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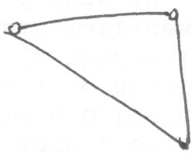
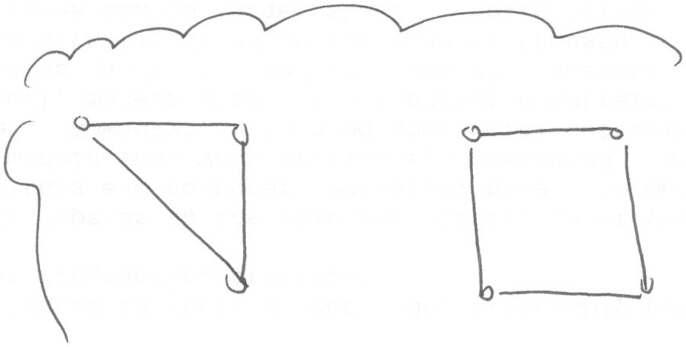
(1) regular = all vertices have same degree

(2)

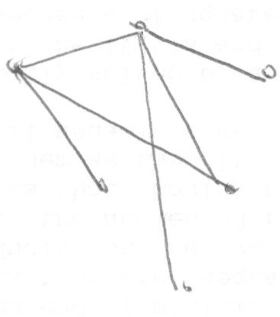




Taken together these two graphs are not a cyclic graph yet it is a regular graph

(3) $d = 3$ $n = 3$ $v = 4$ $e = 6$ $f = 4$ 1st row

$v = 6$ $e = 12$



$\binom{12}{2}\binom{11}{2}\binom{10}{2}\cdots\binom{6}{2} >$

(4)

$v = 6 \quad e = 10$

If G is reguler w/ degree d

$$10 = e = \frac{dv}{2} = \frac{d \cdot 6}{2} = 3 \cdot d \quad \rightarrow\leftarrow.$$

(5) A reguler graph (A graph w/ all vertices of the same degree) must satisfy

$$2e = \sum_{v \in V} \deg(v) = dv$$

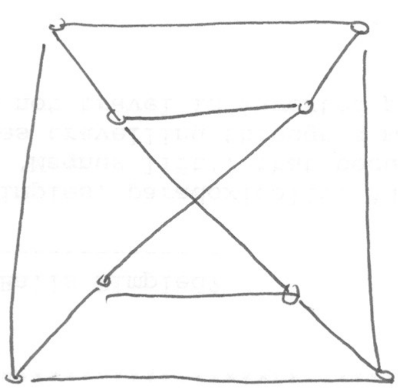
$$e = \frac{dv}{2}$$

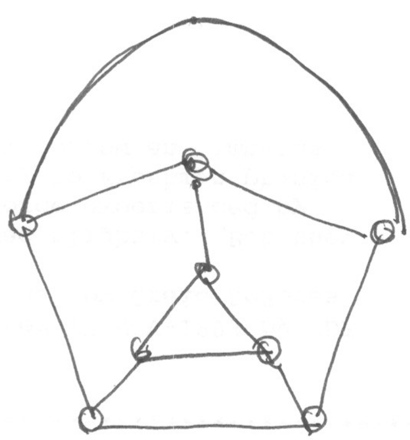
If v is odd

$v = 2k+1 \quad k = 0, 1, \ldots$

For e to be an integer d must be even.

(6)





7

1 vertex

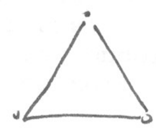
$d=0$

2 vertices

$d=1$

$d=0$

3 vertices

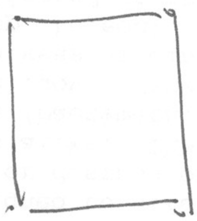


$d=2$

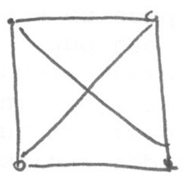
$d=0$

4 vertices

$d=0$



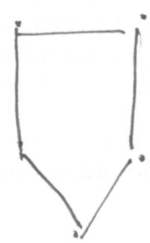
$d=2$



$d=3$

5 vertices

$d=0$

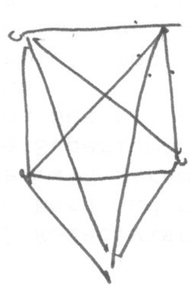


$d=2$

$d=3$ not poss

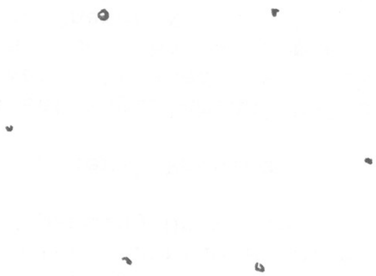
$2e = dv = 3 \cdot 5 = 15$

$\rightarrow\leftarrow$

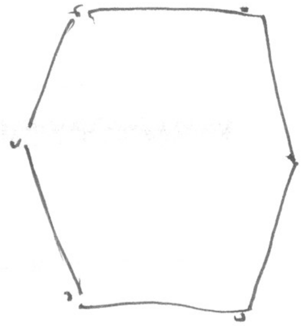


$d=4$

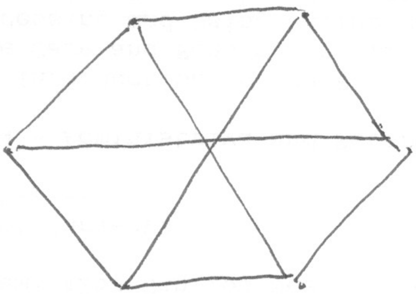
total # of graphs thus far = 12



$d=0$



$d=2$

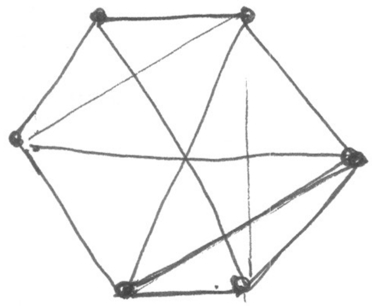


$d=3$

$2e = \cancel{8}\ v\cdot d$

$2e = 3\cdot v = 18$

$e=9.$



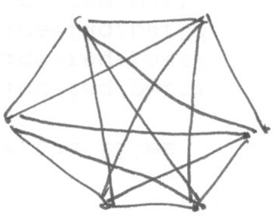
$d=4.$

$2e = 4\cdot 6 = 24$

$e = 12$

$\overset{?}{=} \frac{v(v-1)}{2} = \frac{6(5)}{2} = 15$ No.

not poss.



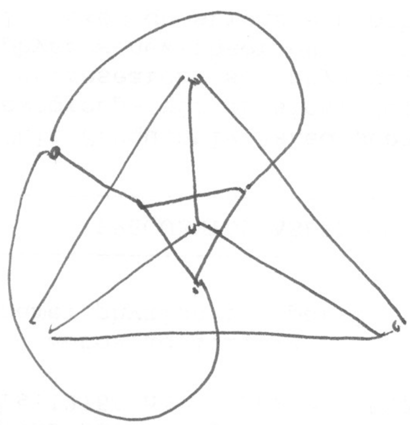
$d=5$

didnt get 20?

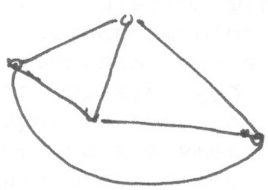
(8) isomorphism of Fig 113 (a) + Fig 112 (a) is obvious

The two graphs in Fig 112 (c) + Fig 113 (c) cannot be isomorphic since the later has a vertex of degree 5 while the former does not.

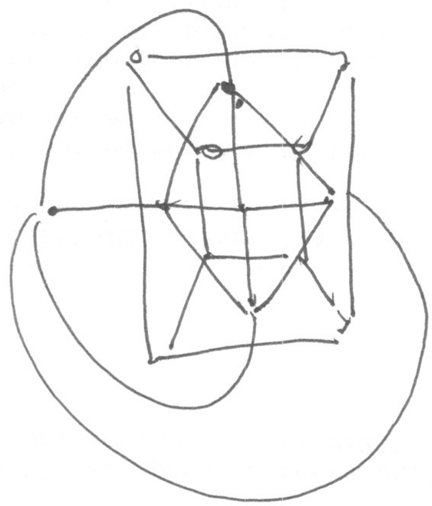
tetrahedron



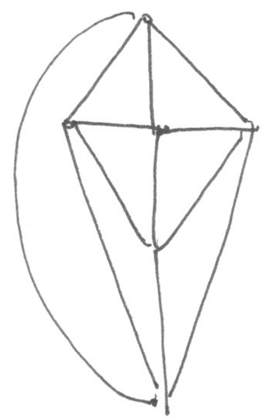
dual



cube



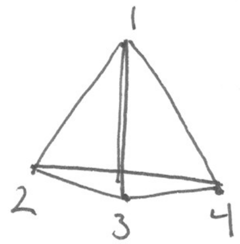
dual

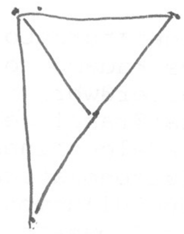


dodecahedron

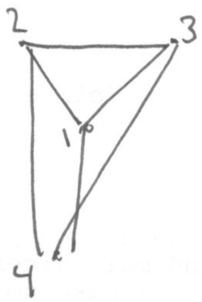
(9) W_v

W_4



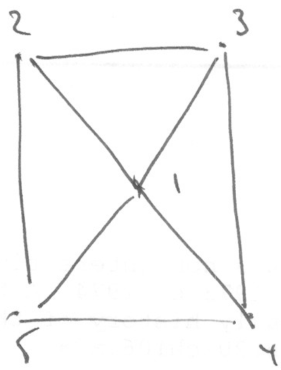


W_4



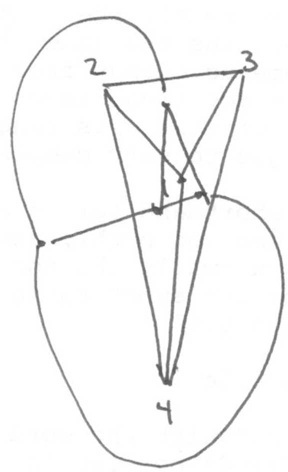
△ wheel

W_5



square wheel

dual of W_4 is



isomorphic.

~~[illegible]~~

1st # of faces = V.

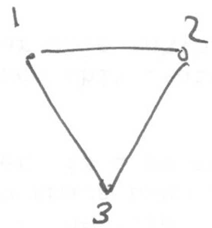
2nd a face corresponds to the 1st vertex

3) ∞ face connects to all V-1 vertexs

4) All V-1 vertexs ....

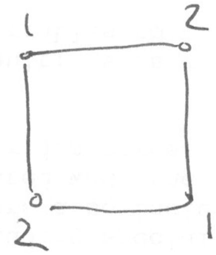
(1) C_v = ~~each~~ cyclic graph on v vertices

has $\chi = 2$ chromatic if v is even



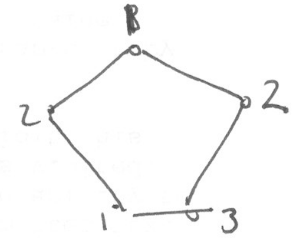
C_3

$\chi = 3$



C_4

$\chi = 2$

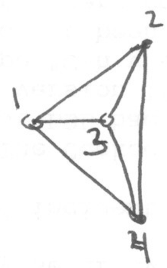


C_5

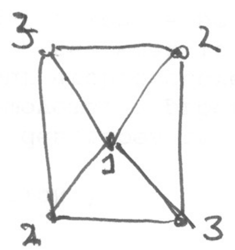
$\chi = 3$

$\chi = 3$ if v is odd

W_v



W_4



W_5

$1 + \chi_{C_{4-1}} = 1 + \chi_{C_3}$

$= 1 + 3 = 4$

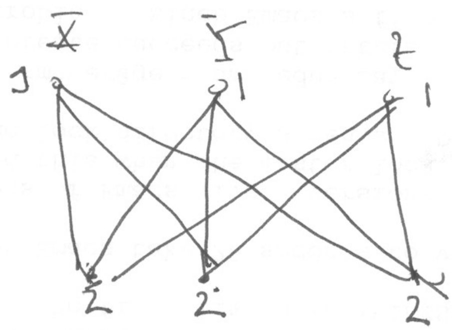
$1 + \chi_{C_{5-1}} = 1 + \chi_{C_4} = 1 + 2 = 3$

Guess ~~$\chi_{W_v} =$~~

$\chi_{W_v} = 1 + \chi_{C_{v-1}}$

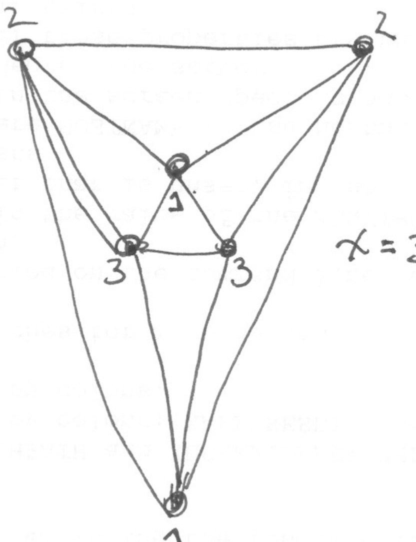
$= 1 + \begin{cases} 2 & v-1 \text{ even} \\ 3 & v-1 \text{ odd} \end{cases}$

UG



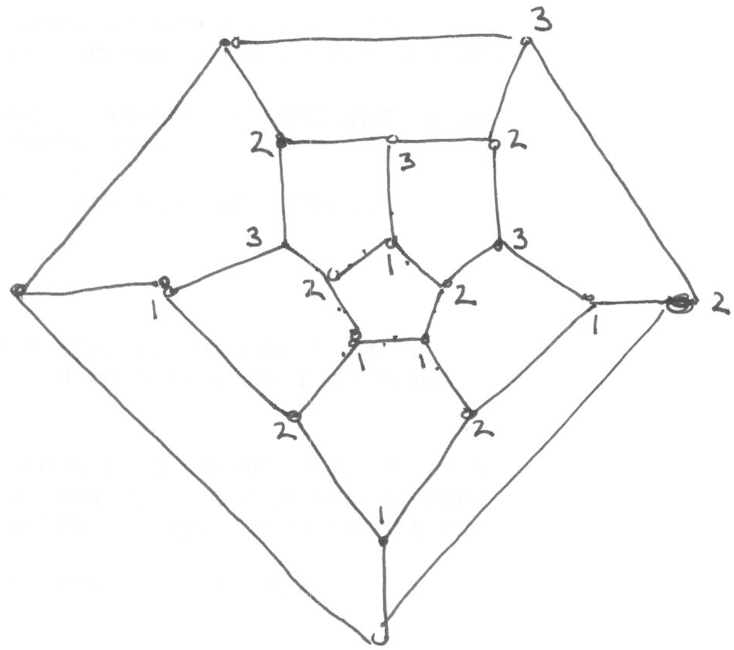
$\chi_{UG} = 2$.

octahedron

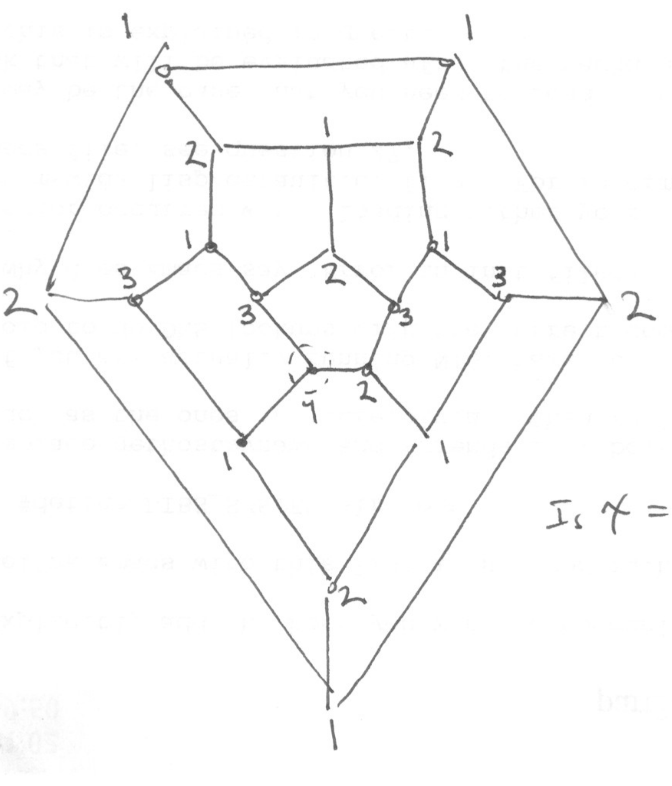


$\chi = 3$.

dodecahedron:



dodecahedron



Is $\chi = 4$?

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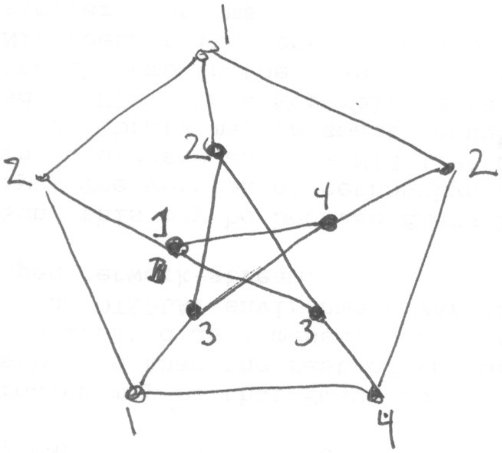
2

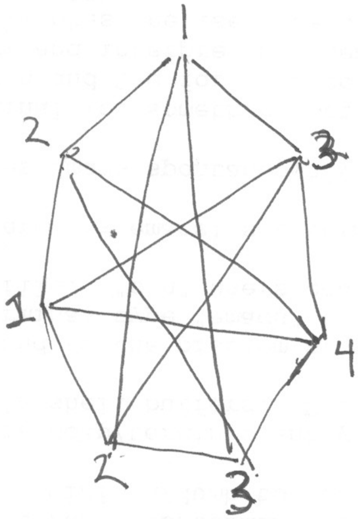
(a)



$\chi = 4$

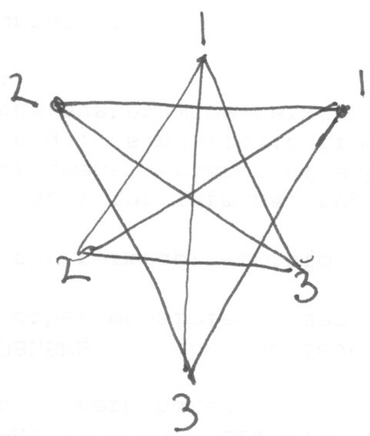
X $\chi = 3$?

(b)



$\chi = 4.$

(c)



$\chi = 3$

(3) Two Color Theorem: All graphs G w/ $\chi = 2$

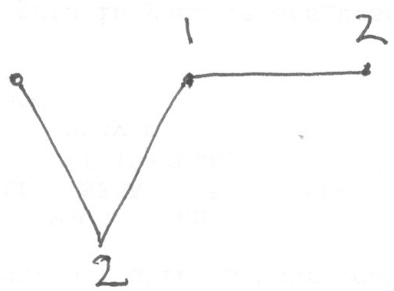
$=$ set all graphs w/ at least 1 edge + no odd cyclic subgraphs.

(i) $\chi = 2$ must have at least 1 edge if not it would be colored w/ $\chi = 1$. $\rightarrow\leftarrow$.

(ii) G cannot have an ~~even~~ odd cycle subgraph for any odd cyclic subgraph requires $\chi \geq 3$. which would be a $\rightarrow\leftarrow$

(only did one direction ...)

(4)



?

(5) $\cancel{\chi\bar{\chi} \geq v}$ $\cancel{\bar{G} = K_v - G}$

$\cancel{\chi_{K_v} = v}$ $\not\geq$

$\cancel{\chi_G \geq 2}$ $\cancel{\chi_{\bar{G}}}$

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(5) $\chi_G = \chi$ $\chi_{K_V} = V$

$$\bar{\chi} = \chi_{\bar{G}} = \chi_{K_V} - \chi_G = V - \chi$$

$$\therefore \chi\bar{\chi} = V\chi - \chi^2$$

~~Sm~~ Assum w.o.l.g ~~#~~ G is a "smaller" graph than $\bar{G}$.

Then χ

$$\bar{\chi} \geq V_1$$

$$\bar{\chi} \geq V_2$$

$$\vdots$$

$$+ \quad \bar{\chi} \geq V_\chi$$

$$\chi\bar{\chi} \geq \sum_{i=1}^{\chi} V_i = V$$

$$\chi + \bar{\chi} \geq 2\sqrt{\chi\bar{\chi}} \geq 2\sqrt{V}.$$

(6) $x\bar{x} = v$ + $x + \bar{x} = 2\sqrt{v}$

$\bar{x} = \frac{v}{x}$.

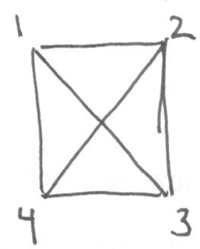
$x + \frac{v}{x} = 2\sqrt{v}$ Then

$x^2 + v - 2\sqrt{v}\,x = 0$

$x^2 - 2\sqrt{v}\,x + v = 0$

$$x = \frac{2\sqrt{v} \pm \sqrt{4v - 4v}}{2} = 2\sqrt{v}$$

Since x must ~~be~~ be an integer pick $v = 4$. $x = 4$.



Thus K_4 will work $x = 4$

$\bar{K}_4$

$=$

N_4

$\bar{x} = 1$

Think also prob v = perfect square will work 4, 9, 16, 25, ... in each case the corresponding K_v is one graph + N_v is the other

(7) K_7 is regular of degree 6, but it is non planar.

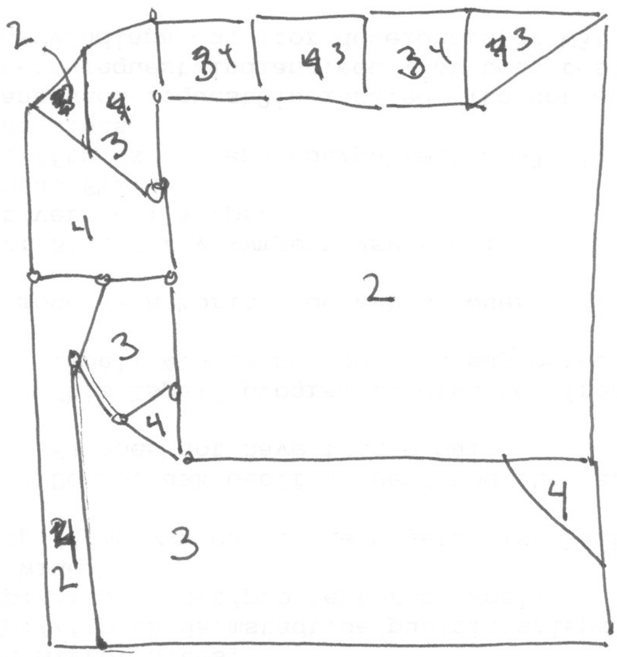
every other graph w/ 7 vertices has at least one vertex ≤ 5.

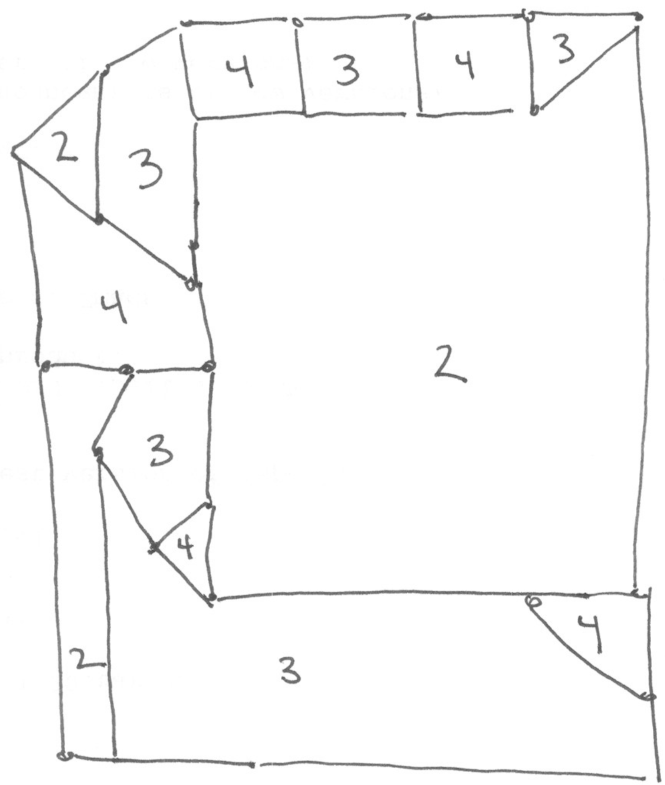
+ in particular every planar graph w/ 7 vertices

?

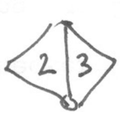
(8) 123 (b)

4 colors only.

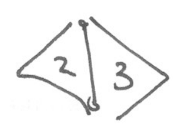




1



1



$$[x] = \lfloor x \rfloor$$

$$\lfloor x \rfloor - m = \lfloor x - m \rfloor \qquad \lfloor x \rfloor \lfloor y \rfloor \leq \lfloor xy \rfloor$$

let $x = p + i \quad y = q + j \quad +$ $\lfloor x \rfloor = p \quad i, j \geq 0.$

$\lfloor y \rfloor = q \quad 0 \leq i < 1$

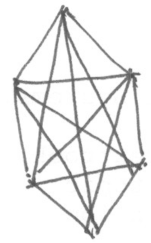
$0 \leq j < 1$

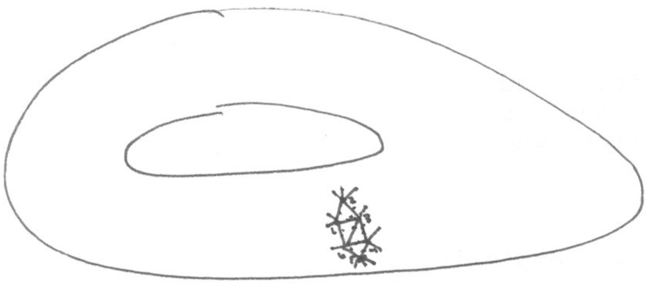
$$\lfloor x \rfloor - m = p - m =$$

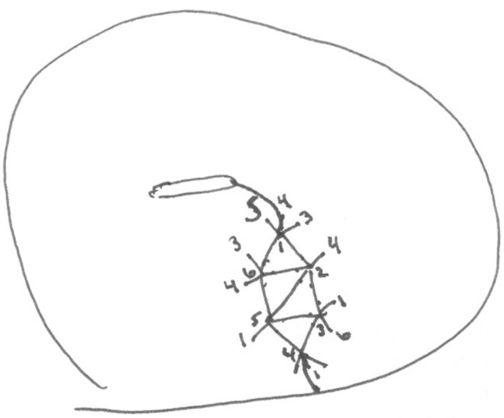
$$\lfloor x \rfloor \lfloor y \rfloor = p \cdot q \overset{?}{\leq} \lfloor (p + i)(q + j) \rfloor = pq \cancel{+ ip +} j$$

$$\lfloor pq + iq + jp + ij \rfloor$$

(1) K_6:







$$v + f - e = 2 - 2g$$

$$g = 1$$

$$v + f - e = 0$$

$v = 6 \quad e = \frac{1}{2}6(5) = 15 \quad f =$

$$6 + f - 15 = 0$$

$$f = 9$$

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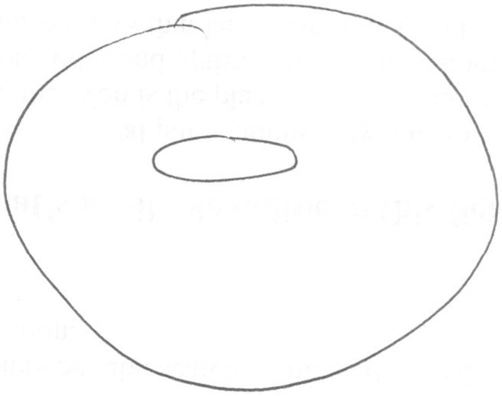
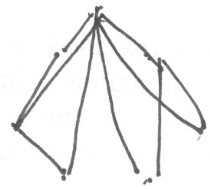
(2)



k_7 :



(3) ?

(4) As G originally was connected & we cut along a ring ~~...~~ duplicating the # of nodes in the ring

?

(8) $k_1 = N_1 \quad g_{k_1} = 0$

$$g = \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil \Bigg|_{v=0} = \lceil 1 \rceil = 1 \rightarrow\leftarrow$$

$k_2 = P_2 \quad g_{k_2} = 0$

$$g = \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil \Bigg|_{v=2} = \left\lceil \frac{(-1)(-2)}{12} \right\rceil = 1$$

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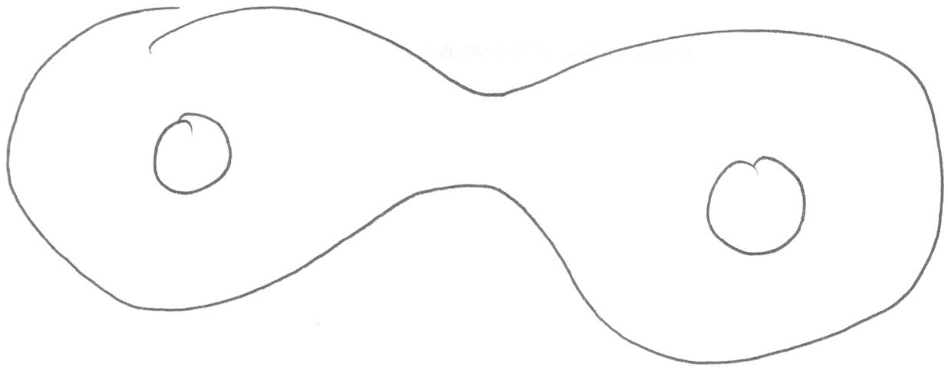
(6) ?



By ~~Cor~~ Corollary 22a: G is connected w/ $v \geq 3$ & g.

$$\left\lceil \frac{1}{6}e - \frac{1}{2}(v-2) \right\rceil \leq g \leq \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$$

$$v = 8 \qquad e = e_{K_7} + 4 = \frac{1}{2}(7)(6) + 4 = 21 + 4 = 25.$$

$$\left\lceil \frac{1}{6}e - \frac{1}{2}(6) \right\rceil \leq g \leq \left\lceil \frac{5(4)}{12} \right\rceil$$

$$\left\lceil \frac{25}{6} - 3 \right\rceil \leq g \leq \left\lceil \frac{20}{12} \right\rceil$$

$$\parallel$$

$$1 \leq g \leq 1 \quad \Rightarrow \quad g = 1$$

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(7) For the complete graphs

$\underline{v}$	$\underline{e} = \frac{1}{2}v(v-1)$	$\underline{g} = \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$	$\underline{f} = 2-2g+e-v$
			$= 2-2\left\lceil \frac{(v-3)(v-4)}{12} \right\rceil + \frac{1}{2}v(v-1) - v$
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

By Corollary 22a pg 168 Trudeau 8-23-02 1

(8) $\left\lceil \frac{1}{6}e - \frac{1}{2}(v-2) \right\rceil \le g \le \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$

$\|$

$\left\lceil \frac{492{,}753}{6} - \frac{1}{2}992 \right\rceil \le g \le \left\lceil \frac{(991)(990)}{12} \right\rceil$

(9) If $L = U$

Then $\frac{1}{6}e - \frac{1}{2}(v-2) = \frac{(v-3)(v-4)}{12}$

$$e = 3(v-2) + \frac{(v-3)(v-4)}{2}$$

$$= \frac{1}{2}\left[6v - \cancel{12} + v^2 - 4v - 3v + \cancel{12}\right]$$

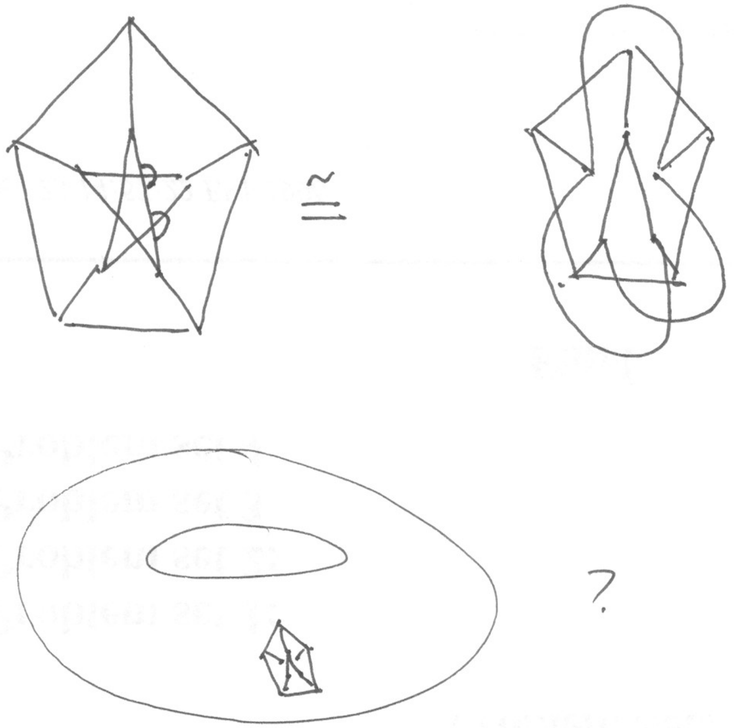
$$= \frac{1}{2}(v^2 - v)$$ ✓

(10) $P(d,n) = \frac{4(g-1)n}{nd - 2d - 2n}$

If $g > 1$

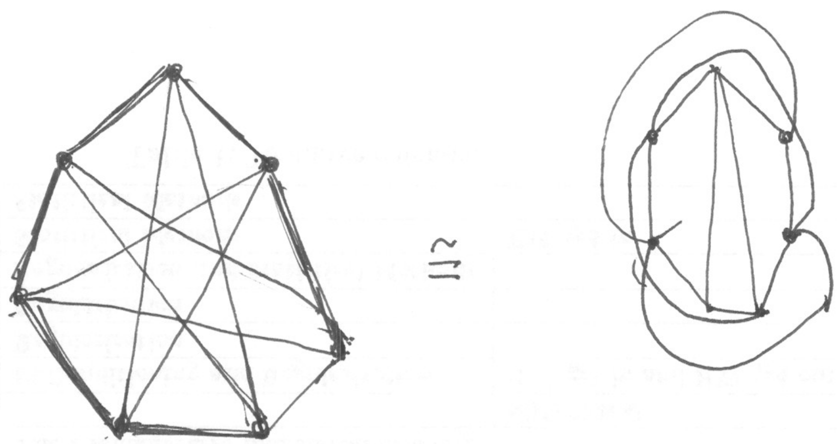
$P(3,7) =$

(11) Fig 126 (a)



?

Fig 126 (b)



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(12) $v = 8+1 = 9$

$e = \frac{1}{2}8(7) + 6 = 34$

According to bound. g.

$$\left\lceil \frac{1}{6}e - \frac{1}{2}(v-2) \right\rceil \leq g \leq \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$$

$$\parallel$$

$$\left\lceil \frac{34}{6} - \frac{7}{2} \right\rceil \leq g \leq \left\lceil \frac{30}{12} \right\rceil$$

$$2 \leq g \leq 3$$

How show $g \neq 2$?

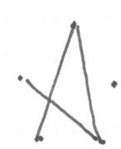
(13) From problem 11 the genus of both these graphs is 1

$\therefore$ By Eulers 2nd thm:

$$v + f - e = 2 - 2g = 0$$

Fig 126 (a)


-2


-2

$v = 10 \quad e = 5 + 5 + 5 = 15.$

$\Rightarrow f = 5.$

Fig 126 (b)

$v = 7 \quad e =$

$$\sum_{v \in V} \deg(v) = 2e$$

$\shortparallel$

$4(7) = 2e \Rightarrow e = 14.$

$v + f - e = 0$

$7 + f - 14 = 0 \Rightarrow f = 7.$

Fig ~~126~~ ~~(b)~~ 143 has genus $= \underline{\underline{3}}$ by problem 12

$v + f - e = 2 - 2g = -4$

$v = 9 \quad e = e_{K_8} + 6 = \frac{1}{2} 8(7) + 6 = 28 + 6 = 34.$

$9 + f - 34 = -4$

$\underbrace{9 + f - 30} - 4 = -4$

$-21 + f = 0 \quad f = 21$

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(14) $\left\lceil \frac{e}{6} - \frac{1}{2}(v-2) \right\rceil \le g \le \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$

1, 1, 2, 3, 5,

$$\left\lceil 3 - \tfrac{1}{2}5 \right\rceil \le g \le \left\lceil \frac{4\cdot 3}{12} \right\rceil = 1$$

$g = 1$.

$$\left\lceil 3 - 2.5 \right\rceil = \left\lceil .5 \right\rceil = 1$$

– – –

$$\left\lceil \frac{52}{6} - \frac{1}{2}(9) \right\rceil \le g \le \left\lceil \frac{8\cdot 7}{6} \right\rceil$$

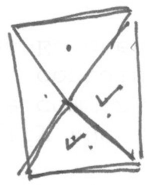
$$= \quad 5 \le g \le 10 \quad ?$$

(15) w/ genus g + $n=3$ # edges bounding every face

$v+f-e=2-2g$

~~B~~ is g=platonic.

$f = \frac{dv}{n}$



$f=?$

~~guess~~ $\cancel{f}=2v$

$\boxed{3f=2e}$

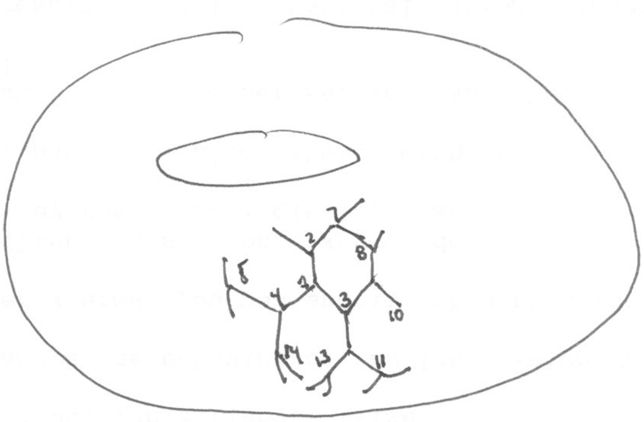
$e = v+f-2+2g$

$= v+\frac{2}{3}e-2+2g$

$\frac{1}{3}e = v-2+2g$

$e = 3(v-2+2g)$

(16) obviously $n=6$ + $d=3$



just draw it on S_1 + one is done

(17) K_7 has $v = 7$. vertices

$e = \frac{1}{2}(7)(6) = 21$ edges

K_7 is connected

$$\left\lceil \frac{1}{6}(21) - \frac{1}{2}5 \right\rceil \le g \le \left\lceil \frac{4 \cdot 3}{12} \right\rceil$$

$$\parallel$$

$$\lceil 1 \rceil \le g \le \lceil 1 \rceil$$

$$\parallel$$

$1 \le g \le 1 \rightarrow g = 1. \Rightarrow K_7$ is ~~p~~ ~~plnt~~

The degree of every vertex is $6 = d$

By Eulers 2nd thm:

$$v + e - f = 2 - 2g$$

$$\parallel$$

$$7 + 21 - f = 0$$

$$\Rightarrow f = 28$$

Then $f = \frac{d(7)}{n}$

$$\Downarrow \quad \Downarrow$$

$$28 = \frac{6 \cdot 7}{n} \Rightarrow 14 = \frac{3 \cdot 7}{n} \Rightarrow 2 = \frac{3}{n} \rightarrow n = 3/2 ?$$

(18) . . . . ordered to read

(19) ?

(20) $d=4$ $n=4$

Square box w/ square hole

pentagonal box w/ pentagonal hole . .

(21) $v=12$ $\chi=10$

By Heawood Coloring Thm:

$$\chi(S_n) = \left\lfloor \frac{7+\sqrt{1+48n}}{2} \right\rfloor = 10$$

$$\Rightarrow \left\lfloor \frac{7+\sqrt{1+48n}}{2} - \frac{20}{2} \right\rfloor = 0$$

$$\Rightarrow \left\lfloor \frac{-13+\sqrt{1+48n}}{2} \right\rfloor = 0$$

$$\Rightarrow 0 \leq \frac{-13+\sqrt{1+48n}}{2} < 1$$

$$0 \le -13 + \sqrt{1+48n} \le 2$$

$$13 \le \sqrt{1+48n} \le 15$$

$$13^2 \le 1+48n \le 15^2$$

$$\frac{13^2-1}{48} \le n \le \frac{15^2-1}{48}$$

$$\| \qquad\qquad \|$$

$$3.5 \le n \le 4.6$$

Since n is an integer. $n=3$ or $n=4$.

$V = \cancel{9}\,12$ $\le d \le 11$

By Eulers 2nd thm: $v + f - e = 2-2g$

$$V + \frac{dv}{n} - \frac{dv}{2} = 2-2g$$

$$2nv + 2dv - ndv = 2(1-g)\cdot 2n$$

$$2nv + (2-n)dv = 4(1-g)n$$

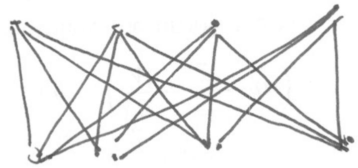
Thus $g \ge 3$ . . .

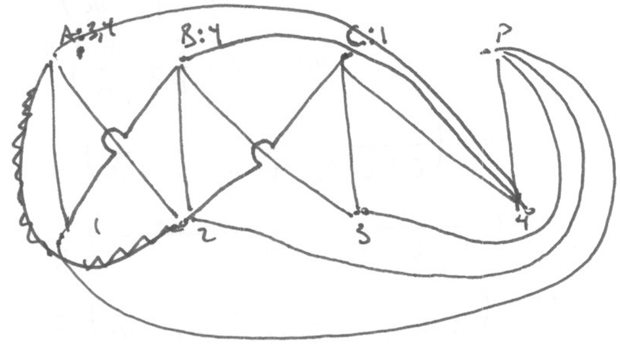
But $\cancel{g \ge \left\lfloor \frac{(v-3)(v-4)}{12} \right\rfloor}$ $g \le \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$

$$\left\lceil \frac{9\cdot 8}{12} \right\rceil = 6.$$

(22) ?

(23) Fig 104 (b)





$v = 8 \quad e =$

$$\sum_{v \in V} \deg(v) = 2e$$

$$\|$$

$$4(8) = 32 = 2e \Rightarrow e = 16.$$

$$\left\lceil \frac{e}{6} - \frac{v-2}{2} \right\rceil \le g \le \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$$

$$\|$$

$$\left\lceil \frac{8}{3} - 3 \right\rceil \le g \le \left\lceil \frac{5 \cdot 4}{12} \right\rceil$$

$$\|$$

$$\cancel{3 \le g}$$

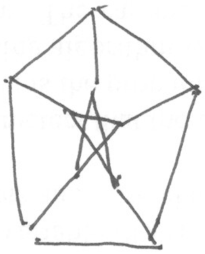
$$\left\lceil -\frac{1}{3} \right\rceil \le g \le \left\lceil 1.66 \right\rceil = 2$$

$$\|$$

0 But I know this graph is not planar since it is ~~an~~ supergraph & an extension of UG. $1 \le g \le 2$.

(24)

Petersen graph



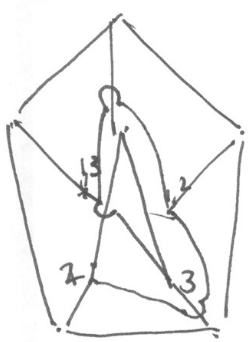
v = 10

e = 5 + 5 + 5 = 15

$$\left\lceil \frac{e}{6} - \frac{1}{2}(v-2) \right\rceil \le g \le \left\lceil \frac{(v-3)(v-4)}{12} \right\rceil$$

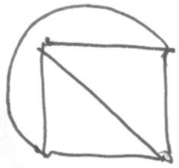
$$\left\lceil \frac{15}{6} - 4 \right\rceil \le g \le \left\lceil \frac{7\cdot 6}{12} \right\rceil$$

$$\lceil -1.5 \rceil \le g \le \lceil 3.5 \rceil$$

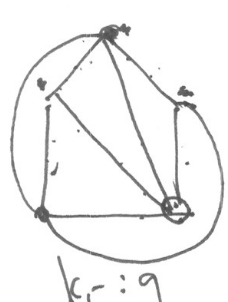


k = 3 ?

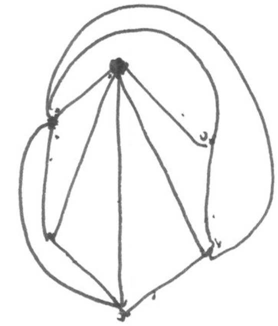
(25)



K_4 : 4



K_5 : 9



K_6 : 6+

K_4 : $v + (v-3) + (v-3)$

3v − 6.

max # edges $\overset{?}{=}$ 3v − 6.

18 − 6 = 12

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(26) ... Follow proof of thm for that corollary...

(1) $\sum_{v \in V} \deg(v) = 2e$

if the # of odd degree vertices did not come in pairs the sum could not be even

(2)(a) If a connected graph has an open Euler walk, then it has exactly 2 odd vertices, conversely if we have exactly 2 odd vertices then graph G has an open Euler walk.

since None of the K_v graphs have exactly 2 odd vertices none of the K_v graphs have an open walk.

~~K_2 has~~ K_2 has an open walk because it has exactly 2 odd vertices.

(b) By Thm 30, If a connected graph has a closed euler walk, then every vertex is even, +

A connected graph has a closed euler walk iff every vertex is even.

The only K_v graph that have every vertex even are ~~K_v~~ when v is odd the each vertex has degree $v-1$.

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~~3~~ (c) open hamiltonian walk requires that the sum of ~~the~~ every pair of vertices of the graph is at least $v-1$. Then G has ~~an~~ open hamiltonian walk.

Since for K_v ~~~~ each vertex has degree $v-1$

each pair of vertexes would have sum $2v-2$

$2v-2 > v-1$

$v > 1$ ✓.

Thus K_v has an open hamiltonian walk $\forall$ K_v $v>1$

(d) By Thm 33 if the sum of the degrees of each pair of vertexes is at least v, Then G has a hamiltonian walk.

The sum of each pair of vertices is $2v-2$

so we have a closed H walk when

$2v-2 > v$

$v>2$ ✓.

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(3) (a) As a Graph G we have 4 vertexes w/ degree 5 of each since an open H path only exists if the total # of ~~vertices~~ ~~is~~ odd vertices is $\underline{\underline{2}}$

Since it is 4 in this example the path cannot be traced.

(b) Has 5 nodes of degrees

6, 6, 4, 4, 4

Since the # of odd vertices is zero the graph has a closed Euler walk + ∴ one can form a Euler walk starting at any vertex

(c) Has 6 vertexes w/ degrees

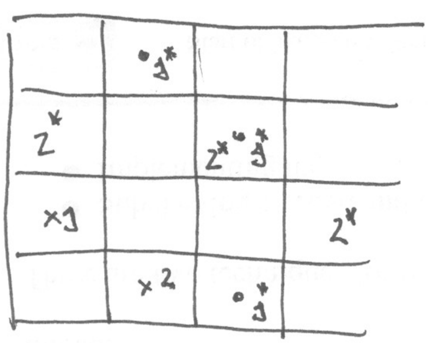
5, 6, 4, 5, 6, 4

Since there are 2 ~~deg~~ vertexes w/ odd degrees this graph does have a ~~Ham~~ Euler walk but only ~~a~~ starting from ~~the~~ either of the odd vertices

(4) To have an Euler walk, if the ~~# of odd vertices is even~~ graph has 0 pairs of odd vertices then the graph has a closed Euler walk. If it has 1 pair of odd vertices then it has an open Euler walk.

For a knights move when the knight is in a corner he can only move to two locations, i.e. the degree of a corner node is 2.

But when the knight is at one down a corner there are 3 locations he can move.



Thus this vertex has degree 3

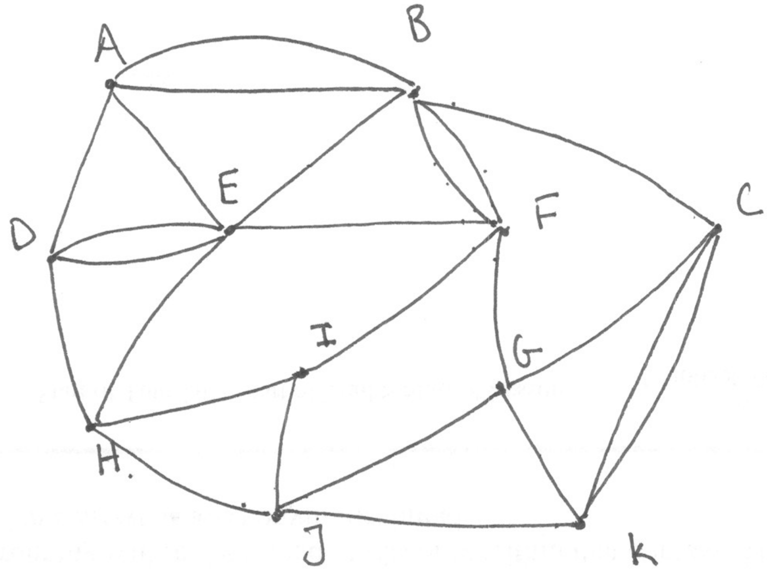
Since there are more than two vertices w/ this property there can be no Euler walk (open or closed)

(5) To have a closed Hamiltonian walk the sum of every pair of vertices must be at least V.

But since I can pick two opposing corners of the chess board & as each has degree 2 when I take ~~....~~ these two as a pair I get a sum of 4 which is not $\geq V$. ?

Thus I can not guaranteed a ~~pair~~ of ~~vertices~~ from this theorm ? closed Hamilton walk

(6)



Thus this multigraph has 11 vertices

F is odd, I is odd since it has exactly 2 odd vertices, then it has an open Euler walk & it must start at vertice F or I & end at vertex I or F.

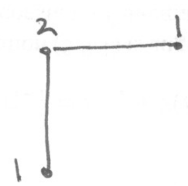
removing the bridge from H to I gives us I of degree 2 now & F still of degree 5.

To have a closed Euler walk we must have all vertices of even degree, to have an open Euler walk must have only 2 odd vertices of odd degree.

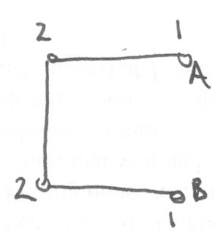
Since we have 1 odd vertex there can be no closed or open Euler walks.

(7)

$v=2$



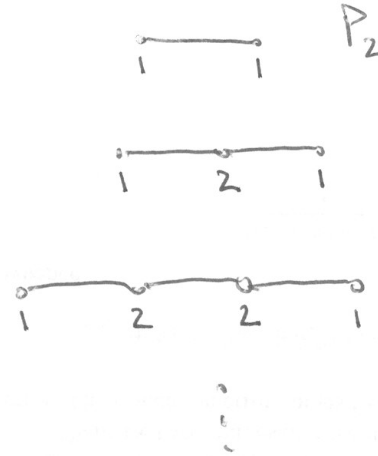
$v=3$



$v=4$

cont pair the two end vertices A & B to get sum ≥ 3. ?

(8)



P_2 $v=2$

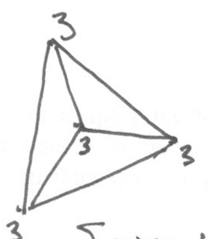
P_3 $v=3$

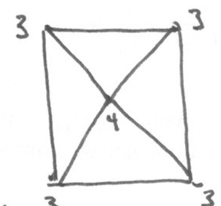
P_4 $v=4$

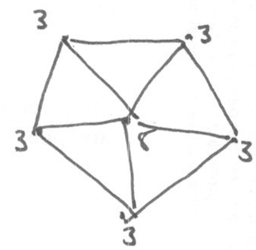
⋮

by inspection these graphs have an open hamiltonian walk

W_4 W_5 W_6







$\mathrm{Min}_{\text{pairs}} \sum \deg(v) = 6 \geq 6$

$\sum$ pairs vertex $= 6 \geq 4$

$\mathrm{Min}_{\text{pairs}} \sum_p \deg(v) = 6 \geq 5$

$\forall$ W_v graph start in the center go to an outside vertex & then walk around the C_{v-1} part of the graph, when we get to the next to last vertex go back to the center completing the Hamiltonian cycle.

(9) By taking C_v + Adding ~~vertices~~ edges only one can obtain any graph w/ a closed hamiltonian path.

Since a super graph must have connectivity greater than its subgraph ~~we~~ ~~see~~ ~~that~~ + since the connectivity of ~~K_v~~ C_v $(v \geq 3)$ is 2, the connectivity of any graph w/ a ~~H~~ closed Hamiltonian path must be greater than 2.

(10) $\{\cdot\} = \lceil \cdot \rceil$

Assume v even the $v = 2m$ $m \in \mathbb{N}$.

so $\lceil \frac{v}{2} \rceil = m$.

~~Seating~~

the degree of ea^i

considering each guest as a vertex of a graph + letting ~~Since~~ ~~the~~ ~~degree~~ ~~of~~ the adjacency relation mean "is an acquaintance" to

if we can find a closed Hamiltonian path ~~listing~~ in G then ~~each~~ sitting the professors in that manner would provide the proper seating.

Since a closed Hamiltonian path is gaurenteed to exist if the sum of every pair of vertexes is $\geq v$. Checking this condition may determine if a feasible seating arrangement is possible.

Since $\forall v_i \neq v_j \quad deg(v_i) + deg(v_j) \geq \left\lceil \frac{v}{2} \right\rceil + \left\lceil \frac{v}{2} \right\rceil$

$$\geq \left\lceil \frac{v}{2} \right\rceil + \left\lfloor \frac{v}{2} \right\rfloor \geq v.$$

∴ such a seating exists

(11) Pick any ~~odd~~ two vertices of odd degree.

Now I have 2n-2 odd vertices

Create a multigraph by adding n edges connecting a pair of ~~[illegible]~~ ~~[illegible]~~ odd vertices. This ~~connects~~ each produces a multigraph (possibly a graph) w/ ~~[illegible]~~ the degree of every vertex even. Thus we have a closed Euler walk existing in this multigraph. Removing each added edge in turn produces an open Euler ~~[illegible]~~ walk that begins & ends. How do I know that I can remove each edge & not break up my open Euler walk?

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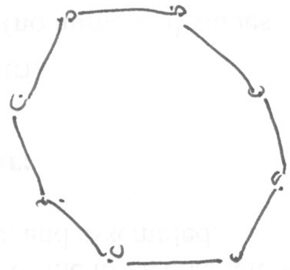
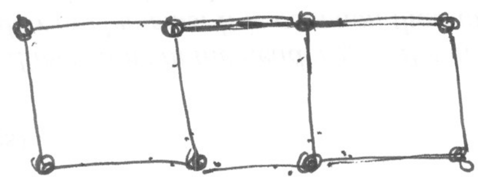
(c) Both a Euler walk + a Hamiltonian walk

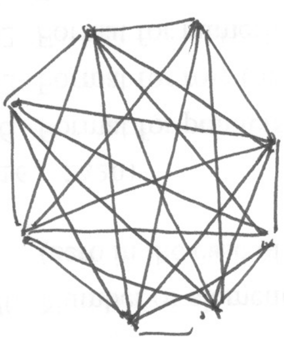


(d) would a disconnected graph work?

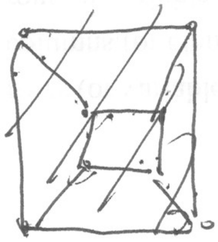


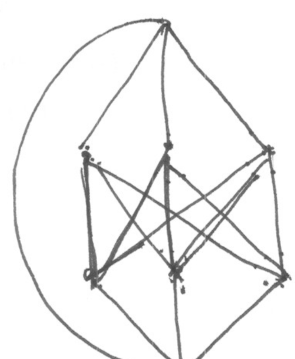
Does not have either a Euler walk or a Hamiltonian walk

(b) K_8 has a Hamiltonian walk but no Euler walk



(a)





Has Euler walk + Hamiltonian walk.

How find graph w/ Euler walk but no Hamilt

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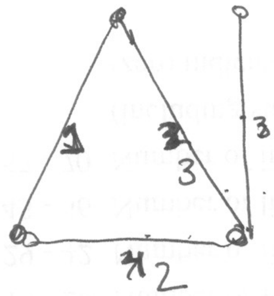
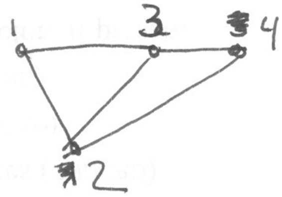
(13)





Try

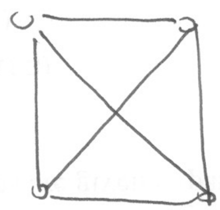
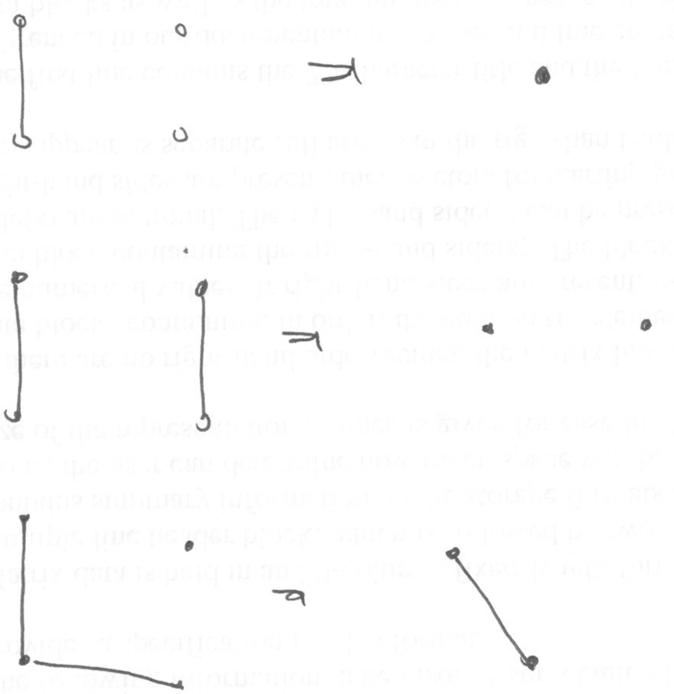
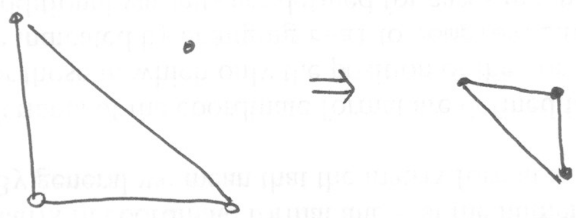
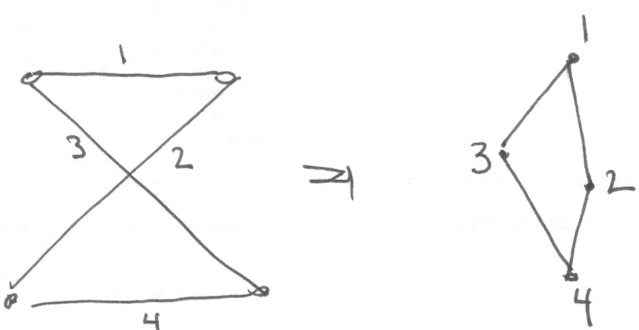
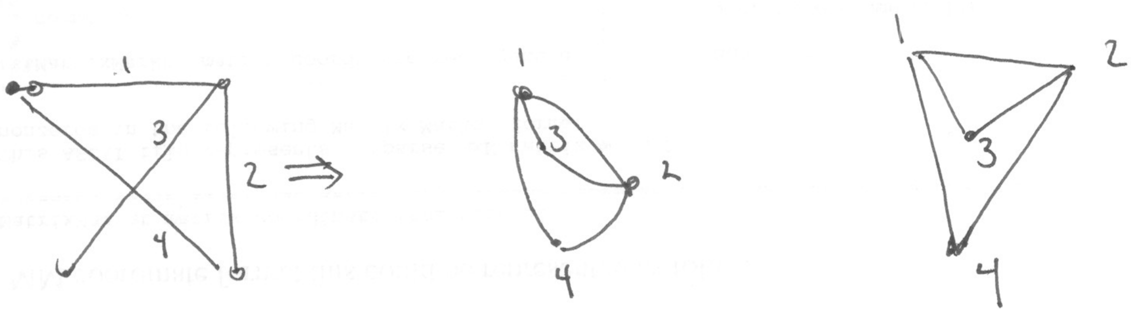


Fig 45:






1
3
2
4
1
3
2
4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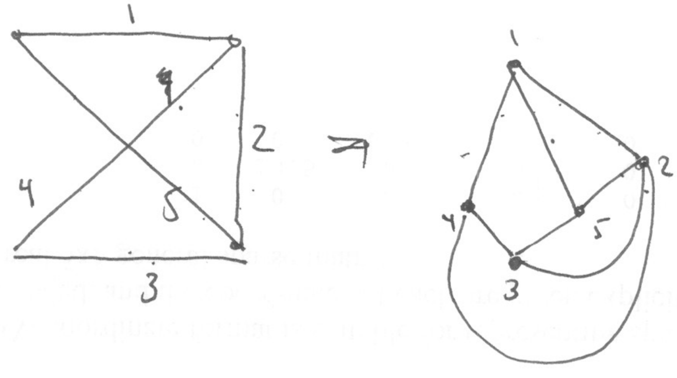
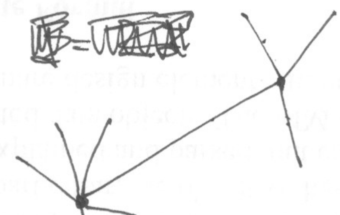


(14) Assume G has a closed Euler walk then the degree of each vertex must be even.

⇒ each edge is incident on an odd # of edges

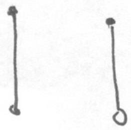


on each side of its edge.

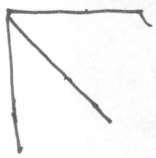
Since both sides have an odd #, the total # of edges incident on any given edge is even. Thus each vertex in the line graph of G is of even degree. Thus $\exists$ a closed Euler path/walk in the line graph of G.

~~Also~~ Also since G has a closed Euler walk, we find that walk can be translated to a closed Hamiltonian path

15

try  ⇒  no

G

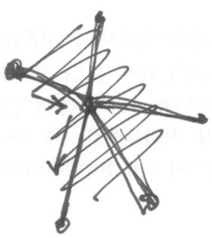
try  ⇒

L(G)

 yes,

G has no closed Euler walk.

16 Consider the closed Hamiltonian walk in G.



? How translate the Hamiltonian ~~walk~~ in G to on in L(G)?

~~G~~ G

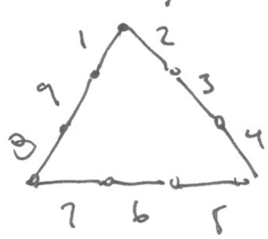
17  ⇒

L(G)

G

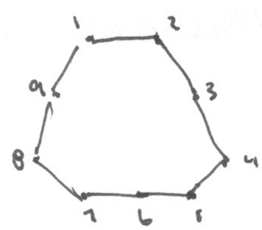
18  ⇒

T(G)



⇒

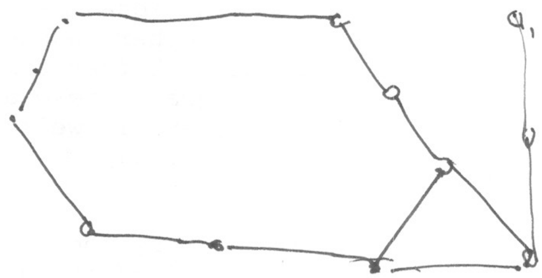
~~E(G)~~ L(T(G))



For Figure 159 (a)  has a ~~closed~~ open Euler walk.

a

$L(T(G))$ is given in Fig 159 (b)

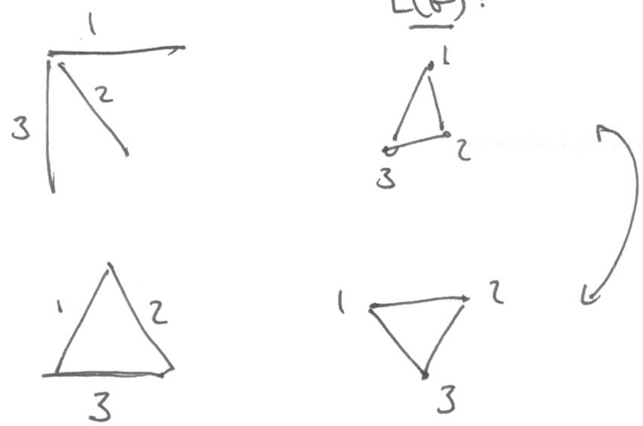


does not have a closed Hamiltonian walk

G: pg 198 Trudeau 8-27-02 1

(19)

L(G):



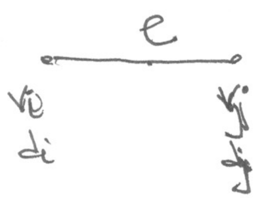
isomorphic

(20)

~~each edge in L(G) has~~

edges from this one vertex must be

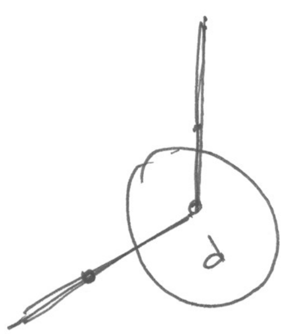
d_v



edges in G = e

vertices in L(G) = e

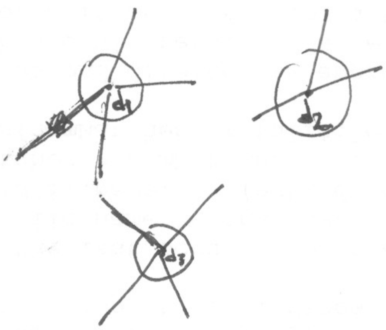
edges in L(G)



d-1 to the degree

$2e_{L(G)} =$

At a vertex of degree d, each edge ~~[illegible]~~ incident has d-1 other edges that must be connected ~~to~~ as edges in L(G)



$d_1(d_1-1)$

↑

d_1 incident edges

+ each edge is ~~[illegible]~~ ~~to~~ adjacent to d_1-1 other edges.

Contributing ~~this~~ ~~many~~ a double counted # of edges in $L(G)$

$$\therefore \sum_{i\in V} d_i(d_i-1) = \sum_{i\in V} d_i^2 - \underset{\overset{\|}{2e}}{\sum_{i\in V} d_i} = 2e_{L(G)}$$

$$\Rightarrow e_{L(G)} = \frac{1}{2}\sum d_i^2 - e.$$

(21) The line graph created from a cyclic graph is obv

C_v has v vertices + v edges

TO be isomorphic to $L(G)$

let G have v vertices + e edges

$v = e$ # vertices in $L(G)$

$e = \frac{1}{2}(d_1^2 + \cdots + d_v^2) - e$

$\Rightarrow \quad v = e = \frac{1}{4}(d_1^2 + \cdots + d_v^2)$

$\Rightarrow \quad v = \frac{1}{4}(d_1^2 + \cdots + d_v^2) \leq \frac{v \cdot \max_{v \in V}(d_v^2)}{4}$

$\Rightarrow \quad 4 \leq \max_{v \in V}(d_v^2)$

But also

$v = \frac{1}{4}(d_1^2 + \cdots + d_v^2) \geq \frac{v(2)^2}{4}$