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from 6.90 get 6.91

Cons Mass $\Rightarrow$ $\rho_t + u\rho_x + \rho u_x = 0$ 6.60

$S_t + u S_x = 0$ 6.62

Mult bottom by ρ & top by S

$\Rightarrow$ $S\rho_t + uS\rho_x + S\rho u_x = 0$

$\rho S_t + \rho u S_x = 0$

Add $(\rho S)_t + \underbrace{(uS\rho_x + S\rho u_x + \rho u S_x)} = 0$

$(\rho S)_t + (\rho u S)_x = 0$

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$$\partial_t \rho + \partial_{x_j}(\rho u_j) = 0 \qquad 6.7$$

$$\partial_t(\rho u) + \partial_{x_j}(\rho u_i u_j - p_{ji}) = 0 \qquad 6.8$$

$$\partial_t\left(\tfrac{1}{2}\rho u_i^2 + \rho e\right) + \partial_{x_j}\left\{\left(\tfrac{1}{2}\rho u_i^2 + \rho e\right)u_j - p_{ji}u_i + q_j\right\} = \rho F_i u_i \qquad 6.9$$

$$T\,dS = de + p\,d(1/\rho) \qquad 6.31$$

~~[illegible]~~ 1D..

$$\partial_t \rho + \partial_x(\rho u) = 0$$

$$\partial_t(\rho u) + \partial_x(\rho u^2 - p_{11}) = 0$$

$$\partial_t\left(\tfrac{1}{2}\rho u^2 + \rho e\right) + \partial_x\left\{\left(\tfrac{1}{2}\rho u^2 + \rho e\right)u - p_{11}u + q_1\right\} = 0$$

$$T\,dS = de + p\,d(1/\rho)$$

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$$\rho_t + (V + \rho V')\rho_x = 0$$

$$\rho_t + V\rho_x + \frac{a^2}{\rho V'}\rho_x = 0$$

try $\rho V' = \frac{a^2}{\rho V'}$

$$V \pm a = -\frac{\rho_t}{\rho_x}$$

$$V = -\left(a + \frac{\rho_t}{\rho_x}\right)$$

$$p = p_0\left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

$$a^2 = \gamma \frac{p}{\rho} = \gamma \frac{1}{\rho} p_0 \left(\frac{\rho}{\rho_0}\right)^{\gamma}$$

Derive eq (6.92)

6.91 $\Rightarrow$ $(\rho S)_t + (\rho u S)_x =$

$$\cancel{\rho_t S} + \rho S_t + \cancel{(\rho u)_x S} + \rho u S_x = 0$$

$$\rho(S_t + u S_x) = 0$$

$$\rho \frac{DS}{Dt} = \frac{\rho}{T}\left(\frac{De}{Dt} + p\frac{D(1/\rho)}{Dt}\right) \quad \text{by 6.31}$$

~~energy w/ 1st two eq can be written~~ $\cancel{\frac{De}{Dt} = \frac{1}{\rho^2}}$

6.8 $\Rightarrow$ $\cancel{\partial_t \rho}\, u + \rho \partial_t u + \cancel{\partial_x(\rho u)}\, u + \partial_x u (\rho u)$

$$- \partial_x P_{11} = 0. \quad \text{/ by cons mass.}$$

$$\Rightarrow \partial_t u + u \partial_x u = \frac{\partial_x P_{11}}{\rho} \quad \text{6.8' cons of mmt.}$$

$$\Rightarrow \frac{Du}{Dt} = \frac{\partial_x P_{11}}{\rho}$$

6.9 $\Rightarrow$ Add $= \partial_x P_{11} \frac{1}{2}u$ Add $=0$ By Mass

$$\frac{1}{2}u\,\underline{\partial_t(\rho u)} + \frac{1}{2}\rho u\,\partial_t u + e\,\underline{\partial_t \rho} + \rho\,\partial_t e \quad \text{Add}$$

$$+ \frac{1}{2}u\,\underline{\partial_x(\rho u^2)} + \frac{1}{2}\rho u^2\,\partial_x u \;\; \text{Add} \;\; + e\,\underline{\partial_x(u\rho)} + u\rho\,\partial_x e$$

$$- P_{11}\partial_x u - u\,\partial_x P_{11} + \partial_x q_1 = 0$$

$\therefore$ gives

$$\rho\frac{De}{Dt} + \frac{1}{2}\rho u\frac{Du}{Dt} + \frac{1}{2}\cancel{u\partial_x} P_{11} = P_{11}\partial_x u + \cancel{u\partial_x} P_{11} - \partial_x q_1$$

|| By mmt eq (page before)

$$\frac{1}{2}\cancel{u\partial_x} P_{11}$$

$$\therefore \quad \frac{De}{Dt} = \frac{1}{\rho}\left(P_{11}\partial_x u - \partial_x q_1\right)$$

$$\therefore \quad \rho \frac{DS}{Dt} = \frac{P}{T}\left(\frac{1}{\rho}\left(p_{11}\partial_x u - \partial_x q_1\right) + P\left(-\frac{1}{\rho^2}\right)\underbrace{\frac{D\rho}{Dt}}_{\overset{\text{Mass}}{=} -\rho \partial_x u} \right)$$

$$= \frac{1}{T}\left(p_{11}\partial_x u - \partial_x q_1 + \frac{p}{\cancel{\rho}}\left(\cancel{\rho}\, \partial_x u\right)\right)$$

$$(\rho S)_t + (\rho u S)_x = \frac{1}{T}\left((p_{11} + p)\partial_x u - \partial_x q_1\right) \qquad \text{eq } 6.92.$$

$$\int_{x_2}^{x_1} \longrightarrow \qquad x_2 < x < x_1$$

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$$\frac{\partial f}{\partial t} + \frac{\partial g}{\partial x} + h = 0 \qquad \text{fns of } (p, \rho, u)$$

$$\partial_t f = \partial_p f \, \partial_t p + \partial_\rho f \, \rho_t + \partial_u f \, u_t$$

$$= \partial_p f \left[-u p_x + a^2 (\rho_t + u \rho_x) \right] + \partial_\rho f \left[-u \rho_x - \rho u_x \right] + \partial_u f \left[\frac{-p_x}{\rho} - u u_x \right]$$

$$= \partial_p f \left[-u p_x + a^2 (-\rho u_x) \right] + \partial_\rho f \left[-u \rho_x - \rho u_x \right] + \partial_u f \left[\frac{-p_x}{\rho} - u u_x \right]$$

$$\partial_x g = \partial_p g \, p_x + \partial_\rho g \, \rho_x + \partial_u g \, u_x$$

<u>Add</u>:

$$\rho_x \left[-u \partial_\rho f + \partial_\rho g \right] + p_x \left[-u \partial_p f - \frac{\partial_u f}{\rho} + \partial_p g \right]$$

$$+ u_x \left[-a^2 \rho \, \partial_p f - \rho \partial_\rho f - u \partial_u f + g_u \right] + h = 0$$

$$t\left(\frac{1}{2t}\int_0^t f[-t-x+2\phi,\tau]\,B'[t+x-\phi]\,d\phi-2f^{(0,1)}[t+x,\tau]+3f[t+x,\tau]\,f^{(1,0)}[t+x,\tau]-\frac{1}{t}\left(\int_0^t f[-t-x+2\phi,\tau]\,d\phi\right)f^{(1,0)}[t+x,\tau]\right)$$

$$\text{eqStar2}=-2f^{(0,1)}[t+x,\tau]+3f[t+x,\tau]\,f^{(1,0)}[t+x,\tau]-\frac{1}{t}\left(\int_0^t f[-t-x+2\phi,\tau]\,d\phi\right)f^{(1,0)}[t+x,\tau]+\frac{1}{2t}\int_0^t f[-t-x+2\phi,\tau]\,B'[t+x-\phi]\,d\phi==0$$

This is different by a sign in the last term...? I don't see how to recongsile the two results...

$$\text{eqStar1}=-2f^{(0,1)}[t-x,\tau]+3f[t-x,\tau]\,f^{(1,0)}[t-x,\tau]-\frac{1}{t}\left(\int_0^t f[-t+x+2\phi,\tau]\,d\phi\right)f^{(1,0)}[t-x,\tau]-\frac{1}{2t}\int_0^t f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi==0$$

To simplify the above I will assume periodicity in f[,τ] and B[ ]. B (and B') should be periodic of period 1 (Real lenght of the interval) and f[,τ] should be periodic of lenght 2 (length of folded interval) But f[+2 ϕ,τ] should be periodic of period 1, because of the 2. Then the first integral term becomes...

ASK! Think should be zero... How know?

$$-\frac{1}{t}\left(\int_0^t f[-t+x+2\phi,\tau]\,d\phi\right)=$$

$$-\frac{1}{t}\left(\int_0^{2t} f[-t+x+\phi,\tau]\,\frac{d\phi}{2}\right)=-\frac{1}{2t}\left(\int_0^{2t} f[-t+x+\phi,\tau]\,d\phi\right)=-\frac{1}{2t}\left(\int_{x-t}^{x+t} f[\phi,\tau]\,d\phi\right)$$

The second integral becomes...

$$\frac{1}{2t}\int_0^t f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi=\frac{1}{2t}\left(\int_0^1 f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi+\int_1^2 f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi+\ldots+\int_{m-1}^m f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi+\int_m^t f[-t+x+2\phi,\tau]\,B'[-t+x+\phi]\,d\phi\right)$$

$g_\rho - u f_\rho = 0$ (1)

$g_p - u f_p - \frac{1}{\rho} f_u = 0$ (2)

$g_u - u f_u - \rho f_\rho - \rho a^2 f_p = 0$ (3) How solve?

get one eq ~~for~~ f only... elliminate g w/ $g_{\rho p} = g_{p\rho}$

$g_{\rho u} = g_{u\rho}$

$g_{pu} = g_{up}$

1st eq $g_{\rho\rho} = g_{\rho\rho}$ gives no new information? $g_{\rho u} = g_{u\rho}$

$\partial_u (1) \Rightarrow g_{\rho u} - f_\rho - u f_{\rho u} = 0$

$\partial_\rho (3) \Rightarrow g_{u\rho} - u f_{u\rho} - f_\rho - \rho f_{\rho\rho} -$

pg ~~172~~ 174 ω

$[x] = x_2 - x_1$

$-U[\rho] + [\rho u] = 0$

$\Rightarrow -U(\rho_2 - \rho_1) + (\rho_2 u_2 - \rho_1 u_1) = 0$

$\Rightarrow -(\rho_2 \underbrace{(U - u_2)}_{v_2} - \rho_1 \underbrace{(U - u_1)}_{v_1}) = 0$

$\Rightarrow [\rho v] = 0$

2nd eq...

$-U[\rho u] + [\rho u^2 + p] = 0$

$-U(\rho_2 u_2 - \rho_1 u_1) + (\rho_2 u_2^2 - \rho_1 u_1^2 + p_2 - p_1) = 0$

~~see [illegible] pg 285. (photo copied & stuck in)~~

~~gets as that $[\rho u^2 + p]$~~ $= 0.$

$\Rightarrow \cancel{U^2 + 2U\rho_2 u_2 - 2U}\rho_1 u_1$

$(\bar{U}-v_2)^2$

$$\rho_2 \bar{U}^2 - 2\bar{U}\rho_2 v_2 + \rho_2 v_2^2 - \rho_2 \bar{U}^2 + \bar{U}\rho_2 v_2$$

$$-\left(\rho_1 \bar{U}^2 - 2\bar{U}\rho_1 v_1 + \rho_1 v_1^2\right) + \rho_1 \bar{U}^2 - \bar{U}\rho_1 v_1$$

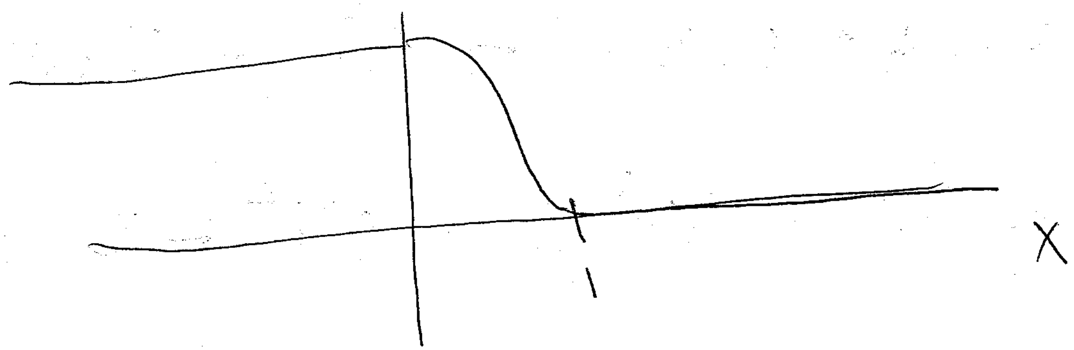
$$+ P_2 - P_1 = 0.$$

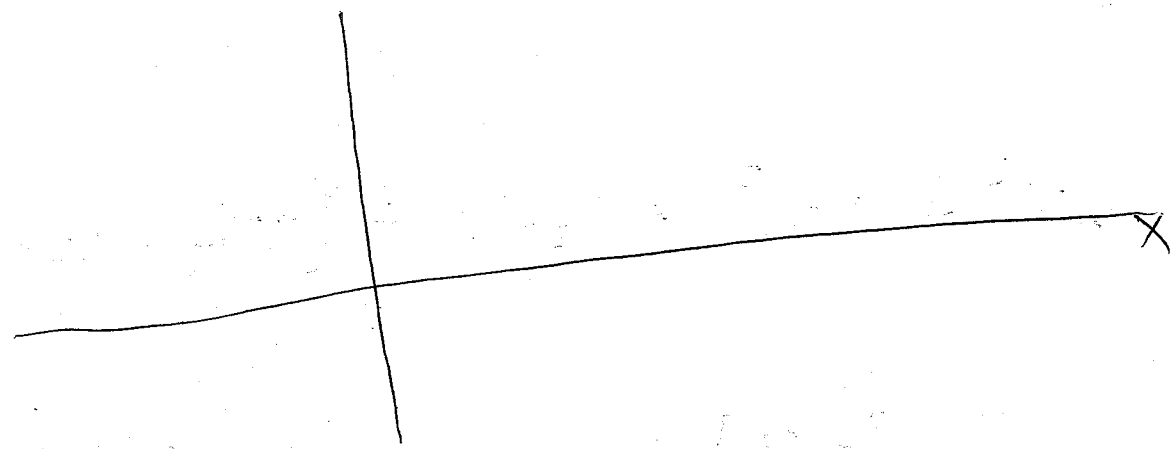
$$\Rightarrow \quad \rho_2 v_2^2 - \rho_1 v_1^2 + P_2 - P_1 - \rho_2 \bar{U}(v_2) + \rho_1 \bar{U} v_1 = 0$$

$$[\rho v^2 + P] - \bar{U}\underbrace{[\rho v]}_{=0} = 0 \quad \text{2nd eq}$$

$$\therefore \quad [\rho v^2 + P] = 0.$$

$u(x,0) = F(\xi)$





Char $\quad x = \xi + F(\xi)t$

Thus $\quad$ char $\quad x = \xi + t \qquad \xi < 0$

$x = \xi + \cos^2\left(\frac{\pi\xi}{2}\right)t \qquad 0 < \xi < 1$

$x = \xi \qquad \xi > 1$

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$$-U\left[\tfrac{1}{2}\rho u^2+\rho e\right]+\left[\left(\tfrac{1}{2}\rho u^2+\rho e\right)u+Pu\right]=0 \qquad h\equiv e+\frac{P}{\rho}$$

$$\Rightarrow -U\left(\tfrac{1}{2}\rho_2 u_2^2+\rho_2 e_2-\tfrac{1}{2}\rho_1 u_1^2-\rho_1 e_1\right)$$

$$+\left(\tfrac{1}{2}\rho_2 u_2^2+\rho_2 e_2\right)u_2+P_2u_2-\left(\tfrac{1}{2}\rho_1 u_1^2+e_1\rho_1\right)u_1-P_1u_1=0$$

w/ 2 other jump conditions

$$-U\left(\rho_2 u_2-\rho_1 u_1\right)+\left(\rho_2 u_2^2+P_2-\rho_1 u_1^2-P_1\right)=0$$

$$-U\left(\rho_2-\rho_1\right)+\left(\rho_2 u_2-\rho_1 u_1\right)=0.$$

$$\Rightarrow \left(\tfrac{1}{2}\rho_2 u_2^2+\rho_2 e_2\right)\left(u_2-U\right)-\left(\tfrac{1}{2}\rho_1 u_1^2+\rho_1 e_1\right)\left(u_1-U\right)-P_1u_1+P_2u_2=0$$

$$\Rightarrow -\left(\tfrac{1}{2}\rho_2 u_2^2+\rho_2 e_2\right)v_2+\left(\tfrac{1}{2}\rho_1 u_1^2+\rho_1 e_1\right)v_1+P_2u_2-P_1u_1=0$$

$$h_1 = e_1 + \frac{P_1}{\rho_1}$$

$$\rho_1 h_1 = \rho_1 e_1 + P_1$$

$\therefore$ for υ's in Above put $\upsilon_i - U + U$

$$\Rightarrow \; -\rho_2 v_2\left(\tfrac{1}{2}\upsilon_2^2\right) + \rho_1 v_1\left(\tfrac{1}{2}\upsilon_1^2\right) - \rho_2 v_2 e_2 + \rho_1 v_1 e_1$$

$$+ P_2(-v_2) + U P_2 - P_1(-v_1) - P_1 U = 0$$

$$\Rightarrow \; -\rho_2 v_2 \tfrac{1}{2}(v_2 - U)^2 + \rho_1 v_1 \tfrac{1}{2}(v_1 - U)^2 - (\rho_2 v_2 e_2 + v_2 P_2)$$

$$+ (\rho_1 v_1 e_1 + v_1 P_1) + U P_2 - U P_1 = 0$$

$$\Rightarrow \; -\rho_2 v_2\left(\tfrac{1}{2} v_2^2 + h_2\right) + \rho_1 v_1\left(\tfrac{1}{2} v_1^2 + h_1\right) + \rho_2 v_2^2 U - \frac{\rho_2 v_2 U^2}{2}$$

$$- \rho_1 v_1^2 U + \frac{\rho_1 v_1}{2} U^2$$

$$\bar{U}(P_2 - P_1) = 0$$

$$\Rightarrow \left[\rho v\left(\tfrac{1}{2}v^2 + h\right)\right]_{1-2} \cancel{+} - \left[\rho v^2\right]_{1-2}\bar{U} - \left[P\right]_{1-2}\bar{U} + \left[\frac{\rho v}{2}\right]_{1-2}\bar{U}^2 = 0$$

$$\Rightarrow \left[\rho v\left(\tfrac{1}{2}v^2 + h\right) - (P + \rho v^2)\bar{U} + \frac{\rho v}{2}\bar{U}^2\right] = 0$$

2 —— 1

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~~get jump relationship for~~

~~e_1~~ for polytropic gas simplify eq 6.95 – 6.97

6.95 same

6.96 sam

$$6.97 \Rightarrow \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}v_2^2 = \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2$$

$$\div\ 6.96 \text{ by } 6.95 \Rightarrow \frac{p_2}{\rho_2 v_2} + v_2 = \frac{p_1}{\rho_1 v_1} + v_1$$

Then ~~6.97 for polytropic gas becomes~~ ..

$\div$ ~~by v_2 both side.~~

$$\cancel{\frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2 v_2} + \frac{v_2}{2} = \frac{\gamma}{\gamma-1}\frac{p_1}{v_2\rho_1} + \frac{1}{2}\frac{v_1^2}{v_2}}$$

GAE

$\cancel{U_1 = a_1 M}$ $\boxed{\bar{U} - a_1 M = U_1}$

$\Rightarrow$ $\;_1 = Y_1 = a_1 M.$

<u>Now:</u> $a_i^2 = \gamma \frac{p_i}{\rho_i} \quad \therefore \quad \frac{p_i}{\rho_i} = \frac{a_i^2}{\gamma}.$

$\Rightarrow$ eqs become $\rho_2 v_2 = \rho_1 v_1$

$$\frac{p_2}{\rho_2 v_2} + v_2 = \frac{p_1}{\rho_1 v_1} + v_1 \Rightarrow \frac{a_2^2}{\gamma v_2} + v_2 = \frac{a_1^2}{\gamma v_1} + v_1$$

& $$\frac{1}{\gamma-1} a_2^2 + \frac{v_2^2}{2} = \frac{1}{\gamma-1} a_1^2 + \frac{v_1^2}{2}.$$

Thus want $\frac{u_2 - u_1}{a_1} = \frac{v_1 - v_2}{a_1} = f(v_1)$ If I can get $v_2 = g(v_1)$

I am done. Thus I have 2 eqs $\frac{a_2^2}{\gamma v_2} + v_2 = \frac{a_1^2}{\gamma v_1} + v_1 = R1$

& $\frac{1}{\gamma-1} a_2^2 + \frac{v_2^2}{2} = \frac{1}{\gamma-1} a_1^2 + \frac{v_1^2}{2} = R2$

& 2 unknowns a_2^2; v_2.

from 1st eq

$$a_2^2 = (R1 - v_2)\, r\, v_2$$

Then put in 2nd eq

$$\frac{1}{r-1}(R1 - v_2)\, r\, v_2 + \frac{v_2^2}{2} = R2$$

$$\Rightarrow \frac{r v_2 R1}{r-1} - \frac{r v_2^2}{r-1} + \frac{v_2^2}{2} = R2$$

$$\Rightarrow v_2^2\left[\frac{1}{2} - \frac{r}{r-1}\right] + \frac{rR1}{r-1} v_2 - R2 = 0.$$

$$\Rightarrow v_2^2\left[\frac{r-1-2r}{2(r-1)}\right] + \frac{rR_1}{r-1} v_2 - R_2 = 0.$$

Mult 1st eq by $\frac{v_2}{2}$

$\Rightarrow \frac{a_2^2}{2r} + \frac{v_2^2}{2} = r1 \cdot \frac{v_2}{2}$ & subtract from the 1st one

$\Rightarrow \left(\frac{1}{r-1} - \frac{1}{2r}\right) a_2^2 + 0 = r2 - r1 \frac{v_2}{2}$

$$-\frac{(r+1)}{2(r-1)} v_2^2 + \frac{r R_1}{r-1} v_2 - R_2 = 0$$

→ v_2 = See Ma outputs...

Then $\frac{u_2 - u_1}{a_1} = \frac{v_1 - v_2}{a_1} =$

$$a_1^4 - 2(2+r) a_1^2 v_1^2 + (2 - r^2) v_1^4$$

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$$h_1 = \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1}$$

$$6.97 \Rightarrow \frac{\gamma}{\gamma-1}\frac{p_1}{\rho_1} + \frac{1}{2}v_1^2 = \frac{\gamma}{\gamma-1}\frac{p_2}{\rho_2} + \frac{1}{2}v_2^2$$

Take eq 6.96 ÷ by 6.95

$$\frac{p_2}{\rho_2 v_2} + v_2 = \frac{p_1}{\rho_1 v_1} + v_1$$

$$\Rightarrow \quad v_2 - v_1 = \frac{p_1}{\rho_1 v_1} - \frac{p_2}{\rho_2 v_2}$$

Mult by $v_2 + v_1$

$$\Rightarrow \quad v_2^2 - v_1^2 = \frac{p_1 v_2}{\rho_1 v_1} - \frac{p_2}{\rho_2} + \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\frac{v_1}{v_2}$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}v_2^2 = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}v_1^2$$

$\rho_1 v_1 = \rho_2 v_2$

$\therefore \quad v_2^2 - v_1^2 = \frac{P_1}{\rho_2} - \frac{P_2}{\rho_2} + \frac{P_1}{\rho_1} - \frac{P_2}{\rho_1}$

$= (P_1 - P_2)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)$ put this in energy from above

then get.

$$\frac{1}{2}(P_1 - P_2)\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right) = \frac{\gamma}{\gamma-1}\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)$$

$$\frac{1}{2}\left(1 - \frac{P_2}{P_1}\right)\left(\frac{1}{1} + \frac{1}{\rho_2/\rho_1}\right) = \frac{\gamma}{\gamma-1}\left(1 - \frac{P_2/P_1}{\rho_2/\rho_1}\right)$$

Solving:
$$\frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \rho_2/\rho_1}$$

or $$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{P_2}{P_1}}{\frac{\gamma+1}{\gamma-1} + \frac{P_2}{P_1}} = \frac{V_1}{V_2}.$$

Energy eq gives ... continues on lines of Liepmann & Puckett pg 40 ...

$u_1 - u_2 =$

Also discussion in Currie pg 345-346.

Question can given $$\frac{P_2 - P_1}{P_1} = \frac{2\gamma}{\gamma+1}(M^2-1)$$

Can we get the other relationships?

$$\frac{P_2}{P_1} = 1 + \frac{2\gamma}{\gamma+1}(M^2-1)$$

Then $$\frac{\rho_2}{\rho_1} = \frac{\frac{\gamma-1+\gamma+1}{\gamma-1} + \frac{2\gamma}{\gamma-1}(M^2-1)}{\frac{\gamma-1+\gamma+1}{\gamma-1} + \frac{2\gamma}{\gamma+1}(M^2-1)} =$$

$$\frac{2\cancel{r} + 2rM^2 - \cancel{2r}}{2r + \frac{2r}{r+1}(r-1)(M^2-1)} = \frac{\cancel{2}rM^2}{\cancel{2}\cancel{r}\left(1 + \frac{r-1}{r+1}(M^2-1)\right)}$$

$$= \frac{M^2(r+1)}{r+1 + (r-1)(M^2-1)} = \frac{(r+1)M^2}{\cancel{r}+1 + rM^2 - \cancel{r} - M^2 + 1}$$

$$= \frac{(r+1)M^2}{(r-1)M^2+2} \quad \checkmark$$

know $\dfrac{V_1}{V_2} = \dfrac{(r+1)M^2}{(r-1)M^2+2} \rightarrow \dfrac{V_2}{a_1}\left(\dfrac{a_1}{V_1}\right) = \dfrac{(r-1)M^2+2}{(r+1)M^2}$, $\dfrac{a_1}{V_1} = \dfrac{1}{M}$

$$\frac{V_2 - V_1}{a_1} = \frac{V_2}{a_1} - M = \frac{(r-1)M^2+2}{(r+1)M^{\cancel{2}}}\cancel{M} - M$$

$$= \frac{(\cancel{r}-1)M^2 + 2 - (\cancel{r}+1)M^2}{(r+1)M}$$

5

$$= \frac{-2m^2+2}{(r+1)m} = \frac{2(1-m^2)}{(r+1)m}.$$

get diff sign.

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1

6.103 from inverting 6.101 $z = \frac{2r}{r+1}(M^2-1) \Rightarrow \left(\frac{r+1}{2r}z+1\right)^{1/2} = M$

then $\frac{u_2-u_1}{a_1} = \frac{2(M^2-1)}{(r+1)M} = \frac{z \;\; \frac{z}{2r}\cancel{(r+1)}}{\cancel{r+1}\left(1+\frac{r+1}{2r}z\right)^{1/2}}$ put 6.103 in 6.99

$$\frac{p_2}{p_1} = \frac{(r+1)M^2}{(r-1)M^2+2} = \frac{(r+1)\left(1+\frac{r+1}{2r}z\right)}{(r-1)\left(1+\frac{r+1}{2r}z\right)+2} = \frac{\left(1+\frac{r+1}{2r}z\right)}{\frac{r-1}{r+1}+\frac{2(r+1)}{r+1}+\frac{r-1}{2r}z}$$

$$= \left(1+\frac{r-1}{2r}z\right)$$

$$\frac{a_2}{a_1} = \frac{\left(2r+(r+1)z-(r-1)\right)^{1/2}\left(r-1+\frac{(r-1)(r+1)}{2r}z+2\right)^{1/2}}{(r+1)\left(1+\frac{r+1}{2r}z\right)^{1/2}}$$

$=$ put $r+1$ into square root both of them.

<u>pg 174 Whitham</u>:

If we have $\frac{p_2}{p_1} + \frac{p_2}{p_1} = 1 + \frac{2\gamma(M^2-1)}{\gamma+1}$

$$= \frac{\gamma+1-2\gamma+2\gamma M^2}{\gamma+1}$$

$$= \frac{1-\gamma+2\gamma M^2}{\gamma+1}$$

Then $\frac{a_2}{a_1} = \left(\frac{p_2}{\rho_2}\cdot\frac{\rho_1}{p_1}\right)^{1/2}$

$$= \left[\frac{\left((\gamma-1)M^2+2\right)}{(\gamma+1)M^2}\cdot\frac{2\gamma M^2-(\gamma-1)}{(\gamma+1)}\right]^{1/2} \checkmark$$

$$\frac{v_2 - v_1}{2a_1} = \frac{\gamma}{\gamma - 1}\,\frac{1}{a_1(v_2 + v_1)}\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_2}\right)$$

$$= \frac{P_1}{\rho_1} - \frac{P_1 + \rho_1 v_1^2 - \rho_2 v_2^2}{\frac{\rho_1 v_1}{\cancel{v_2}}}$$

$$\frac{a_2}{a_1} = \left(\frac{P_2}{\rho_2}\cdot\frac{\rho_1}{P_1}\right)^{1/2} =$$

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$$\frac{a_2}{a_1} = \frac{(1+z)^{1/2}\left(1 + \frac{(r-1)}{2r} z\right)^{1/2}}{\left(1 + \frac{r+1}{2r} z\right)^{1/2}}.$$

$$S = c_v \log \frac{p}{\rho^r}$$

$$\frac{S_2 - S_1}{c_v} = \log\left(\frac{p_2}{p_1}\left(\frac{\rho_2}{\rho_1}\right)^r\right) = \log\left((z+1)\left(\frac{\rho_2}{\rho_1}\right)^r\right)$$

↑ put in

Ma do $\frac{d}{dz}\left(\frac{S_2 - S_1}{c_v}\right) = \cdots$

Math-355_work9.nb

$z + r(z+2) > -1 + 1(-1+2) = 0$ ✓. All terms positive for $r > 1$ & $z = \frac{p_2}{p_1} - 1 > -1$ ✓.

$$\frac{d}{dz}\left(\frac{S_2 - S_1}{C_V}\right) > 0 \quad \forall \; \gamma > 1, \; z > -1.$$

$$\therefore \; \cancel{S_2 - S_1 > 0 \Rightarrow (1+z)\left(\frac{1+\frac{\gamma-1}{2\gamma}z}{1+\frac{\gamma+1}{2\gamma}z}\right)^{\gamma} \to 1}$$

$$\therefore \; \frac{S_2 - S_1}{C_V} \text{ mono} \nearrow \; + \; \text{As} \quad \frac{S_2 - S_1}{C_V}(z=0) = 0$$

$$S_2 - S_1 > 0 \Rightarrow z > 0.$$

$$\Rightarrow P_2 > P_1$$

pg 175 Whitham

$$\frac{d(S_2 - S_1)}{dz} > 0 \qquad \text{check !!?}$$

& if $S_2 - S_1 > 0 \Rightarrow z > 0 \Rightarrow p_2 > p_1$

$$\frac{p_2}{p_1} = \frac{1 + \frac{r+1}{2r} z}{1 + \frac{r-1}{2r} z} > 1 \Rightarrow p_2 > p_1$$

for $$\frac{a_2}{a_1} = \left\{ \frac{(1+z)\left(1 + \frac{r-1}{2r} z\right)}{1 + \frac{r+1}{2r} z} \right\}^{1/2} > \ldots$$

Now $\infty > r > 1$ $\therefore$ $\frac{r-1}{2r} = \frac{1}{2}\left(1 - \frac{1}{r}\right)$ $< \frac{1}{2}$, > 0

$\frac{r+1}{2r} = \frac{1}{2}\left(1 + \frac{1}{r}\right)$ < 1, $> \frac{1}{2}$

Pg 172 W (p, ρ, u)

1

$$\partial_t f = \partial_p f\, \partial_t p + \partial_\rho f\, \partial_t \rho + \partial_u f\, \partial_t u \quad \text{etc for } \partial_x g$$

$$\partial_t p = -up_x + a^2(\rho_t + u\rho_x), \qquad \rho_t = -u\rho_x - \rho u_x \quad \text{(4 terms)}$$

$$\partial_t \rho = -u\rho_x - \rho u_x \quad \text{(2 term)}$$

$$\partial_t u = -\frac{p_x}{\rho} - uu_x \quad \text{(2 term)}$$

$$\partial_x g = \partial_p g\, p_x + \partial_\rho g\, \rho_x + \partial_u g\, u_x$$

Add togeth

$$= \left(\partial_p g + a^2 \cancel{u}\, \partial_p f + -u\partial_p f\right) p_x + \quad -ua^2\cancel{\partial_p f}$$

$$\left(-u\partial_\rho f - \frac{\partial_u f}{\rho} + \partial_\rho g\right) \rho_x +$$

$$+ \left(-a^2 \rho\, \partial_p f - \rho\, \partial_\rho f - u\, \partial_u f + \partial_u g\right) u_x$$

~~Then Adding to $\partial_x g$ we get~~

Thus $\left|\dfrac{(1+z)\left(1+\frac{r-1}{2r}z\right)}{1+\frac{r+1}{2r}z}\right| \le \dfrac{(1)\left(1+z/2\right)}{\min\limits_{1<r<\infty}\left(1+\left(\frac{r+1}{2r}\right)z\right)} = \dfrac{(1)\left(1+z/2\right)}{1+z/2}$

$= 1$

$\Rightarrow \quad a_2 < a_1$

$\dfrac{U_2 - U_1}{a_1} > 0 \quad \Rightarrow \quad U_2 > U_1$ by 6.104

$M > 1$ by 6.103

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1

$$-U\left[\tfrac{1}{2}\rho u^2 + \rho e\right] + \left[\left(\tfrac{1}{2}\rho u^2 + \rho e\right)u + pu\right] = 0$$

$$h = e + \frac{p}{\rho}$$

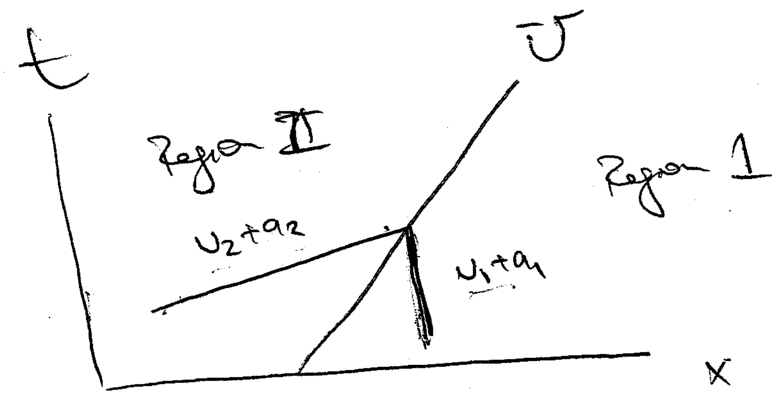
$$\Rightarrow -U\left(\tfrac{1}{2}\rho_2 u_2^2 + \rho_2 e_2 - \tfrac{1}{2}\rho_1 u_1^2 - \rho_1 e_1\right)$$

$$+ \left(\tfrac{1}{2}\rho_2 u_2^2 + \rho_2 e_2\right)u_2 + p_2 u_2 - \left(\tfrac{1}{2}\rho_1 u_1^2 + \rho_1 e_1\right)u_1 - p_1 u_1 = 0$$

$$\Rightarrow \left(\tfrac{1}{2}\rho_2 u_2^2 + \rho_2 e_2\right)(u_2 - U) - \left(\tfrac{1}{2}\rho_1 u_1^2 + \rho_1 e_1\right)(u_1 - U) - p_2 u_2 - p_1 u_1 = 0.$$

$$\Rightarrow -\left(\tfrac{1}{2}\rho_2 u_2^2 + \rho_2 e_2\right)v_2 + \left(\tfrac{1}{2}\rho_1 u_1^2 + \rho_1 e_1\right)(v_1) - p_2 u_2 - p_1 u_1 = 0$$

Pg 125 Whitham



$\Rightarrow$ $\cancel{u_2+a_2 < U < u_1+a_1}$ $\quad u_1+a_1 < U < u_2+a_2$

$$M_1 = \frac{U-u_1}{a_1} > 1 \qquad M_2 = \frac{U-u_2}{a_2} < 1$$

Supersonic when viewed from infront.

subsonic when viewed from behind.

if $U > u_1+a_1 \Rightarrow M > 1$

$$M = \left(1 + \left(\frac{\gamma+1}{2\gamma}\right)z\right)^{1/2} \Rightarrow \left(\frac{\gamma+1}{2\gamma}\right)z > 1$$

$$z > \frac{2\gamma}{\gamma+1}$$

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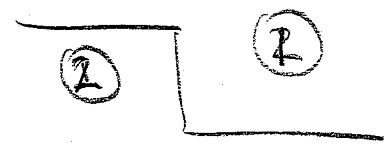

if $U \gg u_1$ $\quad M \gg 1$

$\Rightarrow \quad M \sim \frac{U}{a_1} \quad u_2 \gg u_1$

$$\frac{u_2}{a_1} \sim \frac{2}{r+1}\left(M - \frac{1}{M}\right) \sim \frac{2}{r+1}\frac{U}{a_1}$$

$$\frac{\rho_2}{\rho_1} \sim \frac{r+1}{r-1}$$

$$\frac{p_1}{a_1^2} = \frac{\rho_1}{r}$$

If $p_2 \gg p_1$

$$\frac{p_2}{p_1} \sim \frac{2r}{r+1}\frac{U^2}{a_1^2} \Rightarrow p_2 \sim \frac{2\cancel{r}}{r+1}\frac{\rho_1}{\cancel{r}}U^2.$$

$$\frac{a_2}{a_1} \sim \frac{\sqrt{2r}\,\cancel{M}\,(r-1)^{1/2}\cancel{M}}{(r+1)\cancel{M}} \quad \ldots$$

? Any information?

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$$u_t + c(u)\,u_x = 0 \qquad \text{w/} \quad c = a + U = a_0 + \frac{r+1}{2}\,U.$$

$$U = \frac{c_1 + c_2}{2} = \frac{1}{2}\left(a_1 + u_1 + a_2 + u_2\right)$$

$$= \frac{1}{2}\left(a_0 + \frac{r+1}{2}u_1 + a_0 + \frac{r+1}{2}u_2\right) = a_0 + \frac{r+1}{4}\left(u_1 + u_2\right)$$

From weak shock condition

$$\frac{U - u_1}{a_1} = 1 + \frac{r+1}{4r}z - \frac{(r+1)^2}{32r^2}z^2 + O(z^3)$$

$$\frac{a_2 - a_1}{a_1} = \frac{r-1}{2r}z - \frac{r^2-1}{8r^2}z^2 + \frac{(r-1)(r+1)^2}{16r^3}z^3 + O(z^4)$$

$$\frac{u_2 - u_1}{a_1} = \frac{z}{r} - \frac{r+1}{4r^2}z^2 + \frac{3(r+1)^2}{32r^3}z^3 + O(z^4)$$

Add

$$\frac{1}{2}\left(\frac{a_2 - a_1}{a_1} + \frac{u_2 - u_1}{a_1}\right) = \frac{1}{2r}\left(1 + \frac{r-1}{2}\right)z - \frac{1}{8r^2}\left(\frac{r^2-1}{2} + r+1\right)z^2$$

$$\frac{\bar{U} - u_1}{a_1} = \frac{\left(a_0 + \left(\frac{\gamma-3}{4}\right)u_1 + \frac{\gamma+1}{4}u_2\right)}{a_0 + \frac{\gamma-1}{2}u_1}$$

$$+\frac{(r+1)^2}{32r^3}\left[(r-1)+\frac{3}{2}\right]z^3+O(z^4)$$

$$=\frac{1}{2r}\left(\frac{r+1}{2}\right)z-\frac{1}{8r^2}\left(\frac{r^2-1+2r+2}{2}\right)z^2+\frac{(r+1)^2}{32r^3}\left(r+\tfrac{1}{2}\right)z^3+O(z^4)$$

$$=\frac{r+1}{4r}z-\frac{(r+1)^2}{16r^2}z^2+O(z^3)$$

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$$u_t + c(u)u_x = 0 \quad \text{w/} \quad c = a + u = a_0 + \frac{\gamma+1}{2}u.$$

$$U = \frac{c_1 + c_2}{2} = \frac{1}{2}(a_1 + u_1 + a_2 + u_2)$$

$$= \frac{1}{2}\left(a_0 + \frac{\gamma+1}{2}u_1 + a_0 + \frac{\gamma+1}{2}u_2\right) = a_0 + \frac{\gamma+1}{4}(u_1 + u_2)$$

$$z = \frac{p_2 - p_1}{p_1} \qquad p = \frac{a^2\rho}{\gamma}$$

Then
$$\frac{U - u_1}{a_1} = \frac{a_0}{a_1} + \frac{\gamma+1}{4a_1}(u_1 + u_2) - \frac{u_1}{a_1}$$

$$\therefore z = \frac{a_2^2\rho_2 - a_1^2\rho_1}{a_1^2\rho_1}$$

$$= \frac{a_2^2}{a_1^2}\frac{\rho_2}{\rho_1} - 1$$

But now
$$a = a_0 + \frac{\gamma-1}{2}u$$

$$\therefore a_1 = a_0 + \frac{\gamma-1}{2}u_1$$

$$\therefore \frac{U - u_1}{a_1} = \frac{1}{\left(a_0 + \frac{\gamma-1}{2}u_1\right)}\left[a_0 + \left(\frac{\gamma+1}{4} - 1\right)u_1 + \frac{\gamma+1}{4}u_2\right]$$

Show 6.114 is accurate to 1st order in Z...

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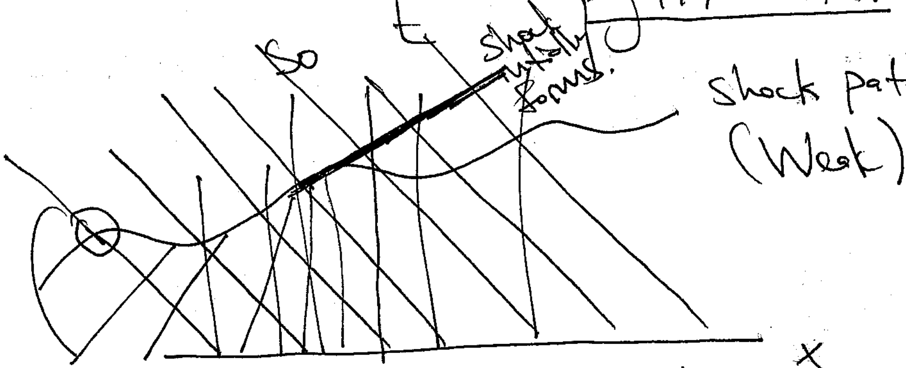

Shock path? (Weak)

U normalized to zoo.

would jump but in weak shock As simple waves we keep ~~the~~ assume the constant accross the shock.

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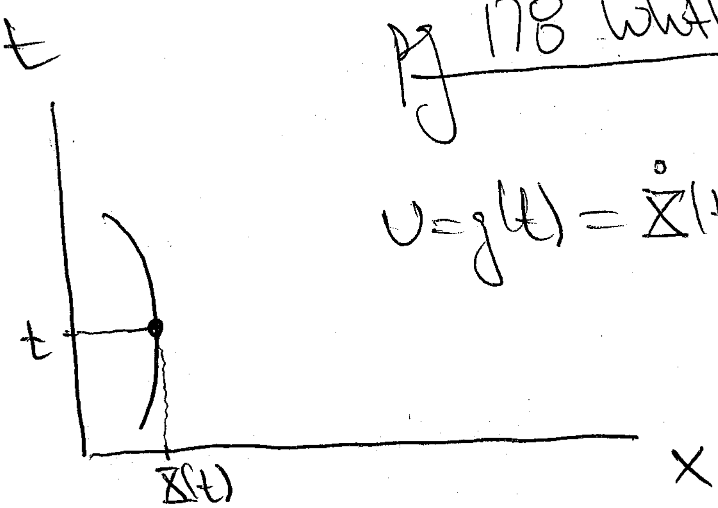
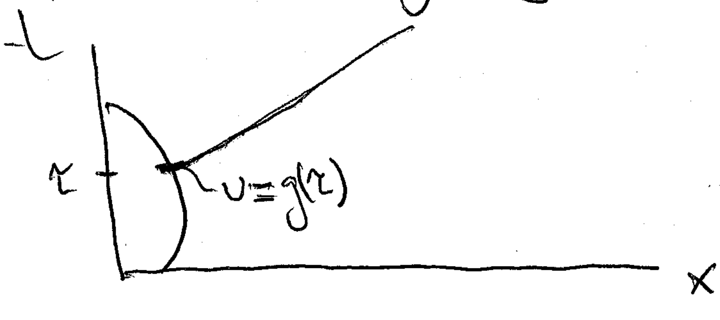


$$U = g(t) = \dot{X}(t) \quad \text{on} \quad x = X(t)$$

g(t) can be given arbitrarily i.e. not related to a given piston or w/ a specific velocity. i.e the signaling problem.



$$\rho = \rho_0 \qquad a = a_0 + \frac{\gamma-1}{2}u$$

$$\therefore\ a_1 = a_0 + \frac{\gamma-1}{2}u_1$$

$$\&\quad a_2 = a_0 + \frac{\gamma-1}{2}u_2$$

$$\therefore\ \frac{1}{2}\left(\frac{a_2-a_1}{a_1} + \frac{u_2-u_1}{a_1}\right) \underset{\text{from above}}{=} \frac{1}{2}\left(\frac{\gamma-1}{2}\frac{(u_2-u_1)}{a_1} + \frac{u_2-u_1}{a_1}\right)$$

$$= \frac{1}{2}\left(\frac{\gamma-1}{2} + \frac{2}{2}\right)\left(\frac{u_2-u_1}{a_1}\right) = \frac{1}{2}\left(\frac{\gamma+1}{2}\right)\left(\frac{u_2-u_1}{a_1}\right)$$

$$=$$

But $\dfrac{u_2-u_1}{a_1} \underset{\text{exact}}{=} \dfrac{z}{\gamma} - \dfrac{\gamma+1}{2\gamma^2}z^2 + O(z^3)$ by weak shock criterion. Pg 176

$\therefore$ Above

$$= \frac{1}{2}\left(\frac{\gamma+1}{2}\right)\left(\frac{z}{\gamma} - \frac{\gamma+1}{2\gamma^2}z^2\right) + O(z^3)$$

$$= \frac{\gamma+1}{4\gamma}z - \frac{(\gamma+1)^2}{8\gamma^2}z^2 + O(z^3)$$

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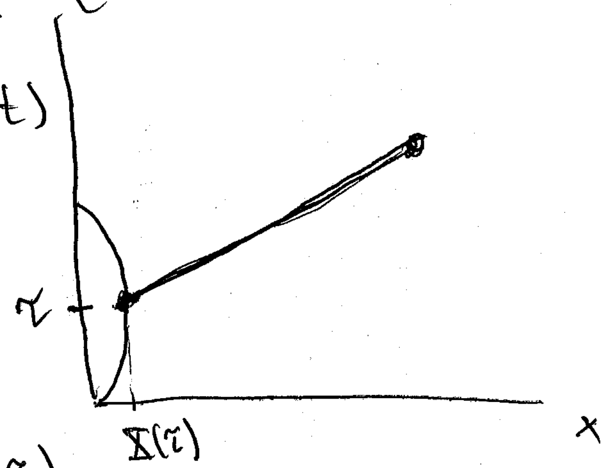
$$u = g(t) = \dot{X}(t) \quad \text{on} \quad x = X(t)$$



$$u = g(\tau)$$

$$x = X(\tau) + \left(a_0 + \frac{\gamma+1}{2} g(\tau)\right)(t-\tau)$$

if $\dot{g}(\tau) > 0$

$$\xi = X(\tau) - \left\{a_0 + \frac{\gamma+1}{2} g(\tau)\right\}\tau$$

$$F(\xi) = a_0 + \frac{\gamma+1}{2} g(\tau)$$

$$\frac{dS}{dt} = a_0 + \frac{\gamma+1}{4}(u_1 + u_2)$$

$u_2 = 0$
$\& \; u_1 = g(\tau)$

$$= a_0 + \frac{\gamma+1}{4}(g(\tau))$$

Now Floor[t]=t–1+h[t] with h[t]<1, thus when limits as t→∞ are taken the only term that remains in the above is...

$$\frac{1}{2}\int_0^1 f[-t+x+2\phi,\tau]\, B'[-t+x+\phi]\, d\phi$$

And finally the equation to be satisfied for equalibrium must be...

$$\text{eqStar} = -2 f^{(0,1)}[t+x,\tau] + 3 f[t+x,\tau]\, f^{(1,0)}[t+x,\tau] + \frac{1}{2}\int_0^1 f[-t-x+2\phi,\tau]\, B'[t+x-\phi]\, d\phi == 0$$

▪ Equivalence of the two equations? Ask Rosales if this is correct... Minus sign in Burghers equation?

We have the following two equations from the analysis earlier...

$$\text{eqStar1} = -2 f^{(0,1)}[t-x,\tau] + 3 f[t-x,\tau]\, f^{(1,0)}[t-x,\tau] - \frac{1}{2}\int_0^1 f[-t+x+2\phi,\tau]\, B'[-t+x+\phi]\, d\phi == 0$$

$$\text{eqStar2} = -2 f^{(0,1)}[t+x,\tau] + 3 f[t+x,\tau]\, f^{(1,0)}[t+x,\tau] + \frac{1}{2}\int_0^1 f[-t-x+2\phi,\tau]\, B'[t+x-\phi]\, d\phi == 0$$

In the first let X=t–x, then we get...

$$\text{eqStar1} = -2 f^{(0,1)}[X,\tau] + 3 f[X,\tau]\, f^{(1,0)}[X,\tau] - \frac{1}{2}\int_0^1 f[-X+2\phi,\tau]\, B'[-X+\phi]\, d\phi == 0$$

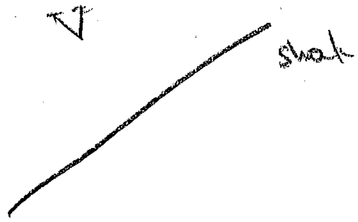
In the second let Y=t+x, then we get...

$$\text{eqStar2} = -2 f^{(0,1)}[Y,\tau] + 3 f[Y,\tau]\, f^{(1,0)}[Y,\tau] + \frac{1}{2}\int_0^1 f[-Y+2\phi,\tau]\, B'[-(-Y+\phi)]\, d\phi == 0$$

Now if B'[–ξ]=B'[ξ], we are done... But this can be made so since our B[x] is only physically defined on the interval 0<x<1. Thus we take the EVEN fourier extension of this function for all reals and we have equivalence of the two equations...

In summary we should solve the following equation to determine the wave propagation in a box 0<x<1

$$f^{(0,1)}[X,\tau] - \frac{3}{2} f[X,\tau]\, f^{(1,0)}[X,\tau] + \frac{1}{4}\int_0^1 f[-X+2\phi,\tau]\, B'[-X+\phi]\, d\phi == 0$$



(6.92) w/ $\partial_t = -V\partial_x$ then

$$(\rho S)_t + (\rho u S)_x = -V\partial_X(\rho S) + (\rho u S)_X = \frac{(p_{11}+p)u_x - q_{1x}}{T}$$

$$\Rightarrow \frac{d}{dX}\{\rho(V-u)S\} = -\frac{(p_{11}+p)u_X - q_{1X}}{T}$$

$$= -\frac{\left(\frac{4}{3}\mu u_X^2 + (\lambda T_X)_X\right)}{T} = -\frac{\frac{4}{3}\mu u_X^2}{T} - \frac{(\lambda T_X)_X}{T}$$

or ~~[illegible]~~

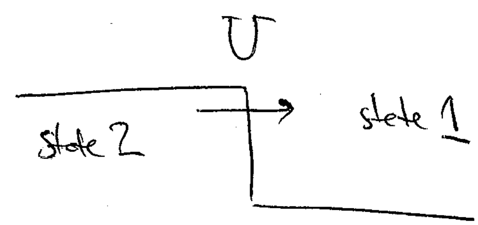
But
$$-\left(\frac{\lambda T_X}{T}\right)_X = -\frac{(\lambda T_X)_X}{T} - \frac{\lambda T_X^2}{T^2}(-1)$$

$$= -\frac{(\lambda T_X)_X}{T} + \frac{\lambda T_X^2}{T^2} \Rightarrow -\frac{(\lambda T_X)_X}{T} = -\left(\frac{\lambda T_X}{T}\right)_X - \frac{\lambda T_X^2}{T^2}$$

$$\therefore \frac{d}{dX}\{\rho(V-u)S\} = -\frac{\frac{4}{3}\mu u_X^2}{T} - \left(\frac{\lambda T_X}{T}\right)_X - \frac{\lambda T_X^2}{T^2}$$

$$\Rightarrow \frac{d}{d\bar{x}}\left\{\rho(U-u)S + \frac{\lambda T_{\bar{x}}}{T}\right\} = -\frac{\left(\frac{4}{3}\mu T u_{\bar{x}}^2 + \lambda T_{\bar{x}}^2\right)}{T^2}$$

$\therefore$



$$\int_{-\infty}^{\infty} \Rightarrow$$

$$\Rightarrow \left\{\rho(U-u)S + \frac{\lambda T_{\bar{x}}}{T}\right\}\Bigg|_{-\infty/2}^{\infty/1} = \int_{-\infty}^{\infty} \frac{4/3\,\mu T u_{\bar{x}}^2 + \lambda T_{\bar{x}}^2}{T^2}\, d\bar{x}$$

But $\left.\dfrac{\lambda T_{\bar{x}}}{T}\right|_1^2 = 0$

$$\therefore \Rightarrow \left\{\rho(U-u)S\right\}\Big|_1^2 = \int_{-\infty}^{\infty} \frac{(4/3)\mu T u_{\bar{x}}^2 + \lambda T_{\bar{x}}^2}{T^2}\, d\bar{x} > 0$$

eq 6.129

$$\rho(-U+u) = A$$

$$\rho V = -A = Q.$$

$$-U\rho u + \rho u^2 + P - \frac{4}{3}\mu u_x = B$$

if $v = U - u \Rightarrow u = U - v$

& the Above ~~[illegible]~~ ... becomes

$$-U\rho(U - v) + \rho(U - v)^2 + P - \frac{4}{3}\mu(-v_x) = B$$

$$\Rightarrow -\cancel{U^2}\rho + U\rho v + \cancel{\rho U^2} - 2\rho U v + \rho v^2 + P + \frac{4}{3}\mu v_x = B$$

$$-\rho U v$$

But $\rho v = Q$ from Above & ∴

$$\Rightarrow -QU + \rho v^2 + P + \frac{4}{3}\mu v_x = B$$

$$\Rightarrow \rho v^2 + P + \frac{4}{3}\mu v_x = P$$

in energy eq ...

$$-U\left(\frac{1}{2}\rho(U - v)^2 + \rho e\right)$$

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$$h = \frac{\gamma}{\gamma-1} RT = c_p T \qquad \rho v = Q$$

$\therefore$ 2nd eq $\quad Qv + R\frac{\rho v T}{v} + \frac{4}{3}\mu v_x = P$

$$\Rightarrow \quad Qv + \frac{RQT}{v} + \frac{4}{3}\mu v_x = P$$

$$\Rightarrow \quad \frac{4}{3}\mu v_x = P - Q\left(v + \frac{RT}{v}\right)$$

But $\quad RT = \frac{\gamma-1}{\gamma} h$

$$\Rightarrow \quad \frac{4}{3}\mu v_x = P - Q\left(v + \frac{\gamma-1}{\gamma}\frac{h}{v}\right)$$

$$+ \quad \left(h + \frac{v^2}{2}\right)Q + \frac{4}{3}\mu v v_x + \lambda\frac{h_x}{c_p} = E$$

$$\frac{\lambda}{c_p} h_x + \frac{4}{3}\mu v v_x = E - Q\left(h + \frac{v^2}{2}\right)$$

$$\therefore \quad \frac{4}{3}\mu v_x = P - Q\left(v + \frac{\gamma-1}{\gamma}\frac{h}{v}\right)$$

$$\frac{\lambda}{c_p} h_x + \frac{4}{3}\mu v v_x = E - Q\left(h + \frac{v^2}{2}\right)$$

if $\quad \dfrac{\lambda}{c_p} = \dfrac{4}{3}\mu$

$$\frac{3}{4} \overset{?}{=} \underbrace{\frac{\mu c_p}{\lambda}}_{.71} = P = \text{Prandtl \#}$$

Then 6.131 $\Rightarrow$

$$\frac{\lambda}{c_p}\left(h_X + v v_X\right) = E - Q\left(\quad \right)$$

$$\parallel$$

$$\frac{4}{3}\mu\left(h + \frac{v^2}{2}\right)_X = E - Q\left(h + \frac{v^2}{2}\right)$$

Right hand side vanishes as $X \to +\infty$

see 3rd shock profile eq

If $h + \frac{v^2}{2} = \frac{E}{Q} \Rightarrow h = \frac{E}{Q} - \frac{v^2}{2}$

6.130 $\Rightarrow$ w/ h put in.

$$\frac{4}{3}\mu V_X = P - Q\left(v + \frac{r-1}{r}\frac{E}{Qv} - \frac{r-1}{r}\frac{v}{2}\right)$$

$$= P - Q\left[\left(\frac{2r - r + 1}{2r}\right)v + \frac{r-1}{r}\frac{E}{Qv}\right]$$

$$= P - Q\left[\frac{r+1}{2r}v + \frac{r-1}{r}\frac{E}{Qv}\right]$$

Notice LHS vanishes as $X \to \pm\infty$

$$\frac{4}{3}\mu V_X = \underbrace{\frac{r+1}{2r}\frac{Q}{v}}_{\text{factor infront of "}v^2\text{" term}}\left[\frac{2r}{r+1}\frac{P}{Q}v - v^2 - \frac{r-1}{\cancel{r}}\frac{E}{Q\cancel{v}}\frac{\cancel{v}}{Q}\frac{2\cancel{r}}{r+1}\right]$$

i.e get quadratic w/ coefficient 1 in front.

$$= \frac{r+1}{2r}\frac{Q}{v}\left[\frac{2rP}{Q(r+1)}v - v^2 - \frac{2(r-1)}{r+1}\frac{E}{Q^2}\right]$$

$$= \frac{r+1}{2r}\frac{Q}{v}\left[-v^2 + \frac{2rP}{Q(r+1)}v - \frac{2(r-1)}{r+1}\frac{E}{Q^2}\right]$$

$$= \frac{\gamma+1}{2\gamma}\frac{Q}{v}(v_1 - v)(v - v_2)$$

$$= \left[v_1 v + v_2 v - v_1 v_2 - v^2 \right]$$

$$\left[-v_1 v_2 + (v_1 + v_2) v - v^2 \right]$$

$$\frac{3Q}{4\mu}\left(\frac{\gamma+1}{2\gamma}\right)(v_1 - v_2)\, X = v_2 \log (v - v_2) - v_1 \log (v_1 - v)$$

$\therefore$ Shock thickness $\propto \frac{4\mu}{3 v_1 \rho_1}\left(\frac{2\gamma}{\gamma+1}\right)\frac{1}{v_1 - v_2}$?

$$= \log \frac{(v - v_2)^{v_2}}{(v_1 - v)^{v_1}}$$

$$e^{[\ \ ]} = \cancel{\left(\frac{v - v_2}{v_1 - v}\right)}$$

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$$\rho_t + u\rho_r + \rho u_r + \frac{j\rho u}{r} = 0$$

$$u_t \qquad u u_r + \frac{1}{\rho} p_r = 0$$

$$\cancel{p_t \quad a^2 \rho_t \qquad + u p_r}$$

in 6.134 put in $\rho_t + u\rho_r = -\rho\left(u_r + \frac{ju}{r}\right)$

Then $p_t + u p_r + a^2 \rho \left(u_r + \frac{ju}{r}\right) = 0$

The last eq becomes

$$p_t + \qquad + a^2 \rho u_r + u p_r + \frac{a^2 \rho j u}{r} = 0$$

$$\Rightarrow \begin{pmatrix} \rho \\ u \\ p \end{pmatrix}_t + \begin{pmatrix} u & \rho & 0 \\ 0 & u & \rho^{-1} \\ 0 & a^2\rho & u \end{pmatrix} \begin{pmatrix} \rho \\ u \\ p \end{pmatrix}_r + \begin{pmatrix} \frac{j\rho u}{r} \\ 0 \\ \frac{a^2 \rho j u}{r} \end{pmatrix} = 0.$$

get for left eivectors

$$l_1 = \begin{pmatrix} -a^2 \\ 0 \\ 1 \end{pmatrix} \qquad \mu_1 = u$$

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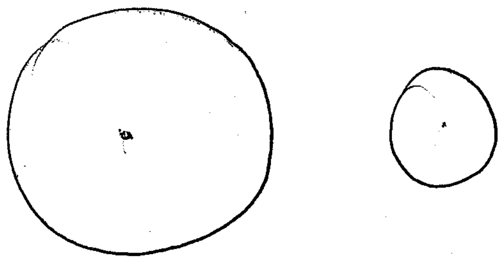

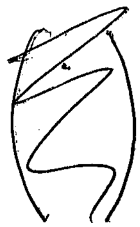

Find left eigenvectors of Above matrix

See Ma output...

$$l_1 = \begin{pmatrix} -a^2 \\ 0 \\ 1 \end{pmatrix} \qquad \mu_1 = U$$

$$l_2 = \begin{pmatrix} 0 \\ -a\rho \\ 1 \end{pmatrix} \qquad \mu_2 = U - a$$

$$l_3 = \begin{pmatrix} 0 \\ a\rho \\ 1 \end{pmatrix} \qquad \mu_3 = U + a$$

Check $\quad l_1 . M = U(-a^2, 0, 1) \quad$ ✓.

Then $l_1^T \longrightarrow$

$$\Rightarrow \quad -a^2 \rho_t + p_t + U(-a^2, 0, 1)\begin{pmatrix} \rho_r \\ u_r \\ p_r \end{pmatrix} +$$

$$+ \quad -a^2 \cancel{\rho U_r} + a^2 \cancel{\rho U_r} = 0$$

$$\Rightarrow \quad -a^2 \rho_t + p_t + U(-a^2 \rho_r + p_r) = 0.$$

$\therefore$ $P - a^2\rho$

$$P_t + uP_r - a^2(\rho_t + u\rho_r) = 0$$

$$\Rightarrow \cancel{P - a^2\rho = \text{constant on } \frac{dr}{dt} = u}$$

$$\frac{dP}{dt} - a^2\frac{d\rho}{dt} = 0 \quad \text{on} \quad \frac{dr}{dt} = u.$$

Then $l_2^T \rightarrow$

$$-a^2\rho u_t + P_t + (u-a)(0, -a^2\rho, 1)\begin{pmatrix} \rho_r \\ u_r \\ P_r \end{pmatrix}$$

$$+ \frac{a^2\rho j u}{r} = 0$$

$$\Rightarrow P_t - a^2\rho u_t + (u-a)(-a^2\rho u_r + P_r) + \frac{a^2\rho j u}{r} = 0$$

$$\Rightarrow [P_t + (u-a)P_r] - a^2\rho[u_t + (u-a)u_r] + \frac{a^2\rho j u}{r} = 0$$

$$\frac{dP}{dt} - a^2\rho\frac{du}{dt} + \frac{a^2\rho j u}{r} = 0 \quad \text{on} \quad \frac{dr}{dt} = u-a$$

$l_3^T \rightarrow$

$$a\rho u_t + p_t + (u+a)(0, a\rho, 1)\begin{pmatrix} p_r \\ u_r \\ p_r \end{pmatrix} + \frac{a^2 \rho j u}{r} = 0$$

$$a\rho u_t + p_t + (u+a)(a\rho u_r + p_r) + \frac{a^2 \rho j u}{r} = 0$$

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$\Rightarrow$ $\cancel{p_t + a\rho u_t}$

$$p_t + (u+a)p_r + a\rho(u_t + (u+a)u_r) + \frac{a^2 \rho j u}{r} = 0$$

$$\rightarrow \quad \frac{dp}{dt} + a\rho\frac{du}{dt} + \frac{a^2 \rho j u}{r} = 0 \quad \text{on} \quad \frac{dr}{dt} = u+a$$

$$= f[t-x,\tau](B[x]-B[-t+x+1])$$

$$-\int_0^t f[-t+x+2\phi,\tau]B'[-t+x+\phi]\,d\phi$$

$$-2t f^{(0,1)}[t-x,\tau]-2f^{(1,0)}[t-x,\tau]\int_0^t B[-t+x+\phi]\,d\phi$$

$$+3t f[t-x,\tau]f^{(1,0)}[t-x,\tau]+f^{(1,0)}[t-x,\tau]\int_0^t f[-t+x+2\phi,\tau]\,d\phi$$

$$-2\int_0^t B[-t+x+\phi]f^{(1,0)}[-t+x+2\phi,\tau]\,d\phi$$

$$+f[t-x,\tau]\int_0^t f^{(1,0)}[-t+x+2\phi,\tau]$$

$$+3\int_0^t f[-t+x+2\phi,\tau]f^{(1,0)}[-t+x+2\phi,\tau]\,d\phi$$

Then

$$t\Big\{-2f^{(0,1)}[t-x,\tau]-2f^{(1,0)}[t-x,\tau]\frac{1}{t}\int_0^t B[-t+x+\phi]\,d\phi$$

$$+3f[t-x,\tau]f^{(1,0)}[t-x,\tau]$$

w/ $\xi = r t^{-n}$ pg 192 Whitham

$$eq1 = \rho(\xi)\left(m r t^{n} + j\, t^{1+m+n}\, u(\xi) + r t^{1+m} u'(\xi)\right)$$

$$- r\left(n r - t^{1+m} u(\xi)\right) \rho'(\xi) == 0.$$

Use $r = \xi t^{n}$

$$\Rightarrow \rho(\xi)\left(m \xi t^{2n} + j\, t^{1+m+n}\, u(\xi) + \xi t^{1+m+n} u'(\xi)\right)$$

$$\int \mathrm{RHS1}(\_, \_, \_)\, d\phi$$

$$\cancel{\frac{2hLuL}{(hL-uL^2)^2}} \int B \cdot B' \, d\phi$$

C_1

$$C_1 \int B \cdot B' \, d\phi + C_2 \int B B' \, d\phi$$

$$+ C_3 f_1 \int B' \, d\phi + C_4 f_1 \int B' \, d\phi$$

$$+ C_5 f_1 \int B' \, d\phi + C_6 f_1 \int B' \, d\phi$$

$$+ C_7 \int f_2 B' \, d\phi + C_8 \int f_2 B'.$$

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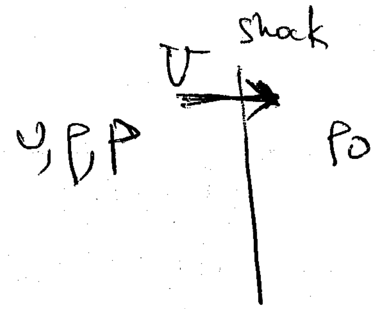

$$[E] = \frac{ML^2}{T^2} \quad [\rho_0] = \frac{M}{L^3}$$

$$\left[\frac{E}{\rho_0}\right] = \frac{L^5}{T^2}$$

$$[R(t)] = L.$$

$$\therefore \quad \left(\frac{E}{\rho_0}\right)\frac{1}{R(t)^5} = k_1 t^{-2}$$

$$\Rightarrow \quad k_2 \frac{E}{\rho_0} t^2 = R^5 \Rightarrow R(t) = k\left(\frac{E}{\rho_0}\right)^{1/5} t^{2/5}$$ eq 6.139

$$\therefore \quad U = \frac{dR}{dt} = k\left(\frac{E}{\rho_0}\right)^{1/5} \frac{2}{5} t^{-3/5}$$

$$\therefore \quad U = \frac{4}{5}\frac{1}{(\gamma+1)}$$

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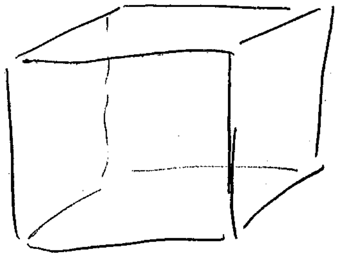


ρ_0 density of air gas ∴ 3 parameters

P

$u = 0$

$$u = \frac{2}{\gamma+1} U \qquad \rho = \frac{\gamma+1}{\gamma-1}\rho_0 \qquad P = \frac{2}{\gamma+1}\rho_0 U^2$$

$$E, \rho_0 \Rightarrow [\rho_0] = \frac{M}{L^3}$$

F·v

$kg \cdot L/s^2 \cdot L/s$

$$[E] = \frac{ML^2}{s^2}$$

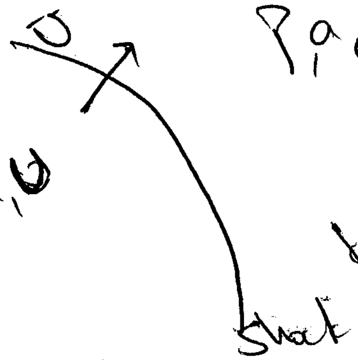
form quantities of dimension

(P, a, ρ) Not good choice as $a = \frac{\gamma P}{\rho}$ & ∴ from P & a we can get ρ.

$$a = \frac{\gamma P}{\rho}$$

(P, a, u) good choice. (Think)

Then



P, a, U

P, a, u

neglect P & a outside shock region compared to those inside shock

keep ρ_0?

$$\frac{hL^{3/2}\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{4\ (hL-uL^2)^2}-$$

$$\frac{\sqrt{hL}\ uL^2\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{uL^4\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{4\sqrt{hL}\ (hL-uL^2)^2}$$

$$E \sim \frac{ML^2}{T^2}$$

$$\rho_0 \sim \frac{M}{L^3}$$

$$\frac{E}{\rho_0} \sim \frac{L^5}{T^2} \sim \text{Thing}$$

$r =$ ~~$R(t)$ $R(t)$ length $\sim t$ time~~

$$\cancel{\frac{R(t)^5}{t^2} \sim \text{const}}$$

$\cancel{R(t)} \sim$

In physical problem $R(t)$ main result of interest.

2 paramet. $R \sim L$ $t \sim T$.

$$\therefore \quad \frac{R(t)^5}{t^2} \sim \frac{E}{\rho_0} \rightarrow \frac{R(t)^5}{t^2} \sim k\frac{E}{\rho_0}$$

$$R(t) \sim k\left(\frac{E}{\rho_0}\right)^{1/5} t^{2/5}$$

$$\frac{uL^3\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ B'\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]}{(hL-uL^2)^2}-$$

$$\frac{hL^{3/2}\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ B'\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{\sqrt{hL}\ uL^2\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ B'\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]}{2\ (hL-uL^2)^2}-$$

$$\frac{hL^2\ f_1^{(0,1)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{(hL-uL^2)^2}+$$

$$\frac{2\,hL\,uL^2\ f_1^{(0,1)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{(hL-uL^2)^2}-\frac{uL^4\ f_1^{(0,1)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{(hL-uL^2)^2}+$$

$$\frac{hL^{3/2}\ B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{2\ (hL-uL^2)^2}-$$

$$\frac{hL\,uL\,B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{(hL-uL^2)^2}-$$

$$\frac{\sqrt{hL}\ uL^2\ B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{uL^3\ B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{(hL-uL^2)^2}-$$

$$\frac{3\,hL^{3/2}\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{4\ (hL-uL^2)^2}+$$

$$\frac{3\sqrt{hL}\ uL^2\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{2\ (hL-uL^2)^2}-$$

$$\frac{3\,uL^4\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{4\sqrt{hL}\ (hL-uL^2)^2}+$$

$$\frac{hL^{3/2}\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{4\ (hL-uL^2)^2}-$$

$$\frac{\sqrt{hL}\ uL^2\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{uL^4\ f_2\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]\ f_1^{(1,0)}\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]}{4\sqrt{hL}\ (hL-uL^2)^2}-$$

$$\frac{hL^{3/2}\ B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{\sqrt{hL}\ uL^2\ B\left[-t\,uL+x+uL\,\phi+\sqrt{hL}\ (-t+\phi)\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{hL^{3/2}\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{4\ (hL-uL^2)^2}-$$

$$\frac{\sqrt{hL}\ uL^2\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{2\ (hL-uL^2)^2}+$$

$$\frac{uL^4\ f_1\left[-t\left(\sqrt{hL}+uL\right)+x,\ \tau\right]\ f_2^{(1,0)}\left[-t\,uL+x-\sqrt{hL}\ (t-2\,\phi),\ \tau\right]}{4\sqrt{hL}\ (hL-uL^2)^2}+$$

If $R(t) = k\left(\frac{E}{\rho_0}\right)^{1/5} t^{2/5}$

Then $\frac{dR}{dt} = k\left(\frac{E}{\rho_0}\right)^{1/5}\frac{2}{5}t^{-3/5} = U.$

Then $u = \frac{2k}{\gamma+1}\left(\frac{E}{\rho_0}\right)^{1/5}\frac{2}{5}t^{-3/5} = \frac{4k}{5(\gamma+1)}\left(\frac{E}{\rho_0}\right)^{1/5}t^{-3/5}$

& $p = \frac{2}{\gamma+1}\rho_0 k^2\left(\frac{E}{\rho_0}\right)^{2/5}\frac{4}{25}t^{-6/5}$

$= \frac{8\rho_0 k^2}{25(\gamma+1)}\left(\frac{E}{\rho_0}\right)^{2/5}t^{-6/5}$

Now:

$\left(\frac{\rho_0}{E}\right)^{\frac{1}{5}}\frac{R}{k} = t^{2/5} \Rightarrow t = \left(\frac{\rho_0}{E}\right)^{\frac{1}{2}}\left(\frac{R}{k}\right)^{5/2}$

$\therefore\ p = \frac{8\rho_0 k^2}{25(\gamma+1)}\left(\frac{E}{\rho_0}\right)^{2/5}\left(\frac{\rho_0}{E}\right)^{-3/5}\left(\frac{R}{k}\right)^{-3}$

$= \frac{8}{25}\frac{\cancel{\rho_0}k^2}{(\gamma+1)}\left(\frac{E}{\cancel{\rho_0}}\right)^{2/5}\left(\frac{E}{\cancel{\rho_0}}\right)^{3/5}\left(\frac{R}{k}\right)^{-3}$

$= \frac{8}{25}\frac{Ek^5}{(\gamma+1)}R^{-3}$

$$\& \quad U = \frac{4k}{5(r+1)}\left(\frac{E}{\rho_0}\right)^{1/5}\left(\frac{\rho_0}{E}\right)^{-3/10}\left(\frac{R}{k}\right)^{-3/2}$$

$$= \frac{4\cancel{k}}{5(r+1)}\left(\frac{E}{\rho_0}\right)^{1/5}\left(\frac{E}{\rho_0}\right)^{3/10}\frac{R^{-3/2}}{\cancel{k}^{-3/2}} = \frac{4}{5(r+1)}\frac{k}{}\left(\frac{E}{\rho_0}\right)^{1/2} R^{-3/2}\, k^{3/2}$$

$$= \frac{4}{5(r+1)}k^{5/2}\left(\frac{E}{\rho_0}\right)^{1/2} R^{-3/2}$$

$$\left[\frac{E}{\rho_0}\right] = \frac{L^5}{T^2} \Rightarrow \quad \frac{T^2}{L^5}\left[\frac{E}{\rho_0}\right] \text{ is dimensionless}$$

Then E

$$\xi = \frac{r}{R(t)} = \frac{r}{k t^{2/5}}\left(\frac{\rho_0}{E_0}\right)^{1/5}$$

$$= \left[\frac{k t^{2/5}}{r}\left(\frac{E_0}{\rho_0}\right)^{1/5}\right]^{-1}$$

$$= \left[\frac{k^5 t^2}{r^5}\left(\frac{E}{\rho_0}\right)\right]^{-1/5}$$

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$$\rho_t + (\rho u)_x = 0$$

$$(\rho u)_t + (\rho u^2 + \tfrac{1}{2}\rho^2)_x = 0$$

$\rho \to h$

$$h_t + (hu)_x = 0 \quad ✓$$

$$h_t u + h u_t + h_x u^2 + 2huu_x + hh_x = 0$$

$$u(-\cancel{h_x}u - \cancel{h}u_x) + u^2\cancel{h_x} + \cancel{h}uu_x + huu_x + hu_t + hh_x = 0$$

$$\Rightarrow \quad u_t + uu_x + h_x = 0$$

where is g??

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2

$$\partial_t P + \partial_x Q = 0$$

$$P_u u_t + P_h h_t + Q_u u_x + Q_h h_x = ?$$

If $Q_u = uP_u + hP_h$ & $Q_h = gP_u + uP_h$

$$\Rightarrow P_u u_t + P_h h_t + uu_x P_u + hu_x P_h + gh_x P_u + h_x u P_h = ?$$

$$\Rightarrow P_u(\underbrace{u_t + uu_x + gh_x}_{=0}) + (\underbrace{h_t + hu_x + h_x u}_{=0})P_h = 0$$

$$\left.\begin{aligned} Q_{uh} &= uP_{uh} + P_h + hP_{hh} \\ Q_{hu} &= gP_{uu} + P_h + uP_{hu} \end{aligned}\right\} \text{subtract} \quad hP_{hh} = gP_{uu}$$

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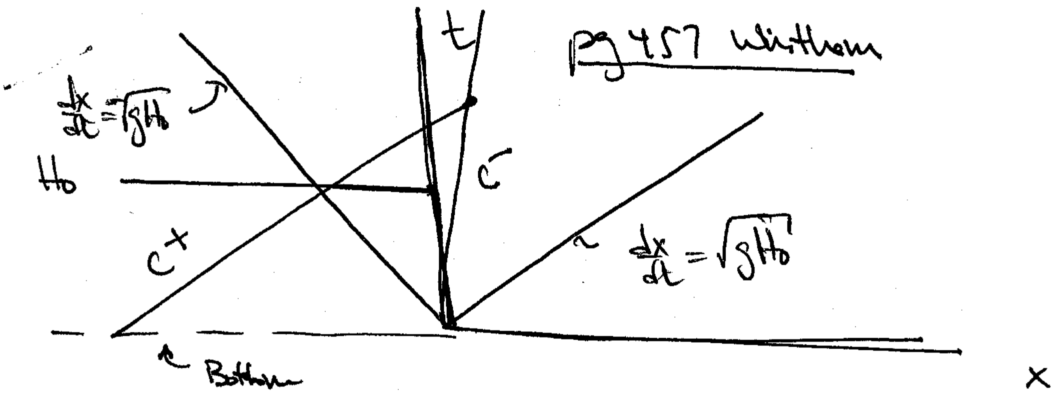


C_+: $U + 2\sqrt{gH} = U + 2\sqrt{gH}\Big|_{t=0}$ on $\frac{dx}{dt} = U + \sqrt{gh}$

$= 0 + 2\sqrt{gH_0}$

$\therefore$ $U + 2\sqrt{gH} = 2\sqrt{gH_0}$ constant

$\frac{x}{t} = U - \sqrt{gh}$

2 eqs. 2 unknowns

Mult by 2 & Add

$$\begin{aligned} U + 2\sqrt{gh} &= 2\sqrt{gH_0} \\ + \quad 2U - 2\sqrt{gh} &= 2(x/t) \\ \hline 3U &= 2\left(\sqrt{gH_0} + (x/t)\right) \end{aligned}$$

$U = \frac{2}{3}\left(\sqrt{gH_0} + (x/t)\right)$ 13.84a

& $\sqrt{gh} = U - \frac{x}{t} = \frac{2}{3}\sqrt{gH_0} + -\frac{1}{3}\left(\frac{x}{t}\right)$

$= \frac{1}{3}\left(2\sqrt{gH_0} - \frac{x}{t}\right)$ 13.84b

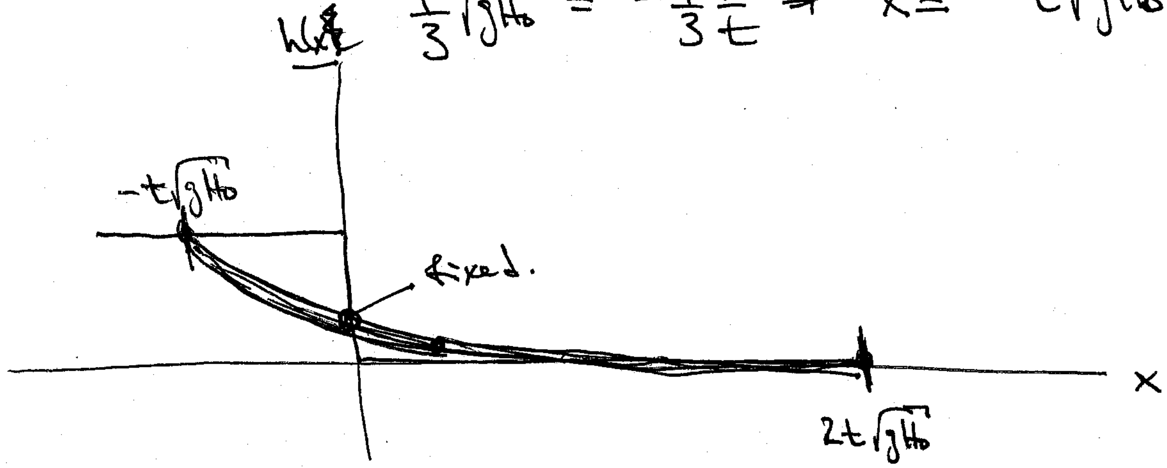
$$h = \frac{1}{9g}\left(2\sqrt{gH_0} - \frac{x}{t}\right)^2$$ head of H_2O

$$h = 0 \Rightarrow \frac{x}{t} = 2\sqrt{gH_0} \Rightarrow x = 2\sqrt{gH_0}\,t$$

tail of H_2O $\quad h = H_0$

$$\Rightarrow \sqrt{gH_0} = \frac{2}{3}\sqrt{gH_0} - \frac{1}{3}\frac{x}{t}$$

$$\frac{1}{3}\sqrt{gH_0} = -\frac{1}{3}\frac{x}{t} \Rightarrow x = -t\sqrt{gH_0}$$



$0 < h < H_0.$ $\qquad t \stackrel{?}{=} \cancel{t}(x)$

Fix t. What is the geometric shape of $h(x)$? This is a parabola..

At $x = 0$ $\quad h = \dfrac{4gH_0}{9g} = \dfrac{4H_0}{9}$

$v = \frac{2}{3}\sqrt{gH_0}$. But one new that along rays $\frac{x}{t}$ the solution is constant $\forall t \Rightarrow$ along $\frac{x}{t} = 0$.

Dam position the solution should be constant $\forall$ time.

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$g = 1$

$$-U(u_1 h_1 - u_2 h_2) + \left(u_1^2 h_1 + \tfrac{1}{2} h_1^2 - u_2^2 h_2 - \tfrac{1}{2} h_2^2\right) = 0$$

$$-U(h_1 - h_2) + (u_1 h_1 - u_2 h_2) = 0$$

$$\|$$

$$-u_2 h_2 = +U(h_1 - h_2) - u_1 h_1$$

$\Rightarrow$ Put in 1st eq'

$$-U(U(h_1 - h_2)) + u_1^2 h_1 + \tfrac{1}{2} h_1^2 - \tfrac{1}{2} h_2^2 - \frac{1}{h_2}\left(U(h_1 - h_2) - u_1 h_1\right)^2 = 0$$

$$-(h_1 - h_2)U^2 + u_1^2 h_1 + \tfrac{1}{2} h_1^2 - \tfrac{1}{2} h_2^2 - \frac{1}{h_2}\left(U^2 (h_1 - h_2)^2 - 2U u_1 h_1 (h_1 - h_2) + u_1^2 h_1^2\right) = 0$$

$$\left[-(h_1-h_2)-\frac{1}{h_2}(h_1-h_2)^2\right]U^2+2u_1\frac{h_1}{h_2}(h_1-h_2)U-\frac{u_1^2h_1^2}{h_2}+\frac{1}{2}h_1^2-\frac{1}{2}h_2^2+u_1^2h_1=0$$

$$U=\frac{-\cancel{2}\frac{u_1h_1}{h_2}(h_1-h_2)\pm\sqrt{\cancel{4}\frac{u_1^2h_1^2}{h_2^2}(h_1-h_2)^2+\cancel{4}\left((h_1-h_2)+\frac{1}{h_2}(h_1-h_2)^2\right)\left(\frac{1}{2}(h_1^2-h_2^2)+u_1^2h_1-\frac{u_1^2h_1^2}{h_2}\right)}}{-\cancel{2}\left((h_1-h_2)+\frac{(h_1-h_2)^2}{h_2}\right)}$$

$$=\frac{-u_1h_1}{h_2}\pm\sqrt{\frac{u_1^2h_1^2}{h_2^2}(h_1-h_2)^2+\frac{1}{2}(h_1-h_2)^2(h_1+h_2)+u_1^2h_1(h_1-h_2)-\frac{u_1^2h_1^2}{h_1}(h_1-h_2)}$$