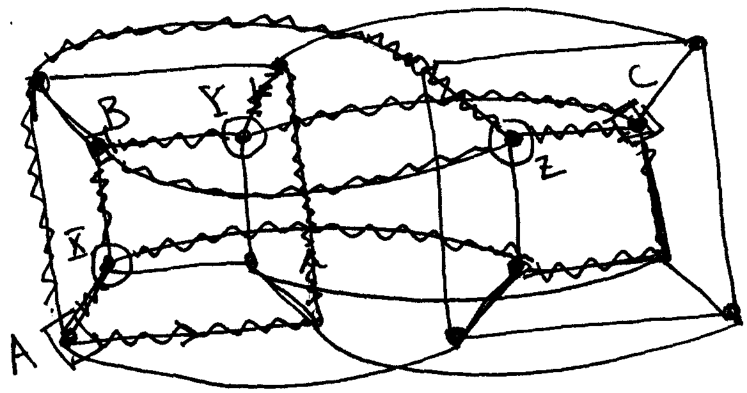


$\cong$



This is planar.

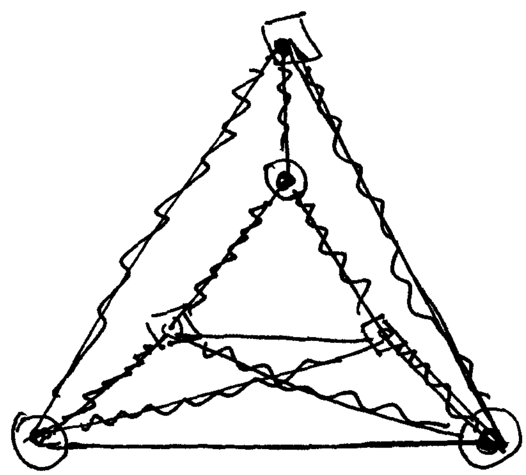
(17) Fig 90 (a)



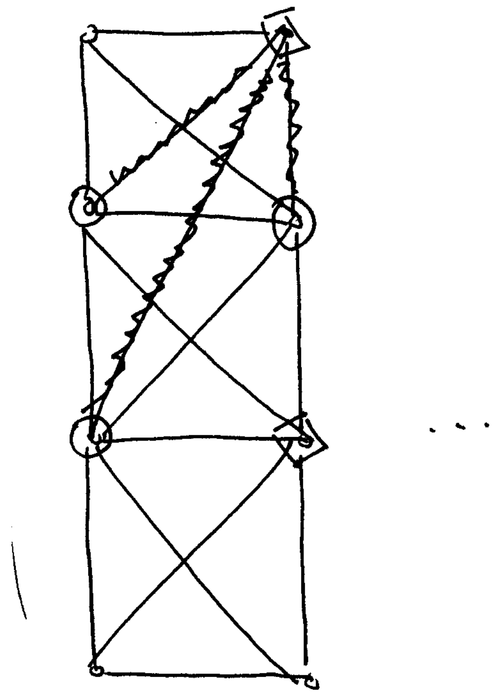
Since this graph is a supergraph of an extension of the Utility graphs it is non planar

Fig 90 (b)

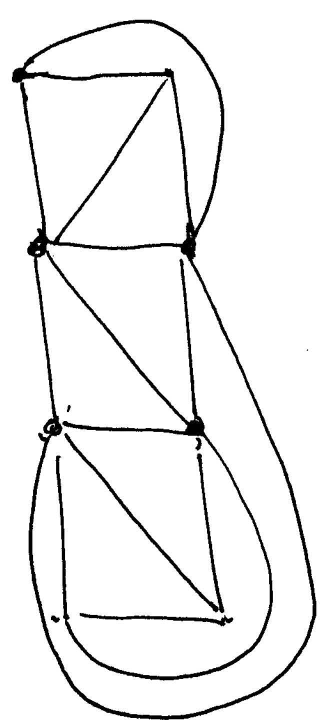
"



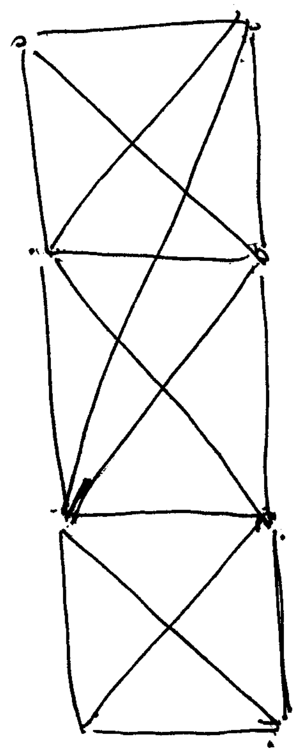
(18-20) Fig 91 (a) Think non-planar.

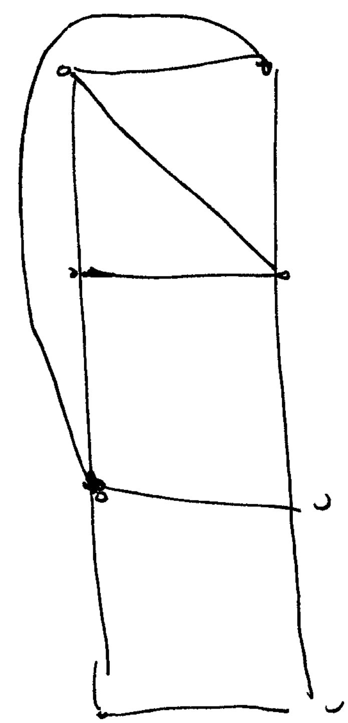


Maybe planar ...

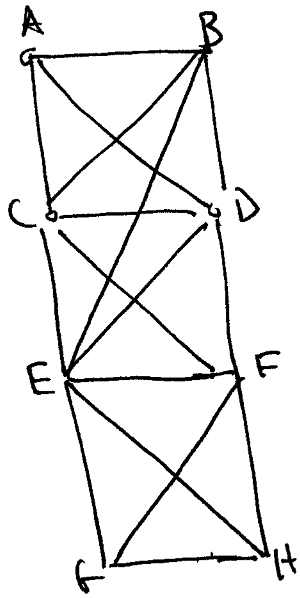


Maybe nonplanar ...



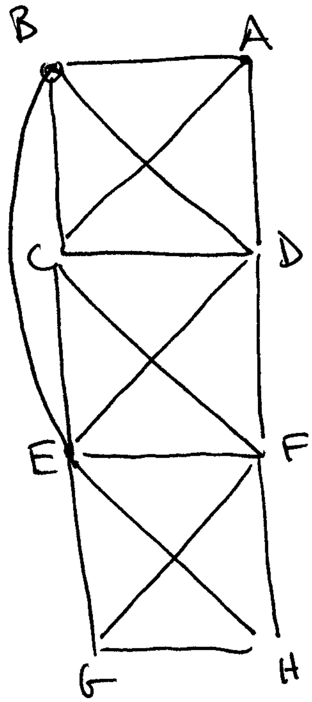


Aus deys plener...

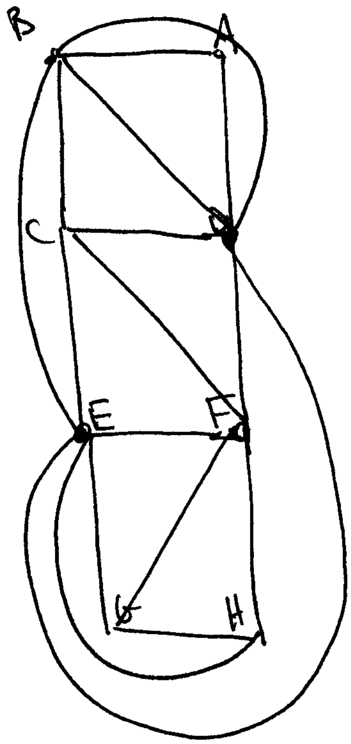


one does gain something by flipping
the one single edge across
2 blocks flips over

≅

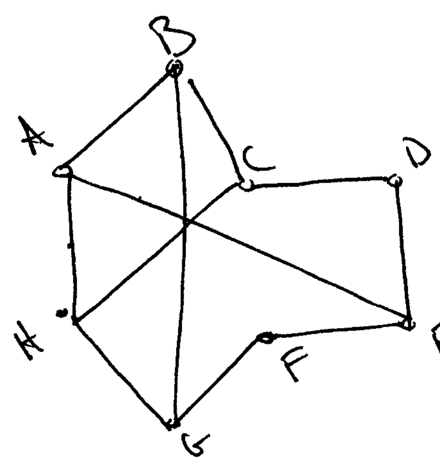


≅

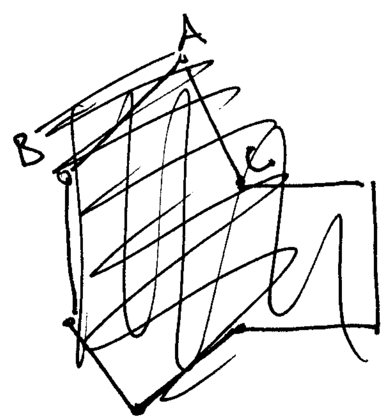


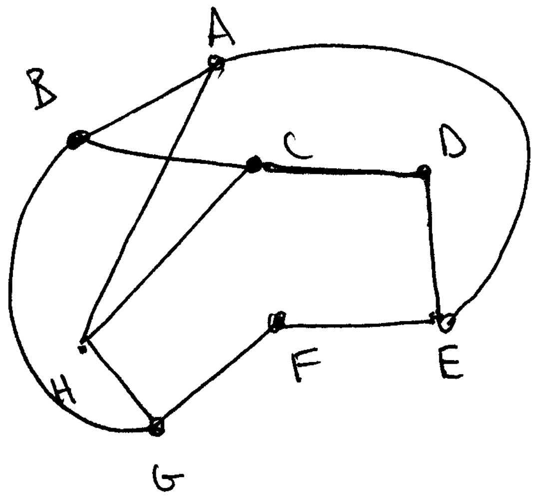
this is planar.

Fig 91 (b)

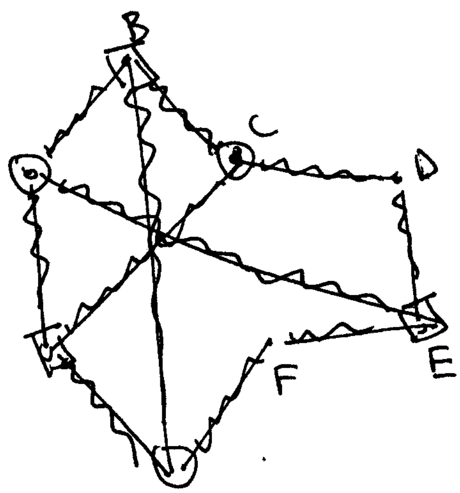


≅



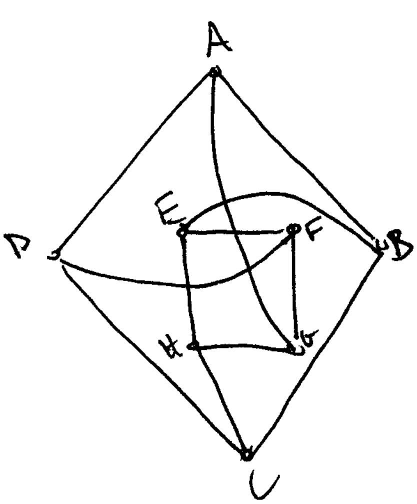


Maybe non planar

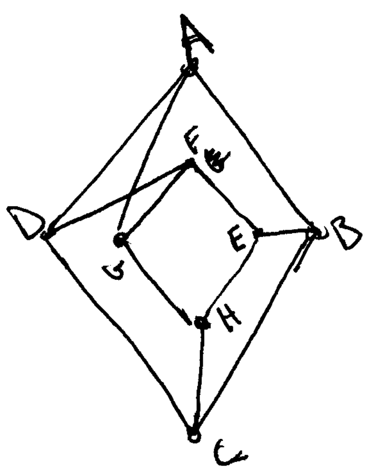


This is non-planar!!

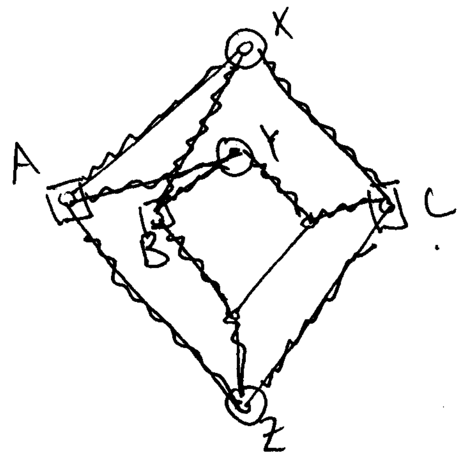
Fig 92: (a)



$\cong$

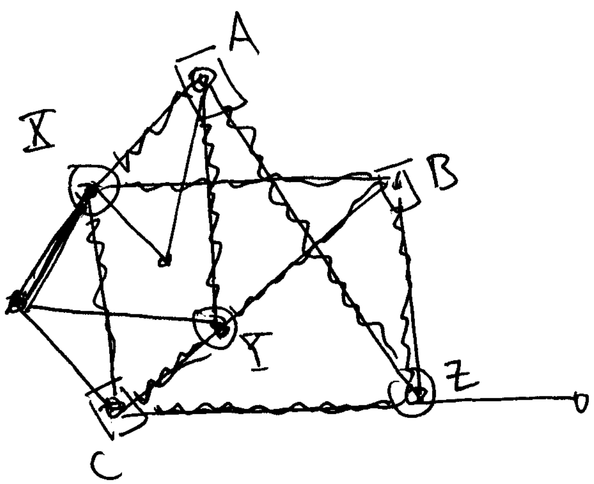


Maybe non-planar.



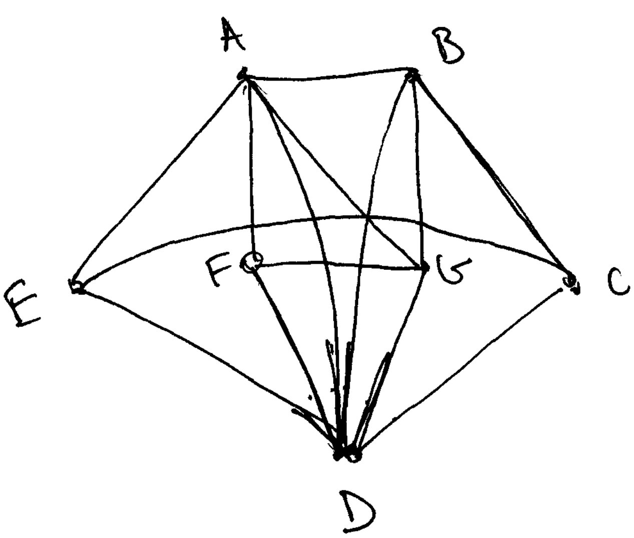
As this ~~cont~~ graph is an extension of a ~~subgraph~~ super graph it is non-planar.

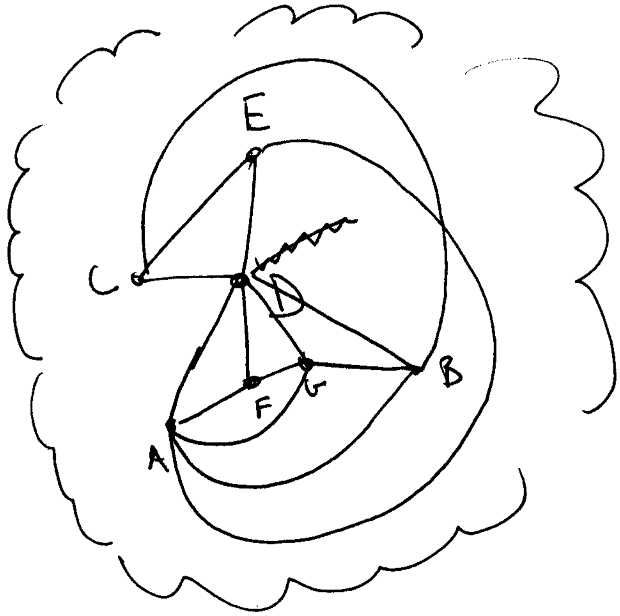
(b)

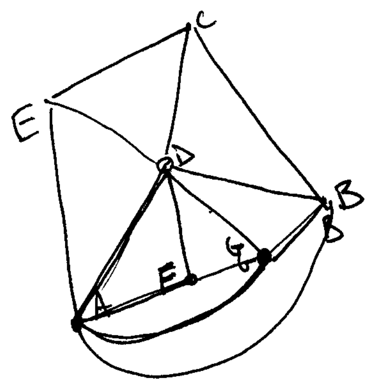


This contains a ~~sub~~ subgraph of UG + ∴ it is non-planar

Fig 93 (a)

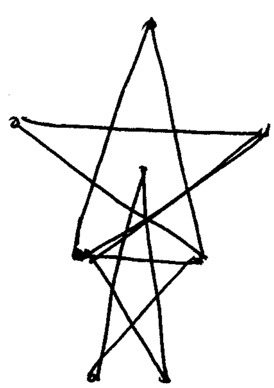




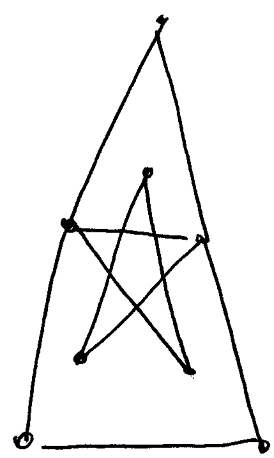


This is plener!!

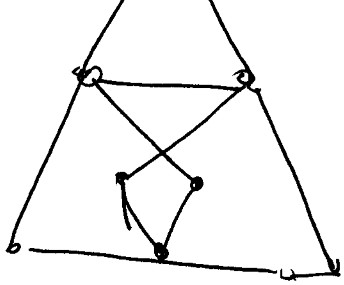
Fig 93 (b)



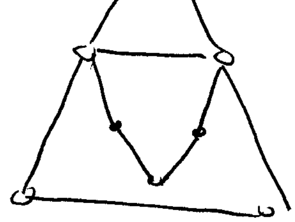
$\cong$



$\cong$



$\cong$



plener.

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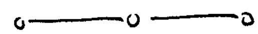
P_3:



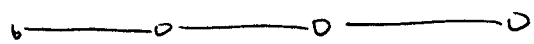
? ?

$2f \le e \le 2v-4$

$\parallel \quad \parallel$

$2 \le 2 \le 2(6)-4$ ✓.

P_4:

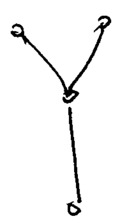


$2f \le e \le 2v-4$

$\parallel$

$2 \le 3 \le 4$ ✓.

T_4



$2f \le e \le 2v-4$

$2 \le 3 \le 4$ ✓.

$f \le \frac{1}{2}e$

$v+f-e \le \frac{1}{2}e + v - e = v - \frac{e}{2}$

$2 \le v - \frac{e}{2} \Rightarrow e \le 2v-4$

K_1:

$2f \le e \le 2v-4$

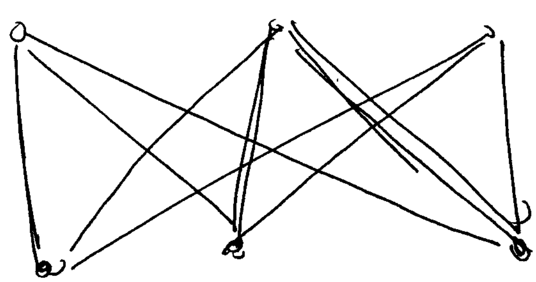
$\parallel$

$2 \le 0 \le$ Not

K_2:

$2f \overset{?}{\le} e \overset{?}{\le} 2v-4$

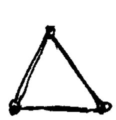
$2 \le 1$ Not true.



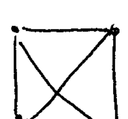
$e \le 2v-4$ if G planar + connected

$9 \le 2(6)-4$

$9 \le 8$ $\rightarrow\leftarrow$.



K_3



K_4

① ~~[scribble]~~ If the statement "every edge of N_1 borders on 2 different faces" were false ⇒ every edge of N_1 borders on ~~[scribble]~~ less than 2 diff faces or more than 2 different faces would have to be true. ⇒

But this cannot be true since N_1 does not have any edges as such & using the law of the ~~extended~~ excluded middle ~~B~~, every edge borders on 2 different faces.

② Errors: 1) ~~[scribble]~~ Statement S is really a conclusion

Given something ⇒ something else holds

S should be a statement . . .

~~The induction hypothesis must be true for k+1 also.~~

~~i.e. given k=1 ~~is true one must be able to show that~~ k=2 is true, this itself cannot be true when we remove~~ the 1st ~~horse~~ . . .

Given A set of 2 horses we cannot remove 1 check the color & remove the other check the color & expect the two colors to match.

. .

(3) ~~e+f=v=2~~

Eulers thm says $v+f-e=2$

for this graph $v=9$ $e=20$

$f=$

using:

$$\sum_{v \in V} \deg(v) = 2e$$

$$\parallel$$

$$4+3+3+8+7+5+4+3+3=40 \Rightarrow e=20$$

$$\therefore \quad 9+f-20=2$$

$$f-11=2$$

$$\underline{\underline{f=13}}$$

(4) Fig 10-1b is a connected graph v.

Assuming that it is planer it must satisfy Eulers relationship

$v+f-e=2$

So counting $V=8$ $f=?$

By Thm 12 if G is planer & connected w/ $v \geq 3$ &

G is not a supergraph of K_3 (Δ) then

$2f \le e \le 2v-4$

in this example

$$\sum \deg(v) = 2e$$

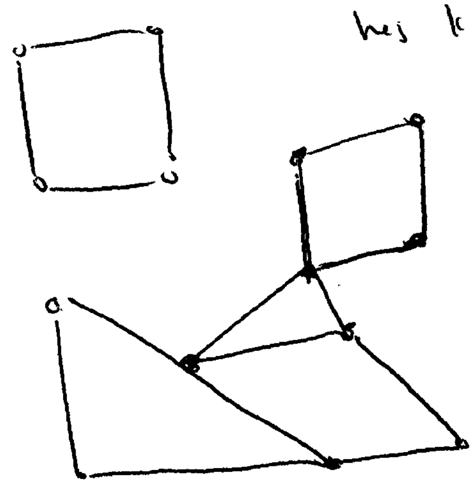
$$\parallel$$

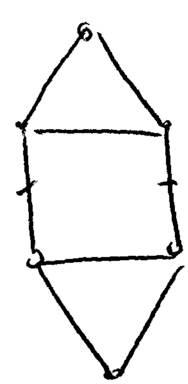
$4(8) = \cancel{8}32 \Rightarrow e = \cancel{8}16$

So $16 \cancel{8} \overset{?}{\le} 2(8)-4 \Rightarrow$ False

5

has $k = 2$ faces





(6) Assume the graph is not connected, this means it exists in 2 or more pieces. As WLOG assume it exists in 2 pieces. Then as each piece satisfies Eulers formula

$$E_1 = v_1 + f_1 - e_1 = 2$$

$$E_2 = v_2 + f_2 - e_2 = 2$$

& for the original graph $v = v_1 + v_2$ ...

$\Rightarrow$ $v + f - e = 4$ $\neq$ what we are told that the graph to begin w/.

... note This is the correct idea but may not be correct because you added the ∞ face in too many times. See next problem

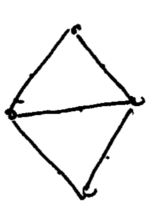
(7) Apply Eulers formula to each graph component.

$$V + f - e = 2$$

Then $v_{total} = \sum v_i$ $\quad e_{total} = \sum e_i$

$f_{total} = \sum(f_i -$

Do an example:

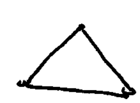


$v=4,\ f=3,\ e=5$

$4+3-5=2$ ✓.

$v=2,\ f=1,\ e=1$

$2+1-1=2$ ✓



$v=3,\ f=2,\ e=3$

$3+2-3=2$ ✓

$v_{total} = 4+2+3 = 9$, $e_{total} = 5+1+3 = 9$, $f_{total} = (\underbrace{1}_{\infty\text{ face}}+2)+(\underbrace{1}_{\infty\text{ face}})+(\underbrace{1}_{\infty\text{ face}}+1)$

Thus $\underset{3\infty\text{ faces}}{f_{total}} = \underset{\uparrow}{3} + \underset{\substack{\uparrow \\ \text{non-}\infty \\ \text{faces}}}{3}$

$\Rightarrow$ We have ~~…~~ counted the ∞ face p times.

$$v_{total} + f_{total} - e_{total} = \cancel{1 + 3 + 3 = 9}$$

$$= \cancel{6}$$

$\therefore f_{total} = \sum_p (f_i - 1)$

$=$

$$= \cancel{\sum v_i + \sum (f_i - 1) - 2\sum e_i =}$$

$+1$

$$\therefore \quad \sum v_i + \underset{\substack{\uparrow \\ \text{to Add back in 1 }\infty \\ \text{face}}}{\sum (f_i - 1) + 1} - \sum e_i = \underbrace{\sum (v_i + f_i - e_i)}_{2p} - \underbrace{\sum 1}_{p} + 1$$

$+1$

$$= \quad 2p - p + 1 = p + 1$$

(8) Corollary 13: If G is planar then G has a vertex of degree ≤ 5.

Checking the degree of vertices of Fig 106 gives that each has degree $= 6$ & thus the graph cannot be planar.

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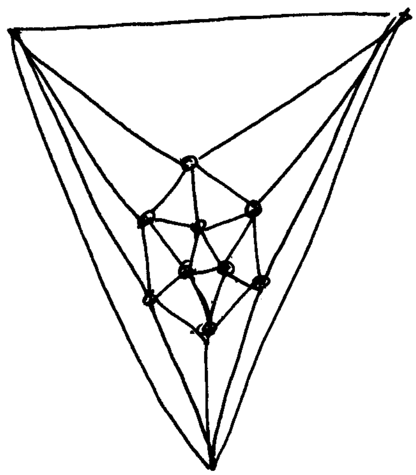
(9)

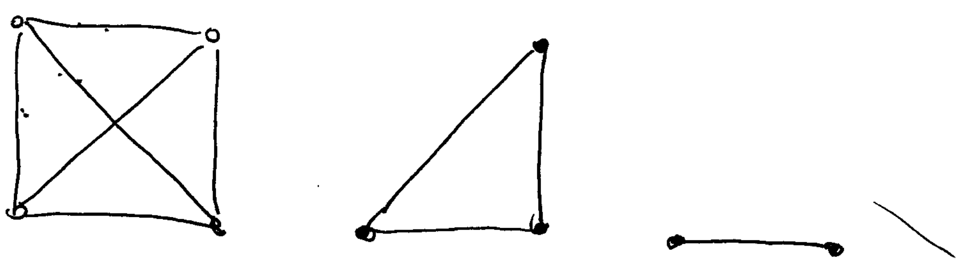


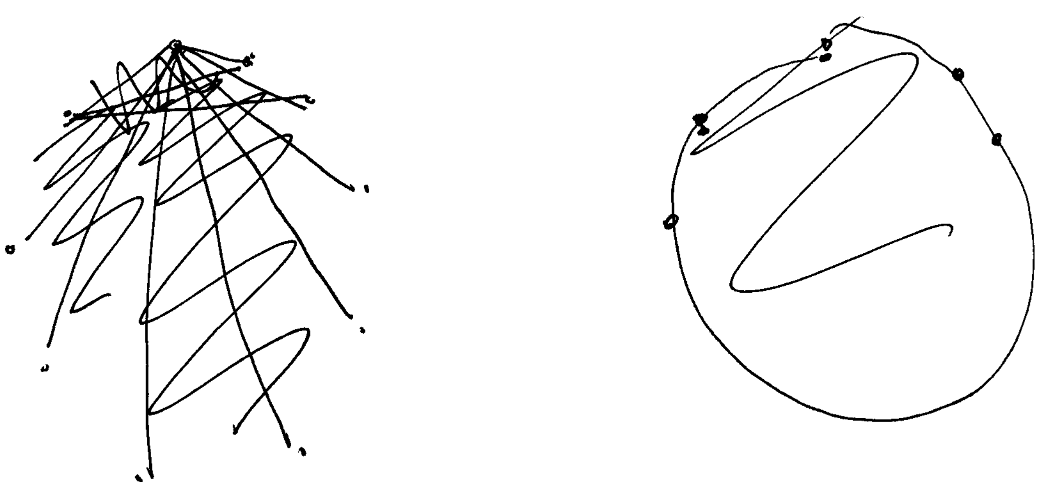
every vertex here is greater than or equal to 5.

(10) Every planar graph w/ $V \geq 4$ has at least 4 vertices w/ degree ≤ 5.

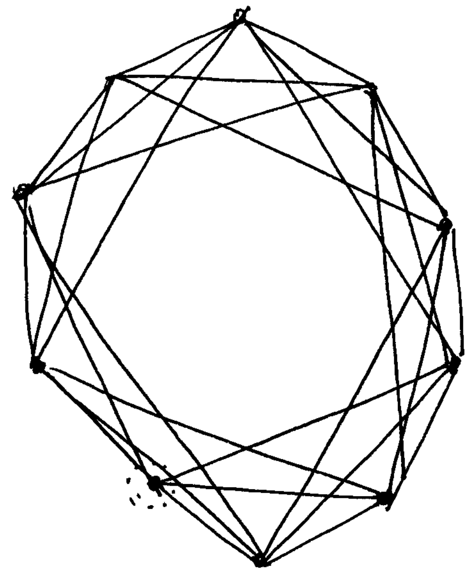
?

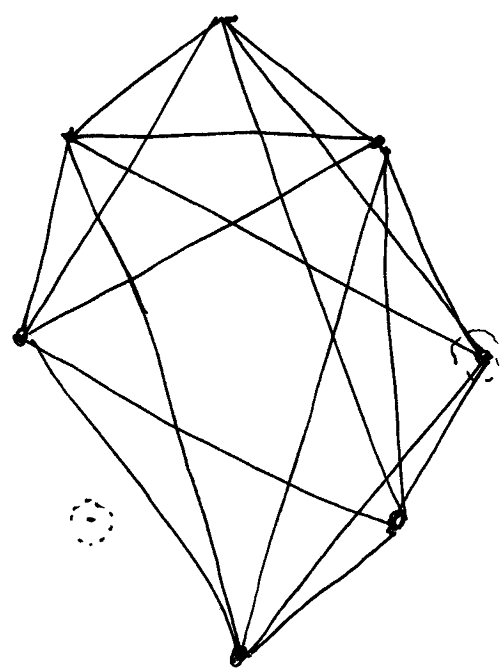
(11)



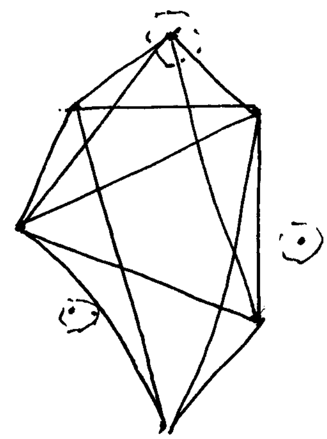


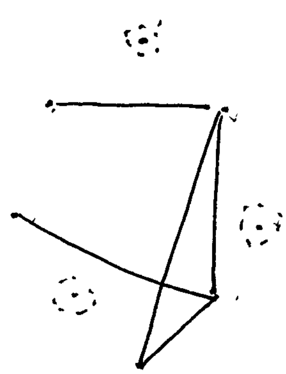
11 cont





⇒



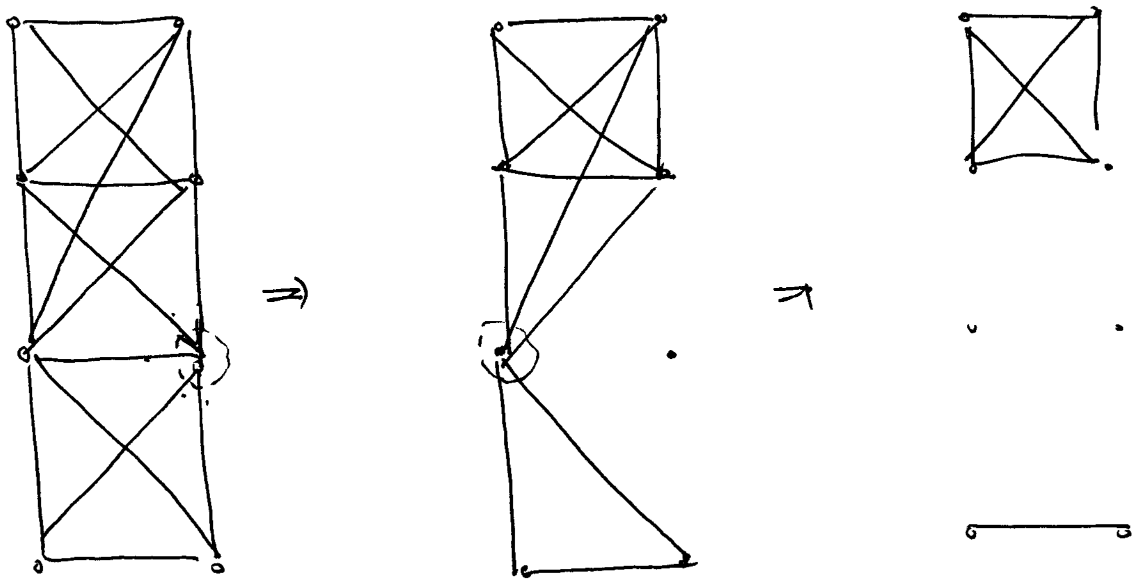


∴ no match not enough vertices.

Fig 91:

(a)

claim
$c=2$



(b)

Claim $c=2$

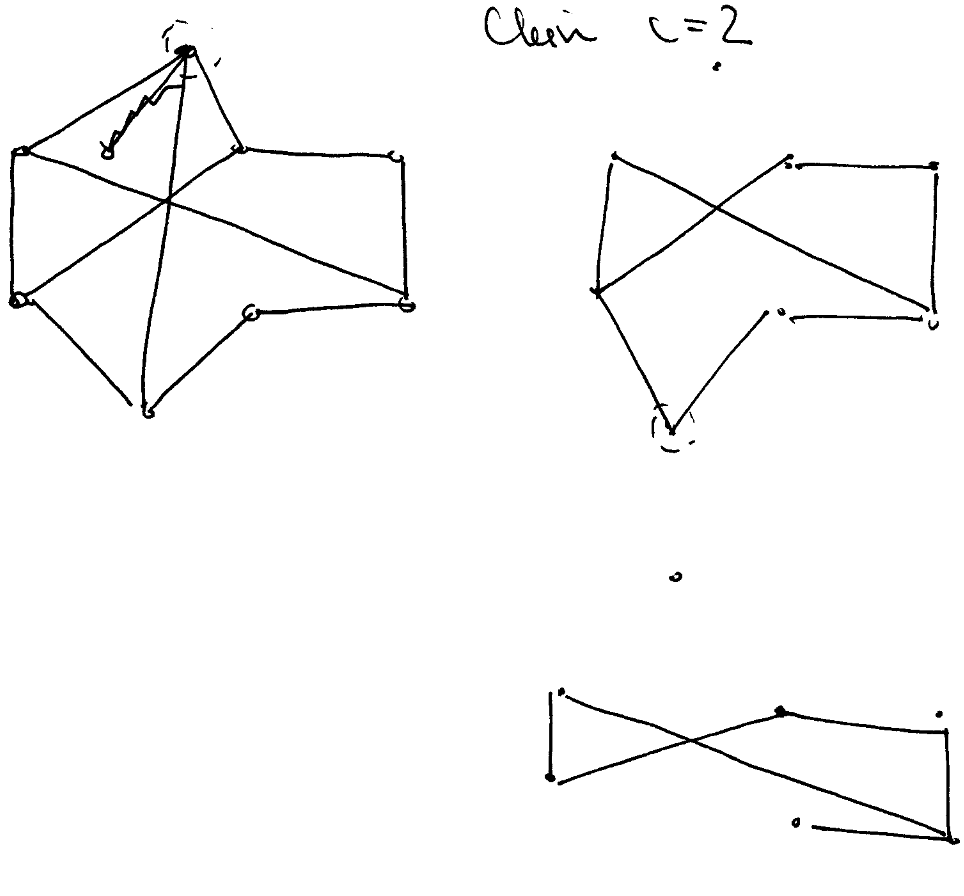
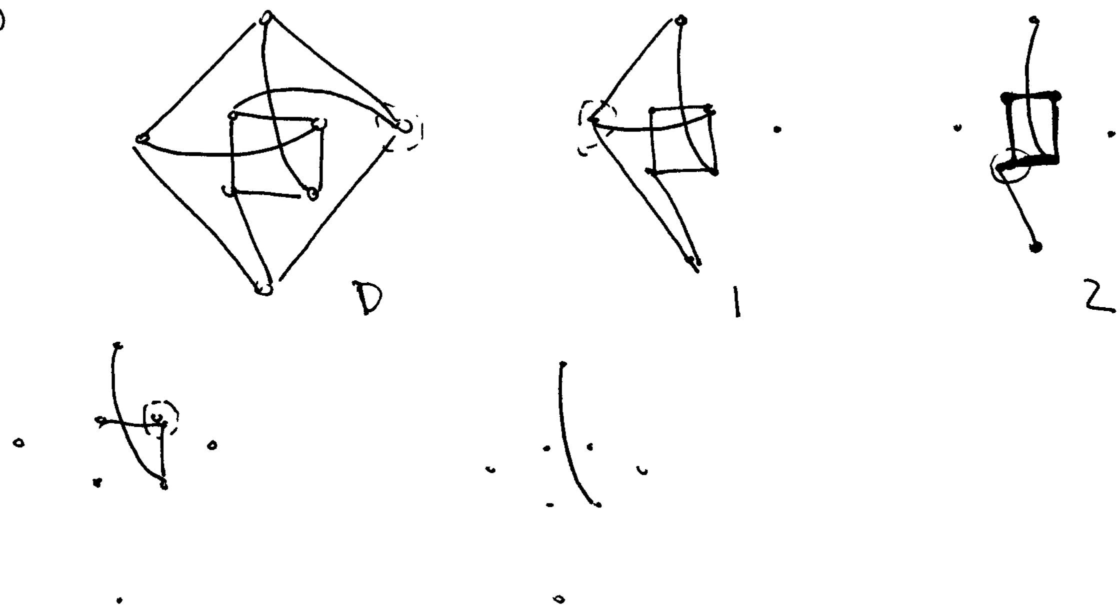


Fig 92:

Claim $c=3$

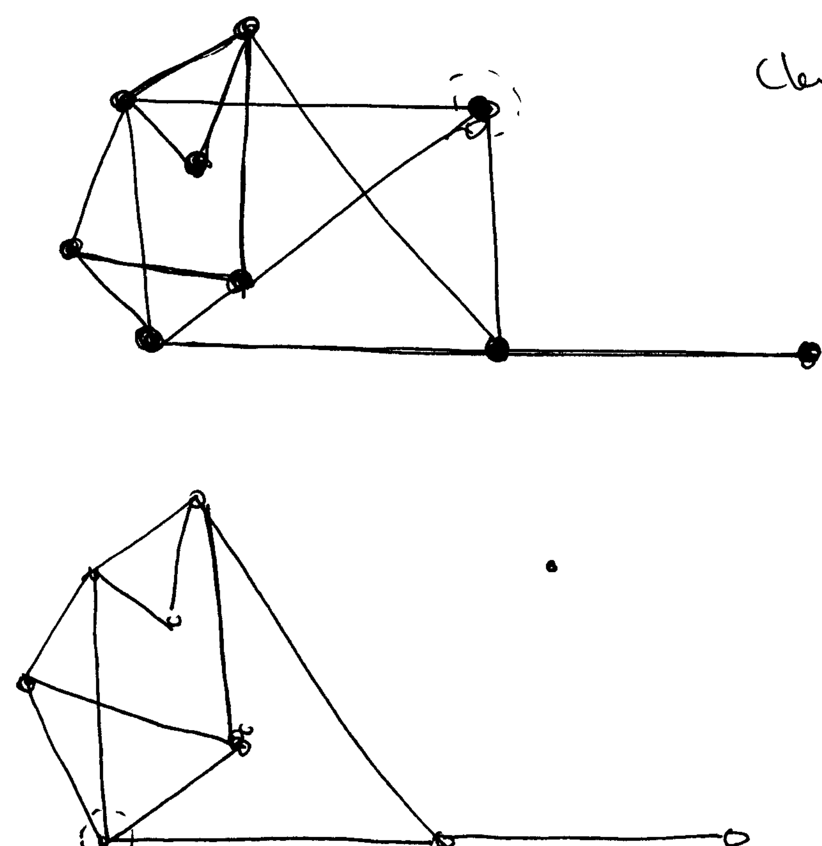
(a)

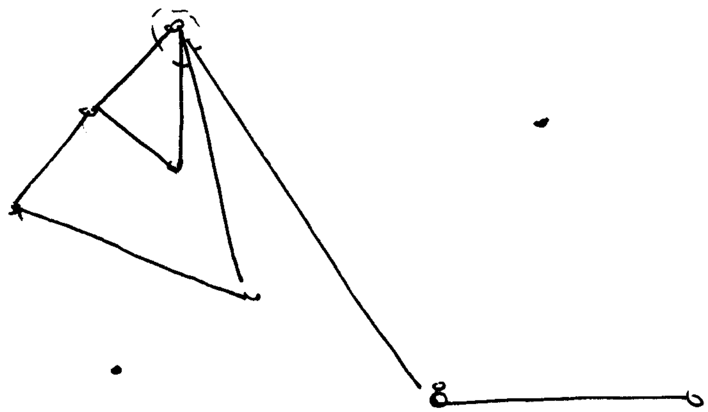


5 is $c=5$?

(b)

Clai $c=\cancel{2}\ 3$





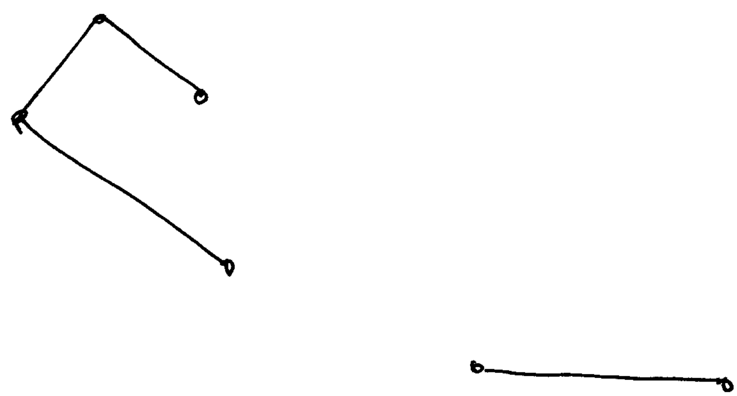
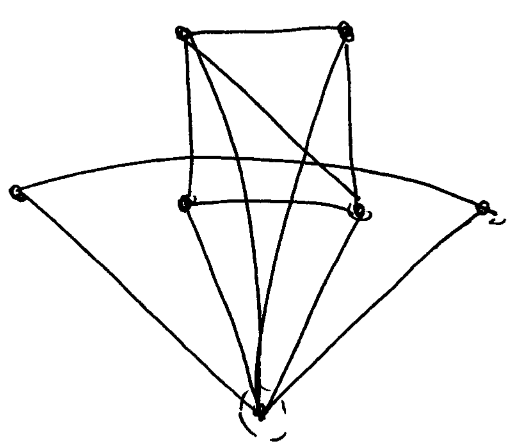
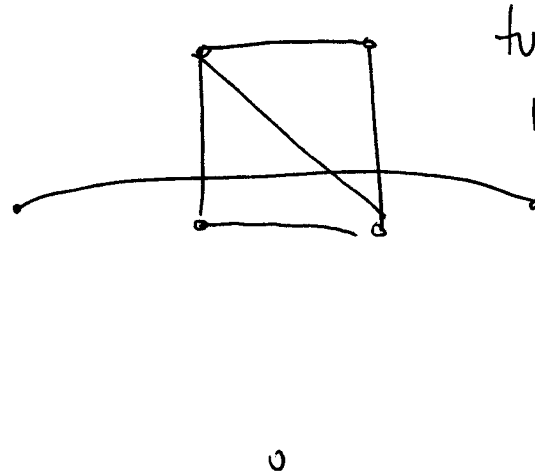


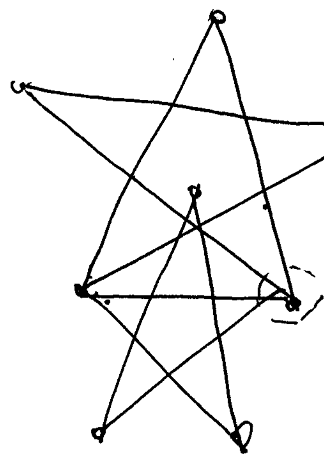
Fig 93: (a) Chain $c = 1$



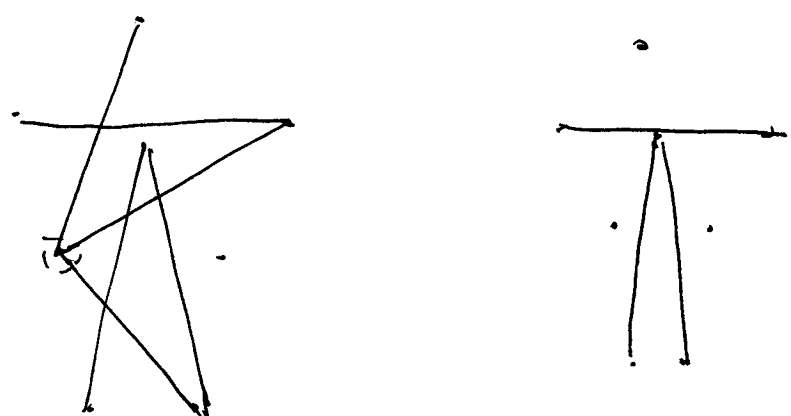


two disconnected pieces

Fig 93: (b)



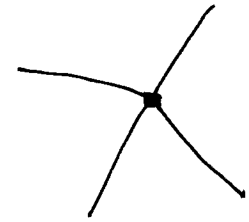
$c = \cancel{3}\, 2$.



two components.

(12) $\frac{2e}{v} = \frac{1}{v}\sum_{v \in V} \deg(v)$ = Average degree of each vertex.

Assume c were greater than $\frac{2e}{v}$, then



$\exists$ a vertex w/ degree less than or equal to $\frac{2e}{v}$, For this vertex we can isolate ~~the point that # edges~~ it by removing all incident edges. Since to remove all edges only requires less than $\frac{2e}{v}$ work the connectivity $c \leq \frac{2e}{v}$

This is in contradiction to the assumption that $c > \frac{2e}{v}$.

(13) $c \leq \frac{2e}{v}$

$3 \leq \frac{14}{v} \leq$

$v \geq 3 \Rightarrow e \leq 3v-6$

$7 \leq 3v-6$

$v \geq \frac{13}{3} = 4.3$

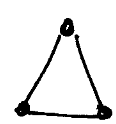
$v \geq 4$

w/ 7 edges we can have at most 8 vertices

$=$ n+1 verts

w/ 3 edges

4 vertices



3 vertices

w/ e edges we can have at least

$e = \frac{v}{2}(v-1) = k(v) = \Theta(v^2)$

$v = k^{-1}(e)$ $v = \Theta(\sqrt{e})$

consider e edges for $e=7$

$e+1$ v.s $\sqrt{e}$ 8 v.s. 2.6

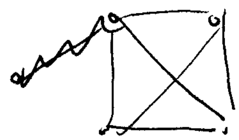
$\therefore$ I claim given e edges

$$k^{-1}(e) < v < e+1$$

~~Thus~~ Now w/ $e=7$ $k^{-1}(7) = ?$

$$14 = v^2 - v \Rightarrow v^2 - v - 14 = 0 \qquad v = \frac{+1 \pm \sqrt{1+4(14)}}{2} = \frac{1 \pm 7.54}{2}$$
$$(\quad)(\quad)$$
$$= 4.27$$

$$k_4 = \tfrac{1}{2}(4)(3) = 6$$
$$k_5 = \tfrac{1}{2}(5)(4) = 10$$

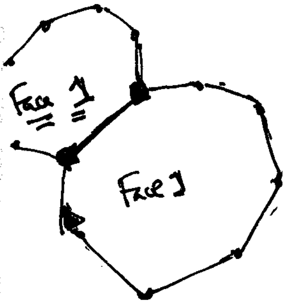


$$3 \leq \frac{14}{v} \leq \frac{14}{4.27} \quad ?$$

(14) Pr: If a planar & connected graph G has the property that the boundary of every face is a cyclic graph, then G is ~~poto~~ polygonal (every edge borders on 2 different faces)

Assume not every edge borders on 2 different faces, For the edge that borders on only one face, find the cyclic graph that is the boundary of this face.

This graph ~~cannot~~ must retrace its steps



?

(15) 11:06 - 11:10.

~~[illegible]~~ If adding one more edge results in a non-planar graph I am guessing

? $e = 3v - 6$

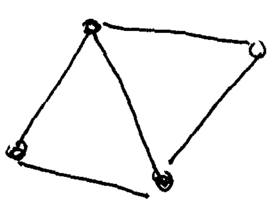
If an edge bordered only 1 face we could add atleast 1 ~~another~~ other edge & not disrupt the planarity of the graph

Thus ~~[illegible]~~ all edges border 2 or more faces, as an edge cannot border 3 or more faces each edge borders 2 faces & the graph is polygonal. If the boundary of every face was not C_3 then adding a diagonal to any C_4 or higher C's would contradict

the ⇒ G must be made up of C_3 graphs

~~[illegible]~~ ~~[illegible]~~ $3f = 2e$? $f = \frac{2}{3}e$

$$V + f - e = 2$$



$$V + \frac{2}{3}e - e = 2$$

$$V - \frac{e}{3} = 2$$

$$e = 3v - 6$$

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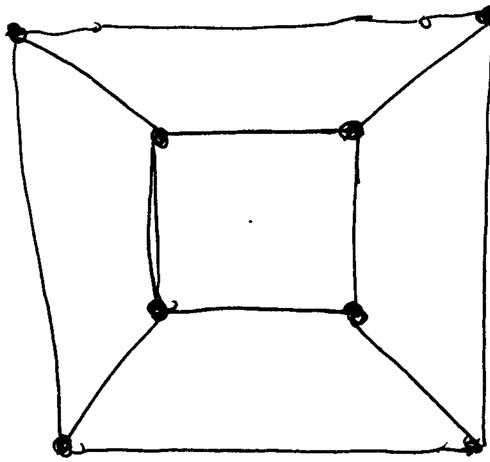
1

(16) Example Graph



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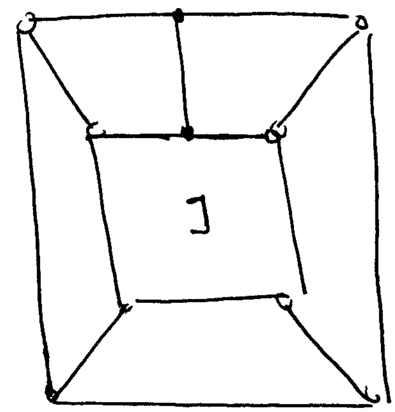
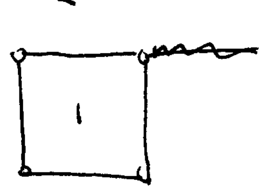


X Not example boundary of face #1 is not C_4.

If every face is bounded by C_4 adding a diagonal to each C_4 face results in C_3 faces. ... would this get us the graph from problem 15?

An edge w/o two faces could not possibly be used as a member of C_4 to surround any given face, thus no faces like this must exist. $\Rightarrow$ G is polygonal

2

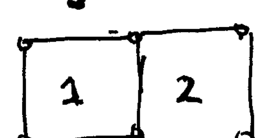


Now

$$4f = 2v \Rightarrow f = \frac{1}{2}v$$

$4(2) = 2v$

3



Thus euler formula =

$$v + f - e = 2$$

$$v + \frac{1}{2}v - e = 2$$

$$\frac{3}{2}v - e = 2$$

$4(3) = 2 \cdot 6. = 2v$

$$e = \frac{3}{2}v - 2$$ why does this not work?

Adding a diagonal edge into every face but the ∞ face gives a graph like problem 15.

$\cancel{e \le 2v-4}$ # edges added $= f-1$

$e = 3v-6$

$\cancel{e+f-1} = \cancel{2v}-4$ from above $4f = 2v$

$f = \frac{1}{2}v$

$\cancel{e + \frac{v}{2} - 1} =$

$\cancel{e} =$

$e + f - 1 = 3v - 6$

$e + \frac{v}{2} - 1 = 3v - 6$

$e = \frac{5}{2}v - 5$

(17) If $c \geq 6$

$+ \quad c \leq \frac{2e}{v}$

$$\Rightarrow \quad 6 \leq c \leq \frac{2e}{v}$$

$$\Rightarrow \quad \frac{2e}{v} \geq 6 \qquad e \geq 3v$$

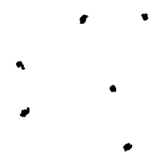
$$\parallel$$

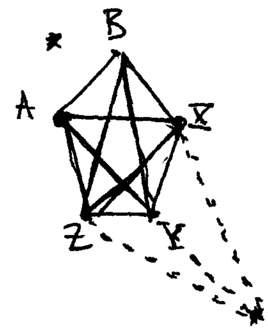
Avg degree of vertex's $\geq 6 \quad \Rightarrow \quad \exists$

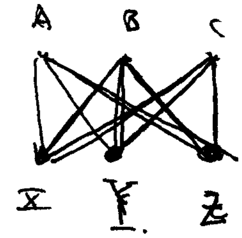
Since $e \geq 3v$ + Thm 11 guarantees that if G is planar + connected w/ $v \geq 3$, then $\frac{3}{2}f \leq e \leq 3v-6$

Thus G cannot be planar + connected

If we assume the graph is connected initially then the graph cannot be planar.



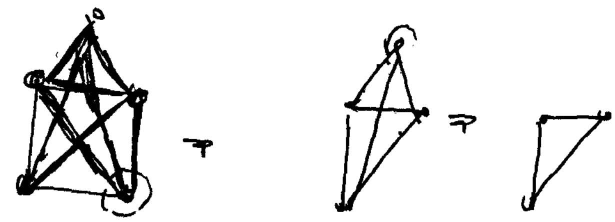




Start w/ the expansion of K_5.

Any expansion of K_5 will have $c \leq 4$.

If I will show that an expansion of K_5.



1 2 3

$C \geq 3$.

Basically I want to show that K_5 almost has a subgraph of UG already that with a few more edges UG will be found

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Lemma 14:

$$2e = \sum_{v \in V} \deg(v) = dv.$$

(all vertices of G)

Since G is a platonic graph $\deg(v) = d \;\; \forall \, v \in V$.

Lemma 15:

As G is a platonic graph it is polygonal & thus every edge borders 2 faces (separates 2 faces)

$$f \cdot n = 2 \cdot e = \sum_{v \in V} \deg(v) = dv$$

$\uparrow$

$n = \#$ edges/face

$$f = \frac{dv}{n}.$$