

$$\rho = \rho(p, t)$$

$\rho$  = mass density,  $t$  = temperature kelvin  
 $p$  = pressure

$$d\rho = \rho_p dp + \rho_t dt$$

$$\frac{1}{\rho} d\rho = \frac{1}{\rho} \frac{\partial \rho}{\partial p} dp + \frac{1}{\rho} \frac{\partial \rho}{\partial t} dt$$

Defn  ~~$\alpha \equiv \frac{1}{\rho} \frac{\partial \rho}{\partial t}$~~   $\alpha \equiv -\frac{1}{\rho} \frac{\partial \rho}{\partial t} \Big|_p = -\frac{1}{\rho} \left( -\frac{M}{V^2} \right) \frac{\partial V}{\partial T} \Big|_p$

$$+ B \equiv \rho \frac{\partial p}{\partial \rho}$$

$$= \frac{1}{V} \frac{\partial V}{\partial T} \Big|_p \quad \text{coefficient of thermal expansion.}$$

$$= \frac{M}{V} \frac{\partial p}{\partial V} \frac{V^2}{(-M)} = -V \frac{\partial p}{\partial V} \Big|_T$$

isothermal Bulk modulus

$$\Rightarrow \frac{1}{\rho} d\rho = -\alpha dt + \frac{1}{B} dp \quad \text{eq 1.13a}$$

$$\Rightarrow \frac{1}{\rho} \left( -\frac{M}{V^2} \right) dV = -\frac{1}{V} dV = -\alpha dt + \frac{dp}{B}$$

$$\Rightarrow -\frac{dV}{V} = -\alpha dt + \frac{dp}{B}$$

$$\frac{dV}{V} = \alpha dt - \frac{dp}{B} \quad \text{eq 1.13b}$$

$$F_{ext} = +M \frac{dH}{dx}$$

$$F_{int} = -M \frac{dH}{dx}$$

$$W = - \int_{-\infty}^a M \frac{dH}{dx} dx = - \int_0^{H(a)} M dH \quad \text{includes mag of system + approach of beam.}$$

$$W' = - \int_a^{-\infty} M(a) \frac{dH}{dx} dx = -M(a) \int_a^{-\infty} \frac{dH}{dx} dx = -M(a) (H(x)) \Big|_a^{-\infty}$$

$$= -M(a)(0 - H(a)) = M(a)H(a)$$

$$W + W' = M(a)H(a) - \int_0^{H(a)} M dH$$

$$= M(a)H(a) - M H \Big|_0^{H(a)} + \int_0^{H(a)} H dM$$

$$= M(a)H(a) - M(a)H(a) - 0 + \int_0^{M(a)} H dM = \int_0^{M(a)} H dM$$

$$dQ = dU + PdV$$

$$dQ = \left. \frac{\partial U}{\partial T} \right|_V dT + \left( P + \left. \frac{\partial U}{\partial V} \right|_T \right) dV$$

$$C_V = \left. \frac{\partial U}{\partial T} \right|_V$$

$$dQ = C_V dT + \left[ P + \left. \frac{\partial U}{\partial V} \right|_T \right] dV$$

$$C_P = \left. \frac{dQ}{dT} \right|_P = C_V + \left[ P + \left. \frac{\partial U}{\partial V} \right|_T \right] \left. \frac{\partial V}{\partial T} \right|_P \quad \text{eq 1.33}$$

$$\left. \frac{\partial U}{\partial V} \right|_T = 0$$

$$C_P = C_V + P \left. \frac{\partial V}{\partial T} \right|_P$$

$$\begin{aligned} PV &= nRT & V &= \text{system volume} \\ P\bar{V} &= RT & \bar{V} &= \text{molar volume} \end{aligned}$$

÷ # moles

$$\bar{C}_P = \bar{C}_V + P \left. \frac{\partial \bar{V}}{\partial T} \right|_P$$

$$\bar{C}_P = \bar{C}_V + R \quad \text{eq 1.35}$$

$$\bar{V} = \frac{RT}{P}$$

$$\left. \frac{\partial \bar{V}}{\partial T} \right|_P = \frac{R}{P}$$

$$dU + PdV = 0$$

$$\left. \frac{\partial U}{\partial T} \right|_V dT + \left[ \left. \frac{\partial U}{\partial V} \right|_T + P \right] dV = 0$$

$$C_V dT + \left[ (C_P - C_V) \left. \frac{\partial T}{\partial V} \right|_P - P + P \right] dV = 0$$

$$C_V dT + (C_P - C_V) \left. \frac{\partial T}{\partial V} \right|_P dV = 0 \quad \text{eq 1.37}$$

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$$\frac{dV}{dt} = \frac{-C_V}{(E_p - C_V)} \left( \frac{\partial V}{\partial t} \right) < 0$$

$$PV = nRT$$

$$\frac{V}{T} = \frac{nR}{P} > 0$$

From 1.13

$$\frac{1}{V} dV = \alpha dT - \frac{dP}{B}$$

$$\alpha \equiv \frac{1}{V} \frac{\partial V}{\partial T} \Big|_P$$

$$-\frac{1}{B} \equiv \frac{1}{V} \frac{\partial V}{\partial P} \Big|_T \Rightarrow B = -V \frac{\partial P}{\partial V} \Big|_T$$

Thus for an ideal gas

pp

$$P\bar{V} = RT$$

$\bar{V}$  = molar volume.

$$B = -\bar{V} \frac{\partial P}{\partial \bar{V}} \Big|_T = +P \quad \text{eq 1.41}$$

isothermal

$$\frac{\partial P}{\partial \bar{V}} \Big|_T = \frac{-RT}{\bar{V}^2} = -\frac{P}{\bar{V}}$$

$$B_{\text{Adiab}} = -\bar{V} \frac{\partial P}{\partial \bar{V}} \Big|_{\text{Adiab}}$$

$$= \gamma P \quad \text{eq 1.42}$$

$$P\bar{V}^\gamma = C \quad \text{by eq 1.40}$$

$$P = C\bar{V}^{-\gamma}$$

$$\frac{\partial P}{\partial \bar{V}} \Big|_{\text{Ad}} = -\gamma C\bar{V}^{-\gamma-1}$$

$$= -\gamma \bar{V}^{-1} P$$

$$= -\gamma \frac{P}{\bar{V}}$$

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① ~~Molar~~  $M_{N_2} = 28 \text{ g/mol}$   
 $M_{O_2} = 32 \text{ g/mol}$

$$P_{\text{total}} = P_1 + P_2 \quad \text{w/} \quad P_1 = \frac{n_1 RT}{V} \quad P_2 = \frac{n_2 RT}{V}$$

$$= \frac{(n_1 + n_2) RT}{V} = n_{\text{total}} \frac{RT}{V}$$

$$P_{\text{total}} V = n_{\text{total}} RT$$

$$x_{O_2} = \frac{n_{O_2}}{n_{O_2} + n_{N_2}} = ?$$

$$P_{\text{total}} = 10^6 \text{ dynes/cm}^2 \quad T = 20^\circ\text{C} + 273.15 = 293.15^\circ\text{K}$$

$$\rho_{\text{Air}} = 1.189 \cdot 10^{-3} \text{ gm/cm}^3 = \frac{M_{\text{total}}}{V} = \frac{n_{O_2} M_{O_2} + n_{N_2} M_{N_2}}{V}$$

$$\Rightarrow V = \frac{n_{O_2} M_{O_2} + n_{N_2} M_{N_2}}{\rho_{\text{Air}}}$$

$$\Rightarrow \frac{P_{\text{total}}}{\rho_{\text{Air}}} (n_{O_2} M_{O_2} + n_{N_2} M_{N_2}) = n_{\text{total}} RT \quad \div \text{ by } n_{\text{total}}$$

$$\Rightarrow \frac{P_{\text{total}}}{\rho_{\text{Air}}} (x_{O_2} M_{O_2} + x_{N_2} M_{N_2}) = RT$$

$$x_{O_2} + x_{N_2} = 1$$

$$x_{N_2} = 1 - x_{O_2}$$

$$\frac{P_{\text{total}}}{P_{\text{total}}} (X_{O_2} M_{O_2} + M_{N_2} + X_{O_2} M_{N_2}) = RT$$

$$(M_{O_2} - M_{N_2}) X_{O_2} + M_{N_2} = \frac{P_{\text{total}}}{P_{\text{total}}} RT$$

$$X_{O_2} = \frac{\frac{P_{\text{total}}}{P_{\text{total}}} RT - M_{N_2}}{M_{O_2} - M_{N_2}}$$

$$R = 8.314 \text{ J/mol} \cdot \text{K}$$

$$= 8.314 \cdot 10^7 \frac{\text{ergs}}{\text{mol} \cdot \text{K}}$$

$$\left[ \frac{P_{\text{total}}}{P_{\text{total}}} \right] = \frac{M/L^3}{M/L^3} = \frac{M}{L^3} \cdot \frac{L^3}{M} = \frac{L^2}{L^2} = \frac{1}{\text{velocity}^2}$$

~~1N = 10^5 dyne~~

$$1N = 10^5 \text{ dyne}$$

$$1J = 1N \cdot 1m$$

$$= 10^5 \text{ dynes} \cdot 10^2 \text{ cm}$$

$$= 10^7 \text{ dyne cm}$$

$$= 10^7 \text{ ergs}$$

$$\left[ \frac{P_{\text{total}}}{P_{\text{total}}} RT \right] = \frac{L^2}{L^2} \cdot K \cdot \frac{M}{L^3} \frac{1}{\text{mol}} = \frac{M}{\text{mol}}$$

$$\therefore X_{O_2} = \frac{\left( \frac{1.189 \cdot 10^{-3}}{10^6} \right) (8.314 \cdot 10^7) (293.15) - 28}{32 - 28} = .2447$$

$$X_{N_2} = .75527$$

$$(2) B(0^\circ\text{C}) = 2.032 \cdot 10^4 \text{ atm}$$

for  $\text{H}_2\text{O}$ 

$$B(100^\circ\text{C}) = 2.137 \cdot 10^4 \text{ atm}$$

$$\alpha(1 \text{ atm}) = 4.33 \cdot 10^{-4} \text{ deg}^{-1}$$

$$\alpha(1000 \text{ atm}) = ?$$

$$\frac{1}{V} dV = \alpha dT - \frac{dP}{B}$$

Assume  $\alpha = \alpha(V)$  only  
 $B = B(T)$ 

$$\frac{dV}{V} = \alpha(V) dT - \frac{dP}{B(T)}$$

Don't know. Can you do it assuming ideal gas eq of state?

$$\alpha_{\text{I.G.}} = \frac{1}{V} \frac{\partial V}{\partial T} \Big|_P = \frac{1}{V} \frac{\partial}{\partial T} \left( \frac{RT}{P} \right) \Big|_P = \frac{R}{VP} = T^{-1}$$

$$B_{\text{I.G.}} = P$$

$$\frac{dV}{V} = T^{-1} dT - \frac{dP}{P} \quad \text{can be integrated}$$

$$\ln V = \ln T - \ln P + C? \quad \text{what have I done wrong?}$$

 $V \cdot P = C \cdot T$  Nothing new eq. of state.

$$\alpha(1000 \text{ atm}) = \frac{PV}{R}$$

$$\alpha(1000 \text{ atm}) = 4.58 \cdot 10^{-4} \text{ deg}^{-1} \quad \text{B/W}$$

Last Idea:  $B_f$  &  $\alpha$  are linearly related

$$\Rightarrow f = \frac{B_f}{B_i} = \frac{2.137}{2.032} = 1.05$$

$$\text{Then } \beta \alpha_f = (1.05) \alpha_i = 1.05 \cdot 4.33 \cdot 10^{-4} = 4.553 \cdot 10^{-4}.$$

close !! . Damn.

How do this problem the correct way?

$$(3) \quad \alpha = \frac{1}{V} \frac{\partial V}{\partial T} \bigg|_P = T^{-1} \quad \text{see prob 2.}$$

$$(4) \quad \text{isoth} \quad W = \int_{P=3\text{atm}}^{P=1\text{atm}} P dV = \int_{P_1}^{P_2} \frac{RT}{V} dV$$

$$W > 0$$

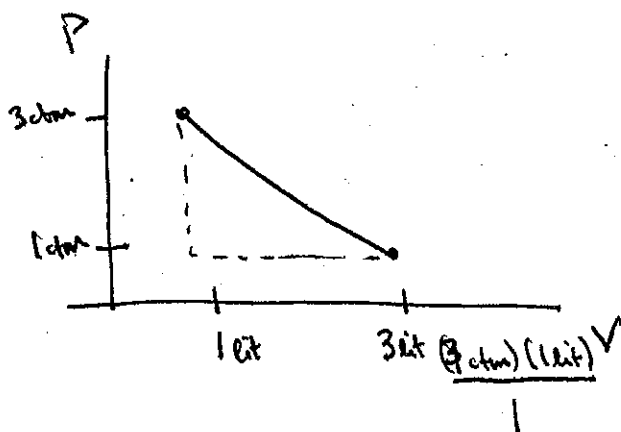
$$= RT \ln \frac{V_2}{V_1} = RT \ln \left( \frac{P_1}{P_2} \right)$$

$$\frac{P_1}{P_2} \approx 1$$

$$= P_1 V_1 \ln \left( \frac{P_1}{P_2} \right)$$

Why is sign wrong?

$$W_{\text{iso}} = (3\text{atm})(1\text{liter}) \ln \left( \frac{3}{1} \right) = 3.29 \text{ atm} \cdot \text{liters} \\ = 333.9 \text{ J.}$$



$$W = (2\text{atm}) \frac{1}{2} (2\text{ lit}) = 2\text{ atm} \cdot \text{liters} \\ = 405 \text{ J.}$$

⑤

$$I = \frac{C}{T} H$$

~~$$H_1 = 10000 \text{ oe}$$~~

$$H_2 = 0 \text{ oe}$$

$$C = .1^\circ$$

$$T = T(T)$$

1st law

$$dT = dT(T) = dQ + dW$$

$$= dQ - H dI$$

How do we know the sign?

Assuming isothermal change & w/  $T = T(T)$   $dT = 0$ 

$$\Rightarrow dQ = H dI = \frac{T}{C} I dI$$

$$Q = \frac{T}{C} \int_{I_i}^{I_f} I dI = \frac{T}{C} \frac{I^2}{2} \Big|_{I_i}^{I_f} = \frac{T}{C} \frac{C^2}{2T^2} H^2 \Big|_{H_i}^{H_f} = \frac{C}{2T} (H_f^2 - H_i^2)$$

Then

$$(a) Q_{300K} = \frac{.1}{2(300)} (0 - (10^5)^2) = -1.66 \cdot 10^4 \text{ oe}^2$$

$$\frac{1.66 \cdot 10^4}{C} = .01$$

~~$$C = 1.66 \cdot 10^6$$~~

$$C = 1.66 \cdot 10^6$$

$$(b) Q_{40K} = \frac{.1}{2(4)} (0 - (10^4)^2) = -1.25 \cdot 10^6 \text{ oe}^2$$

~~$$1.25 \cdot 10^6 \cdot 10^2$$~~

$$\text{Now } 1 \text{ oe} = \text{oersted} = \frac{\text{Gauss}}{\text{Mo}} = 79.577472 \frac{\text{A}}{\text{m}} \sqrt{\frac{1}{C}} = .753$$

⑥ Pv:

$$\frac{\left. \frac{\partial V}{\partial p} \right|_T}{\left. \frac{\partial V}{\partial p} \right|_{\text{adiabatic}}} = \frac{C_p}{C_v}$$

From 1st law  
 & system capable  
 of only PV work

$$dU = dQ - p dV$$

undergoing a Adiabatic transformation

$$dU = -p dV$$

~~$$\left. \frac{\partial U}{\partial p} \right|_T = -p \left. \frac{\partial V}{\partial p} \right|_T$$~~

$$C_p \equiv \left. \frac{dU}{dp} \right|_{\text{Adia}} = -p \left. \frac{dV}{dp} \right|_{\text{Adia}}$$

~~Again from~~  ~~$dU = dQ - p dV$~~  ~~Why~~

$$C_v = \left. \frac{\partial Q}{\partial T} \right|_V = \left. \frac{\partial U}{\partial T} \right|_V$$

$$dU = dQ - p dV$$

$$\left. \frac{\partial U}{\partial T} \right|_V = \left. \frac{\partial Q}{\partial T} \right|_V$$

Per the third  $\rho = \rho(p, T)$

$\rho = \text{mass density}$

$$\Rightarrow \frac{m}{V} = \rho(p, T)$$

$$\Rightarrow \underline{V = V(p, T)}$$

$$= B_T$$

$$\left. \frac{\partial x}{\partial y} \right|_z = \frac{1}{\left. \frac{\partial y}{\partial x} \right|_z}$$

$$\left. \frac{\partial x}{\partial y} \right|_z \left. \frac{\partial y}{\partial z} \right|_x = 1$$

$$\frac{C_p}{C_v} = 1 +$$

$$dQ = \left. \frac{\partial U}{\partial T} \right|_V dT + \left\{ \left( \frac{\partial U}{\partial V} \right)_T + P \right\} dV$$

$$dQ = C_V dT + \left\{ \left. \frac{\partial U}{\partial V} \right|_T + P \right\} dV$$

$$\left. \frac{dQ}{dT} \right|_P = C_V + \left\{ \left. \frac{\partial U}{\partial V} \right|_T + P \right\} \left. \frac{dV}{dT} \right|_P$$

(6)

Q1

$$\frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial V}{\partial P}\right)_{\text{Adiab}}} = \frac{C_p}{C_v}$$

$$\left(\frac{\partial H}{\partial P}\right)_P = \left(\frac{\partial Q}{\partial P}\right)_P$$

$$dH = dU + PdV + VdP$$

$$H = U + PV$$

$$dU = dH + PdV$$

$$dU = dQ + dW$$

$$dU = \cancel{dQ} - PdV \Rightarrow dU = -PdV$$

$$\left(\frac{\partial U}{\partial P}\right)_{\text{Adiab}} = -P \left(\frac{\partial V}{\partial P}\right)_{\text{Adiab}}$$

$$\left(\frac{\partial V}{\partial P}\right)_{\text{Adiab}} = -\frac{1}{P} \left(\frac{\partial U}{\partial P}\right)_{\text{Adiab}}$$

$$= -\frac{1}{P} \frac{\partial U}{\partial}$$

$$C_p = \left(\frac{\partial U}{\partial T}\right)_P = \left(\frac{\partial H}{\partial T}\right)_P = \left(\frac{\partial Q}{\partial T}\right)_P$$

$$C_v = \left(\frac{\partial U}{\partial T}\right)_V = \left(\frac{\partial Q}{\partial T}\right)_V$$

How show? How work of  $\left(\frac{\partial V}{\partial P}\right)_{\text{Adiab}}$  Variables?

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⑦ eq 1.14

$$\frac{dl}{l} = \beta dt + \frac{1}{\eta Y_t} df$$

 $l$  = wire length $\eta$  = cross section $\beta$  = coefficient of linear expansion $t$  = temperature $Y_t$  = isothermal Young's modulus $f$  = force.

At constant temp

$$(a) \quad \frac{dl}{l} = \frac{1}{\eta Y_t} df \quad \Rightarrow \quad df = \frac{\eta Y_t}{l} dl$$

integrate both sides  ~~$f = \eta Y_t \ln l + C$~~  Treating all quantities asconstants  $\rightarrow$   ~~$f$~~  integrating both sides gives:

$$\frac{l_{\text{final}} - l_0}{l_0} = \frac{1}{\eta Y_t} (f_{\text{final}} - f_0)$$

$$\Rightarrow f(l) = \frac{\eta Y_t}{l_0} (l - l_0) \quad \left\{ f \propto l \right.$$

Then

$$dw = f(l) dl$$

$$W = \int_{l_1}^{l_2} f(l) dl = \frac{\eta Y_t}{l_0} \int_{l_1}^{l_2} (l - l_0) dl$$

$$= \frac{\eta Y_t}{l_0} \left[ \frac{(l_2 - l_0)^2}{2} - \frac{(l_1 - l_0)^2}{2} \right]$$

$$\therefore W = \left[ \frac{l_0^2 f_2^2}{\gamma^2 \gamma_t^2} - \frac{l_0^2 f_1^2}{\gamma^2 \gamma_t^2} \right] \frac{\gamma \gamma_t}{2 l_0} = \frac{l_0}{2 \gamma \gamma_t} (f_2^2 - f_1^2)$$

(b) If  $l$  &  $\gamma$  are not constants but  $\gamma$  cross section is related to length  $l$  by  $\gamma l = \Omega$  constant

$$\frac{dl}{l} = \frac{1}{\Omega \gamma_t} df$$

$$\gamma = \frac{\Omega}{l}$$

$$\frac{dl}{l^2} = \frac{df}{\Omega \gamma_t}$$

$$-\frac{1}{l} \Big|_{l_0}^{l_1} = \frac{1}{\Omega \gamma_t} (f_1 - f_0)$$

$l_0 = \text{equilibrium length.}$

$$= -\left(\frac{1}{l_1} - \frac{1}{l_0}\right) = \frac{f_1}{\Omega \gamma_t} \rightarrow f(l) = \Omega \gamma_t \left(\frac{1}{l_0} - \frac{1}{l}\right)$$

$$\rightarrow f(l) = \Omega \gamma_t \left(\frac{1}{l_0} - \frac{1}{l}\right)$$

Then work  $W = \int_{l_1}^{l_2} f(l) dl = \Omega \gamma_t \int_{l_1}^{l_2} \left(\frac{1}{l_0} - \frac{1}{l}\right) dl$

$$= \frac{\Omega \gamma_t}{l_0} (l_2 - l_1) - \Omega \gamma_t \ln \frac{l_2}{l_1}$$

$$= \underbrace{\Omega \gamma_t \left[ \frac{l_2}{l_0} - \ln l_2 \right]}_{\text{Final state}} - \underbrace{\Omega \gamma_t \left[ \frac{l_1}{l_0} - \ln l_1 \right]}_{\text{initial state}}$$

Final state

initial state

(c)

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If  $l_{12} \approx l_0 + \delta l_{12}$  Then

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} + O(x^4)$$

$$W = \sum Y_t \left[ \frac{l_0 + \delta l_{12}}{l_0} - \ln(l_0 + \delta l_{12}) \right] - "$$

$$= \sum Y_t \left[ 1 + \frac{\delta l_{12}}{l_0} - \ln(l_0 (1 + \frac{\delta l_{12}}{l_0})) \right] - "$$

$$= \sum Y_t \left[ 1 + \frac{\delta l_{12}}{l_0} - \ln l_0 - \ln(1 + \frac{\delta l_{12}}{l_0}) \right] - "$$

$$= \sum Y_t \left[ 1 + \frac{\delta l_{12}}{l_0} - \ln l_0 - \left[ -\frac{\delta l_{12}}{l_0} - \frac{\delta l_{12}^2}{l_0^2} \right] \right]$$

$$\text{From } f(l) = \sum Y_t \left( \frac{1}{l_0} - \frac{1}{l} \right) = \frac{\sum Y_t}{l_0 l} (l - l_0) = \frac{q(l) Y_t}{l_0} (l - l_0)$$

Same exact expression as before :-

$$= \frac{q(l) Y_t}{l_0} (l - l_0)$$

assuming  $l \approx l_0$   $q(l) \approx q(l_0) = q_0$  constant.

The expression above should be ~~try to~~ ~~expressed~~ able to be manipulated into the one from part (a).

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$$\textcircled{8} \quad \beta = 13 \cdot 10^{-6} \text{ deg}^{-1} \quad Y_t = 2 \cdot 10^{12} \text{ dynes/cm}^2$$

$$q = 100 \text{ cm}^2$$

$$T_0 = 10^\circ \text{C}$$

$$T_1 = 35^\circ \text{C}$$

From the 1st law applied to this system

$$\frac{dl}{l} = \beta dt + \frac{1}{q Y_t} dF$$

the constraints that the ~~the~~ endpoints are fixed gives us that  $dl = 0$  no movement in

length

I will assume  ~~$\beta, q, Y_t$~~   $\beta, q, Y_t$  do not depend on temperature.

$$\Rightarrow -\beta dt = \frac{1}{q Y_t} dF$$

$$= -\beta(T_1 - T_0) = \frac{1}{q Y_t} (F_1 - \cancel{F_0}^0)$$

$$F_1 = -\beta q Y_t (T_1 - T_0)$$

$$F_1 = -(13 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1})(100 \text{ cm}^2)(2 \cdot 10^{12} \text{ dynes/cm}^2)(25^\circ \text{C})$$

$$= -6.5 \cdot 10^{10} \text{ dynes}$$

$$1 \text{ ton} =$$

$$1 \text{ dyne} = \text{force} = \frac{\text{cm}}{\text{s}^2} \text{ g}$$

$$g(\text{cgs}) = 980 \text{ cm/s}^2$$

$$\text{dynes} = ? \text{ tons}$$

$$\frac{1 \text{ ton}}{g} \pm \text{units of force} = (1 \text{ ton})(g)$$

$$1 \text{ ton} = 2000 \text{ lbs} = 907.18474 \text{ kg} = 907.18474 \cdot 10^3 \text{ g}$$

$$\therefore 1 \text{ ton} = 907.18474 \cdot 10^3 \text{ g} \quad (980 \text{ cm/s}^2)$$

As unit of  
force.

$$= 8.89 \cdot 10^8 \text{ dynes}$$

$$\therefore 1 \text{ dyne} = 1.124 \cdot 10^{-9} \text{ tons (force)}$$

$$\therefore F = -6.5 \cdot 10^{10} \text{ dynes} = 7.311 \cdot 10^1 \text{ tons (force)}$$

$$= 73 \text{ tons.}$$

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(9) Given  $F(x, y, z) = 0 \Rightarrow \exists$  relationship between  $x, y, \& z$ .

let use assume we can solve for  $z$  say  $\Rightarrow$  Application of ~~theorem~~ implicit theorem.

$$F(x, y, z) \Leftrightarrow z = z(x, y)$$

$$dz = \frac{\partial z}{\partial x} \Big|_y dx + \frac{\partial z}{\partial y} \Big|_x dy$$

Then to evaluate  $\frac{\partial x}{\partial y} \Big|_z \Rightarrow dz = 0$

$$0 = \frac{\partial z}{\partial x} \Big|_y dx + \frac{\partial z}{\partial y} \Big|_x dy$$

$$\frac{\partial z}{\partial x} \Big|_y \frac{\partial x}{\partial y} \Big|_z = - \frac{\partial z}{\partial y} \Big|_x \Rightarrow \frac{\partial x}{\partial y} \Big|_z \cdot \frac{\partial y}{\partial z} \Big|_x \cdot \frac{\partial z}{\partial x} \Big|_y = -1 \quad \text{last eq to show.}$$

Q: I don't understand why this didn't work for a method to evaluate

$\frac{\partial x}{\partial y} \Big|_z = ?$  I would have gotten

$$\frac{\partial x}{\partial y} \Big|_z = - \frac{\partial x}{\partial z} \Big|_y \cdot \frac{\partial z}{\partial y} \Big|_x$$

Another method since  $z$  is fixed as a constant  $z_0$

$$z = z(x, y) = z_0 \Rightarrow x = x(y)$$

Another application of the implicit theorem.

$$dz = 0 = \frac{\partial z}{\partial x} \Big|_y dx + \frac{\partial z}{\partial y} \Big|_x dy$$

$$\frac{\partial x}{\partial y} \Big|_z = - \frac{\partial z}{\partial y} \Big|_x \cdot \frac{\partial y}{\partial z} \Big|_x$$

$$z(x(y), y) = z_0 \quad + \quad z(x, y(z)) = z_0$$

$$\frac{dz}{dy} \Rightarrow z_x \frac{dx}{dy} \Big|_z + z_y = 0 \quad + \quad z_x + z_y \frac{dy}{dx} \Big|_z = 0$$

$$\textcircled{B} \quad \frac{dx}{dy} \Big|_z = - \frac{z_y}{z_x} \quad \frac{dy}{dx} \Big|_z = - \frac{z_x}{z_y}$$

$$\therefore \frac{dx}{dy} \Big|_z \cdot \frac{dy}{dx} \Big|_z = +1$$

(10) Van der Waals' Equation of state:

~~$$(p + \frac{a}{\bar{v}^2})(\bar{v} - b) = RT$$~~

$$(p + \frac{a}{\bar{v}^2})(\bar{v} - b) = RT$$

Then the virial expansion

$$\frac{p\bar{v}}{RT} = 1 + \frac{B(T)}{\bar{v}} + \frac{C(T)}{\bar{v}^2} + \frac{D(T)}{\bar{v}^3}$$

1st coefficient is  $B(T)$  while 2nd coefficient is  $C(T)$

From ~~Eq~~ Van der Waal

$$p = \frac{RT}{(\bar{v} - b)} - \frac{a}{\bar{v}^2}$$

So that

$$\frac{p\bar{v}}{RT} = \frac{\bar{v} RT}{RT(\bar{v} - b)} - \frac{a\bar{v}}{\bar{v}^2 RT}$$

$$= \frac{\bar{v}}{\bar{v} - b} - \frac{a}{\bar{v} RT}$$

$$= \frac{1}{1 - b/\bar{v}} - \frac{a}{\bar{v} RT}$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

expanding R.H.S for large  $\bar{v}$  gives

$$\frac{p\bar{v}}{RT} = -\frac{a}{\bar{v} RT} + \sum_{k=0}^{\infty} \left(\frac{b}{\bar{v}}\right)^k$$

$$= -\frac{a}{\bar{v} RT} + 1 + \frac{b}{\bar{v}} + \frac{b^2}{\bar{v}^2} + \frac{b^3}{\bar{v}^3} + O\left(\frac{1}{\bar{v}^4}\right)$$

$$= 1 + \left(b - \frac{a}{RT}\right) \frac{1}{\bar{v}} + \frac{b^2}{\bar{v}^2} + \frac{b^3}{\bar{v}^3} + O\left(\frac{1}{\bar{v}^4}\right)$$

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$$(11) \quad p(\bar{V} - b) = RT$$

 $\bar{V}$  = molar specific volume. $\beta$  = Bulk modulus $\alpha$  = thermal expansion coefficientFrom eq 1.13 b we get expanding  $dV$  in terms of  $T$  &  $P$ 

$$\frac{1}{\bar{V}} \left( \frac{\partial \bar{V}}{\partial T} \Big|_P dT + \frac{\partial \bar{V}}{\partial P} \Big|_T dP \right) = \alpha dT - \frac{1}{\beta} dP$$

$$\Rightarrow \frac{1}{\bar{V}} \frac{\partial \bar{V}}{\partial T} \Big|_P = \alpha \quad + \quad \frac{1}{\bar{V}} \frac{\partial \bar{V}}{\partial P} \Big|_T = -\frac{1}{\beta} \Rightarrow \beta = -\bar{V} \frac{\partial P}{\partial \bar{V}} \Big|_T$$

$$\alpha = \frac{1}{\bar{V}} \frac{\partial \bar{V}}{\partial T} \Big|_P \quad + \quad \beta = -\bar{V} \frac{\partial P}{\partial \bar{V}} \Big|_T$$

$$\text{Thus } \bar{V} = \frac{RT}{P} + b$$

$$\text{So } \frac{\partial \bar{V}}{\partial T} \Big|_P = \frac{R}{P} \quad \frac{\partial \bar{V}}{\partial P} \Big|_T = -\frac{RT}{P^2}$$

So

$$\alpha = \frac{1}{\left(\frac{RT}{P} + b\right)} \frac{R}{P} \quad + \quad \beta = + \left(\frac{RT}{P} + b\right) \left(\frac{RT}{P^2}\right)^{-1} = \left(\frac{RT}{P} + b\right) \left(\frac{P^2}{RT}\right)$$

For a perfect gas  $b=0$ 

$$\alpha = \frac{R}{P \left(\frac{RT}{P}\right)} = \frac{1}{T} \quad + \quad \beta = P$$

$$\text{Thus } \alpha = \frac{1}{\frac{R}{P}(T + \frac{bP}{R})} \frac{R}{P} = \frac{1}{(T + \frac{bP}{R})}$$

$$+ B = P + \frac{bP^2}{RT}$$

$$\text{So } \alpha = \frac{1}{T} \frac{1}{1 + \frac{bP}{TR}} = \cancel{\frac{1}{T}} \frac{1}{1 + \frac{bP}{TR}}$$

$$= \frac{1}{T} \left( 1 - \frac{bP}{TR} + O(b^2) \right)$$

$$= \frac{1}{T} - \frac{bP}{T^2 R} + O(b^2)$$

since  $b > 0$

The Bk mode increases relative to that in an ideal gas.

The coefficient of ~~volume~~ thermal expansion decreases relative to an ideal gas.

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(12)  $\gamma = 1.4$  Air  $n = 1$  mole

$$T_i = 291^\circ \text{K}$$

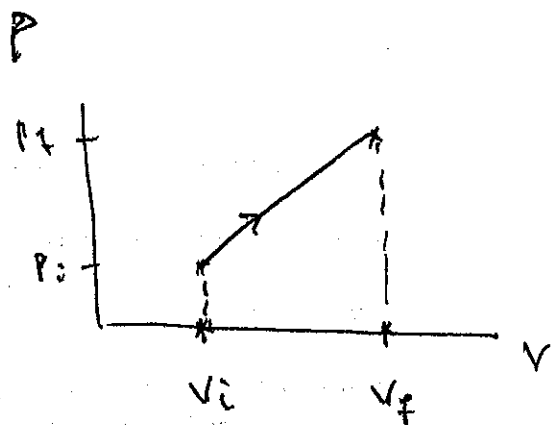
$$T_f = 319^\circ \text{K}$$

$$\bar{V}_i = 21 \text{ liters}$$

$$\bar{V}_f = 22 \text{ liters}$$

$$P_i = 115.2 \cdot 10^3 \text{ Pa}$$

$$P_f = \cancel{117.9 \cdot 10^3 \text{ Pa}} \\ 120.5 \cdot 10^3 \text{ Pa}$$



$PV = nRT$  & due to  $C_p + C_v$  constants  $\Rightarrow \frac{C_p}{C_v} = \gamma$

$$\Rightarrow T \bar{V}^{\gamma-1} = \text{const}$$

$$= T_i \bar{V}_i^{\gamma-1} = T_f \bar{V}_f^{\gamma-1}$$

$$= (291)(21)^{.4} = 983.5$$

} valid for Adiabatic change only!!

$$P_i = \frac{(1)(8.314 \text{ J/mol} \cdot \text{K})(291^\circ \text{K})}{21 \text{ liters}}$$

$$1 \text{ liter} = 10^{-3} \text{ m}^3$$

$$= 115.2 \cdot 10^3 \frac{\text{J}}{\text{m}^3}$$

$$1 \text{ J} = \frac{\text{m}^2 \text{ kg}}{\text{s}^2}$$

$$= 115.2 \cdot 10^3 \text{ Pa}$$

$$\therefore \frac{1 \text{ J}}{\text{m}^3} = \frac{\frac{\text{m kg}}{\text{s}^2}}{\text{m}^3} = \frac{\text{F}}{\text{m}^2} = P_{\text{press}}$$

Now  $\Delta W = P dV$   
 $-\Delta U = +W = \int_{V_i}^{V_f} P dV = \int_{V_i}^{V_f} \frac{P_i V_i^\gamma}{V^\gamma} dV = P_i V_i^\gamma \int_{V_i}^{V_f} V^{-\gamma} dV$  X

wrong:  
we are told the transformation takes place

$$W = P_i V_i^{\gamma} \left( \frac{V^{-\gamma+1}}{-\gamma+1} \right) \bigg|_{V_i}^{V_f}$$

X along a straight line in P-V plane:

$$= \frac{P_i V_i^{\gamma}}{1-\gamma} \left( \frac{1}{V_f^{-\gamma+1}} - \frac{1}{V_i^{-\gamma+1}} \right) = 111.5 \text{ J}$$

Adiabatic transformation only!!

$$\therefore W = \int_{V_i}^{V_f} \left[ P_i + \left( \frac{P_f - P_i}{V_f - V_i} \right) (V - V_i) \right] dV$$

$$= P_i (V_f - V_i) + \frac{1}{2} \left( \frac{P_f - P_i}{V_f - V_i} \right) \left[ (V_f - V_i)^2 - 0 \right]$$

$$= P_i (V_f - V_i) + \frac{1}{2} (P_f - P_i) (V_f - V_i)$$

$$= (V_f - V_i) \left[ \frac{1}{2} (P_f + P_i) \right] = \frac{1}{2} (P_f + P_i) (V_f - V_i)$$

w/ numbers

$$= 117.85 \text{ J}$$

Then  $\Delta U = \Delta Q - \Delta W$

$$\Rightarrow \Delta U = \Delta Q - \Delta W$$

$$\Delta U = U_f - U_i =$$

To calculate  $\Delta U$  follow a transformation between state i + f that is adiabatic. Such a transformation could be obtained by simply following the gas w/ its constant V law equation of state (Hawkins?)

This gives  $\Delta T = 117.5^\circ\text{C}$  which cannot be correct.

How properly to this? For example: How to I compute the internal energy w/o knowing  $C_p$  or  $C_v$ ?

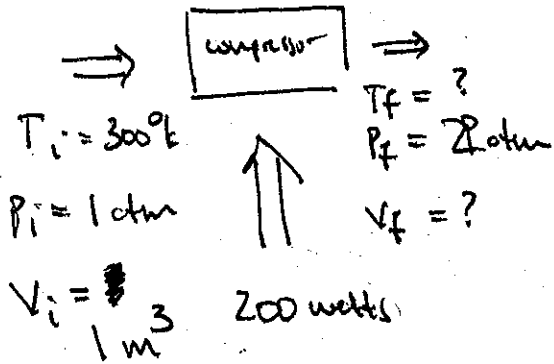
~~$$117.85 = \Delta Q -$$~~

$$-111.5 = \Delta Q - 117.85$$

$$\Delta Q = 117.85 - 111.5 = 6.35^\circ\text{C}.$$

Why is  $\Delta Q$  so large for a ~~few~~  $\Delta T$  of  $30^\circ\text{K}$ .

(13)



$$\gamma = \frac{4}{10} = \frac{2}{5}$$

Adiabatic  $\Rightarrow$  No heat flows into or out of compressor

$$\Delta U = \cancel{Q} - \Delta W$$

reversible  $\Rightarrow$  Don't see when this is explicitly used?  
in 1st law

$$\Delta U = -\Delta W$$

Assume 200 J are used to change the initial state to the final state through applying work on the gas through eq \*.

Then From eq  $P\bar{V}^\gamma = \text{const.}$  eq 1.40 +  $\bar{P}\bar{V} = \bar{P}T$   
 $\bar{V}^\gamma = \frac{\bar{R}T^\gamma}{P^\gamma}$   
 Valid for adiabatic flows.

$$\frac{\bar{R}T^\gamma}{P^\gamma} = \text{const.}$$

$$T^\gamma P^{1-\gamma} = \text{const.}$$

$$\Delta U = -\Delta W$$

$$200 \text{ J/s} = \int_{V_i}^{V_f} P dV$$

$$\begin{aligned}
 &= - \int_{V_i}^{V_f} \frac{P_i V_i^\gamma}{V^\gamma} dV = \frac{-P_i V_i^\gamma}{1-\gamma} \left( V_f^{1-\gamma} - V_i^{1-\gamma} \right) \\
 &= \frac{P_i}{\gamma-1} \left( V_f \left( \frac{V_i}{V_f} \right)^\gamma - \right)
 \end{aligned}$$

$$\Delta U = -\Delta W = - \int_{V_i}^{V_f} p dV = - p(V_f - V_i) + \int_{P_i}^{P_f} V dp$$

$$\Delta U = -p(V_f - V_i) + \int_{P_i}^{P_f} V dp$$

Assuming an adiabatic transformation:

$$T^r P^{1-r} = \text{constant} \quad \Rightarrow \quad T_i^r P_i^{1-r} = \dots = T_f^r P_f^{1-r}$$

$$\Rightarrow T_f = T_i \left( \frac{P_i}{P_f} \right)^{\frac{1-r}{r}} = T_i \left( \frac{1}{2} \right)^{\frac{-1.4}{1.4}}$$

$$= T_i \left( \frac{1}{2} \right)^{-\frac{2/5}{7/5}} = T_i \left( \frac{1}{2} \right)^{-2/7} = T_i 2^{2/7} \quad \checkmark \text{ correct.}$$

$$= 368.7^\circ \text{K}$$

Also  $P_i V_i^r = P_f V_f^r$

$$\Rightarrow V_f = V_i \left( \frac{P_i}{P_f} \right)^{\frac{1}{r}} = V_i \left( \frac{1}{2} \right)^{\frac{1}{7/5}} = V_i \left( \frac{1}{2} \right)^{5/7}$$

$$= \cancel{.609} \quad .609 V_i$$

$$W = -\frac{P_i V_i^r}{1-r} (V_f^{1-r} - V_i^{1-r})$$

Solve for  $V_f$

$$= -\frac{P_i}{1-r} \left( V_f \left( \frac{V_i}{V_f} \right)^r - V_i \right)$$

But From  $P_i V_i^r = P_f V_f^r$  we get

$$\frac{V_i^r}{V_f^r} = \frac{P_f}{P_i}$$

$$\therefore W = -\frac{P_i}{1-r} \left( V_f \frac{P_f}{P_i} - V_i \right) = \frac{1}{(r-1)} [V_f P_f - P_i V_i]$$

$$\therefore \frac{(r-1)W + P_i V_i}{P_f} = V_f$$

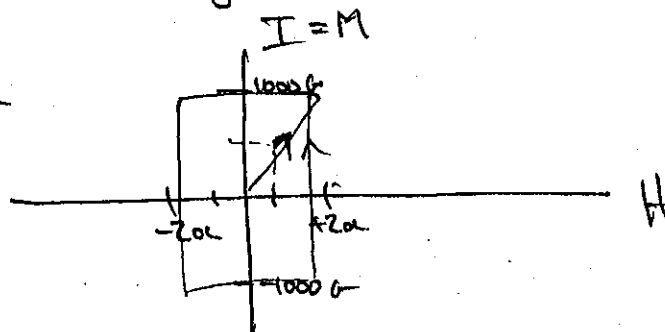
$$V_f = \frac{V_0}{2} + \frac{(r-1)W}{2}$$

$$(14) C = 3.2 \times 10^{-3} \text{ cm} \cdot \text{deg}$$

$\pm 2 \text{ oe} \sim$  unit of magnetic field.

$\pm 1000$  gauss induced induction

$$f = \text{Frequency} = 60 \text{ cycles/sec}$$



$$\Delta T = \Delta Q - \Delta W$$

//

$$\Delta T = \Delta Q - H \Delta M$$

This assuming an adiabatic transformer (60 cycles/sec is prob. fast enough to eliminate any heat transformer)

$$\Delta T = -H \Delta M \quad \text{per unit vol. of sample}$$

$$\Rightarrow C_T \Delta T = -H \Delta M$$

$$\Rightarrow C_T \Delta T = - \oint H \Delta M = 0 \quad \text{as magnetic field is not great enough to cause hysteresis?}$$

$$\text{w/ } H = 1 \text{ oe.}$$

when amplitude = 3 oe

$$\text{Then } \oint H \Delta M \approx (2 \text{ oe})(1000 \text{ G}) = 2000 \text{ G oe per second}$$

$$\text{So that } \Delta T (\text{per second}) = - \frac{2000 \text{ G oe}}{C_T} \quad \text{convert Gauss into primary units + oe into primary units +}$$

when amplitude = 50 oe no difference is felt for this mag is large enough to cause permanent hysteresis.

(15)

$$I = n \mu L \left( \frac{\mu H}{kT} \right)$$

Pg 25 Weinert

12-07-01

$$L(x) = \coth(x) - \frac{1}{x}$$

 $n = \# \text{ atoms per Vol}$ 
 $\mu = \text{mag. mom}$ 
 $H \ll \frac{kT}{\mu} \quad \text{Then}$ 

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} \approx$$

From Pg 42 Goldstein + Pyrk

$$\coth(x) = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + O(x^5)$$

$$= \frac{1}{x} + \sum_{k=1}^{\infty} \frac{2^{2k} B_{2k}}{(2k)!} x^{2k-1}$$

$$x^2 \ll \pi^2$$

$$\text{So } I \approx n \mu \left( \frac{1}{x} + \frac{x}{3} - \frac{x}{x} \right)$$

$$x = \frac{\mu H}{kT}$$

$$= \frac{n \mu}{3} \frac{\mu H}{kT} = \frac{n \mu^2 H}{3kT}$$

$$= \frac{\left( \frac{n \mu^2}{3k} \right) H}{T}$$

$$C = \frac{n \mu^2}{3k}$$

(16) Van der Waals

$$\left(P + \frac{a_1}{\bar{V}^2}\right)(\bar{V} - b_1) = RT$$

$$\downarrow \left(P + \frac{a_2}{\bar{V}^2}\right)(\bar{V} - b_2) = RT$$

$$\text{w/ } a_1 \neq a_2 \quad \downarrow \quad b_1 \neq b_2$$

$$\text{let } P = P_0 P' \quad \bar{V} = \bar{V}_0 V' \quad \text{molar volume.}$$

$$T = T_0 T'$$

Then we get

$$\left(P_0 P' + \frac{a}{\bar{V}_0^2 \bar{V}'^2}\right)(\bar{V}_0 V' - b) = RT_0 T' \quad \div T_0$$

$$\frac{1}{T_0} \left(P_0 P' - \frac{a}{\bar{V}_0^2 \bar{V}'^2}\right) \bar{V}_0 (V' - \frac{b}{\bar{V}_0}) = RT'$$

$$\Rightarrow \left(\frac{P_0 \bar{V}_0}{T_0} P' - \frac{a}{\bar{V}_0 T_0 \bar{V}'^2}\right) (V' - \frac{b}{\bar{V}_0}) = RT'$$

$$\text{pref } \bar{V}_0 = b \quad \& \quad T_0 \Rightarrow$$

$$\frac{a}{\bar{V}_0 T_0} = 1 = \frac{a}{b T_0} = 1$$

$$\Rightarrow T_0 = \frac{b}{a}$$

12-07-02

Then pick

$$\frac{P_0 V_0}{T_0} = 1$$

$$P_0 = \frac{T_0}{V_0} = \frac{b/a}{b} = \frac{1}{a}$$

$$\therefore V_0 = b, \quad T_0 = b/a, \quad P_0 = 1/a$$

(17)  $C_p = C_0 + C_1 T \frac{\text{cal}}{\text{deg} \cdot \text{mol}}$   $C_0 = 8.62$   
 $C_1 = 0.002$

(a) Calculate  $P \sim V$  under adiabatic conditions From eq in prob #6

$$\frac{\left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial V}{\partial P}\right)_{\text{adiab}}} = \frac{C_p}{C_v}$$

Assuming  $C_p - C_v = R$  This knowing one of  $C_p$  or  $C_v$  is enough to

determine the other

$$C_v = C_p - R$$

$$\left(\frac{\partial T}{\partial V}\right)_{\text{Adiabatic}} = -\left(\frac{C_p - C_v}{C_v}\right) \frac{T}{V}$$

$$= -\left(\frac{C_p}{C_v} - 1\right) \frac{T}{V} = -\left(\frac{C_p}{C_p - R} - 1\right) \frac{T}{V}$$

$$= -\left(\frac{C_p - C_p + R}{C_p - R}\right) \frac{T}{V}$$

$$= \left(\frac{R}{R - C_p}\right) \frac{T}{V} = \frac{T}{\left(1 - \frac{C_p}{R}\right) V}$$

$$R = 8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}} \approx (\text{cal}) \quad \text{J} = \text{energy} = \text{cal}$$

to compute  $\frac{C_p}{R}$  the  $\left(\frac{\partial T}{\partial V}\right)_{\text{Adiabatic}} = \frac{T}{\left(1 - C_p - C_1 T\right) V} \Rightarrow \frac{\partial T}{\partial V}$

$$(17) \quad C_p = C_p^0 + C_p^1 T$$

$$P\bar{V} = RT$$

$$C_p - C_v = R, \quad C_v = C_p - R$$

$$(a) \quad C_v dT + (C_p - C_v) \frac{dT}{dV|_p} dV = 0$$

$$\Rightarrow (C_p^0 + C_p^1 T - R) dT + R \frac{dT}{dV|_p} dV = 0$$

$$\Rightarrow (C_p^0 - R + C_p^1 T) dT + R \frac{T}{V} dV = 0$$

$$\frac{(C_p^0 - R + C_p^1 T)}{TR} dT + \frac{dV}{V} = 0$$

$$\left( \frac{C_p^0 - R}{TR} + \frac{C_p^1}{R} \right) dT + \frac{dV}{V} = 0$$

$$P = P(T)$$

$$\frac{C_p^0 - R}{R} \ln T + \frac{C_p^1}{R} T + \ln V = C_1$$

$$C_1 = \left( \frac{C_p^0 - R}{R} \right) \ln T + \frac{C_p^1}{R} \ln e^T + \ln V = C_1$$

$$T^{\frac{C_p^0 - R}{R}} e^{\frac{C_p^1 T}{R}} V = C_2$$

$$T^{\frac{C_p^0 - R}{R}} e^{\frac{C_p^1 T}{R}} \frac{RT}{P} = C_2$$

$$\frac{T^{\frac{C_p^0 - R}{R}} e^{\frac{C_p^1 T}{R}}}{P} = C_3$$

$$PV = nRT$$

$$T = \frac{PV}{nR}$$

$$\frac{dT}{dV|_p} = \frac{P}{nR}$$

$$= \left( \frac{V}{T} \right)^{-1}$$

(b)  $T > 0 \quad \bar{V} > b$

$$P = \frac{RT}{\bar{V}-b} \exp\left[-\frac{a}{RT\bar{V}}\right]$$

(c) Given  ~~$T > T_c$~~ ,  $T > T_c$  +  $\forall P$

$$\bar{V} = \frac{RT}{P} \left[ 1 + \frac{B(T)}{\bar{V}} + \frac{C(T)}{\bar{V}^2} + O(\bar{V}^{-3}) \right] \quad \text{give approximations to the value of } \bar{V}.$$

Now  $p(\bar{V}) = \frac{C_1}{\bar{V}-C_2} \exp\left[-\frac{C_3}{\bar{V}}\right]$

w/  $C_1 > 0, C_2 > 0, C_3 > 0$  ~~taking~~ taking typical values for  $C_1, C_2, C_3$  gives

$$C_1 = RT = (8.314 \text{ J/mol}\cdot\text{K})(300^\circ\text{K}) = 2.49 \cdot 10^3 \text{ J/mol}$$

$$C_2 = b = 3.64 \cdot 10^{-5}$$

$$C_3 = \frac{a}{RT} = \frac{.1358 \text{ J}}{2.49 \cdot 10^3}$$

$$= 5.45 \cdot 10^{-5}$$

} for air

pg 26 wenn

$$(18) \quad \left. \frac{\partial P}{\partial V} \right|_T = 0 \quad + \quad \left. \frac{\partial P}{\partial T} \right|_T = 0$$

Vander Waals eqns  $\rightarrow$

$$\left(p + \frac{a}{\bar{v}^2}\right)(\bar{v} - b) = RT$$

$$\left. \frac{\partial}{\partial V} \right|_T \Rightarrow \left( \left. \frac{\partial P}{\partial V} \right|_T + \frac{-2a}{\bar{v}^3} \right)(\bar{v} - b) + \left(p + \frac{a}{\bar{v}^2}\right)(1) = 0$$

$$\left. \frac{\partial P}{\partial V} \right|_T = \frac{2a}{\bar{v}^3} - \frac{\left(p + \frac{a}{\bar{v}^2}\right)}{\bar{v} - b} \stackrel{!}{=} 0 \quad (1)$$

$$+ \left. \frac{\partial^2 P}{\partial \bar{v}^2} \right|_T = -\frac{6a}{\bar{v}^4} - \frac{\left(\left. \frac{\partial P}{\partial V} \right|_T - \frac{2a}{\bar{v}^3}\right)}{(\bar{v} - b)} + \frac{\left(p + \frac{a}{\bar{v}^2}\right)}{(\bar{v} - b)^2}$$

use eq (1)

$$= -\frac{6a}{\bar{v}^4} + \frac{\cancel{p + \frac{a}{\bar{v}^2}}}{(\bar{v} - b)^2} - \frac{\cancel{p + \frac{a}{\bar{v}^2}}}{(\bar{v} - b)^2} =$$

$$= -\frac{6a}{\bar{v}^4} + \frac{p + \frac{a}{\bar{v}^2}}{(\bar{v} - b)^2} + \frac{p + \frac{a}{\bar{v}^2}}{(\bar{v} - b)^2}$$

$$= -\frac{6a}{\bar{v}^4} + \frac{2\left(p + \frac{a}{\bar{v}^2}\right)}{(\bar{v} - b)^2} \stackrel{!}{=} 0 \quad (2)$$

2 eqs 2 unknowns  $p + V$  solving eq (1) for

$p$  gives:

$$\frac{2a}{\bar{V}^3}(\bar{V}-b) - \frac{a}{\bar{V}^2} = p$$

$$\Rightarrow p = \frac{2a}{\bar{V}^2} - \frac{2ab}{\bar{V}^3} - \frac{a}{\bar{V}^2} = \frac{a}{\bar{V}^2} - \frac{2ab}{\bar{V}^3} = \underline{\underline{\frac{a}{\bar{V}^2}(1 - \frac{2b}{\bar{V}})}}$$

$$\Rightarrow p = \frac{a}{\bar{V}^2}(1 - \frac{2b}{\bar{V}}) \quad \text{putting into eq (2) gives:}$$

$$-\frac{6a}{\bar{V}^4} + \frac{2(\frac{2a}{\bar{V}^2} - \frac{2ab}{\bar{V}^3})}{(\bar{V}-b)^2} = 0$$

$$\Rightarrow -\frac{6a}{\bar{V}^4} + \frac{4a(\cancel{\bar{V}}-b)}{\cancel{\bar{V}}^3(\bar{V}-b)^2} = 0$$

$$\Rightarrow -\frac{3\cancel{a}}{\bar{V}} + \frac{2\cancel{a}}{(\bar{V}-b)} = 0 \quad \Rightarrow +3(\bar{V}-b) = 2\bar{V}$$

$$\boxed{\bar{V} = 3b}$$

$$\text{Then } p = \frac{a}{9b^2}(1 - \frac{2b}{3b}) = \frac{a}{9b^2}(\frac{1}{3}) = \underline{\underline{\frac{a}{27b^2}}}$$

$$\omega/ \quad p_c = \frac{a}{27b^2} + \bar{V}_c = 3b \quad \text{The EOS gives}$$

$$\begin{aligned} T_c &= \frac{1}{R} \left( \frac{a}{27b^2} + \frac{a}{9b^2} \right) (3b-b) = \frac{a}{9Rb^2} \left( \frac{1}{3} + 1 \right) 2b \\ &= \frac{a}{9Rb^2} \left( \frac{4}{3} \right) 2b = \underline{\underline{\frac{8a}{27Rb}}} \end{aligned}$$

Then

$$\frac{P_c \bar{V}_c}{RT_c} = \frac{(\frac{a}{27b^2})(3b)}{R(\frac{8a}{27b})} = \frac{\frac{3}{b}}{\frac{8}{b}} = \frac{3}{8}$$

Derive EOS

$$p(\bar{V}-b) = RT \exp\left(-\frac{a}{RT\bar{V}}\right)$$

$$\frac{\partial}{\partial \bar{V}} \Big|_T \Rightarrow$$

$$\frac{\partial p}{\partial \bar{V}} (\bar{V}-b) + p(1) = RT \exp\left(-\frac{a}{RT\bar{V}}\right) \left(\frac{a}{RT\bar{V}^2}\right)$$

$$\Rightarrow \frac{\partial p}{\partial \bar{V}} \Big|_T = \frac{1}{(\bar{V}-b)} \left[ \frac{a}{\bar{V}^2} \exp\left(-\frac{a}{RT\bar{V}}\right) - p \right]$$

$$= \frac{1}{(\bar{V}-b)} \left[ \frac{ap(\bar{V}-b)}{RT\bar{V}^2} - p \right] = \frac{ap}{RT\bar{V}^2} - \frac{p}{(\bar{V}-b)}$$

$$\frac{\partial^2 p}{\partial \bar{V}^2} \Big|_T = \frac{a}{RT\bar{V}^2} \frac{\partial p}{\partial \bar{V}} - \frac{2ap}{RT\bar{V}^3} - \frac{\partial p}{\partial \bar{V}} + \frac{p}{(\bar{V}-b)^2}$$

$$= \frac{a}{RT\bar{V}^2} \left[ \frac{ap}{RT\bar{V}^2} - \frac{p}{\bar{V}-b} \right] - \frac{2ap}{RT\bar{V}^3} - \frac{1}{(\bar{V}-b)} \left[ \frac{ap}{RT\bar{V}^2} - \frac{p}{\bar{V}-b} \right] + \frac{p}{(\bar{V}-b)^2}$$

=

$$(19) \quad p(\bar{V}-b) = RT \exp \left[ -\frac{a}{RT\bar{V}} \right] =$$

(a) Virial coefficients

$$\frac{p\bar{V}}{RT} = 1 + \frac{B(T)}{\bar{V}} + \frac{C(T)}{\bar{V}^2} + O(\frac{1}{\bar{V}^3})$$

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$\frac{p\bar{V}}{RT} = \frac{\bar{V}}{(\bar{V}-b)} \exp \left[ -\frac{a}{RT\bar{V}} \right]$$

$$\frac{1}{1-x} = \sum_{k=0}^{\infty} x^k$$

$$= \frac{\bar{V}}{\bar{V}-b} \left[ 1 - \frac{a}{RT\bar{V}} + \frac{a^2}{2R^2T^2\bar{V}^2} - \frac{a^3}{6R^3T^3\bar{V}^3} + O(\frac{1}{\bar{V}^4}) \right]$$

$$= \frac{1}{1-b/\bar{V}} \left[ 1 - \frac{a}{RT\bar{V}} + \frac{a^2}{2R^2T^2\bar{V}^2} - \frac{a^3}{6R^3T^3\bar{V}^3} + O(\frac{1}{\bar{V}^4}) \right]$$

$$= \left( 1 + \frac{b}{\bar{V}} + \frac{b^2}{\bar{V}^2} + \frac{b^3}{\bar{V}^3} + O(\frac{1}{\bar{V}^4}) \right) \left( 1 - \frac{a}{RT\bar{V}} + \frac{a^2}{2R^2T^2\bar{V}^2} - \frac{a^3}{6R^3T^3\bar{V}^3} + O(\frac{1}{\bar{V}^4}) \right)$$

$$= 1 - \frac{a}{RT\bar{V}} + \frac{a^2}{2R^2T^2\bar{V}^2} + O(\bar{V}^{-3})$$

$$+ \frac{b}{\bar{V}} - \frac{ba}{RT\bar{V}^2} + \frac{b^2}{\bar{V}^2}$$

$$= 1 + \left( \frac{a}{RT} + b \right) \frac{1}{\bar{V}}$$

$$+ \left( \frac{a^2}{2R^2T^2} - \frac{ba}{RT} \right) \frac{1}{\bar{V}^2} + O(\bar{V}^{-3})$$

Thus

$$B(T) = \frac{a}{RT} + b$$

$$C(T) = \frac{a^2}{2R^2T^2} - \frac{ba}{RT}$$

$$\rightarrow P = \frac{T^{\frac{C_p^0}{R}} e^{\frac{C_p^1}{R} T}}{C_3}$$

$$R = 8.314 \text{ J}$$

$$\frac{C_p^0}{R} = \frac{8.62 \frac{\text{cal}}{\text{K} \cdot \text{mol}}}{8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}} = \frac{4.18(8.62)}{8.314} = 4.33$$

$$1 \text{ J} = \frac{1}{4.18} \text{ cal} = .2392 \text{ cal}$$

$$1 \text{ cal} = 4.18 \text{ J}$$

$$\frac{C_p^1}{R} = \frac{2 \cdot 10^{-3} \frac{\text{cal}}{\text{K} \cdot \text{mol}}}{8.314 \frac{\text{J}}{\text{mol} \cdot \text{K}}} = \frac{2 \cdot 10^{-3} (4.18)}{8.314} = 1.005 \cdot 10^{-3}$$

(b) Polyatomic gas  $PV^r = \text{constant}$

$$PV = nRT$$

$$P(nR)^r \frac{T^r}{P^r} = C_1$$

$$P^{1-r} T^r = C_2 \Rightarrow P T^{\frac{r}{1-r}} = C_3$$

$$\cancel{P^{r-1} T^{-r}} \neq C_3 \Rightarrow C_4 = \frac{T^{\frac{r}{r-1}}}{P}$$

For a polyatomic gas  $r = \frac{5}{3}$  or  $\frac{7}{3}$

$$r = 1.4 = \frac{14}{10} = \frac{7}{5}$$

$$1 \leq r \leq \frac{5}{3}$$

$$\frac{r}{r-1} = \frac{\frac{5}{3}}{\frac{2}{3}} = \frac{5}{2}$$

$$\text{Then } \frac{r}{r-1} = \frac{\frac{7}{5}}{\frac{2}{5}} = \frac{7/5}{2/5} = \frac{7}{2} = 3.5$$

(20)

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$$\alpha = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P$$

$$\beta = -V \left. \frac{\partial P}{\partial V} \right|_T$$

$$-\frac{1}{\beta} = \frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T$$

$$= \frac{\partial V}{\partial P} \bigg|_T$$

$$\beta = -V \left. \frac{\partial P}{\partial V} \right|_T$$

$$\frac{1}{T} \left( 1 + \frac{3a}{\sqrt{T}^2} \right) = \frac{1}{V} \left. \frac{\partial V}{\partial T} \right|_P \quad (1)$$

$$\frac{P}{\left( 1 + \frac{a}{\sqrt{T}^2} \right)} = -V \left. \frac{\partial P}{\partial V} \right|_T \quad (2)$$

(a) compatible  $\Rightarrow \frac{\partial^2 V}{\partial P \partial T} = \frac{\partial^2 V}{\partial T \partial P}$

From (1)  $\left. \frac{\partial V}{\partial T} \right|_P = \frac{V}{T} \left( 1 + \frac{3a}{\sqrt{T}^2} \right)$

so  $\frac{\partial^2 V}{\partial P \partial T} \bigg|_P = \frac{1}{T} \left. \frac{\partial V}{\partial P} \right|_T \left( 1 + \frac{3a}{\sqrt{T}^2} \right) + \frac{V}{T} \left( -\frac{3a}{\sqrt{T}^2} \right) \left. \frac{\partial V}{\partial P} \right|_T$

From (2)  $\left. \frac{\partial V}{\partial P} \right|_T = -\frac{V}{P} \left( 1 + \frac{a}{\sqrt{T}^2} \right)$

so  $\frac{\partial^2 V}{\partial T \partial P} \bigg|_P = -\frac{1}{P} \left. \frac{\partial V}{\partial T} \right|_P \left( 1 + \frac{a}{\sqrt{T}^2} \right) - \frac{V}{P} \left( -\frac{a}{\sqrt{T}^2} \right) \left. \frac{\partial V}{\partial T} \right|_P$

$$-\frac{V}{P} \left( -\frac{2a}{\sqrt{T}^3} \right)$$

Thus simplifying these results gives

$$\begin{aligned}
 \frac{\partial^2 \bar{V}}{\partial p|_T \partial T|_p} &= \cancel{\frac{1}{T} \left( 1 + \frac{3a}{\bar{V} T^2} \right)} \\
 &= \frac{1}{T} \left( 1 + \frac{3a}{\bar{V} T^2} - \frac{3a}{\bar{V} T^2} \right) \frac{\partial \bar{V}}{\partial p|_T} = \frac{1}{T} \frac{\partial \bar{V}}{\partial p|_T} \\
 &= \frac{1}{T} \left( \frac{-\bar{V}}{p} \left( 1 + \frac{a}{\bar{V} T^2} \right) \right) \\
 &= -\frac{\bar{V}}{Tp} \left( 1 + \frac{a}{\bar{V} T^2} \right)
 \end{aligned}$$

$$\begin{aligned}
 \downarrow \frac{\partial^2 \bar{V}}{\partial T|_p \partial p|_T} &= -\frac{1}{p} \left[ 1 + \frac{a}{\bar{V} T^2} - \frac{a}{\bar{V} T^2} \right] \frac{\partial \bar{V}}{\partial T|_p} + \frac{2a}{p T^3} \\
 &= \cancel{\frac{1}{p} \left( 1 + \frac{2a}{\bar{V} T^2} \right)} \\
 &= -\frac{1}{p} \frac{\partial \bar{V}}{\partial T|_p} + \frac{2a}{p T^3} \\
 &= -\frac{1}{p} \left( \frac{\bar{V}}{T} \left( 1 + \frac{3a}{\bar{V} T^2} \right) \right) + \frac{2a}{p T^3} \\
 &= -\frac{\bar{V}}{Tp} \left( 1 + \frac{a}{\bar{V} T^2} \right) - \frac{2a}{p T^3} + \frac{2a}{p T^3} \quad \text{same!!}
 \end{aligned}$$

(b) Since  $\alpha$  &  $\beta$  are consistent. Assuming  $\bar{V} = \bar{V}(T, P)$  w/  $\bar{V}$  = specific molar volume.

$$d\bar{V} = \left. \frac{\partial \bar{V}}{\partial T} \right|_P dT + \left. \frac{\partial \bar{V}}{\partial P} \right|_T dP$$

$$\alpha \equiv \frac{1}{\bar{V}} \left. \frac{\partial \bar{V}}{\partial T} \right|_P \Rightarrow \left. \frac{\partial \bar{V}}{\partial T} \right|_P = \alpha \bar{V}$$

$$\beta \equiv -\bar{V} \left. \frac{\partial P}{\partial \bar{V}} \right|_T \Rightarrow \left. \frac{\partial \bar{V}}{\partial P} \right|_T = -\frac{\bar{V}}{\beta}$$

$$d\bar{V} = \alpha \bar{V} dT - \frac{\bar{V}}{\beta} dP$$

$$\Rightarrow \left. \frac{\partial \bar{V}}{\partial P} \right|_T = -\frac{\bar{V}}{\beta}$$

using definitions of  $\alpha$  &  $\beta$

$$d\bar{V} = \frac{\bar{V}}{T} \left(1 + \frac{3a}{\bar{V}T^2}\right) dT - \frac{\bar{V}}{P} \left(1 + \frac{a}{\bar{V}T^2}\right) dP$$

$$\Rightarrow \left. \frac{d\bar{V}}{dT} \right|_P = \frac{\bar{V}}{T} \left(1 + \frac{3a}{\bar{V}T^2}\right) + \left. \frac{d\bar{V}}{dP} \right|_T = -\frac{\bar{V}}{P} \left(1 + \frac{a}{\bar{V}T^2}\right)$$

$$\cancel{\text{cancel}} = \frac{\bar{V}}{T} + \frac{3a}{T^3} = -\frac{\bar{V}}{P} - \frac{a}{PT^2}$$

~~Eq (1)~~  
Solve eq (1) 1st

$$\left. \frac{d\bar{V}}{dT} \right|_P - \frac{\bar{V}}{T} = \frac{3a}{T^3}$$

Solving homogeneous eq gives:

$$\frac{d\bar{V}}{\bar{V}} = \frac{dT}{T} \Rightarrow \bar{V} = C_1 T$$

$$\ln \bar{V} = \ln T + C$$

$$\bar{V} = C_2 T$$

To solve the inhomogeneous eq try a solution of the

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form  $\bar{V}(T) = \frac{C_3}{T^2}$  Then

$$-2\frac{C_3}{T^3} - \frac{C_3}{T^3} = \frac{3a}{T^3} \Rightarrow -3C_3 = 3a \Rightarrow \underline{\underline{C_3 = -a}}$$

$$\therefore \bar{V}(T) = C_2 T - \frac{a}{T^2} \quad \text{where } C_2 = C_2(P).$$

Putting this hypothesis into eq (2) gives

$$\frac{dC_2}{dP} T = -\frac{1}{P} \left( C_2 T - \frac{a}{T^2} \right) - \frac{a}{P T^2} = -\frac{C_2 T}{P}$$

$$\frac{dC_2}{dP} = -\frac{C_2}{P} \Rightarrow \frac{dC_2}{C_2} = -\frac{dP}{P} \Rightarrow \ln C_2 = -\ln P + C_4$$
$$\Rightarrow C_2 = \frac{C_5}{P}$$

Thus finally,

$$\bar{V}(T, P) = \frac{C_5 T}{P} - \frac{a}{T^2} \quad \text{or} \quad P \left( \bar{V} + \frac{a}{T^2} \right) = C_5 T$$

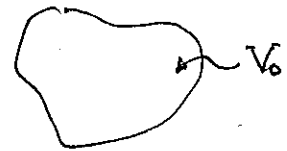
How do I know  $C_5$  (a constant independent of  $T$  &  $P$ ) is equal to  $R$ ?

- Claim  
 (2) Gas 1 obeys equation of state w/  $r_1$   
 Gas 2 obeys " " " " " " " "  $r_2$

$$\gamma_1 = \frac{C_{p1}}{C_{v1}} \text{ is a constant.}$$

or  $C_p + C_v$  are constants

$$pV^{r_1} = \text{constant} = p_0 V_0^{r_1}$$



mix 2 gases at mole fractions  $f_1$  &  $f_2$  respectively,  
 $\Rightarrow n_1 + n_2$  are # moles of gas 1+2, then  $f_1 = \frac{n_1}{n_1+n_2}$ ,  $f_2 = \frac{n_2}{n_1+n_2}$ .

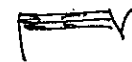
Assuming Dalton's law of partial pressure:

$$P = P_1 + P_2 = \frac{C_1}{\bar{V}^{r_1}} + \frac{C_2}{\bar{V}^{r_2}} = \frac{P_0 \bar{V}_0^{r_1}}{\bar{V}^{r_1}} + \frac{P_0 \bar{V}_0^{r_2}}{\bar{V}^{r_2}}$$

$$= \frac{P_0 n_1 V_0^{r_1}}{n_1}$$

$$\bar{V} = \frac{V}{n}$$

$$\left( \frac{1}{r-1} = \frac{f_1}{r_1-1} + \frac{f_2}{r_2-1} \right)$$



$$P = P(V)$$

$$\Rightarrow P = C V^{-r}$$

$$P = P_1 + P_2 = \frac{P_0 V_0^{r_1}}{V^{r_1}} + \frac{P_0 V_0^{r_2}}{V^{r_2}} = P_0 \left( \left( \frac{V_0}{V} \right)^{r_1} + \left( \frac{V_0}{V} \right)^{r_2} \right)$$

How do you solve this problem?

(22)  $P\bar{V} + G(\bar{V}) = \Pi U$

$U$  = energy per unit mole.

$$[P\bar{V}] = \frac{N \cdot L^3}{L^2 (\# \text{ moles})} = N \cdot L = \text{energy.}$$

Show  $\Rightarrow \Pi = \frac{B\alpha}{C_V}$

$$U = \frac{1}{\Pi} (P\bar{V} + G(\bar{V}))$$

$$B = -\bar{V} \left. \frac{\partial P}{\partial \bar{V}} \right|_T \quad \alpha = \frac{1}{\bar{V}} \left. \frac{\partial \bar{V}}{\partial T} \right|_P$$

$C_V = ?$

$$C_V = \frac{\partial U}{\partial T} = \frac{\partial}{\partial T} \left( \frac{\partial U}{\partial T} \right)_V$$

Is this not correct? or is

$$C_V = \left. \frac{\partial U}{\partial T} \right|_{\bar{V}} = \frac{\bar{V}}{\Pi} \left. \frac{\partial P}{\partial T} \right|_{\bar{V}}$$

$(P, T, \bar{V}) \quad C_V = \frac{C_V}{\bar{V}} ?$

Use  $\left. \frac{\partial x}{\partial y} \right|_z \left. \frac{\partial y}{\partial z} \right|_x \left. \frac{\partial z}{\partial x} \right|_y = -1$

w/  $x = P$   
 $y = T$   
 $z = \bar{V}$

$$\left. \frac{\partial P}{\partial T} \right|_{\bar{V}} \cdot \left. \frac{\partial T}{\partial \bar{V}} \right|_P \cdot \left. \frac{\partial \bar{V}}{\partial P} \right|_T = -1$$

$$\left. \frac{\partial P}{\partial T} \right|_{\bar{V}} = \frac{-1}{\left. \frac{\partial T}{\partial \bar{V}} \right|_P \left. \frac{\partial \bar{V}}{\partial P} \right|_T}$$

$$= -\frac{\bar{V}}{T} \cdot \frac{\partial P}{\partial \bar{V}} \cdot \left. \frac{\partial \bar{V}}{\partial P} \right|_T$$

$$= -(\bar{V}\alpha) \left( \frac{B}{-\bar{V}} \right) = \alpha B$$

$\therefore C_V = \frac{\bar{V}}{\Pi} \alpha B \Rightarrow$

$$\frac{B\alpha}{\left( \frac{C_V}{\bar{V}} \right)} = \Pi \quad \checkmark$$

(23)

 $\beta$  coeff of linear expansion of zero tension.PV:

$$\gamma = \frac{\partial T}{\partial \epsilon} \Big|_{\text{Adia}} = -\frac{1}{\beta} \left[ \frac{Y_s}{Y_t} - 1 \right]$$

 $T = \text{temperature, Not tension.}$ 

For a 1D wire  $\epsilon \equiv \frac{\Delta l}{l}$

looking for  $\frac{\partial T}{\partial \epsilon} \Big|_{\text{Adia}} = \frac{\partial T}{(\partial l/l)} = \swarrow$   
 $\uparrow$   
 By eq 1.14

$$1 = \frac{\beta \Delta T}{(\Delta l/l)} + \frac{1}{\gamma Y_t} \frac{\partial T}{\partial \epsilon} \Big|_{\square?} = \frac{\partial T}{\partial \epsilon} \Big|_{\square?} = \left( 1 - \beta \left( \frac{\partial T}{\partial \epsilon} \right) \right) \gamma Y_t$$

$$\frac{\Delta l}{l} = \beta \Delta T + \frac{1}{\gamma Y_t} \Delta F \quad \div \quad \frac{\Delta l}{l} = \Delta \epsilon$$

$$1 = \beta \frac{\partial T}{\partial \epsilon} + \frac{1}{\gamma Y_t} \frac{\partial T}{\partial \epsilon}$$

$\uparrow$   
 solve for  $\frac{\partial T}{\partial \epsilon} = \frac{1}{\beta} \left[ 1 - \frac{1}{\gamma Y_t} \frac{\partial T}{\partial \epsilon} \right] = -\frac{1}{\beta} \left[ \frac{1}{\gamma Y_t} \frac{\partial T}{\partial \epsilon} - 1 \right]$

define  $Y_s \equiv \frac{1}{\beta} \frac{\partial T}{\partial \epsilon} = \frac{\partial P}{\partial \epsilon}$

Youngs modulus  $\equiv ?$  Then

$$\gamma = -\frac{1}{\beta} \left[ \frac{Y_s}{Y_t} - 1 \right]$$