Pg 21 Whitham

$x = \xi + F(\xi)t$

$\rho_t = f'(\xi)\xi_t \qquad \rho_x = f'(\xi)\xi_x$

$1 = \xi_x + F'(\xi)\xi_x t \qquad 1 = \xi_x(1 + F'(\xi)t)$

$0 = \xi_t + F'(\xi)\xi_t t + F(\xi) = F(\xi) + [1 + F'(\xi)t]\xi_t$

$$\rho_t = -\frac{f'(\xi)F(\xi)}{(1+F'(\xi)t)} \qquad \rho_x = \frac{f'(\xi)}{1+F'(\xi)t}$$

$$\therefore \quad \rho_t + \underset{\overset{\|}{c(\rho)}}{F(\xi)}\rho_x = 0 \ \checkmark$$

$F(\xi) = c(f(\xi))$

$t = -\frac{1}{F'(\xi)}$

$\therefore$ Break occ w/ $F'(\xi) < 0$
+ $|F'(\xi)|$ largest

$x = \xi + F(\xi)t$

+ $x = \xi + d\xi + \underset{}{\overset{F(\xi) + F'(\xi)d\xi}{\overset{=}{F(\xi + d\xi)}}}t$

$\delta\xi \to 0 \qquad F'(\xi)d\xi$

(over)

Pg 37 Whitham

$$U = \frac{Q(\rho_2) - Q(\rho_1)}{\rho_2 - \rho_1} \qquad \rho_2 = \rho_1 + h(\rho_1)$$

$$U(h) = \frac{Q(\rho_1 + h) - Q(\rho_1)}{h} = \frac{\cancel{Q(\rho_1)} + Q'(\rho_1)\cancel{h} + \frac{Q''(\rho_1)h^2}{2} + \cdots \cancel{-Q(\rho_1)}}{\cancel{h}}$$

$$= Q'(\rho_1) + \frac{Q''(\rho_1)}{2}(\rho_2 - \rho_1)$$

$$+ O(\rho_2 - \rho_1)^2$$

$$\rho_t + c(\rho)\rho_x = 0$$

$$\rho_2 \triangleq \rho_1 + \underline{h(\rho_1)}$$

$$C(p_2) = Q'(p_2) = Q'(p_1 + h)$$

$$= Q'(p_1) + Q''(p_1)h + O(h^2)$$

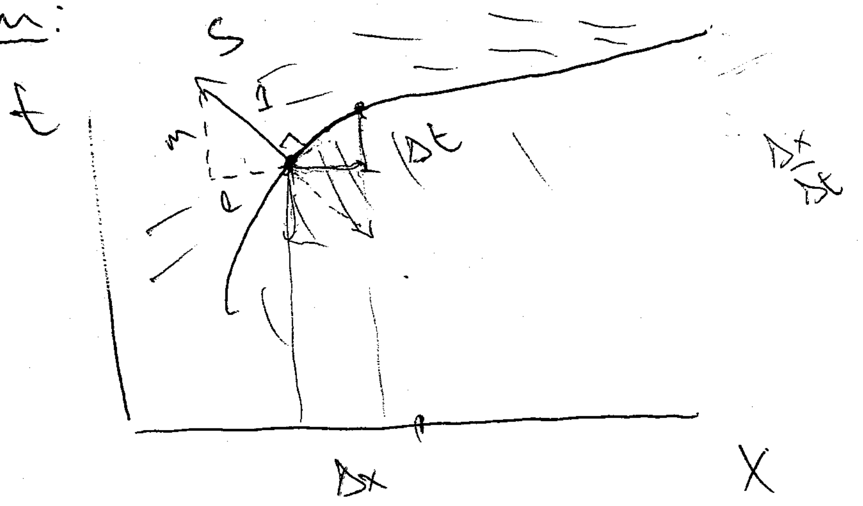
$$= C(p_1) + (p_2 - p_1)Q''(p_1) + O(h^2)$$

Thus $\quad (p_2 - p_1)Q''(p_1) = C(p_2) - C(p_1) - O(p_2 - p_1)^2$

Thus $\quad \sigma = c_1 + \frac{1}{2}c_2 - \frac{1}{2}c_1 + O(p_2 - p_1)^2$

$$= \frac{1}{2}(c_1 + c_2) + O(p_2 - p_1)^2$$

pg 40 Whitham:



$[\rho]l + [Q]m = 0$

$\frac{l}{m} + \frac{[Q]}{[\rho]} = 0$

$\therefore\ U = \frac{\Delta x}{\Delta t} = -\left(\frac{l}{m}\right)$

$\therefore\ U = \frac{[Q]}{[\rho]}$

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$$U = \frac{Q(\rho_2) - Q(\rho_1)}{\rho_2 - \rho_1} = \frac{\alpha(\rho_2^2 - \rho_1^2) + B(\rho_2 - \rho_1)}{\rho_2 - \rho_1} = \alpha(\rho_2 + \rho_1) + B$$

$$= \frac{1}{2}\left[2\alpha\rho_2 + B + 2\alpha\rho_1 + B\right]$$

$$= \frac{1}{2}\left[c_1 + c_2\right]$$

$c = Q'(\rho)$

$F(\xi) = c = c(f(\xi)) = Q'(f(\xi))$

$\rho_t + \underset{c(\rho)}{}\ \rho_x = 0 \Rightarrow c_t + c\,c_x = 0$

$C'(\rho)$

$$\int_{``x"} \left[ \cancelto{0}{\phi\rho\Big|_{``t"}} - \int_{``t"} \phi_t \rho \, dt \right] dx + \int_{``t"} \left[\cancelto{0}{Q(\rho)\phi\Big|_{``x"}} - \int_{``x"} Q(\rho)\phi_x \, dx \right] dt$$

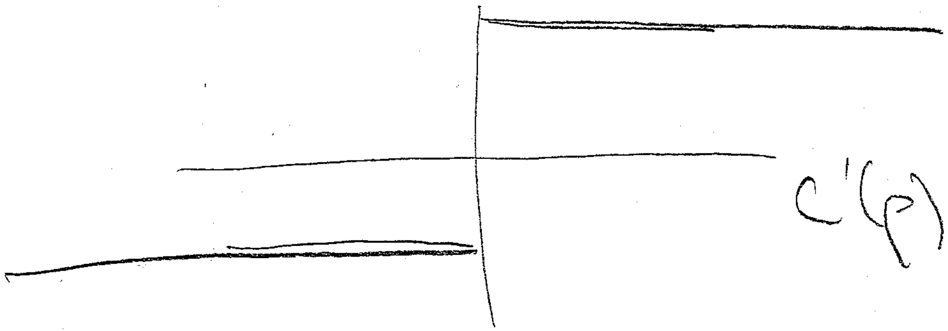
$$= - \iint_{R} \left(\rho \phi_t + Q(\rho) \phi_x \right) dx \, dt$$

<u>Pg 44 Whitham</u>

If st cons. eq lt $\frac{\partial(f)}{\partial t} + \frac{\partial(g)}{\partial x} = 0$

$$U = \frac{[g]}{[f]}$$

$$\therefore \quad U = \frac{[\frac{1}{2}c^2]}{[c]} = \frac{1}{2}(c_1 + c_2)$$



$c'(\rho) > 0 \quad \forall \rho$

$c_2 > c_1 \Rightarrow c_2 - c_1 > 0$

$$\iint_R (\rho \phi_t + Q(\rho)\phi_x)\, dx\, dt = 0$$

$$\iint_R (\phi \rho_t + (Q(\rho))_x \phi)\, dx\, dt =$$ [illegible]

"x"

pg 45 Whitham

$$\dot{s}(t) = \frac{1}{2}\left[(1+tF_1')\dot{\xi}_1 + F_1 + (1+tF_2')\dot{\xi}_2 + F_2\right]$$

Bt $t = \dfrac{-(\xi_1-\xi_2)}{F_1-F_2}$

$$\& \quad \dot{s}(t) = \frac{1}{2}(F_1+F_2)$$

$$\therefore (F_1+F_2)(F_1-F_2) = \cancel{Ft}\,\{\left((F_1-F_2)+(\xi_2-\xi_1)F_1'\right)\dot{\xi}_1$$

$$+ F_1(F_1-F_2) + \left((F_1-F_2)+(\xi_2-\xi_1)F_2'\right)\dot{\xi}_2$$

$$+F_2(F_1-F_2)$$

$$\Rightarrow \cancel{F_1}^2 - \cancel{F_2}^2 = \cancel{F_1}^2 - \cancel{F_1}F_2 + \cancel{F_1F_2} \qquad -\cancel{F_2}^2$$

$$0 = F_1\dot{\xi}_1 - F_2\dot{\xi}_1 + -(\xi_1-\xi_2)F_1'\dot{\xi}_1 + F_1\dot{\xi}_2 - F_2\dot{\xi}_2 - (\xi_1-\xi_2)F_2'\dot{\xi}_2$$

$$= -(\xi_1-\xi_2)(F_1'\dot{\xi}_1 + F_2'\dot{\xi}_2) + F_1(\dot{\xi}_1+\dot{\xi}_2) - F_2(\dot{\xi}_1+\dot{\xi}_2)$$

$$= \frac{1}{2}\{F_1'\dot{\xi}_1 + F_2'\dot{\xi}_2\}(\xi_1-\xi_2) + \frac{1}{2}(F_1+F_2)(\dot{\xi}_1-\dot{\xi}_2)$$

$$= \frac{1}{2}(F_1+F_2)(\dot{\xi}_1-\dot{\xi}_2) + \frac{1}{2}F_1(\dot{\xi}_1+\dot{\xi}_2) - \frac{1}{2}F_2(\dot{\xi}_1+\dot{\xi}_2)$$

$$= \frac{1}{2}\Big[F_1(\dot{\xi}_1 - \cancel{\dot{\xi}_2}) + F_2(\cancel{\dot{\xi}_1} - \dot{\xi}_2) + F_1(\dot{\xi}_1 \cancel{+\dot{\xi}_2}) - F_2(\cancel{\dot{\xi}_1} + \dot{\xi}_2)\Big]$$

$$= \frac{1}{2}\left[2F_1\dot{\xi}_1 - 2F_2\dot{\xi}_2\right] = F_1\dot{\xi}_1 - F_2\dot{\xi}_2$$

Integrate wrt t

$$\int^{t} (F_1' \dot{\xi}_1 + F_2' \dot{\xi}_2)(\xi_1 - \xi_2)\, dt \neq (\xi_1 - \xi_2)(F_1 + F_2)$$

$$\frac{d}{dt}\left[\left(F(\xi_1) + F(\xi_2) \right) (\xi_1 - \xi_2) \right]$$

$$= \left(F_1' \dot{\xi}_1 + F_2' (\dot{\xi}_2) \right) (\xi_1 - \xi_2) + (F_1 + F_2)(\dot{\xi}_1 - \dot{\xi}_2)$$

$\therefore$ LHS is

$$\frac{1}{2} \frac{d}{dt} (*) = \frac{d}{dt} \int_{\xi_1}^{\xi_2} F(\xi)\, d\xi$$

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$$q = Q(\rho) - \nu\rho_x \qquad v = V(\rho) - \frac{\nu}{\rho}\rho_x$$

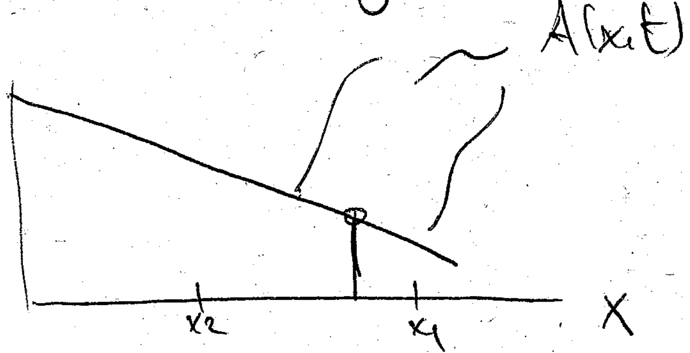
$$\rho = \rho_0 \qquad v = v_0 = V(\rho_0)$$

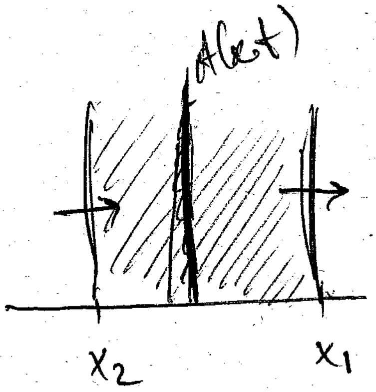
$$\rho = \rho_0 + r \qquad v = v_0 + \omega$$

$$\Rightarrow \omega_t + v_0\omega_x = -\frac{1}{\tau}\left(\cancel{v_0} + \omega - \cancel{V(\rho_0)} - V'(\rho_0)r + \frac{\nu r_x}{\rho_0}\right)$$

$$\Rightarrow \tau(\omega_t + v_0\omega_x) = -\left\{\omega - V'(\rho_0)r + \frac{\nu r_x}{\rho_0}\right]$$

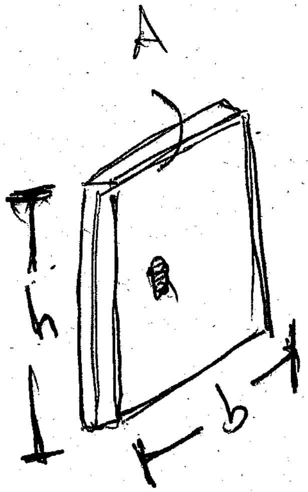
<u>pg 80 Whitham</u>





$$\frac{d}{dt}\int_{x_2}^{x_1} A(x,t)\,dx + q(x_1,t) - q(x_2,t) = 0$$

$$= -q(x_1,t) + q(x_2,t)$$



smooth sol

$$\hookrightarrow \quad \frac{\partial A}{\partial t} + \frac{\partial q}{\delta x} = 0$$

$$q = Q(A, x)$$

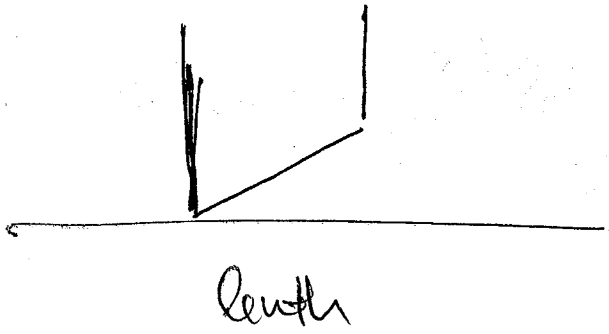
$$\Rightarrow \quad \frac{\partial A}{\partial t} + \frac{\partial Q}{\partial A}\frac{\partial A}{\delta x} + \frac{\partial Q}{\delta x} = 0$$

$$dA = b\,dh$$

$$\Rightarrow \quad \frac{\partial A}{\partial t} + \frac{\partial Q}{\delta A}\frac{\partial A}{\delta x} = -\frac{\partial Q}{\delta x}$$

$$c = \frac{\partial Q}{\delta A} \cong \frac{\partial Q}{\partial (bh)} = \frac{\partial Q}{b\partial h}$$

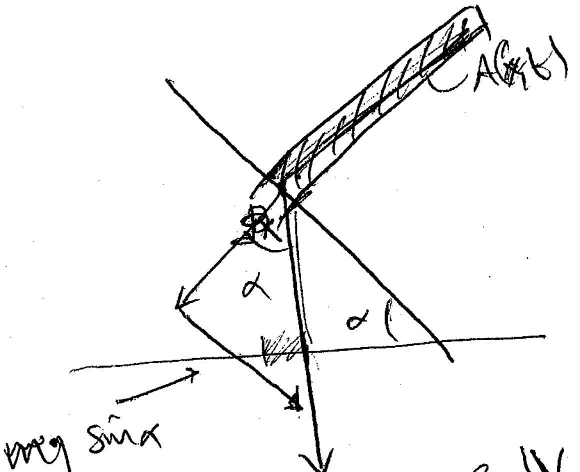
$v = \frac{q}{A}$



$\rho_0 C_f P v^2$

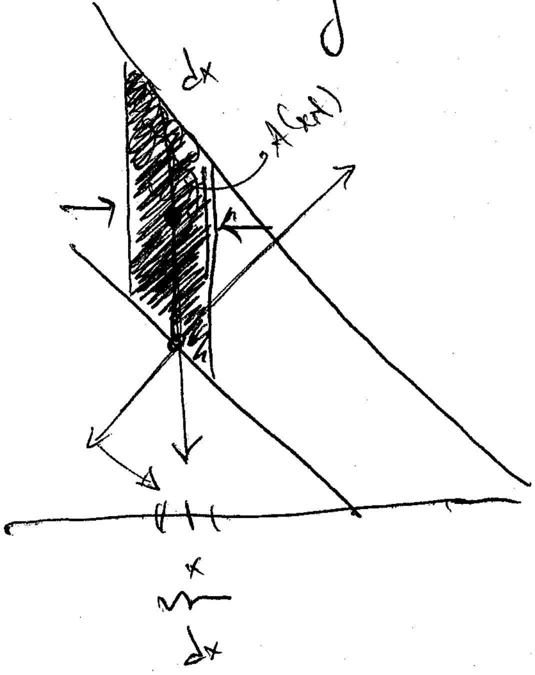
C_f frictional force for H_2O on River bank

$P \sim$ fraction of cross sectional area that is wetted.



$mg = \rho_0 dV g$

$= \rho_0 A(x,t)$



$\rho_0 g A \sin\alpha$ / lenth

complete balance gives

$$\cancel{\rho_0} C_f P v^2 = \cancel{\rho_0} g A \sin\alpha$$

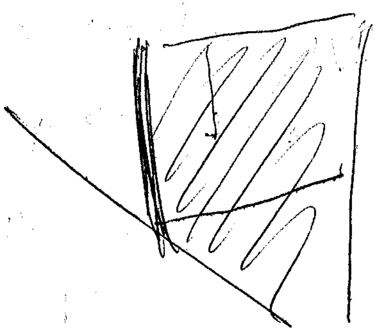
$$\Rightarrow v = \sqrt{\frac{A}{P} \frac{g \sin\alpha}{C_f}}$$

$$J_y^0(u_h) = \int_\Omega k_0 \nabla \Big(\sum_{i=N+}$$

$$Q = vA$$

$$q = \underline{Q(A,x)} = vA = \sqrt{\frac{A^3 g \sin\alpha}{P C_f}}$$

$P \sim \underline{f(A)}$



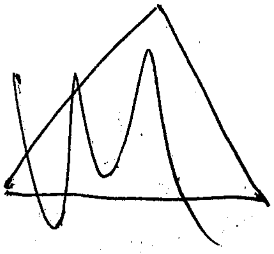
$$v \sim A^{1/2} \qquad Q \sim A^{3/2}$$

P ~ perimeter measures

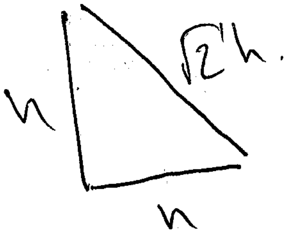
$$c = \frac{d}{dA}(Q)$$

$$= \frac{d}{dA}(vA) = v + A\frac{dv}{dA} = v + A\frac{d}{dA}\left(\sqrt{\frac{A}{P}\frac{g\sin\alpha}{C_f}}\right)$$

$$= v + \frac{A}{2\sqrt{A}}\sqrt{\frac{g\sin\alpha}{PC_f}} = v + \frac{v}{2} = \frac{3}{2}v.$$



$$A = \frac{1}{2}h^2$$



$$P = (2+\sqrt{2})h$$

$$h \sim O(A^{1/2}) \Rightarrow P \sim O(A^{1/2}).$$

(h) $$J_1^0(u_h) = k_0 \int_{\Gamma_E} \frac{\partial u_h}{\partial n} d\ell = k_0 \int_{y=0}^{1} \frac{\partial u_h}{\partial(-x)} dy + k_0 \int_{x=0}^{1} \frac{\partial u_h}{\partial y} dx$$ 1

$$= -k_0 \int_{y=0}^{1} \frac{\partial u_h}{\partial x} dy + k_0 \int_{x=0}^{1} \frac{\partial u_h}{\partial y} dx$$

for $p=1$

$$\cong -k_0 \sum_{i=1}^{N_s+1} \left(\frac{U_{h(N_s+1+i)} - 1}{h} \right) h + k_0 \sum_{\substack{i=N_s+1, 2N_s+1, \dots \\ (N_s+1)^2}} \left(\frac{1 - U_{h\,i-1}}{h} \right) h$$

~~All Nodes~~ in ~~2nd column~~

~~p=1~~
~~i=1,2~~

$$= -k_0 \sum_{i=1}^{N_s+1} U_{h(N_s+1+i)} + k_0(N_s+1) + k_0(N_s+1) - k_0 \sum_{\substack{i=N_s+1, 2N_s+1, \dots \\ (N_s+1)^2}} U_{h\,i-1}$$

$$= 2k_0(N_s+1) - k_0 \Big[\underbrace{\sum_{\substack{i=1,2,\dots \\ i=N_s+1, 2N_s+1, \dots}}^{N_s+1} U_h}_{\text{sum of some of } U_h \text{ values.}} \Big]$$

Very easy to implement for $p=2$ ~~Nothing~~ No difficulties arise.

if $P \propto A^{1/2}$

$\Rightarrow V \propto \left(\frac{A}{A^{1/2}}\right)^{1/2} = A^{1/4}$

$\Rightarrow n = 1/4$. if $C_f \propto A^{-1/3}$

$\Rightarrow V \propto \left(\frac{A}{A^{1/2} A^{-1/3}}\right)^{1/2} =$

$= (A^{1/2} \cdot A^{1/3})^{1/2} = (A^{5/6})^{1/2} = A^{5/12}$

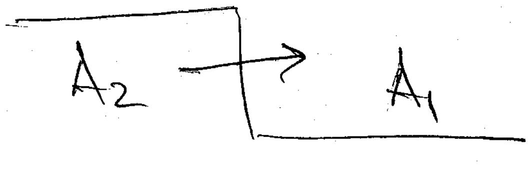
if we just have Mannings law. $C_f \propto A^{-1/3}$

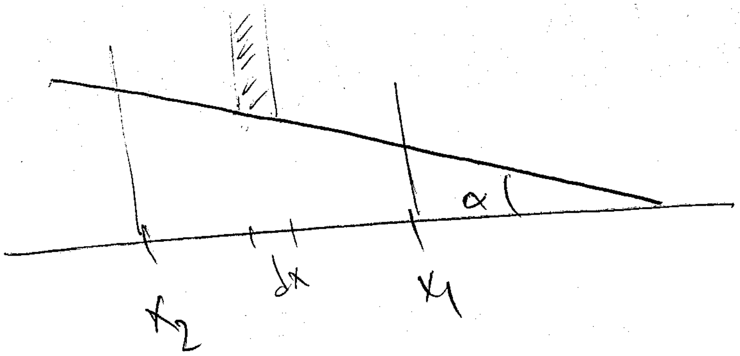
$V \propto \left(\frac{A}{A^{-1/3}}\right)^{1/2} = (A^{4/3})^{1/2} = A^{2/3}$ $\quad n = 2/3$

$$C = V + A\frac{dV}{dA} = V + A\frac{d}{dA}(A^n)$$

$$= V + nA^n = V(1+n)$$

C(A) ↑ ~~&~~ As A ↑





$$g'S = C_f \frac{v^2}{h} \quad w/ \quad h_t + (hv)_x = 0$$

$$\Rightarrow v = \left(\frac{g'S}{C_f} h\right)^{1/2}$$

---

do Stability analysis. Say from constant state

$v = v_0$
$h = h_0$ w/ $g'Sh_0 = C_f v_0^2$

$$C_f \frac{v_0^2}{h_0} = g'S$$ required for solution.

let $v = v_0 + w$
$h = h_0 + \eta$ $\Rightarrow$

$$\eta_t + (h_0 v_0 + h_0 w + v_0 \eta + \eta w)_x = 0$$

$$\eta_t + h_0 w_x + v_0 \eta_x = 0.$$

~~$h_0 w_t + v_0 \eta_t + ((h_0^2 + 2h_0\eta + \eta^2)$~~

$$w_t + (v_0 + w)(w_x) + g'\eta_x = g'S - \frac{C_f(v_0+w)^2}{h_0+\eta}$$

$$= g'S - \frac{C_f(v_0^2 + 2v_0 w + \cancelto{0}{w^2})}{h_0(1+\eta/h_0)}$$

$$= g'S - \frac{C_f}{h_0}(v_0^2 + 2v_0 w)\sum_{k=0}^{\infty}(-1)^k\frac{\eta^k}{h_0^k}$$

$$= g'S - \frac{C_f}{h_0}v_0^2\sum_{k=0}^{\infty}\frac{(-1)^k\eta^k}{h_0^k} - 2\frac{v_0 w C_f}{h_0}\sum_{k=0}^{\infty}\frac{(-1)^k\eta^k}{h^k}$$

$$= \cancel{g'S} - \cancel{\frac{C_f v_0^2}{h_0}}\left[1 - \frac{\eta}{h_0} + \cdots\right] - 2\frac{v_0 w C_f}{h_0}\left[1 + \cdots\right]$$

$$= \quad + \frac{C_f v_0^2\eta}{h_0^2} - \frac{2v_0 C_f}{h_0}w.$$

$$\Rightarrow \quad w_t + v_0 w_x + g'\eta_x = \frac{C_f v_0^2}{h_0^2}\eta - \frac{2v_0 C_f}{h_0}w$$

$$C_f\frac{v_0^2}{h_0} = g'S \; ; \quad \frac{1}{v_0}\left[\frac{v_0^2 C_f}{h_0}\right]$$

& $w_t + V_0 w_x + g'\eta_x = g'S\left[\frac{\eta}{h_0} - \frac{2}{V_0}w\right]$

elliminate w & write an eq for η, by considering

$$(\partial_t + V_0\partial_x)\eta + (h_0\partial_x)w = 0$$

& $$\left(-\frac{g'S}{h_0} + g'\partial_x\right)\eta + \left(\partial_t + V_0\partial_x + \frac{2g'S}{V_0}\right)\cancel{W} = 0$$

Then $$\left[\qquad\qquad\right]\begin{pmatrix}\eta\\ w\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

Multiply 1st eq by $\partial_t + V_0\partial_x + \frac{2g'S}{V_0}$

& 2nd eq by $h_0\partial_x$ to get

$$\left(\partial_t + V_0\partial_x + \frac{2g'S}{V_0}\right)(\partial_t + V_0\partial_x)\eta + (\qquad)h_0\partial_x w = 0$$

$$(h_0\partial_x)\left(-\frac{g'S}{h_0} + g'\partial_x\right)\eta + (h_0\partial_x)\left(\partial_t + V_0\partial_x + \frac{2g'S}{V_0}\right)W = 0$$

Sbtract $\Rightarrow$

$$\Rightarrow \left(\partial_t^2 + v_0\partial_t\partial_x + v_0\partial_{xt} + v_0^2\partial_x^2 + \frac{2g'S}{v_0}\partial_t + \frac{2\cancel{v_0}g'S}{\cancel{v_0}}\partial_x\right)\eta$$

$$+ \left(\frac{g'\cancel{v_0}S}{\cancel{v_0}}\partial_x - g'h_0\partial_x^2\right)\eta = 0$$

$\Rightarrow \Big(\partial_t^2 + 2v_0\partial_{xt} +$ [illegible] $\partial_x +$ [illegible] ∂_t

$$+ (v_0^2 - g'h_0)\partial_x^2 + \frac{2g'S}{v_0}\partial_t + (3g'S)\partial_x\Big)\eta = 0$$

$$\left(\partial_t^2 + 2v_0\partial_{xt} + (v_0^2 - g'h_0)\partial_x^2\right)\eta$$

$$+ \frac{2g'S}{v_0}\left(\partial_t + \underbrace{\frac{3}{2}v_0}_{c_0}\partial_x\right)\eta = 0.$$

$$\left(\partial_t + (v_0 + \sqrt{g'h_0})\partial_x\right)\left(\partial_t + (v_0 - \sqrt{g'h_0})\partial_x\right)\eta$$

$$+ \frac{2g'S}{v_0}\left(\partial_t + c_0\partial_x\right)\eta = 0$$

Stability condition

$$c_- < \frac{3v_0}{2} < c_+$$

$$\Rightarrow \qquad v_0 - \sqrt{g'h_0} < \frac{3v_0}{2} < v_0 + \sqrt{g'h_0}$$

$$-\sqrt{g'h} < \frac{v_0}{2} < \sqrt{g'h}$$

$$-2\sqrt{gh'} < v_0 < 2\sqrt{g'h}$$

But if $(\partial_t + c_+\partial_x)(\partial_t + c_-\partial_x) = 0.$

$$\Rightarrow (\partial_t + c_0\partial_x)\eta = 0$$

consider $\underline{3.38}$ $\quad h = h_0 + \eta$

$v = v_0 + w.$

$$\eta_t + v_0\eta_x + h_0 w_x = 0$$

w/ $v_0 + w = \left(\frac{g'S}{C_f}\right)^{1/2} h_0^{1/2}\left(1 + \eta/h_0\right)^{1/2} = \left(\quad\right)^{1/2} h_0^{1/2} \sum_{k=0}^{\infty} \binom{1/2}{k} \eta^k/h_0^k.$

Now just $k=0$ + $k=1$

$$= \left(\quad\right)^{1/2}\left[1 + \frac{1}{2}\frac{\eta}{h_0}\right]$$

But: we requir $V_0 = \left(\frac{g'S}{C_f}\right)^{1/2} h_0^{1/2}$

$\therefore \Rightarrow \omega = \frac{1}{2}\frac{1}{\sqrt{h_0}}\left(\frac{g'S}{C_f}\right)^{1/2}\eta$

Then in lin. version of 3.38

$$\Rightarrow \eta_t + V_0\eta_x + \frac{\sqrt{h_0}}{2}\left(\frac{g'S}{C_f}\right)^{1/2}\eta_x = 0$$

$$\Rightarrow \eta_t + \left(V_0 + \frac{V_0}{2}\right)\eta_x = 0$$

$$\Rightarrow \eta_t + c_0\eta_x = 0 \quad \text{w/} \quad c_0 = \frac{3V_0}{2} \quad ✓$$

Stability Again

$V_0 < 2\sqrt{g'h_0}$ of $V_0 = \left(\frac{g'S}{C_f}\right)^{1/2} h_0^{1/2}$

$$\Rightarrow \left(\frac{g'S}{C_f}\right)^{1/2} \cancel{h_0^{1/2}} < 2\sqrt{g'\cancel{h_0}}$$

$$\frac{S}{C_f} < 4 \Rightarrow S < 4C_f.$$

$$k_0 \int_{\Gamma_E} \frac{\partial u_n}{\partial n}\, d\ell = k_0 \int_{y=0}^{1} \frac{\partial u_n}{\partial(-x)}\, dy + k_0 \int_{x=0}^{1} \frac{\partial u_n}{\partial(y)}\, dx$$

$$= -k_0 \int_{y=0}^{1} \frac{\partial u_n}{\partial x}\, dy + k_0 \int_{x=0}^{1} \frac{\partial u_n}{\partial y}\, dx \quad \Big\} \text{ for } p=1$$

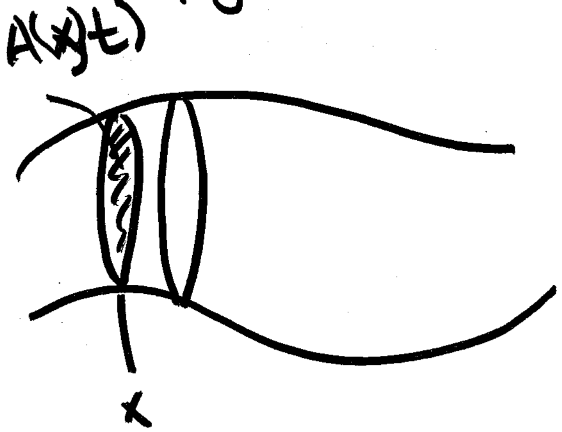
$$\cong -k_0 \sum_{i=1}^{pN_s+1} \left(\frac{u_{n(N_s+1+i)} - \underset{\substack{\| \\ 1}}{u_{n(i)}}}{\bar h}\right) \cdot \bar h + k_0 \sum_{\substack{i=1 \\ i=pN_s+1,\ 2pN_s+1,\ 3pN_s+1,\ \dots \\ (pN_s+1)^2}}^{\cancel{(N_s+1)}} \frac{\left(u_{n(i)} - u_{n\,i-1}\right)}{\bar h} \cdot \bar h$$

$$= -k_0 \sum_{i=1}^{N_s+1} u_{n(N_s+1+i)} + k_0(pN_s+1)$$

$$+ k_0\left[pN_s+1\right] - k_0 \sum_{i=pN_s+1,\ 2pN_s+1,\ \dots} u_{n\,i-1}$$

$$= 2k_0(N_s+1) - k_0\Big[\sum_i u_{n\,i}$$

Pg 80 Whitham



1) Flow in ~~[illegible]~~ flexable tubing?

2) Flow in River?

$$q = Q(A, x)$$

$$\frac{\partial q}{\partial x} = \frac{\partial Q}{\partial A}\cdot\frac{\partial A}{\partial x} + \frac{\partial Q}{\partial x}$$ in D.E

Force friction $\rho_0 C_f P v^2$; $\frac{kg}{m^3}\cdot \frac{m^2}{s^2} C_f P$

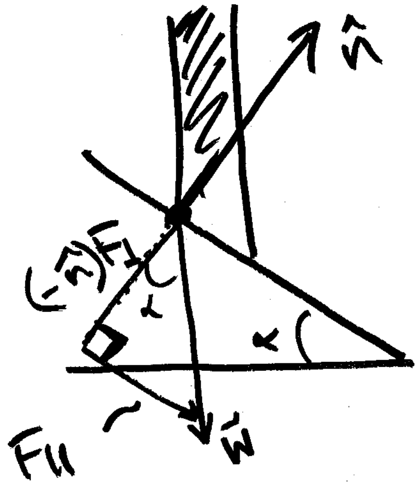
$$= \frac{kg}{m s^2} C_f P$$

$[F] = kg\, m/s^2$

if $P \sim \cancel{A}\ x$

$\Rightarrow$ + $[P] = \cancel{m^2}\ m$

Then ~~kg m/s²~~ $\underbrace{[\rho_0 C_f P v^2] = \frac{kg}{s^2}[C_f]}_{\text{force/length.}}$



~~W~~

$$F_\parallel = W\sin\alpha$$

$\therefore \quad F_{\parallel} = g\rho_0 A\sin\alpha$ per unit length

Thus on ballance

$$g\cancel{\rho_0} A\sin\alpha = \cancel{\rho_0} C_f P v^2$$

$$v = \sqrt{\frac{gA\sin\alpha}{C_f P}} = \frac{1}{A}$$

$$Av = q = Q(A,x)$$

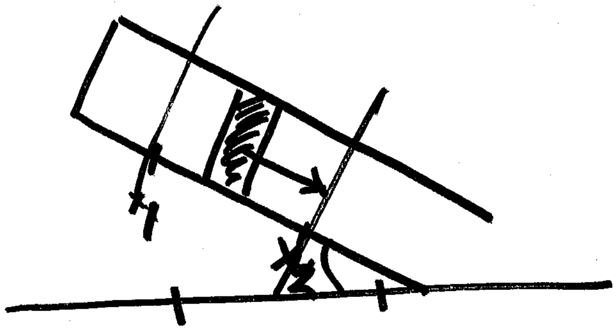
$$\Rightarrow \quad Q(A,x) = \sqrt{\frac{gA^3\sin\alpha}{C_f P}}$$

$$c = \frac{\partial Q}{\partial A} = \frac{\partial}{\partial A}(Av) = v + A\frac{\partial v}{\partial A}$$

$$= v + A\left(\frac{1}{2}\left(\cancel{gA}\,\frac{g\sin\alpha}{C_f P}\right)^{1/2} A^{-1/2}\right)$$

$$= v + \frac{1}{2}(v) = \frac{3}{2}v.$$

Broad Rect Channel

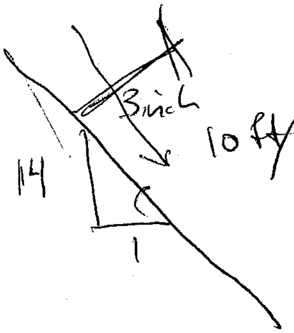


$$\int_{x_1}^{x_2} hv \, dx$$

$$\frac{d}{dt}\int_{x_1}^{x_2} \rho h A v + \left[hv^2\right]\Big|_{x_2}^{x_1} +$$

Idea is: given a problem & you want
to simplify it in a kinematic
wave approximation i.e. you want to get 1 equation
for only one variable. Then taking the 2nd
equation Using the terms that
are not differentiated one can sometimes
get an approximation to produce a simple
wave. Idea then is to test the stability
of this approximation. Then linearize both
equations & combine them into one equation
that is now second order. Then use stability criteria
for this second order equation to determine something
about the original.

Pg 86 Whitham



$$\frac{v_0}{\sqrt{g h_0}} = \frac{10\,\text{ft/s}}{\sqrt{\underset{g'}{\cancel{32\,\text{ft/s}^2}} \cdot 3/12\,\text{ft}}}$$

$$\simeq 3.5$$

$$S/C_f = \frac{v_0^2}{h_0 g'}$$

$$= \frac{(10\,\text{ft/s})^2}{3/12\,\text{ft}\; g'}$$

$$g' = 32\,\text{ft/s}^2 \cos(\tan^{-1}(\tfrac{14}{1}))$$

$$= \cdots$$

$$S = \tan\alpha = 14/1$$

$$\Rightarrow \boxed{\frac{14}{C_f} = 12.5}$$

$h = h(X)$; $v = v(X)$; $X = x - Ut$

using eq 3.37

$$\Rightarrow \quad h_X(-U) + v h_X + h v_X = 0.$$

$$\& \quad -U v_X + v v_X + g' h_X = g' S - \frac{C_f v^2}{h}$$

$$\therefore \quad \partial_t \longrightarrow \text{"} -U \partial_x \text{"}$$

$$(v-U)h_{\underline{X}} + hv_{\underline{X}} = 0$$

$$+\ (v-U)v_{\underline{X}} + g' h_{\underline{X}} = g' S - \frac{C_f v^2}{h}$$

$$\boxed{\begin{array}{l} \partial_t \rightarrow \text{"} -U \underset{\substack{\uparrow \\ \text{Identity operator}}}{I} \text{"} \\ \partial_x \rightarrow \text{"} -U \partial_t \text{"} \end{array}}$$

$$\rightarrow\ -U h_{\underline{X}} + (hv)_{\underline{X}} = 0$$

$$\Rightarrow\ (hv - Uh)_{\underline{X}} = 0$$

$$hv - Uh = B.$$

$$\Rightarrow\ h(v-U) = B$$

for $\underline{X} \rightarrow \infty$ $\Rightarrow$ $\left.\begin{array}{l} x \rightarrow +\infty \\ t \rightarrow -\infty \end{array}\right\}$ uniform state v_1, h_1

$\underline{X} \rightarrow -\infty$

$t = -\infty$

$\Rightarrow$ $\Rightarrow$ $\left.\begin{array}{l} t \rightarrow +\infty \\ x \rightarrow -\infty \end{array}\right\}$ uniform state v_2, h_2.

$$\Rightarrow\ B = h_1(v_1 - U) = h_2(v_2 - U)$$

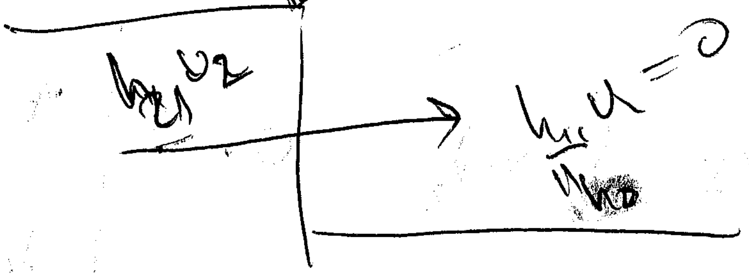
$$\Rightarrow\ h_1 v_1 - h_1 U = h_2 v_2 - h_2 U$$

$\Rightarrow$ RH shock condition...

$h_t + (hv)_x = 0$

$-S(h_1 - h_2) + (h_1 v_1 - h_2 v_2) = 0.$

$\boxed{S=1}$



P Shock in
Still H_2O

$(n-m)u + (n+m)c = 0$

$\frac{u}{c} = -\frac{(n+m)}{n-m} = \frac{n+m}{m-n}$

Check

v_2 $u_2 > u_1$
$h_2 > h_1$ ✓

$\Rightarrow$ $v_{\underline{X}}, h_{\underline{X}} \xrightarrow{\underline{X}\to\pm\infty}$? How does go to zero?

If: $g'S - C_f\frac{v_1^2}{h_1} = g'S - C_f\frac{v_2^2}{h_2}$

$$v_1^2 = \frac{S}{C_f}g'h_1 \;;\; v_2^2 = \frac{S}{C_f}g'h_2$$

w/ $B = h_2(U - v_2)$

& $B = h_1(U - v_1)$

Mult 1st by h_1 & 2nd by h_2 & subtract.

$\Rightarrow$ $B(h_1 - h_2) =$ ~~[illegible]~~ $= -h_1h_2v_2 + h_1h_2v_1$

$$B = \frac{(v_1 - v_2)h_1h_2}{h_1 - h_2} = \left(\frac{S}{C_f}g'\right)^{1/2}\left[\frac{h_1^{1/2} - h_2^{1/2}}{h_1 - h_2}\right]h_1h_2$$

$$= \left(\frac{S}{C_f}g'\right)^{1/2}\frac{h_1h_2}{h_1^{1/2} + h_2^{1/2}}$$

solving 2nd eq for U gives

$$U = \frac{v_2h_2 - v_1h_1}{h_2 - h_1} = \left(\frac{g'S}{C_f}\right)^{1/2}\left[\frac{h_2^{3/2} - h_1^{3/2}}{h_2 - h_1}\right]$$

[illegible]und $v. \Rightarrow U - B/h = V.$

Then $(-B/h)\frac{d}{dX}\left(-\frac{B}{h}\right) + g'\frac{dh}{dX} = g'S - C_f\frac{(U-B/h)^2}{h}$

$$\frac{-B^2}{h}\frac{1}{h^2}\frac{dh}{dX} + g'\frac{dh}{dX} = g'S - C_f\frac{(Uh-B)^2}{h^3}$$

$$\Rightarrow \left(\frac{-B^2}{h^3}+g'\right)\frac{dh}{dX} = g'S - \frac{C_f(Uh-B)^2}{h^3}$$

$\Rightarrow \frac{dh}{dX} =$ [illegible]

$$= -\frac{\frac{(B-Uh)^2 C_f - g'Sh^3}{\cancel{h^3}}}{\frac{1}{\cancel{h^3}}(g'h^3-B^2)}$$

~~$h^3(g'h^3$~~

$\Rightarrow \frac{dh}{dX} = -$

$$\Rightarrow \frac{dh}{dX} = -\frac{[(B-Uh)^2C_f - g'h^3S]}{(g'h^3-B^2)}$$

if $\frac{dh}{dX} = 0$ when $X \to \pm\infty$

$\Rightarrow$ ~~$g'h^3S$~~ [illegible]

$$(B-\bar{U}h)^2 C_f - g' h^3 S = 0$$

$h_1 + h_2$ Must be roots

Pg 88 Whitham

3.50 $$\frac{dh}{dx} = -\frac{(B-Uh)^2 C_f - g'h^3 S}{g'h^3 - B^2}$$

$$(B-Uh_1)^2 C_f - g'h_1^3 S = 0$$

$$\& \ (B-Uh_2)^2 C_f - g'h_2^3 S = 0$$

Another Root

$$-g'h^3 S + (B-Uh)^2 C_f = 0$$

$$-g'h^3 S + (B^2 - 2BUh + U^2h^2) C_f = 0$$

$$-g'Sh^3 + U^2 C_f h^2 - 2BUC_f h + B^2 C_f = 0$$

$$\Rightarrow h^3 - \frac{U^2 C_f}{g'S} h^2 + \frac{2BUC_f}{g'S} h - \frac{B^2 C_f}{g'S} = 0$$

The $h_1 h_2 h_3 = + \dfrac{B^2 C_f}{g'S}$

$$h_3 = \frac{C_f B^2}{S g' h_1 h_2}$$

w/ B from 3.48

$$\Rightarrow H = \frac{\cancel{C_f}}{\cancel{S}\cancel{g'}\cancel{h_1 h_2}} \left(\frac{\cancel{g'}\cancel{S}}{\cancel{C_f}}\right) \frac{h_1^{\cancel{2}} h_2^{\cancel{2}}}{(h_1^{1/2} + h_2^{1/2})^2}$$

$$= \frac{h_1 h_2}{(h_1^{1/2} + h_2^{1/2})^2}$$

$$\Rightarrow \quad H < h_1, h_2$$

$$\frac{dh}{dx} = - \frac{+\cancel{g'} S \left[-h^3 - \frac{U^2 C_f}{g' S} h^2 + \frac{2BUC_f}{g' S} h - \frac{B^2 C_f}{g' S} \right]}{\cancel{g'} \left(h^3 - B^2/g'\right)}$$

$$= + \frac{S \left[h^3 + \frac{U^2 C_f}{g' S} h^2 - \frac{2BUC_f}{g' S} h + \frac{B^2 C_f}{g' S} \right]}{h^3 - B^2/g'}$$

$$= \frac{S(h - h_1)(h - h_2)(h - H)}{h^3 - B^2/g'}$$

$$\frac{dh}{dx} = -\frac{S(h_2 - h)(h - h_1)(h - H)}{h^3 - B^2/g'}$$

3.46 ⇒ $B = h(U - u)$.

$$g'h^3 - B^2 = g'h^3 - (U-u)^2 h^2 = h^2 \left[g'h - (U-u)^2 \right].$$

Sign of denominator is

$$= h^2 \left[\sqrt{g'h} - (U-u) \right] \cdot \left[\sqrt{g'h} + (U-u) \right]$$

$u < U$.

$x^2 + ax + b = (x - x_1)(x - x_2) = x^2 - (x_1 + x_2)x + x_1x_2$

$a = x_1 + x_2$

$b = x_1x_2$

Pg 97 Whitham

$$c_t + cc_x = \cancel{0}\ \nu c_{xx}$$

$$c = -2\nu \frac{\phi_x}{\phi}$$

1st let $c = \psi_x$

$$\Rightarrow \quad \psi_{xt} + \psi_x \psi_{xx} = \nu \psi_{xxx}$$

$$\Rightarrow \quad \psi_{xt} + \left[\frac{(\psi_x)^2}{2}\right]_x = \nu \psi_{xxx}$$

$\int dx$

$$\psi_t + \frac{\psi_x^2}{2} = \nu \psi_{xx}$$

let $\psi = -2\nu \log \phi$ $\quad \left[c = \psi_x = \frac{-2\nu}{\phi} \phi_x \right]$

Then $\psi_t = -2\nu \frac{\phi_t}{\phi}$

$$\psi_x = -2\nu \frac{\phi_x}{\phi} \; ; \; \psi_{xx} = -2\nu \frac{\phi_{xx}}{\phi} + 2\nu \frac{\phi_x^2}{\phi^2}$$

$$\Rightarrow \quad -2\nu \frac{\phi_t}{\phi} + \frac{1}{2} 4\nu^2 \frac{\phi_x^2}{\phi^2} = \nu\left(-2\nu \frac{\phi_{xx}}{\phi} + 2\nu \frac{\phi_x^2}{\phi^2}\right)$$

$$-2\nu \frac{\phi_t}{\phi} + 2\nu^2 \cancel{\frac{\phi_x^2}{\phi^2}} = -2\nu^2 \frac{\phi_{xx}}{\phi} + 2\nu^2 \cancel{\frac{\phi_x^2}{\phi^2}}$$

$$\Rightarrow \quad \phi_t = \nu \phi_{xx}$$

Pg 105 ~~Whittacker~~ Whitham

$$g \approx \frac{1}{1+\exp\{\log(2\sqrt{\pi}R)\}\exp\{2R(z-1)\}} \quad 4.36 \qquad z \sim 1$$

$$= \frac{1}{1+\exp\{2R(z-1)+\log(2\sqrt{\pi}R)\}}$$

$$R = \frac{A}{2\nu} \qquad z = \frac{x}{\sqrt{2At}}$$

$$= \frac{1}{1+\exp\left\{\frac{A}{\nu}\left(\frac{x}{\sqrt{2At}}-1\right)+\log\left(\overset{2\sqrt{\pi R}}{\cancel{\quad}}\right)\right\}}$$

$$= \frac{1}{1+\exp\left\{\frac{A}{\sqrt{2At}\,\nu}\left(x-\sqrt{2At}\right) + \frac{1}{2}\log(4\pi R)\right\}}$$

$$\Rightarrow g^{-1} = 1+\exp\left\{\frac{\sqrt{A}}{\sqrt{2t}\,\nu}\left(x-\sqrt{2At}\right)+\frac{1}{2}\log\left(\frac{2\pi A}{\nu}\right)\right\}$$

$$g^{-1}-1 = \exp\left\{\frac{1}{2\nu}\sqrt{\frac{2A}{t}}\left(x-\sqrt{2At}\right)+\frac{1}{2}\log(\cdots)\right\}$$

(4.23) $$c = c_1 + \frac{(c_2 - c_1)}{1 + \exp\left\{ \frac{c_2 - c_1}{2\nu}(x - Ut) \right\}}$$

$$U = \frac{c_1 + c_2}{2} \qquad x = \sqrt{2At}$$

If $c_2 - c_1 = \sqrt{\frac{2A}{t}}$ $$\frac{dx}{dt} = U = \frac{\sqrt{2A}}{2\sqrt{t}} = \underline{\sqrt{\frac{A}{2t}}}$$

(4.23) $\Rightarrow$ $$c = c_1 + \frac{\sqrt{\frac{2A}{t}}}{1 + \exp\left\{ \frac{1}{2\nu}\sqrt{\frac{2A}{t}}\left(x - \sqrt{\frac{At}{2}}\right) \right\}}$$?

(4.36) $$g \approx \frac{1}{1 + 2\sqrt{\pi R}\, e^{2R(z-1)}} \qquad z \approx 1.$$

$\Rightarrow$ $g \approx$

$$g \sim \frac{1}{2\sqrt{R}} \frac{e^{R(1-z^2)}}{\sqrt{\pi} + e^{R} \int_{2\sqrt{R}}^{\infty} e^{-\xi^2} d\xi}$$

$z \approx 1$

$$\Rightarrow g \sim \frac{1}{2\sqrt{R}} \frac{e^{R}}{\sqrt{\pi} + e^{R} \underbrace{\int_{0}^{\infty} e^{-\xi^2} d\xi}_{\frac{\sqrt{\pi}}{2}}} = \frac{1}{2\sqrt{R}} \frac{e^{R}}{\left(\sqrt{\pi} + \frac{\sqrt{\pi}}{2} e^{R}\right)}$$

- Transition layer $R \gg 1 \Rightarrow \underline{O(R^{-1/2})}$.

$$g \sim \frac{1}{2\sqrt{R}}\,\frac{e^{R(1-z^2)}}{\sqrt{\pi} + e^{R}\int_{z\sqrt{R}}^{\infty} e^{-\xi^2}\,d\xi} = \frac{e^{-Rz^2}}{2\sqrt{R}\left(\sqrt{\pi}\,e^{-R} + \int_{z\sqrt{R}}^{\infty} e^{-\xi^2}\,d\xi\right)}$$

if $z \approx 0$ + $R \gg 1$ if $z = O(R^{-1/2})$

Then this is the order relation between z + R i.e. The two variables are not independent but infact depend on each other.

Then $Rz^2 \sim O(1)$

$$z\sqrt{R} \sim R^{-1/2}\sqrt{R} \sim O(1).$$

$$\therefore\; g \sim \frac{e^{-Rz^2}}{2\sqrt{R}\left(\int_{z\sqrt{R}}^{\infty} e^{-\xi^2}\,d\xi\right)} \sim \frac{e^{-\frac{\cancel{A}}{2\nu}\frac{x^2}{(2\cancel{A}t)}}}{2\sqrt{\frac{A}{2\nu}}\int_{\frac{x}{\sqrt{2\cancel{A}t}}\cancel{2}\left(\frac{\cancel{A}}{2\nu}\right)^{1/2}}^{\infty} e^{-\xi^2}\,d\xi}.$$

$$= \frac{e^{-x^2/4\nu t}}{\frac{\sqrt{2A}}{\sqrt{\nu}}\int_{\frac{x}{\sqrt{4\nu t}}}^{\infty} e^{-\xi^2}\,d\xi} \Rightarrow c = \sqrt{\frac{\cancel{2A}}{t}}\;\frac{e^{-x^2/4\nu t}}{\frac{\cancel{\sqrt{2A}}}{\sqrt{\nu}}\int_{x/\sqrt{4\nu t}}^{\infty} e^{-\xi^2}\,d\xi}$$

$$c = \sqrt{\frac{\nu}{t}}\,\frac{e^{-x^2/4\nu t}}{\int_{x/\sqrt{4\nu t}}^{\infty} e^{-\xi^2}\,d\xi}$$

$$\frac{d}{dt}\int_{-\infty}^{\infty} c\,dx = \int_{-\infty}^{\infty} c_t\,dx = \int_{-\infty}^{\infty}\left(\nu c_{xx} - \left(\tfrac{1}{2}c^2\right)_x\right)dx$$

$$= \left|\nu c_x - \tfrac{1}{2}c^2\right|_{-\infty}^{\infty} = 0.$$

<u>Pg 107 Whitham</u>

$$c = \psi_x = -2\nu\frac{\phi_x}{\phi}. \qquad \phi_t = \nu\phi_{xx}$$

$$\phi_x = \sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}\cdot\left(\frac{-x}{2\nu t}\right)$$

$$\phi_{xx} = \left(\frac{-x}{2\nu t}\right)^2\sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t} - \frac{1}{2\nu t}\left(\sqrt{\frac{a}{t}}\right)e^{-x^2/4\nu t}$$

$$\phi_t = -\frac{1}{2}\frac{\sqrt{a}}{t^{3/2}}\,e^{-x^2/4\nu t} + \sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}\cdot\left(\frac{x^2}{4\nu t^2}\right)$$

$\phi_t = \nu\phi_{xx}$ ✓.

$$C = \frac{-2\nu\phi_x}{\phi} = \frac{+\cancel{2}\cancel{\nu}\left(x/\cancel{2}\cancel{\nu}t\right)\sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}}{1+\sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}}$$

$$= \left(\frac{x}{t}\right)\frac{\sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}}{1+\sqrt{\frac{a}{t}}\,e^{-x^2/4\nu t}}.$$

$$\int_{-\infty}^{\infty} e^{-\eta^2/4\nu t}\,d\eta = \int_{-\infty}^{\infty} e^{-\eta^2}\,d\eta\,(4\nu t)^{1/2} = (4\nu t)^{1/2}\pi^{1/2}.$$

pg 115 Whitham:

5.4

$$\Rightarrow \quad m_j\left(T' \frac{\partial u_j}{\partial t} + X' \frac{\partial y}{\partial x}\right) + l_j b_j = 0$$

$$\parallel$$

$$m_j T' \frac{\partial u_j}{\partial t} + m_j X' \frac{\partial y}{\partial x} + l_j b_j = 0$$

$\parallel$ should be $\parallel$

$$l_i A_{ij} \frac{\partial u_j}{\partial t} + l_i a_{ij} \frac{\partial u_j}{\partial x} + l_j b_j = 0$$

$$\therefore \quad l_i A_{ij} = m_j T' \quad \& \quad l_i a_{ij} = m_j X'$$

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✓ ✓

$$l_1 v_t + l_1 c v_x + l_2 w_t + l_2 c w_x = 0$$

Matching coeff

$$l_2 c = -l_1$$

$$l_1 c = -r l_2$$

$$\Rightarrow \begin{pmatrix} l_1 + c l_2 \\ c l_1 \quad r l_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

must to have non trivial l_1, l_2

$$\begin{vmatrix} 1 & c \\ c & r \end{vmatrix} \overset{\text{must}}{=} 0$$

$r = c^2$. $\quad c = +\sqrt{r}, \; l_1 \qquad , l_2 = 1$

l_2

Pg 118 What

$$l_1(u_t + cu_x) + l_2(v_t + cv_x) = 0$$

$$l_1 u_t + cl_1 u_x + l_2 v_t + cl_2 v_x = 0$$

Match coeff

$cl_1 = l_2$

$cl_2 = -l_1$

$cl_1 = 0$ As no V_t term

$\Rightarrow l_1 = 0$.

or $c = 0$, $l_1(u_t - v_x) + l_2(u_x - v) = 0$

$c = 0$, l_1 & $l_2 = 0$

$c \neq 0$. $l_1 = 0$

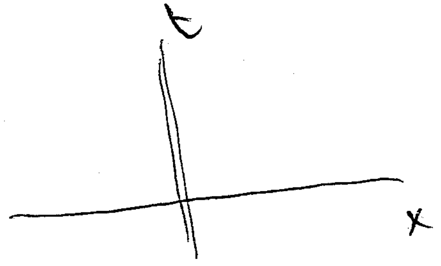
$$|A_{ij} x' - a_{ij} T'| = 0 \qquad A_{ij} \frac{\partial v_j}{\partial t} + a_{ij} \frac{\partial v_j}{\partial x} + l_i b_i = 0$$

$$\|$$

$$\underset{\sim}{A} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \partial_t u \\ \partial_t v \end{pmatrix} \qquad \underset{\sim}{a} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \partial_x u \\ \partial_x v \end{pmatrix}$$

$$\therefore \quad |\underset{\sim}{A} X' - \underset{\sim}{a} T'| = \begin{vmatrix} X' & T' \\ -T' & 0 \end{vmatrix} = 0$$

$$\Rightarrow \quad T'^2 = 0$$ No movement in T direction



$\Rightarrow$ [illegible]

$$\Rightarrow \quad \bar{l} = (0, l_2) \qquad l_2 \neq 0.$$

Ex 3: $\underset{\sim}{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$\left| \underset{\sim}{A} \bar{X}' - \underset{\sim}{a} T' \right| = \begin{vmatrix} \bar{X}' & 0 \\ 0 & -T' \end{vmatrix} = 0$$

$$\Rightarrow \bar{X}' T' = 0$$

Ex 4:

v, r, ω

$\underset{\sim}{A} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $\underset{\sim}{a} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -r & 0 \end{pmatrix}$

$$\begin{vmatrix} -T' & 0 & 0 \\ \bar{X}' & 0 & 0 \\ 0 & -rT' & \bar{X}' \end{vmatrix} = 0$$

$$-T'\underbrace{\begin{vmatrix} 0 & 0 \\ -r & X' \end{vmatrix}}_{\overset{\shortparallel}{0}} - X'(0) + 0 = 0 \quad \forall X', T'.$$

Pg 134 Whitham

$$h_t + vh_x + hv_x = 0$$

$$v_t + vv_x + g'h_x = g'S - C_f \frac{v^2}{h}$$

$\xi = x - ct$ & let $h = h_0 + \xi h_1(t) + \frac{1}{2}\xi^2 h_2(t) + \dots$

$$v = v_0 + \xi v_1(t) + \frac{1}{2}\xi^2 v_2(t) + \dots$$

$$h_t = -ch_1(t) + \xi \dot{h}_1(t) + \xi(-c)h_2(t) + \frac{\xi^2}{2}\dot{h}_2(t) + \dots$$

$$h_x = h_1(t) + \xi h_2(t) + \dots$$

$$v_t = -cv_1(t) + \xi \dot{v}_1(t) + \xi(-c)v_2(t) + \frac{\xi^2}{2}\dot{v}_2(t) + \dots$$

$$v_x = v_1(t) + \xi v_2(t) + \dots$$

$$h_t + vh_x + hv_x = -ch_1(t) + \xi\left(\dot{h}_1(t) - ch_2(t)\right) + \xi^2 \frac{\dot{h}_2(t)}{2}$$

$$+ \left(v_0 + \varepsilon v_1 + \frac{\varepsilon^2}{2} v_2 + \cdots\right)\left(h_1 + \varepsilon h_2 + \cdots\right)$$

$$+ \left(h_0 + \varepsilon h_1 + \frac{\varepsilon^2}{2} h_2 + \cdots\right)\left(v_1 + \varepsilon v_2\right) = 0$$

$$\Rightarrow \quad - - - - \quad + v_0 h_1 \quad + \varepsilon\left(v_1 h_1 + h_2 v_0\right) + \varepsilon^2\left(h_2 v_1 + \frac{h_1 v_2}{2}\right) + \cdots$$

$$+ h_0 v_1 + \varepsilon\left(h_1 v_1 + h_0 v_2\right) + \varepsilon^2\left(\frac{v_1 h_2}{2} + h_1 v_2\right) + \cdots$$

$$\Rightarrow O(1) \qquad -ch_1 + v_0 h_1 + h_0 v_1 = 0$$

$$\Rightarrow O(\varepsilon) \qquad \dot{h}_1 - ch_2(t) + v_1 h_1 + h_2 v_0 + h_1 v_1 + h_0 v_2 = 0$$

$$\dot{h}_1 + 2 v_1 h_1 + h_0 v_2 + h_2 (v_0 - c) = 0$$

$O(\xi^2)$ Not reliable because didn't keep $O(\xi^2)$ terms in h_x + v_x

In 2nd eq:

$$-cv_1 + \xi\dot{v}_1 + \xi(-c)v_2 + \cdots$$

$$+ \left(v_0 + \xi v_1 + \frac{\xi^2}{2}v_2 + \cdots\right)\left(v_1 + \xi v_2 + \cdots\right) \quad \Rightarrow v_0v_1 + \xi(v_2v_0 + v_1^2) + \cdots$$

$$+ \quad g'h_1 + g'\xi h_2 \quad + \cdots$$

$$= g'S - C_f\frac{\left(v_0 + \xi v_1 + \frac{\xi^2}{2}v_2 + \cdots\right)^2}{h_0 + \xi h_1 + \frac{\xi^2}{2}h_2 + \cdots} \simeq g'S - C_f\frac{\left(v_0^2 + 2\xi v_1v_0 + \cdots\right)}{h_0 + \xi h_1 + \frac{\xi^2}{2}h_2 + \cdots}$$

$$\simeq g'S - \frac{C_f}{h_0}\left(v_0^2 + 2\xi v_1v_0 + \cdots\right)$$

$$+ \frac{C_f h_1 (v_0^2)}{h_0^2} \, \xi$$

$\therefore$ $\underline{O(1)}$

$$-Cv_1 + v_0 v_1 + g' h_1 = g'S - \frac{C_f}{h_0} v_0^2 = 0$$

By Assumption that v_0, h_0 constant state sol.

$$\Rightarrow \quad v_1(v_0 - c) + g' h_1 = 0$$

$\underline{O(\xi)}$

$$\dot{v}_1 \overset{\wedge}{+} v_2 v_0 + v_1^2 + g h_2 = -2\frac{C_f v_1 v_0}{h_0} + \frac{C_f h_1 v_0^2}{h_0^2}$$

$$\underbrace{+ -Cv_2}_{v_2(v_0 - c)}$$

But $\dfrac{C_f v_0^2}{h_0} = g'S$

$$= -2\frac{v_1}{v_0} g'S + \frac{h_1}{h_0} g'S$$

$$= g'S\left(-2\frac{v_1}{v_0} + \frac{h_1}{h_0}\right)$$

$$\frac{(v_0-c)^2}{g'} \cancel{v_1} + h_0 \cancel{v_1} = 0$$

—

Mult 2nd eq in 5,46 by (v_0-c)

$$g'(v_0-c)h_2 + (v_0-c)^2 v_2 + (v_0-c)\left[\dot{v}_1 + v_1^2 + g'S\left(\frac{2v_1}{v_0} - \frac{h_1}{h_0}\right)\right] = 0$$

Mult 1st eq in 5.46 by g'

$$g'(v_0-c)h_2 + h_0 g' v_2 + g'\left[\dot{h}_1 + 2v_1 h_1\right] = 0$$

<u>Subtract</u>:

$$g'\left[\dot{h}_1 + 2v_1 h_1\right] + (c-v_0)\left\{\dot{v}_1 + v_1^2 + g'S\left(\frac{2v_1}{v_0} - \frac{h_1}{h_0}\right)\right\} = 0$$

Bt:

$$\frac{h_0}{V_0 - c} = -\frac{h_1}{V_1} \qquad \text{eq } 5.45$$

$\therefore$ get $\dot{h}_1 + 2 v_1 h_1 + \frac{h_1}{V_1}$

$$g'\dot{h}_1 + \frac{2g'}{h_0}h_1^2(c-v_0) + ((c-v_0)\left[\frac{c-v_0}{h_0}\frac{dh_1}{dt} + \frac{(c-v_0)^2}{h_0^2}h_1^2\right.$$

$$\left. + g'S\left(\frac{2(c-v_0)h_1}{h_0v_0} - \frac{h_1}{h_0}\right)\right)\Bigg] = 0$$

$$\Rightarrow \underset{\underset{\displaystyle 2g'}{\displaystyle \|}}{\left[g' + \frac{(c-v_0)^2}{h_0}\right]}\frac{dh_1}{dt} + 2g'(c-v_0)\frac{h_1^2}{h_0} + g'(c-v_0)\frac{h_1^2}{h_0}$$

$$+ Sg'(c-v_0)\underset{\underset{\displaystyle -3\frac{h_1}{h_0}\Big]}{\displaystyle \|}}{\left[\frac{2ch_1}{h_0v_0} - \frac{2h_1}{h_0} - \frac{h_1}{h_0}\right]}$$

$$\Rightarrow \frac{dh_1}{dt} + \frac{3}{2}(c-v_0)\frac{h_1^2}{h_0} + \frac{S}{v_0}(c-v_0)\left[c - \frac{3}{2}v_0\right]\frac{h_1}{h_0}$$

$$\frac{(v_0-c)^2}{g'}\cancel{v_1} + h_0\cancel{v_1} = 0$$

Take 1st eq in 5.46 + mult by (v_0-c) Both sides

$$\Rightarrow (c-v_0)^2 h_2 + (v_0-c)h_0 v_2 + (v_0-c)\left[\dot{h}_1 + 2v_1h_1\right] = 0$$

take 2nd eq in 5.46 + mult by h_0

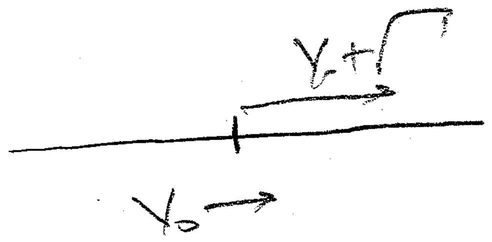
$$g'h_0h_2 + h_0(v_0-c)v_2 + h_0\left[\dot{v}_1 + v_1^2 + g'S\left(\frac{2v_1}{v_0} - \frac{h_1}{h_0}\right)\right] = 0$$

Now subtract to get $(\div$ by $v_0-c)$

$$\dot{h}_1 + 2v_1h_1 - \frac{h_0}{(v_0-c)}\left[\dot{v}_1 + v_1^2 + g'S\left(\frac{2v_1}{v_0} - \frac{h_1}{h_0}\right)\right] = 0$$

$$\frac{dh_1}{dt} = -\frac{3}{2}(c-v_0)\frac{h_1^2}{h_0}$$

If downstream wave $C = V_0 + \sqrt{g h_0}$



$$\frac{dh_1}{dt} = -\frac{3}{2}\frac{\sqrt{g'}}{\sqrt{h_0}}\, h_1^2$$

Upstream waves break if $(C = V_0 - \sqrt{g' h_0}$

$$\frac{dh_1}{dt} = +\frac{3}{2}\frac{\sqrt{g'}}{\sqrt{h_0}}\, h_1^2$$

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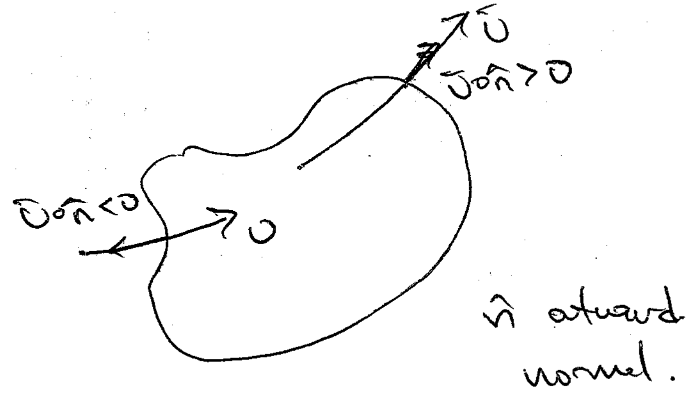
$P_i = P_{ji} \ell_j$ $P_{ji} \hat{=} \sigma_{ji}$ in other books

P_{ji} ith comp. of force/A on elt whos Area has normal in jth dir.



$$\frac{d}{dt}\int_V \rho \, dV = -\int_S \rho \bar{u} \circ \hat{n} \, dS$$

$$\frac{d}{dt}\int_V \rho u_i \, dV = -\int_S \rho u_i \bar{u}\circ\hat{n} \, dS + \int_V \rho F_i \, dV + \int_S P_i \hat{n} \, dS$$

$$\frac{d}{dt}\int_V \left(\tfrac{1}{2}\rho u_i^2 + \rho e\right) dV = -\int_S \left(\left(\tfrac{1}{2}\rho u_i^2 + \rho e\right)\bar{u}\circ\hat{n} - P_i u_i + \ell_i q_i\right) dS$$

$$+ \int_V \rho F_i u_i \, dV$$

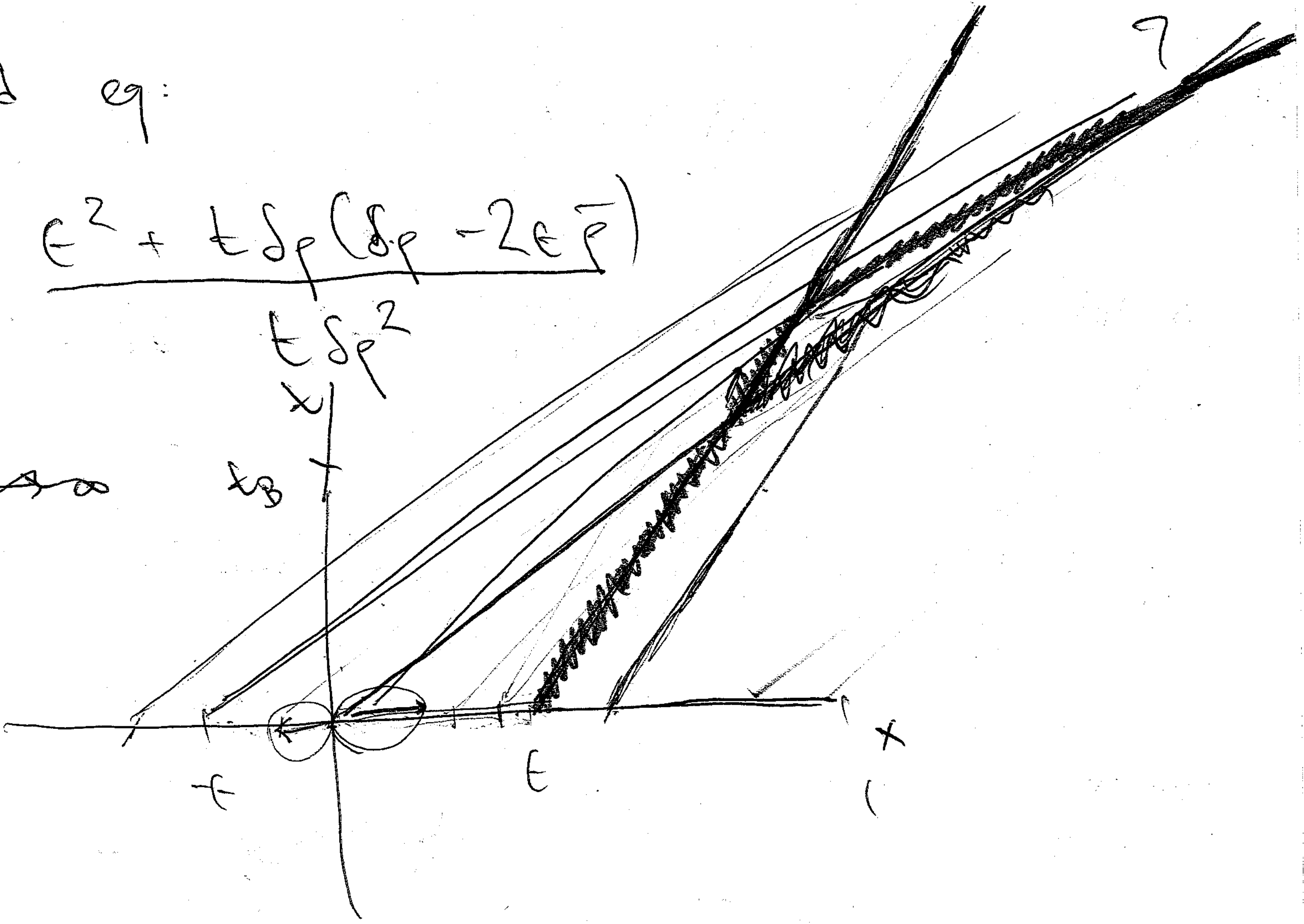
put in second eq:

get $x = \frac{\epsilon^2 + t\delta_p(\delta_p - 2\epsilon\bar{p})}{t\delta_p^2}$

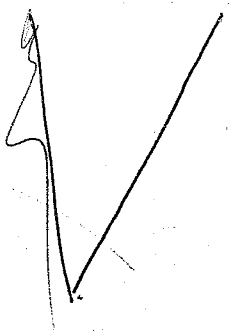
Note As ~~$\epsilon \to \infty$~~ t_B

~~K~~ $\longrightarrow \epsilon$

$x =$



$\mathring{s}(t)$



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cons ang mmt.

$p = m\dot{r}$ lin. mmt.

$\bar{l} = \bar{r} \times \bar{p}$ = ang mmt.

$\bar{\tau} = \bar{r} \times \bar{F}$.

$$\frac{d}{dt}\int_V (\bar{r} \times \rho\bar{u})_i \, dV$$

$$= -\int (\bar{r} \times \rho\bar{u})_i \underset{\overset{\shortparallel}{u_j l_j}}{\bar{u} \cdot \hat{n}} \, dS + \int_V \rho(\bar{r} \times \bar{F})_i \, dV + \boxed{\int_S (\bar{r} \times \bar{P})_i \cdot \hat{n} \, dS}$$

wrong

$\bar{P} = (P_{1i} l_i)\hat{i} + (P_{2i} l_i)\hat{j}$
$+ (P_{3i} l_3)\hat{k}$.

Assume correct for $i = 3$.

$(\bar{r} \times \rho\bar{u})_i = \rho(\bar{r} \times \bar{u})$

$= \rho(vx - uy)$

$\therefore \int_V$

get eq for envelope of intersecting charac 6

~~$x = \xi + F(\xi)$~~

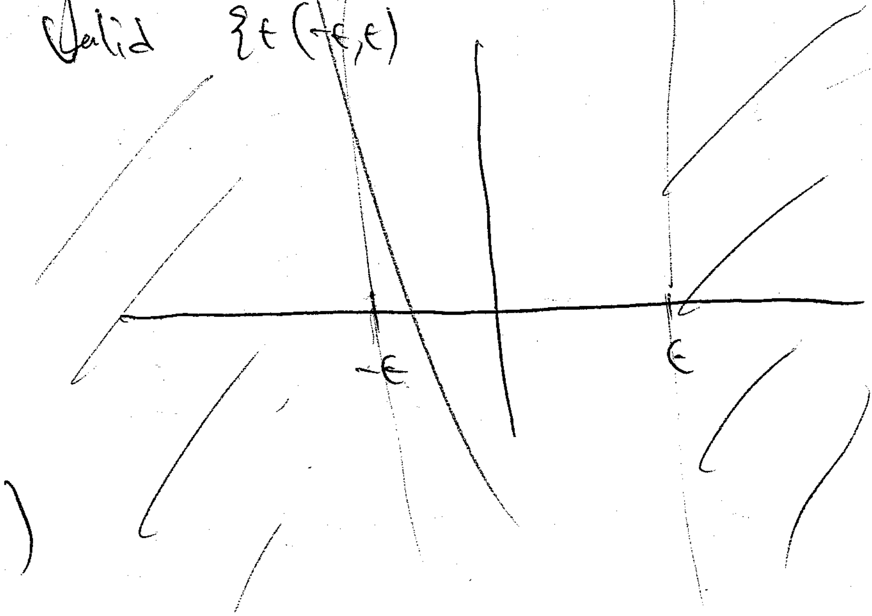
$$F'(\xi) = \begin{cases} 0 & \xi < -\epsilon \\ -\frac{\delta\rho}{\epsilon}\left(\bar{\rho} + \frac{\delta\rho}{2\epsilon}\xi\right) & -\epsilon < \xi < \epsilon \\ 0 & \xi > \epsilon \end{cases}$$

$$0 = 1 + F'(\xi) t$$

$$x = \xi + F(\xi) t$$

$$F(\xi) = \begin{cases} 1 - \hat{\rho}_2^2 & \xi < -\epsilon \\ 1 - \left(\bar{\rho} + \frac{\delta\rho}{2\epsilon}\xi\right)^2 & -\epsilon < \xi < \epsilon \\ 1 - \hat{\rho}_1^2 & \xi > \epsilon . \end{cases}$$

$$t = \frac{-1}{\boxed{F'(\xi)}} \qquad F'(\xi) \neq 0. \quad \text{Valid } \xi \in (-\epsilon, \epsilon)$$



Then

$$t\left(-\frac{\delta\rho}{\epsilon}\left(\bar{\rho} + \frac{\delta\rho}{2\epsilon}\xi\right)\right) = -1$$

$$\xi = + \frac{2\epsilon(\epsilon - t\,\delta\rho\,\bar{\rho})}{t\,\delta\rho^2}$$

$$\int_V \partial_t\left((\bar{r}\times\rho\bar{u})_i\right) dV + \int \partial_{x_j}\left((\bar{r}\times\rho\bar{u})_i u_j\right) dV$$

$$\rho u_j (u_2 x_1 - u_1 x_2)$$

$$= \int_V \rho(\bar{r}\times\bar{F})_i \, dV + \int_S (\bar{r}\times\bar{P})_i \, l_i \, dS$$

$i=3.$

$$F_2 x_1 - x_2 F_1$$

$$\int_V \partial_{x_i}(\bar{r}\times\bar{P})_i \, dV.$$

∂_{x_j}

All rest:

$$\partial_t\left(\rho(\cancel{8}\, u_2 x_1 - u_1 x_2)\right) + \partial_{x_j}\left((\rho x_1 u_2 - \rho x_2 u_1) u_j + \left[\quad \right]\right)$$

? surface term

$$= \rho(x_1 F_2 - x_2 F_1)$$

Surface term $\quad - \int_V \partial.$

$$(\bar{r} \times \bar{P})_3 = x_1 P_2 - x_2 P_1 = x_1 P_{j2} \ell_j - x_2 P_{j1} \ell_j$$

$$= (x_1 P_{j2} - x_2 P_{j1}) \ell_j$$

Thus $\displaystyle \int_S (\bar{r} \times \bar{P})_3 \, dS = \int_S (x_1 P_{j2} - x_2 P_{j1}) \ell_j \, dS$

$$\int \partial_{x_j} (x_1 P_{j2} - x_2 P_{j1}) \, dV.$$

6.8 for $i=2$

$$\partial_t(\rho u_2) + \partial_{x_j}(\rho u_2 u_j - P_{j2}) = \rho F_2$$

& $i=3$

Then eq 6.11 becom

$$\partial_t x_1 \cdot \rho u_2 + x_1 \partial_t \rho u_2 + \frac{\partial(x_1 u_j)}{\partial x_j} \rho u_2 + x_1 \partial_{x_j}(\rho u_2 u_j)$$

Then 6.8 $i=1$ also we get

$$\underbrace{\cancel{\partial_t x_1}}_{\to 0 \text{ why??}}(\rho u_2) - \underbrace{\cancel{\rho u_1 \partial_t x_2}}_{\to 0 \text{ why}} + \rho u_2 \partial_{x_j}(x_1 u_j) - \rho u_1 \partial_{x_j}(x_2 u_j)$$

$$+ \underset{P_{12} = \delta_{j1}}{\underset{\|}{P_{j2}\partial_{x_j}x_1}} - \underset{\delta_{j2} = -P_{21}}{\underset{\|}{P_{j1}\partial_{x_j}x_2}} = 0.$$

$\partial_{x_j}(x_i v_j)$

$\partial_{x_1}(x_1 v_1) + \partial_{x_2}(x_1 v_2) + \partial_{x_3}(x_1 v_3)$

$x_1 (\partial_{x_j} v_j) + v_j \partial_{x_j} x_1$ oth term.

$$v_j \partial_{x_j} x_1 = \delta_{j1} = v_1$$

$-(x_2(\partial_{x_j} v_j) + v_2)$

But cont gives

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$$\rho v_2 \left(\underset{\overset{?}{=}\, v_1}{\partial_t x_1} + \partial_{x_j}(x_1 v_j) \right) - \rho v_1 \left(\underset{\overset{?}{=}\, v_2}{\partial_t x_2} + \partial_{x_j}(x_2 v_j) \right)$$

Ask

$\partial_t x_1 \overset{?}{=} v_1$

or $\partial_t x_1 = 0$

$$=$$

$$\rho v_2 \left[\underset{= v_1}{\delta_{j1} v_j} + \underline{x_1}\, \partial_{x_j} v_j \right] - \rho \underline{v_1} \left(\underset{= v_2}{\delta_{j2} v_j} + \underline{x_2}\, \partial_{x_j}(v_j) \right)$$

$$\partial_t \rho + v_j \partial_{x_j} \rho + \rho \underbrace{\partial_{x_j} v_j}_{} = 0$$

$$\partial_{x_j} v_j = -\frac{1}{\rho} \frac{D\rho}{Dt}$$

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$$h = e + \frac{p}{\rho} = \frac{1}{2}\alpha R T + RT$$

$$= \left(\frac{1}{2}\alpha + 1\right) RT$$

Also $h = \frac{\gamma}{\gamma - 1}\frac{p}{\rho}$ so if $h = \left(\frac{1}{2}\alpha + 1\right)\frac{p}{\rho}$

$$\Rightarrow \quad \frac{\gamma}{\gamma-1} = \cancel{\frac{\gamma-1}{\gamma-1}} + \frac{1}{\gamma-1} = \frac{1}{2}\alpha + \cancel{1}$$

$$\Rightarrow \quad \gamma = 1 + \frac{2}{\alpha}$$

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Use full form of eqs of motion for gas dynamics.

Mass: $\frac{D\rho}{Dt} + \rho \nabla\circ \vec{u} = 0.$

Mmt: $\rho \frac{Du_i}{Dt} + \frac{\partial p}{\partial x_i} = \rho F_i$

Then 6.27 becomes.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_j) = 0$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial}{\partial x_j}(\rho u_i u_j) + \frac{\partial p}{\partial x_i} = \rho F_i$$

$$\cancel{\frac{1}{2}\frac{\partial \rho}{\partial t} u_i^2}$$

$$\cancel{\frac{\partial \rho}{\partial t}\left(\frac{1}{2}u_i^2 + e\right) + \rho\frac{\partial}{\partial t}\left(\frac{1}{2}u_i^2+e\right)}$$

$$\cancel{+ \frac{\partial}{\partial x_j}\left\{\left(\frac{1}{2}\rho u_i^2 + \rho e + p\right)u_j\right\} = \rho F_i u_i}$$

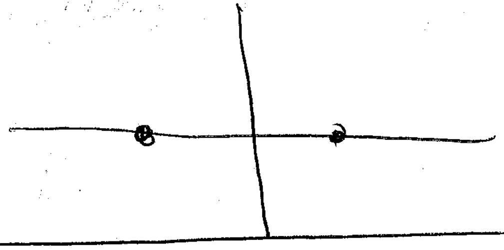
~~Put in Mass~~

$$\cancel{\left(\frac{1}{2}u_i^2 + e\right)\left(-\frac{\partial}{\partial x_j}(\rho u_j)\right) + \rho}$$

$$\Rightarrow \frac{\partial \rho}{\partial t}\cdot e + \rho\frac{\partial e}{\partial t} + \frac{1}{2}\frac{\partial}{\partial t}(\rho u_i)\cdot u_i$$

$$+ \frac{1}{2}\rho u_i \frac{\partial u_i}{\partial t}$$

$$Ai(-x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(-xu+u^3/3)}\,du$$



$$-x+u^2=0$$

$$u=\pm\sqrt{x}$$

$$\sim \frac{1}{2\pi}\int_{-\sqrt{x}-\epsilon}^{-\sqrt{x}+\epsilon}$$

$\underline{8}$

$$\rho_t + (\rho u)_x = 0 \qquad \text{Mass}$$

~~[illegible]~~

$$\frac{d}{dt}\int_{x_1}^{x_2}(u\rho)\,dx = -\left(u^2\rho\right|_{x_1}^{x_2} + p\Big|_{x_1}^{x_2}$$

No heat cond, Nonviscid

$$(u\rho)_t + (u^2\rho)_x \mp p_x = 0$$

$$(u\rho)_t + (u^2\rho - p)_x = 0$$

Energy:

$$\frac{d}{dt}\iiint_{V(t)}\left(\tfrac{1}{2}\rho u^2 + e\rho\right)dV = -\iint_S \bar{u}\cdot\hat{n}(\ldots\ldots)$$

$+\iint_S (-p\,\bar{u}\cdot\hat{n})\,dS$ work done by ~~[illegible]~~ force

$+\iint_{S(t)} \cancel{\bar{q}\cdot\hat{n}\,dS}^{\,0}$ $\qquad \bar{q}$ = flux of ~~[illegible]~~ heat

$$+\frac{\partial}{\partial x_j}\left\{\left(\frac{1}{2}\rho u_i^2+\rho e+p\right)u_j\right\} = \rho F_i u_i$$

put in mass & mmt for time der.

$$-\cancel{\frac{\partial}{\partial x_j}(\rho u_j)}e + \overset{\checkmark}{\rho\frac{\partial e}{\partial t}} + \frac{u_i}{2}\left[\overset{\checkmark}{\rho F_i} - \overset{\checkmark\checkmark}{\frac{\partial p}{\partial x_i}} - \cancel{\frac{\partial}{\partial x_j}(\rho u_i u_j)}\right]$$

$$+\frac{1}{2}\rho u_i\frac{\partial u_i}{\partial t} + \cancel{\frac{\partial}{\partial x_j}(\rho u_j)}\cdot e + \overset{\checkmark}{\rho u_j\frac{\partial e}{\partial x_j}} + \overset{\checkmark\checkmark}{\frac{\partial p}{\partial x_j}}u_j$$

$$+p\frac{\partial u_j}{\partial x_j} + \cancel{\frac{\partial}{\partial x_j}\left(\frac{1}{2}\rho u_i u_j\right)}u_i + \frac{1}{2}\rho u_i u_j\frac{\partial u_i}{\partial x_j} = \overset{\checkmark}{\rho F_i u_i}$$

" $\frac{1}{2}\rho u_i u_j \cdot u_i$ "

$$\Rightarrow \rho\frac{\partial e}{\partial t} + \rho u_j\frac{\partial e}{\partial x_j} + \frac{1}{2}u_j\frac{\partial p}{\partial x_j} + [illegible]$$

$$+\frac{1}{2}\rho u_i\frac{\partial u_i}{\partial t} + p\frac{\partial u_i}{\partial x_i} + \frac{1}{2}\rho u_i u_j\frac{\partial u_i}{\partial x_j} = \frac{\rho F_i u_i}{2}$$

——— look at mmt eq:

$$\frac{\partial \rho}{\partial t}\cdot u_i + \frac{\partial}{\partial x_j}(\rho u_i u_j) + \frac{\partial p}{\partial x_i} = \rho F_i$$

$$+\rho\frac{\partial u_i}{\partial t}$$

$$\bar{q} = -k\nabla T$$

$$\iint \bar{U}\circ\bar{P} \qquad + \iiint_{V(t)} \rho\,\bar{f}\circ\bar{v}\,dV$$

[illegible]$=$

$$P_i = -p\,[illegible]_{ij} +$$

$$\sigma_{ij} = -p\delta_{ij} + \cancelto{0}{\lambda\delta_{ij}\frac{\partial u_k}{\partial x_k}} + \mu\cancelto{0}{\left(\frac{\partial u_i}{\partial x_j}\right.} +$$

$$\boxed{P_i = \sigma_{ij}\hat{n}_j}$$

$$E_t + (vE + vp)_x = 0$$

Put in mass

$$+\rho\frac{\partial u_i}{\partial t}$$

$$\Rightarrow -\cancel{\frac{\partial}{\partial x_j}(\rho u_j)}u_i + \frac{\partial}{\partial x_j}(\rho u_i u_j) + \frac{\partial p}{\partial x_i}{}_{\wedge} = \rho F_i$$

$$\|$$

$$\Rightarrow \qquad \cancel{\frac{\partial}{\partial x_j}(\rho u_j)}u_i + \rho u_j\frac{\partial u_i}{\partial x_j}$$

$$\rightarrow \qquad \rho\frac{\partial u_i}{\partial t} + \rho u_j\frac{\partial u_i}{\partial x_j} + \frac{\partial p}{\partial x_i} = \rho F_i$$

take product w/ u_i + ½

$$\underbrace{\frac{1}{2}\rho u_i\frac{\partial u_i}{\partial t}}_{**} + \frac{\rho u_j u_i}{2}\frac{\partial u_i}{\partial x_j} + \frac{u_i}{2}\frac{\partial p}{\partial x_i} = \frac{\rho u_i F_i}{2}$$

put in.

$$\rho\frac{\partial e}{\partial t} + \rho u_j\frac{\partial e}{\partial x_j} + \cancel{\frac{1}{2}u_j\frac{\partial p}{\partial x_j}} + p\frac{\partial u_i}{\partial x_i} + \cancel{\frac{1}{2}\rho u_i u_j\frac{\partial u_i}{\partial x_j}}$$

$$+\cancel{\frac{\rho u_i F_i}{2}} - \cancel{\frac{u_i}{2}\frac{\partial p}{\partial x_i}} - \cancel{\frac{\rho u_j u_i}{2}\frac{\partial u_i}{\partial x_j}} = \cancel{\frac{\rho u_i F_i}{2}}$$

$$\rho\frac{De}{Dt} + p\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{D\rho}{Dt} = \rho_t + (\bar{u}\cdot\nabla)\rho$$

$$\frac{D\rho}{Dt} + \rho\nabla\cdot\bar{u} = 0$$

$$\frac{De}{Dt} + \frac{p}{\rho}\frac{\partial u_i}{\partial x_i} = 0$$

$$\Rightarrow \quad \nabla\cdot\bar{u} = -\frac{1}{\rho}\frac{D\rho}{Dt}$$

$$\frac{De}{Dt} - \frac{p}{\rho^2}\frac{D\rho}{Dt} = 0.$$

As $\quad de + p\,d(\frac{1}{\rho}) = TdS$

$$-\frac{p}{\rho^2}d\rho = TdS$$

$$T\frac{DS}{Dt} = 0.$$

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$$T\frac{DS}{Dt} = 0 \quad \Rightarrow \quad \frac{DS}{Dt} = 0$$

if $\quad p = p(\rho, S)$

$$\frac{Dp}{Dt} = \frac{\partial p}{\partial \rho}\Big|_S \frac{D\rho}{Dt} + \cancelto{0}{\frac{\partial p}{\partial S}\Big|_\rho \frac{DS}{Dt}} \quad \text{By Above}$$

$$\equiv a^2 \frac{D\rho}{Dt}.$$

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$$e = C_v T \quad \& \quad C_p - C_v = R$$

$$\gamma - 1 = \frac{R}{C_v}$$

$$= C_v \frac{p}{\rho R} = \frac{p}{\rho} \frac{1}{\gamma - 1}$$

$$T\,dS = de + p\,dV = C_v\,dT + p\,dV$$

$$dS = C_v \frac{dT}{T} + \frac{p}{T} dV = C_v \frac{dT}{T} + R\rho\,dV = C_v \frac{dT}{T} + R\rho\,d(1/\rho)$$

$$\Rightarrow \quad S = C_v \log T + R \int \frac{1}{\left(\frac{1}{\rho}\right)} d(1/\rho)$$

$$= C_v \log T + R \log 1/\rho + S_0$$

$$= C_v \log T - R \log \rho + S_0 \qquad R = C_p - C_v$$

$$= C_v \left[\log T\rho \right] - C_p \log \rho + S_0$$

$$C_p = \gamma C_v$$

$$= C_v \log T\rho - C_v \log \rho^{\gamma} + s_0$$

$$= C_v \log \frac{T}{\rho^{\gamma-1}} + s_0 = C_v \log\left(\frac{p}{\rho^{\gamma-1}\rho}\right) + s_0'$$

$$= C_v \log\left(\frac{p}{\rho^{\gamma}}\right) + s_0'$$

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1

$$\frac{DS}{Dt} = 0 \quad \Rightarrow \quad \frac{Dp}{Dt} - a^2 \frac{D\rho}{Dt} = 0.$$

$$p = p(\rho, S)$$

$$dp = \partial_\rho p \, d\rho + d_S p \, dS \overset{=0.}{}$$

$$\|$$

$$\therefore \quad \frac{Dp}{Dt} = a^2 \frac{D\rho}{Dt} \qquad a^2 \equiv \partial_\rho p.$$

$\Leftarrow$ eq $d_S p \, dS = 0$

As $d_S p = 0 \Rightarrow$ p ind. of S ~~← 3rd law~~ ~~thermo~~

~~because we could change S of the system~~

$$p = p(\rho)$$

Don't know why $d_S p \neq 0$ is excluded as a case. but if so $dS \equiv 0$ ∴ $\frac{DS}{Dt} \equiv 0$

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$P(\rho) = \frac{P_0}{\rho_0^{\gamma}}\rho^{\gamma}$ $\qquad c_0^2 = \frac{\gamma P_0}{\rho_0}$

~~$P - P_0 = P_0\left(\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right) = \frac{\gamma P_0}{\rho_0}\left(\frac{\rho^{\gamma}}{\gamma \rho_0^{\gamma - 1}} - 1\right)$~~

~~c_0^2~~

[illegible] $=$ $\frac{P}{\rho^{\gamma}} = \frac{P_0}{\rho_0^{\gamma}}$ $\qquad P = P_0 + P'$

$\rho = \rho_0 + \rho'$

$$\frac{P_0 + P'}{(\rho_0 + \rho')^{\gamma}} = \frac{P_0}{\rho_0^{\gamma}}\left(\frac{1 + P'/P_0}{(1 + \rho'/\rho_0)^{\gamma}}\right) = \cancel{\frac{P_0}{\rho_0^{\gamma}}}$$

$$\Rightarrow \cancel{1} + \frac{P'}{P_0} = \cancel{1} + \gamma \rho'/\rho_0$$

$$\Rightarrow P' = \frac{\gamma P_0}{\rho_0}\rho'$$

$$\Rightarrow (P - P_0) = c_0^2(\rho - \rho')$$

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$$\frac{D\rho}{Dt} + \rho \nabla \cdot \vec{u} = 0$$

$$\rho \frac{Du_i}{Dt} + \frac{\partial p}{\partial x_i} = \rho F_i$$

$$\frac{DS}{Dt} = 0$$

$$\frac{Dp}{Dt} = a^2 \frac{D\rho}{Dt}$$

$$a^2 = \frac{\partial p}{\partial \rho}\Big|_S$$

expand in taylor About $\rho = \rho_0$

$$\Rightarrow \overset{a_0^2}{\overset{\parallel}{\frac{\partial p}{\partial \rho}\Big|_{S,\rho_0}}} + \frac{\partial^2 p}{\partial \rho^2}\Big|_S (\rho - \rho_0) + \frac{1}{2!}\frac{\partial^3 p}{\partial \rho^3}(\rho-\rho_0)^2 + O((\rho-\rho_0)^3)$$

If $p = p(\rho)$

let $p = p_0 + p'$

$\rho = \rho_0 + \rho_0'$

$u = u'$

$$\frac{Dp'}{Dt} = a_0^2 \frac{D\rho'}{Dt}$$

or $Dp' = a_0^2 D\rho'$

$\Rightarrow \quad p - p_0 = a_0^2(\rho - \rho_0)$

$$\frac{D(\rho_0+\rho')}{Dt} + (\rho_0+\rho')\frac{\partial u_j}{\partial x_j} = 0$$

$$\partial_t(\rho_0+\rho') + (\vec{u}'\cdot\nabla)(\rho_0+\rho') + (\rho_0+\rho')\frac{\partial u_j'}{\partial x_j} = 0$$

$$\partial_t \rho' + \cancelto{0}{(\vec{u}'\cdot\nabla)\rho_0} + \rho_0\frac{\partial u_j'}{\partial x_j} = 0 \qquad 6.53.$$

$$\rho_0\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_i}(p_0+p') = 0$$

$$\rho_0\frac{\partial u_i}{\partial t} + \frac{\partial p'}{\partial x_i} = 0 \qquad 6.54$$

$$\frac{\partial u_i}{\partial t} = -\frac{1}{\rho_0}\frac{\partial p'}{\partial x_i}$$

$$u_i' = u_i^{(0)}(\bar{x}) + \frac{\partial}{\partial x_i}\frac{1}{\rho_0}\int_0^t (p-p_0)\,dt$$

$$u_i = \frac{\partial \phi}{\partial x_i}$$

Then $$\rho_0 \frac{\partial}{\partial t} \frac{\partial}{\partial x_i} \phi + \frac{\partial}{\partial x_i} (p - p_0) = 0$$

$$\frac{\partial}{\partial x_i} \left(\rho_0 \frac{\partial}{\partial t} \phi + p - p_0 \right) = 0$$

$$\rho_0 \frac{\partial \phi}{\partial t} + p - p_0 = C(t)$$

Arb constant can be Absorbed in ϕ

~~Why $C(t) \equiv 0$~~ ??

i.e $C(t) = \int^t C'(t)$

$\partial_t \left(\phi - \int^t C'(t) \right) = -\frac{1}{\rho_0}(p - p_0)$

$$\Rightarrow \quad p - p_0 = -\rho_0 \frac{\partial \phi}{\partial t}$$

As $$p - p_0 = a_0^2 (\rho - \rho_0)$$

$$\rho - \rho_0 = -\frac{\rho_0}{a_0^2} \frac{\partial \phi}{\partial t}$$

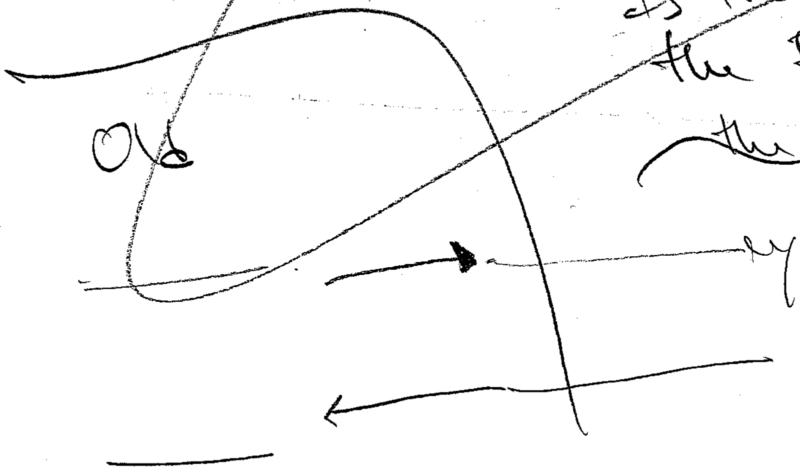
Prob 2: Ask 1) # roots of poly $\dot{x} = P(x,y) = 0$

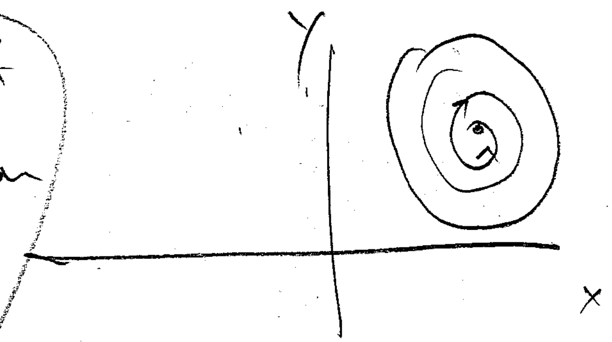
$\dot{y} = Q(x,y) = 0$

deg P, deg Q ≥ 2.

Is there a bound k of the form $k = d^q$ on the # of limit cycles for All dynamical Systems.

(2) Old





(3) No.

Then $$\frac{\partial \rho}{\partial t} = -\frac{\rho_0}{a_0^2} \partial_t^2 \phi$$

$$+ \rho_0 \frac{\partial u_j}{\partial x_j} = \rho_0 \partial_{x_j}^2 \phi$$

$\therefore$ Summing $$0 = -\partial_t^2 \phi + a_0^2 \partial_{x_j}^2 \phi$$

$$\therefore \partial_t^2 \phi = a_0^2 \partial_{x_j}^2 \phi$$

Wave eq

$\underline{u_i}$

$$\partial_{x_i}^2 u_i = \partial_{x_i}^3 \phi = \partial_{x_i}(\partial_{x_i}^2 \phi) = \frac{1}{a_0^2} \partial_{x_i}(\partial_t^2 \phi)$$

$$= \frac{1}{a_0^2} \partial_t^2 (\partial_{x_i} \phi) = \frac{1}{a_0^2} \partial_t^2 \phi$$

$$\Rightarrow \partial_t^2 \phi = a_0^2 \partial_{x_i}^2 \phi.$$

Prob 3:

∂x

$$y_i^{n+1} = y_i^n + \frac{\Delta t}{2}\left(f(y_i^n) + f(y_i^{n+1})\right)$$

$f = u u_x$

$$u_j^{n+1} = u_j^n + \frac{\Delta t}{2\Delta x}\left[u_j^n\left(\frac{u_{j+1}^n - u_j^n}{\Delta x}\right) + u_{j-1}^{n+1}\frac{(u_{j0} - u_{j-1}^{n+1})}{\Delta x}\right)$$

$$= \frac{\Delta t}{2\Delta x}\left[u_j^n u_{j+1} \quad u_{j-1}^{n+1}(\Delta u\right.$$

$u_j^{n+1} = u_j^n + \Delta u$

$u_i^{n+1} = u_i^n + \Delta u$

$\Delta u = L(\Delta u) \quad (\Delta u^2)$

Wave for p.

$$\partial_t^2 p = -\rho_0 \partial_t^3 \phi = -\rho_0 \partial_t(\partial_t^2 \phi)$$

$$= -\rho_0 \partial_t(a_0^2 \partial_{x_i}^2 \phi) = \partial_{x_i}^2(-\rho_0 a_0^2 \partial_t \phi)$$

$$= a_0^2 \partial_{x_i}^2(-\rho_0 \partial_t \phi)$$

$$= a_0^2 \partial_{x_i}^2(p)$$

Wave for ρ

$$\partial_t^2 \rho = -\frac{\rho_0}{a_0^2} \partial_t(\partial_t^2 \phi) = -\frac{\rho_0}{\cancel{a_0^2}} \partial_t(\cancel{a_0^2} \partial_{x_i}^2 \phi) = -\rho_0 \partial_{x_i}^2(\partial_t \phi)$$

$$= \partial_{x_i}^2(-\rho_0 \partial_t \phi) = \partial_{x_i}^2(a_0^2 \rho) = a_0^2 \partial_{x_i}^2 \rho .$$

$\frac{\partial p}{\partial x_i} = 0 \qquad i = 1,2$ <u>pg 159 Whitham</u>

$\frac{\partial p}{\partial x_3} = -\rho g$

$\Rightarrow \frac{dp}{dz} = -\rho g$ If $p = p_0(z) + p_1(\bar{x}t)$

$\rho = \rho_0 + \rho_1(\bar{x}t)$

$u = (0 0 w)$

$\Rightarrow \frac{dp_0}{dz} = -\rho_0 g$

For Vertical prop. of plane wave

$$\frac{D\rho}{Dt} + \rho \frac{\partial u_j}{\partial x_j} = 0$$

$$\Rightarrow \partial_t \rho + (\bar{u}\cdot\nabla)\rho + \rho \nabla\cdot\bar{u} = 0$$

$$\Rightarrow \partial_t \rho_1 + w \partial_z \rho_0 + \rho_0 \partial_z w = 0.$$

1st eq

$$\rho \frac{Du_i}{Dt} + \frac{\partial p}{\partial x_i} = \rho F_i$$

$$\rho_0 \partial_t w + \cancelto{0}{\rho_0 w \partial_z w} + \partial_z p_0 +$$

$$\partial_z p_1 = -\rho_0 g + -\rho_1 g$$

$$0 = 0$$

$$0 = 0.$$

2nd eq

$$\frac{Dp}{Dt} - a_0^2 \frac{D\rho}{Dt} = 0$$

$= [\rho_0][g]$?

$$\Rightarrow \partial_t p + (w\partial_z)p - a_0^2(\partial_t \rho + w\partial_z \rho) = 0$$

$$\Rightarrow \partial_t p_1 + w\partial_z p_0 - a_0^2(\partial_t \rho_1 + w\partial_z \rho_0) = 0.$$

3rd eq.

---

pg 160 know $a_0^2 = \left.\frac{dp}{d\rho}\right|_{p_0, \rho_0}$

$$\therefore [a_0^2] = \frac{[p_0]}{[\rho_0]}$$

$\therefore$ eq 6.58 gives $\frac{[p_0]}{[L]} = +[\rho_0][g]$

$$\therefore [L] = +\left(\frac{[p_0]}{[\rho_0]}[g]\right)^{-1} = +\frac{[a_0^2]}{[g]}$$

If $P_1 = O(\epsilon P_0)$

L $\frac{dP_0}{dz} = O\left(\frac{P_0}{L}\right)$ $P_{1z} = O\left(\frac{\epsilon P_0}{\lambda}\right)$

← P_0 changes on length scales O(L) ↑ P_1 changes on length scales o(λ)

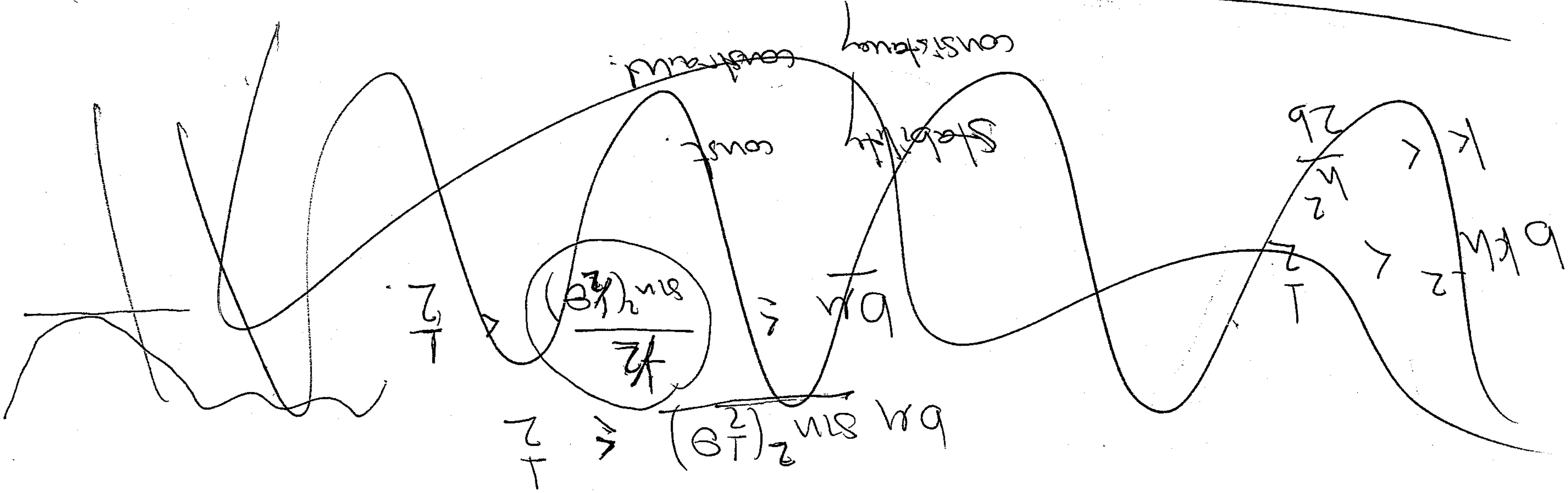
z in P_0 varies over length scales of L

z in P_1 " " " " λ. (wave length of ossilation).

Now $\frac{\lambda}{L} \sim 10^{-4}$

$O\left(\frac{P_0'}{P_{1z}}\right) \sim O\left(\frac{P_0/L}{\epsilon P_0/\lambda}\right) \sim O\left(\frac{\lambda/L}{\epsilon}\right) \sim O(1)$

↑ can be.



$bkh^{-2} < \frac{1}{2}$

$k < \frac{h^2}{2b}$

$b\mu \sin^2(\frac{1}{2}\theta) \le \frac{1}{2}$

$b\mu \le \frac{1/2}{\sin^2(\frac{1}{2}\theta)} < \frac{1}{2}.$

Stability const.

consistancy constraint.

$$\frac{p_0'}{p_{1z}} \sim O\left(\frac{p_0/L}{\epsilon p_0/\lambda}\right) = O\left(\frac{\lambda}{\epsilon L}\right) \sim O\left(\frac{10^{-4}}{\epsilon}\right) \sim O(1)$$

Thus p_0' may be greater than p_{1z} if $\epsilon < 10^{-4}$ As it may be

$$\rho_0 w_t + \cancel{p_0'} + p_{1z} = -\cancel{\rho_0} g - \rho_1 g$$

Then get ~~$O(\epsilon)$~~

Claim All terms $O(\epsilon)$ $\quad \rho_1 g, w p_0', w p_0'$

Assume ~~$\rho_1 = O(\epsilon \rho_0)$~~

Thus $\rho_1 g = O(\epsilon \rho_0 g)$ is ~~$O(\epsilon)$~~

$p_{1z} = O\left(\frac{\epsilon p_0}{\lambda}\right)$

$\rho_0 w_t =$ [illegible] $O(\epsilon)$ ~~A w~~ is

$$O\left(\frac{\rho_0 \epsilon}{a_0/\lambda}\right) \sim O\left(\frac{\rho_0 \epsilon}{\lambda a_0}\right)$$

eqs of 6.59 are almost eqs 6.53, 6.54. 9

but canceling eqlbrium value we get

$$\partial_t \rho_1 + \underline{\underline{w \partial_z \rho_0}} + \rho_0 \partial_z w = 0$$

not in 6.53, 54

$$\rho_0 \partial_t W + \partial_z P_1 = - \underline{\underline{\rho_1}} g$$

not in 6.53, 54

get what 3rd eq in 6.59 should be

$$+ \frac{Dp}{Dt} - a^2 \frac{D\rho}{Dt} = 0 \Rightarrow \frac{D(p_0 + p_1)}{Dt} - a_0^2 \frac{D(\rho_0 + \rho_1)}{Dt} = 0$$

$$\Rightarrow \partial_t(P_1) + \cancelto{0}{(v_0 \cdot \nabla)} P_0 - a_0^2 \left(\partial_t \rho_1 + \cancelto{0}{(v_0 \cdot \nabla)} \rho_0 \right) = 0$$

$$\partial_t P_1 - a_0^2 \partial_t \rho_1 = 0$$

Thus, 3rd eq 6.59

$$\partial_t P_1 + \underline{\underline{W \partial_z P_0}} - a_0^2 (\partial_t \rho_1 + \underline{\underline{W \partial_z \rho_0}}) = 0$$

not there not there

thus want to show $O(w) =$

$$O(\partial_t \rho_1) = \frac{O(\epsilon \rho_0)}{(\lambda/a_0)} = O\left(\frac{\epsilon \rho_0}{\lambda/a_0}\right) \quad (1)$$

$$O(\rho_0 \partial_z w) = O\left(\frac{\rho_0 \epsilon}{\lambda}\right) \quad (2)$$

$$O(w \partial_z \rho_0) = O\left(\frac{\epsilon \rho_0}{L}\right) \quad (3) \quad \therefore \quad \frac{(3)}{(1)} \sim O\left(\frac{1/L}{\lambda/a_0}\right) \sim O\left(\frac{a_0}{\lambda L}\right)$$

want to show $\frac{(3)}{(1)}$ small.

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~~$\rho_1 g$, $w\rho_0'$, $w\rho_0'$~~

$$\rho_{1t} + w\rho_0' + \rho_0 w_z = 0 \longrightarrow \rho_{1zt} = -(w\rho_0')_z - (\rho_0 w_z)_z$$

$$\rho_0 w_t + p_{1z} = -\rho_1 g$$

$$p_{1t} + w\underline{p_0'} - a_0^2(\rho_{1t} + w\rho_0') = 0$$

∂t

$$\rho_0 w_{tt} + p_{1zt} = -\rho_{1t} g$$

∂z

$$p_{1zt} + w_z\underline{p_0'} + w\underline{p_0''} - a_0^2(\rho_{1zt} + w_z\rho_0' + w\rho_0'') = 0$$

$$\rho_0 w_{tt} + a_0^2\left(-(w\cancel{\rho_0'})_z - (\rho_0 w_z)_z + (\cancel{w\rho_0'})_z\right) - (w\underline{p_0'})_z$$

$$= -\rho_{1t} g$$

$$\rho_0 w_t + P_{1z} = -\rho_1 g$$

∂_t

$$\Rightarrow \rho_0 w_{tt} + P_{1zt} = -\rho_{1t} g \qquad *$$

$+$

$$P_{1t} + wP_0' - a_0^2(\rho_{1t} + w\rho_0') = 0$$

$\partial_z \rightarrow$ $\qquad **$

$$P_{1zt} + (wP_0')_z - (a_0^2)_z(\rho_{1t} + w\rho_0') - a_0^2(\rho_{1tz} + (w\rho_0')_z) = 0$$

Putting $(**)$ in $*$ we get

$$\rho_0 w_{tt} + (a_0^2)_z(\rho_{1t} + w\rho_0') + a_0^2(\rho_{1tz} + (w\rho_0')_z) - (wP_0')_z = -\rho_{1t} g$$

Now ρ_{1tz} from Mass eq $\quad \rho_{1t} + w\rho_0' + \rho_0 w_z = 0$

$$\rho_{1tz} + (w\rho_0')_z + (\rho_0 w_z)_z = 0$$

$$\omega_{tt} = a_0^2 \frac{(\rho_0 \omega_z)_z}{\rho_0} + \frac{(\omega \underline{P}_0')_z}{\rho_0} - \frac{\rho_{1t} g}{\rho_0}$$

$$= a_0^2 \omega_{zz} + \frac{a_0^2 \rho_0' \omega_z + (\omega \underline{P}_0')_z - \rho_{1t} g}{\rho_0}$$

$\therefore$

$$\rho_0 \omega_{tt} + a_0^2\left(-\cancel{(\omega\rho_0')_z} - (\rho_0\omega_z)_z + \cancel{(\omega\rho_0')_z}\right)$$

$$+ (a_0^2)_z\left(\rho_{1t} + \cancel{\omega\rho_0'}\right) - (\omega\rho_0')_z = -\rho_{1t}\, g$$

$$\shortparallel$$

$$-\cancel{\omega\rho_0'} - \rho_0\omega_z$$

$$= \rho_0\omega_{tt} - a_0^2\left(\rho_0\omega_{zz} + \rho_{0z}\,\omega_z\right) + (a_0^2)_z\left(-\rho_0\omega_z\right) - (\omega\rho_0')_z$$

$$= -\rho_{1t}\, g$$

$$= \omega_{tt} - a_0^2\,\omega_{zz} - a_0^2\,\frac{\rho_{0z}\omega_z}{\rho_0} - \omega_z (a_0^2)_z - \frac{\omega_z\rho_0'}{\rho_0} - \omega\frac{\rho_0''}{\rho_0}$$

$\searrow \; \swarrow$ two combine to give

$$-\frac{(a_0^2\rho_0)'\,\omega_z}{\rho_0} = -\frac{\rho_{1t}\, g}{\rho_0}$$

$$\therefore \quad w_{tt} = a_0^2 w_{zz} + \frac{(a_0^2 \rho_0)' w_z}{\rho_0} + \frac{w_z P_0' + w P_0'' \cdot - \rho_{1t} g}{\rho_0}$$

$$+ \; (w P_0')_z - \rho_{1t} g \quad P_0' = \rho_0 g \Big]$$

$$(w P_0')_z = (w \rho_0 g)_z$$

$$= w_z \rho_0 g + w \rho_{0z} g \overset{?}{=} -g \rho_{1t}$$

$$\therefore \quad w_{tt} = a_0^2 w_{zz} + \frac{(a_0^2 \rho_0)' w_z}{\rho_0}$$

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Isothermal: a_0^2 const

~~Tuesday~~ : ~~10:00~~
Wed 1:00 - 1:30

? $P = \rho RT$? why not $a_0^2 = RT$?

$a_0^2 = \frac{dP}{d\rho} = \gamma RT$ const if isentropic

$$\frac{dp_0}{dz} = -\rho_0 g$$

$$\frac{d\rho_0 RT}{dz} = -\rho_0 g$$

why γ ? only if also isentropic

Assuming isentropic flow.

$$\frac{d\rho_0}{\rho_0} = -\frac{g}{RT_0}\,dz$$

$$\Rightarrow \rho_0(z) = \rho_0(0)\,e^{-z/H}$$

$\cancel{\frac{RT_0}{g}} \in \cancel{\frac{a_0^2}{\gamma g}}$

$$w/ \quad H = \frac{RT_0}{g} = \frac{a_0^2}{\gamma g}.$$

If $z \ll H$ $\quad \rho_0(z) \simeq \rho_0(0)\left[1 + O\left(-\frac{z}{H}\right) + \dots\right]$

$$\frac{\omega^2}{a_0^2} = k^2 + \frac{1}{4}\frac{1}{H^2} \qquad k = \frac{2\pi}{\lambda}$$

$a_0 =$

$$\frac{\omega^2}{a_0^2 k^2} = 1 + \frac{1}{4}\frac{1}{H^2 k^2}$$

$$= 1 + \frac{1}{4}\frac{1}{(2\pi)^2\left(\frac{H}{\lambda}\right)^2} = 1 + \frac{1}{4}\frac{1}{(2\pi)^2}\left(\frac{\lambda}{H}\right)^2$$

if $\frac{\lambda}{H} \ll 1$ $\qquad \frac{\omega^2}{a_0^2 k^2} \simeq 1$ $\quad$ result

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$$\frac{1}{\rho_0}\partial_z P_0 = \frac{1}{\rho_0}\frac{dP_0}{d\rho_0}\cdot\frac{d\rho_0}{dz} \quad \text{(By chain rule)}$$

$$= \frac{a_0^2}{\rho_0}\frac{d\rho_0}{dz}$$

But ~~$P_0 = \rho_0^r C_1$~~ $\therefore$ ~~$\rho_0 = C_2 P_0^{1/r}$~~ I want $\rho_0 = \rho_0(a_0^2)$

~~$\frac{d\rho_0}{dz} = \frac{C_2}{r}\frac{P_0^{1/r}}{P_0}\frac{dP_0}{dz} = \frac{\rho_0}{r P_0}\frac{dP_0}{dz}$~~

As $P_0 = \rho_0^r C_1$ $\quad \rho_0 = C_2 P_0^{1/r}$ $\quad r\frac{P_0}{\rho_0} = a_0^2$

$$\therefore \rho_0 = C_2\left(\frac{a_0^2\rho_0}{r}\right)^{1/r} = \frac{C_2}{r^{1/r}} a_0^{2/r}\rho_0^{1/r} = C_3 a_0^{2/r}\rho_0^{1/r}$$

$$\therefore \rho_0^{\frac{r-1}{r}} = C_3 a_0^{2/r}$$

$$\log \rho_0 = \frac{r}{r-1}\cdot\frac{2}{r}\log a_0 + C_4 = \frac{2}{r-1}\log a_0 + C_4$$

$p_0 + p_1 = C\rho_0^r\left(1 + r\frac{\rho_1}{\rho_0} + \cdots\right)$

$p_0 = C\rho_0^r$ ∴ $p_1 = C\rho_0^{r-1} r \rho_1$

$\frac{dp_0}{dz} = rC\frac{d\rho_0}{dz}\rho_0^{r-1}$

$=$

$\frac{d\rho_0}{dz} = d$

$\frac{dp_0}{d\rho_0} \cdot \frac{d\rho_0}{dz}$

$\parallel$

a_0^2

$a_0^2 = \frac{dp}{d\rho} = \frac{dp_0}{dz} \cdot \frac{dz}{d\rho}$

$\cancel{\frac{dp_0}{dz}} = 2a_0\frac{da_0}{dz} = \frac{dp}{dz\,d\rho}$

$= \frac{d}{d\rho}\left(\frac{dp}{dz}\right)$

$= \frac{d}{d\rho}\left(-g\rho\right)$

$-g$

Thus $\frac{1}{\rho_0}\frac{d\rho_0}{dz} = \frac{d}{dz}\log\rho_0 = \frac{d}{dz}\left[\frac{2}{\gamma-1}\log a_0\right]$

$$= \frac{2}{\gamma-1}\frac{1}{a_0}\frac{da_0}{dz}$$

$\therefore \quad \frac{a_0^2}{\rho_0}\frac{d\rho_0}{dz} = \frac{2}{\gamma-1}a_0\frac{da_0}{dz}.$

As $\partial_z p_0 = -g\rho_0$

$\frac{1}{\rho_0}\partial_z p_0 = -g$

& we have cycle of equality.

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$$\frac{1}{\rho_0}\frac{dp_0}{dz} = -\frac{\cancel{\rho_0} g}{\rho_0} = -g$$

But $a_0^2 = \frac{\gamma p_0}{\rho_0} \Rightarrow \frac{d}{dz}(a_0^2\rho_0) = \gamma \partial_z p_0 = -\gamma \rho_0 g$

$$\therefore \quad -g = \frac{1}{\gamma\rho_0}\partial_z(a_0^2\rho_0)$$

$$= \frac{\cancel{\rho_0}\, 2 a_0 \partial_z a_0}{\gamma \cancel{\rho_0}} + \frac{a_0^2 \partial_z \rho_0}{\gamma\rho_0}$$

$$= \frac{2}{\gamma} a_0 \partial_z a_0$$

$$\frac{dp_0}{dz} = \frac{1}{\gamma}\Big[\partial_z$$

But $\gamma p_0 =$

$$\gamma\frac{p_0}{\rho_0} = a_0^2$$

$$p_0 = \frac{\rho_0 a_0^2}{\gamma}$$

$$2 a_0 \, da_0 = (r-1)(-g) \, dz$$

$$a_0^2(z) = a_0^2(0) - (r-1) g z .$$

freq

$$\approx O(\theta) \Rightarrow O(z) \simeq \frac{a_0^2(0)}{(r-1)g}$$

$$W = Ae^{z/2H}\cos(kz-\omega t)$$

$$W_{tt} = -(-\omega)^2 w. \qquad W$$

$$W_{zz} = -k^2 W.$$

Plugging in one gets (Done on Ma.)

$$\frac{1}{\rho_0}\frac{dp_0}{dz} = \frac{1}{\rho_0}a_0^2\frac{d\rho_0}{dz} \qquad \left(\frac{Dp}{Dt} = a^2\frac{D\rho}{Dt}\right)$$

But also $p \propto \rho^r$

$$(p_0+p_1+\dots) = c(\rho_0+\rho_1+\dots)^r$$

$$= \text{[illegible]} + c\rho_0^r\left(1+\frac{\rho_1}{\rho_0}+\dots\right)^r$$

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1

know $p(\rho, S) = P$ fn

can invert to get $S = S(P, \rho)$ ~~under~~ when can we do this??

$\therefore \quad \partial_t S = \partial_P S \cdot \partial_t P + \partial_\rho S \, \partial_t \rho$

$\partial_x S = \partial_P S \, \partial_x P + \partial_\rho S \, \partial_x \rho$

$$P_0 = P(\rho_0, S_0)$$

$$dP = \frac{\partial P}{\partial \rho} d\rho + \frac{\partial P}{\partial S} dS$$

put in 6.62 ÷ by $\partial_P S$

$$\partial_t P + u \partial_x P + \left(\frac{\partial_\rho S}{\partial_P S} \right) \left(\partial_t \rho + u \partial_x \rho \right) = 0$$

|| show $-a^2$ ≠

$$\frac{\left.\frac{\partial S}{\partial \rho}\right|_P}{\left.\frac{\partial S}{\partial P}\right|_\rho} =$$

Maxwell relation:

want to show

$$\frac{1}{\rho_0}\frac{d\rho_0}{dz} = -g$$

$$\frac{1}{\rho_0}\Big($$

$$2a_0\frac{da_0}{dz} = -g(\gamma-1)$$

$$a_0^2(z) = a_0^2(0) - (\gamma-1)gz.$$

$$\rho_0 =$$

$$\frac{da_0^2}{dz} = P''(\rho_0)\frac{d\rho_0}{dz}$$

$$2a_0\frac{da_0}{dz} =$$

$$\frac{Dp}{Dt} - a_0^2(z)\frac{D\rho}{Dt} = 0$$

1st ord

$$\Rightarrow \frac{\partial p_0}{\partial t} - a_0^2(z)\frac{\partial \rho_0}{\partial t} = 0.$$

1:30 - 3:30

$$\frac{\left.\frac{\partial S}{\partial \rho}\right|_P}{\left.\frac{\partial S}{\partial P}\right|_\rho} = - \left.\frac{dP}{d\rho}\right|_S$$

Constant S

$$\Rightarrow \quad dS = 0 = \left.\frac{\partial S}{\partial P}\right|_\rho dP + \left.\frac{\partial S}{\partial \rho}\right|_P d\rho$$

$$\Rightarrow \quad \cancel{\frac{\left.\frac{\partial S}{\partial P}\right|_\rho}{\left.\frac{\partial S}{\partial \rho}\right|_P}} \quad \frac{\left.\frac{\partial S}{\partial \rho}\right|_P}{\left.\frac{\partial S}{\partial P}\right|_\rho} = -\left.\frac{dP}{d\rho}\right|_S \quad \checkmark$$

yea!!

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D

$l_1 = l_2$

$$\Rightarrow p_t + u p_x - a^2(\rho_t + u \rho_x) = 0 \qquad \Leftrightarrow 6.63.$$

$l_2 \neq 0$

$l_1 = a^2 \qquad l_2^2 = l_1 = a^2 \qquad l_2 = \pm a$

$$p_t + (u \pm a) p_x \pm \rho a (u_t + u u_x) + \rho a^2 u_x$$

$$+ 0 = 0$$

$\rho a \{ u_t + u u_x$

$\pm a u_x$

$$p_t + (u \pm a) p_x \pm \rho a \{ u_t + (u \pm a) u_x \}$$

$$= 0$$

$$C_+ : \frac{dx}{dt} = u + a \qquad \frac{dp}{dt} + \rho a \frac{du}{dt} = 0$$

$$C_- : \frac{dx}{dt} = u - a \qquad \frac{dp}{dt} - \rho a \frac{du}{dt} = 0$$

$$\frac{dS}{dt} = 0 \qquad P : \frac{dx}{dt} = u.$$

$\rho_0, u_0 = 0,$

$\frac{dp}{dt} + \rho_0 a_0 \frac{du}{dt} = 0$ on $C_+ : \frac{dx}{dt} = a_0$

$\frac{dp}{dt} - \rho_0 a_0 \frac{du}{dt} = 0$ on $C_- : \frac{dx}{dt} = -a_0$

$\frac{ds}{dt} = 0$ on $P : \frac{dx}{dt} = 0$

$$(p-p_0) + \rho_0 a_0 (u - 0) = F(x - a_0 t)$$

$$(p-p_0) - \rho_0 a_0 u = G(x + a_0 t)$$

$$S - S_0 = H(x)$$

Adding $2(p-p_0) = F(x-a_0t) + G(x+a_0t)$

Subtracting $2\rho_0 a_0 u = F(x-a_0t) - G(x+a_0t)$

$$u = \frac{F(x-a_0t)}{2\rho_0 a_0} - \frac{G(x+a_0t)}{2\rho_0 a_0}$$

& $\frac{p-p_0}{\rho_0 a_0} = \frac{F(x-a_0t)}{2\rho_0 a_0} + \frac{G(x+a_0t)}{2\rho_0 a_0}$

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l_1 6.60 $\quad l_1\rho_t + l_1 \underset{\sim}{u}\rho_x + l_1 \underline{\underline{\rho}} u_x = 0$

l_2 6.61 $\quad l_2 \underline{\rho u_t} + l_2 \underline{\underline{\rho u u_x}} + l_2 \underline{p_x} = 0$

ΔEE 6.63 $\quad \underline{p_t} + \underline{u p_x} - a^2(\rho_t + u\rho_x) = 0$

$$p_t + (u + l_2)p_x + l_2\rho(\underline{\underline{u_t}} + u u_x) + l_1 \underline{\underline{\rho}} u_x + l_1 \underset{\sim}{u}\rho_x - a^2(\rho_t + u\rho_x)$$
$$+ l_1\rho_t = 0$$

$$\Downarrow$$

$$(l_1 - a^2)(\rho_t + u\rho_x) = 0. \checkmark$$

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$$\frac{dp}{dt} + \rho a \frac{du}{dt} = 0 \Rightarrow a^2\frac{d\rho}{dt} + \rho a \frac{du}{dt} = 0$$

$$\|$$

$$\frac{a}{\rho}\frac{d\rho}{dt} + \frac{du}{dt} = 0$$

$$dt \to \quad \frac{a}{\rho}d\rho + du = 0$$

2

$$\int \frac{a(\rho)\,d\rho}{\rho} + v = \text{const}$$

$p = k\rho^{r}$

$\log p = r\log\rho + \log k \quad \frac{r-1}{2}$

$\frac{1}{p}\frac{dp}{d\rho} = \frac{r}{\rho}$

$\frac{dp}{d\rho} = \frac{r}{\rho}p = rk\rho^{r-1}$

$\frac{d\rho}{\rho} = d(\log\rho) = d\left(\frac{2}{r-1}\log a\right)$

$= \frac{2}{r-1}\frac{da}{a}$

$\int a\frac{d\rho}{\rho} = \frac{2}{r-1}\int da = \frac{2}{r-1}a$.

1) Show
2) get [illegible]
3)

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$2\log a = (r-1)\log\rho + \log kr$

$\frac{2}{r-1}\log a = \log\rho + \frac{\log kr}{r-1}$