Check eq 2-3 for pg 21 wilde
n=4.

$\cancel{x_k}=$

$$x_k = \frac{(1+\epsilon)}{3}\left\lfloor \frac{k+1}{2} \right\rfloor - \left\{ \left\lfloor \frac{k+1}{2} \right\rfloor - \left\lfloor \frac{k}{2} \right\rfloor \right\}(\epsilon)$$

$$x_1 = \frac{1+\epsilon}{3}(1) - \{1-0\}\epsilon = \frac{1+\epsilon}{3} - \epsilon$$

$$x_2 = \frac{1+\epsilon}{3}(1) - \{1-1\}\epsilon = \frac{1+\epsilon}{3}$$

$$x_3 = \frac{1+\epsilon}{3}(2) - \{2-1\}\epsilon = \frac{2}{3}(1+\epsilon) - \epsilon$$

$$x_4 = \frac{1+\epsilon}{3}(2) - \{2-2\}\epsilon = \frac{2}{3}(1+\epsilon)$$

$\Rightarrow$ $x_1 = \frac{1}{3} - \frac{2}{3}\epsilon$

$x_2 = \frac{1}{3} + \frac{1\epsilon}{3}$

$x_3 = \frac{2}{3} - \frac{1}{3}\epsilon$

$x_4 = \frac{2}{3} + \frac{2}{3}\epsilon$

pg 39 Wilde 02-14-02

Lattice Search:

Given k points to search over

Assume $k+1 = F_n$ A Fibonacci #. for some n

Then map these k points to $1, 2, \ldots, k$.

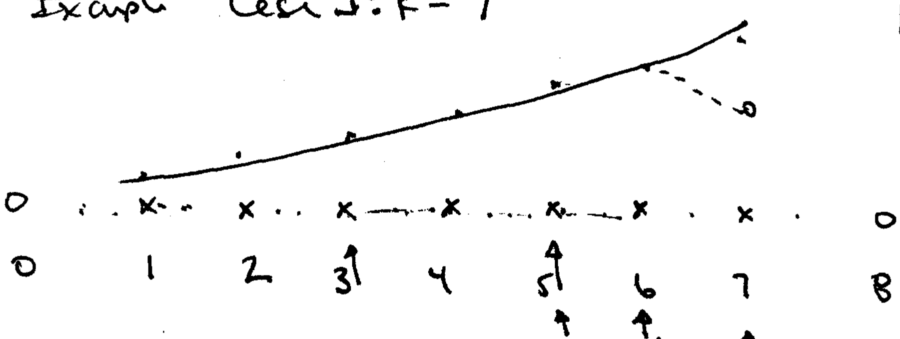
including the points $0 + k+1$ we have $k+2$ pts

w/ $k+2-1$ cells $\Rightarrow$ $k+1$ is a Fibonacci #

Example case 1: $k=7$

Monotone increasing fn:

$7+1=8=F_5$



$f(3) < f(5) \Rightarrow$ New interval $(3,8)$ of length 5. 1 pair comparison.

$f(5) < f(6) \Rightarrow$ New interval $(5,8)$ of length $3 = 2+1$ 2 pair comp

$f(6) < f(7) \Rightarrow$ New interval $(6,8)$ of lngt $2 = 1+1$ 3 pair comp

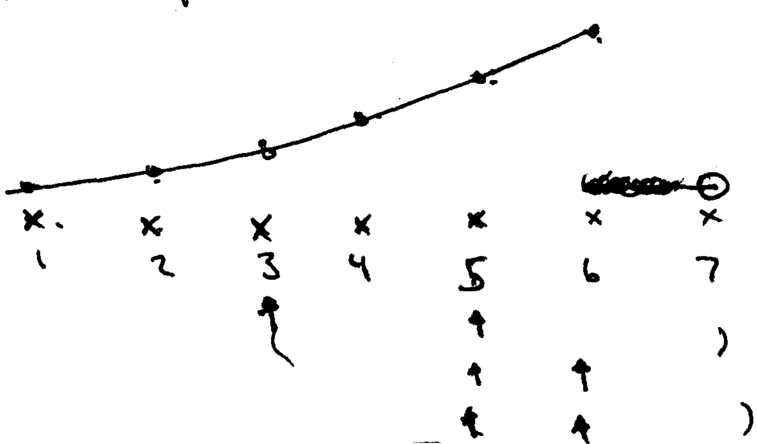
$f(7)$ what do I test w/? Must be $f(6)$ but how program this?

Example #2: $k=6$

Monotone increasing

Add Additional pts

$F_0=1$, $F_1=1$, $F_2=2$, $F_3=3$, $F_4=5$, $F_5=8$, $F_6=13$, $F_7=21$



exps =

$\underline{f(3) > f(5)}$ $\underline{f(3) < f(5)}$

$k =$

$\cancel{l = 3}$ $\cancel{F =}$

$l = 3 = F_3$ $\cancel{r = 6 =} F$

$r = 5 = F_4$

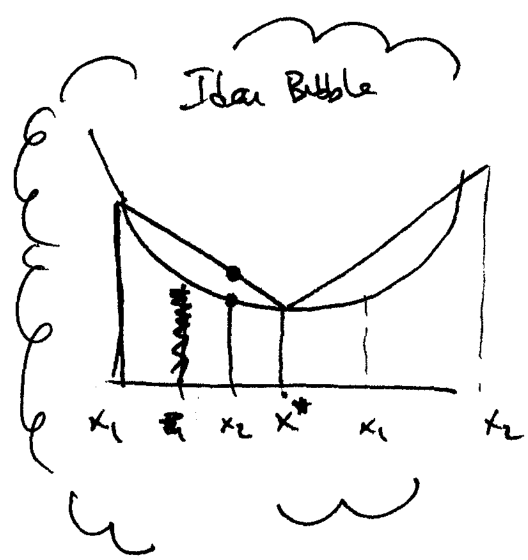
$l = 5 = F_4$ $r = 6 = F$

PS 5 Wilde

(1) f concave $\Rightarrow$ $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$.

Now given a Global minimum of $f(x)$ on an Interval I. called x^*. If f is convex pf that f is unimodal on I.

Pf: ~~Since~~ Let $x_1 < x_2 < x^*$. Since x^* is a global min of f on I know $f(x_1) > f(x^*)$.



Now consider the line segment from x_1 to x^*. By the convexity of f.

$f(\alpha x_1 + (1-\alpha)x^*) \leq \alpha f(x_1) + (1-\alpha) f(x^*)$
$\forall\ \alpha \in [0,1]$.

Since x_2 is between x_1 + x^* $\exists$ α_2 $\ni$

$x_2 = \alpha_2 x_1 + (1-\alpha_2)x^*$ + thus

$f(x_2) = f(\alpha_2 x_1 + (1-\alpha_2)x^*) \leq \alpha_2 f(x_1) + (1-\alpha_2) f(x^*)$

$= f(x^*) + \alpha_2 (f(x_1) - f(x^*))$

Since $f(x_1) > f(x^*)$

$\leq f(x^*) + (1)(f(x_1) - f(x^*)) = f(x_1)$

$\therefore$ As $f(x_2) \leq f(x_1)$ we have half of the unimodal condition.

Let $x^* < x_1 < x_2$

let $\alpha_1 \ni \quad x_1 = \alpha_1 x^* + (1-\alpha_1) x_2$

Then $f(x_1) = f(\alpha_1 x^* + (1-\alpha_1)x_2) \le \alpha_1 f(x^*) + (1-\alpha_1) f(x_2)$

$= f(x_2) + \alpha_1 (f(x^*) - f(x_2)) = f(x_2) - \alpha_2 (f(x_2) - f(x^*))$

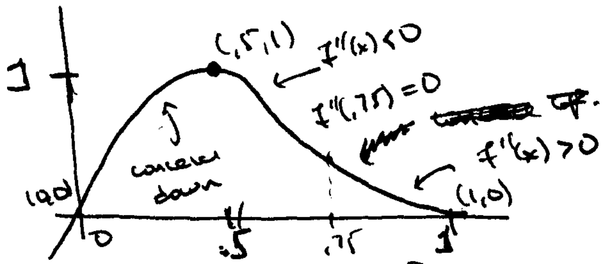
since $f(x_2) > f(x^*)$

$< f(x_2) - 0(f(x_2) - f(x^*)) = f(x_2)$

$\therefore \quad f(x_1) < f(x_2) \quad \checkmark.$

P3 S7 Wilde

(2) Idea is that to have the fn change curvature in the 2nd half of the interval.



Thus pick several points + impose conditions on $f(x)$

$\therefore$ $f(x) = a + bx + cx^2 + dx^3$

$$f(0)=0 \Rightarrow a=0$$

$$f(.5) = f(1/2) = \frac{b}{2} + \frac{c}{4} + \frac{d}{8} = 1 \qquad (1)$$

$$f(1)=0 \Rightarrow b+c+d=0 \qquad (2)$$

$$= f'' = 2c + 6dx$$

$$f''(3/4) = 2c + 6d\left(\frac{3}{4}\right) = 0$$

$$2c + \frac{9}{2}d = 0 \qquad d = -\frac{4}{9}c \quad \text{put in eqs (1) + (2)}$$

in eq (1)

$$b + \frac{c}{2} + \frac{d}{4} = 1 \rightarrow b + \frac{c}{2} + \frac{-4}{4(9)}c = 1$$

$$b + \frac{c}{2} - \frac{c}{9} = 1 \Rightarrow b + \frac{7}{18}c = 1$$

in eq (2)

$$b + c - \frac{4}{9}c = 0 \Rightarrow b + \frac{5}{9}c = 0$$

$$= \begin{pmatrix} 1 & 7/18 \\ 1 & 5/9 \end{pmatrix}\begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} b \\ c \end{pmatrix} = \frac{1}{\frac{5}{9} - 7/18}\begin{pmatrix} \begin{vmatrix} 1 & 7/18 \\ 0 & 5/9 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \end{pmatrix} = \frac{1}{(3/18)}\begin{pmatrix} 5/9 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{5/9}{3/18} \\ -6 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} b \\ c \end{pmatrix} = \begin{pmatrix} \frac{\cancel{18}^{2}}{\cancel{9}} \cdot \frac{5}{3} \\ -6 \end{pmatrix} = \begin{pmatrix} \frac{10}{3} \\ -6 \end{pmatrix}$$

$$\therefore \quad d = -\frac{4}{9}c = -\frac{4}{\cancel{9}_3}(\cancel{-6}^{2}) = +\frac{8}{3}$$

$$f(x) = \frac{10}{3}x - 6x^2 + \frac{8}{3}x^3$$

PS 5? Wilde

(3)

(a)

x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7
3.1	4.2	5.0	6.5	7.1	8.8	9.5	10.5
1.0	1.5	3.2	3.0	2.6	2.3	1.9	1.4
y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7

Looking for a maximum we see that it is at x_2

$I_1 = 1.9$

$I_2 = 2.3$

$I_3 = 2.1$

$I_4 = 2.3$

$I_5 = 2.4$

$I_6 = 1.7$

(b) $I_{\text{cont max}} = I_2 = 2.3$

(c) $I_{\max} = 2.4$

pg 87 Wilde

(4) Interval explored is $2 \le x \le 12$ $\epsilon = .3$

w/ 8 experiments. # of experiments is even. Place all exps at location

$$x_k = \frac{(1+\epsilon)\lfloor \frac{k+1}{2} \rfloor}{(\frac{n}{2})+1} - \left\{ \left\lfloor \frac{k+1}{2} \right\rfloor - \left\lfloor \frac{k}{2} \right\rfloor \right\} \epsilon \qquad 1 \le k \le 8$$

~~Thus the exps are placed at~~ Now

$$\cancel{x_k} \quad \frac{1+\epsilon}{\frac{n}{2}+1} = \frac{1.3}{5} = .26$$

Then the 8 exps are placed "relatively" at:

$x' = a + (b-a)x$

$x \in (0,1)$

$x' \in (a,b)$

$$x_1 = .26(1) - \{1-0\}(.3) =$$

$$x_2 = .26(1) - \{1-1\}(.3)$$

$$x_3 = .26(2) - \{2-1\}(.3)$$

$$x_4 = .26(2) - \{2-2\}(.3)$$

$$x_5 = .26(3) - \{3-2\}(.3)$$

$$x_6 = .26(3) - \{3-3\}(.3)$$

$$x_7 = .26(4) - \{4-3\}(.3)$$

$$x_8 = .26(4) - \{4-4\}(.3)$$

$$X_k = \frac{(1+\epsilon)}{5}\left\lfloor\frac{k+1}{2}\right\rfloor - \left\{\left\lfloor\frac{k+1}{2}\right\rfloor - \left\lfloor\frac{k}{2}\right\rfloor\right\}\epsilon$$

$$x_1 = \left(\frac{1+\epsilon}{5}\right)(1) - \{1-0\}\epsilon = \frac{1+\epsilon}{5} - \epsilon = \frac{1}{5} - \frac{4}{5}\epsilon$$

$x_1 < 0$ unless $\frac{1}{5} - \frac{4}{5}\epsilon > 0$

$$1 - 4\epsilon > 0 \Rightarrow \epsilon < \frac{1}{4} = .25$$

or ϵ is .3?

$$\Delta x = \frac{1+\epsilon}{(\frac{4}{2})+1}\left\{\left\lfloor\frac{k+2}{2}\right\rfloor - \left\lfloor\frac{k+1}{2}\right\rfloor\right\}$$

$$- \left\{\left\lfloor\frac{k+2}{2}\right\rfloor - \left\lfloor\frac{k+1}{2}\right\rfloor - \left\lfloor\frac{k+1}{2}\right\rfloor + \left\lfloor\frac{k}{2}\right\rfloor\right\}\epsilon$$

$$= \frac{1+\epsilon}{(\frac{4}{2})+1}\left\{\left\lfloor\frac{k+1}{2}+\frac{1}{2}\right\rfloor - \left\lfloor\frac{k+1}{2}\right\rfloor\right\}$$

$$- \left\{\left\lfloor\frac{k+1}{2}+\frac{1}{2}\right\rfloor - \left\lfloor\frac{k+1}{2}\right\rfloor - \left\lfloor\frac{k}{2}+\frac{1}{2}\right\rfloor + \left\lfloor\frac{k}{2}\right\rfloor\right\}\epsilon$$

$$= \cancel{\frac{1+\epsilon}{(\frac{4}{2})+1}\{} 1$$

Now if k is even $k = 2p$

$$\left\lfloor\frac{k+1}{2}+\frac{1}{2}\right\rfloor = \left\lfloor\frac{2p+1}{2}+\frac{1}{2}\right\rfloor = p+1$$

~~so~~ + $\left\lfloor \frac{k+1}{2} \right\rfloor = \left\lfloor \frac{2p+1}{2} \right\rfloor = p$

So $\left\lfloor \frac{k+1}{2} + \frac{1}{2} \right\rfloor - \left\lfloor \frac{k+1}{2} \right\rfloor = 1$

If k is odd $k = 2p+1$

$$\left\lfloor \frac{k+1}{2} + \frac{1}{2} \right\rfloor = \left\lfloor \frac{2(p+1)}{2} + \frac{1}{2} \right\rfloor = p+1$$

$$\left\lfloor \frac{k+1}{2} \right\rfloor = \left\lfloor \frac{2(p+1)}{2} \right\rfloor = p+1$$

So $\left\lfloor \frac{k+2}{2} \right\rfloor - \left\lfloor \frac{k+1}{2} \right\rfloor = 1$ or 0

$\therefore$

$$\Delta x \leq \frac{1+\epsilon}{\frac{n}{2}+1}(1) - \{0 - 0\}\epsilon = \frac{1+\epsilon}{\frac{n}{2}+1}$$

$\therefore$ requirement that $\Delta x \geq 0$ it is necessary that

$$\frac{1+\epsilon}{\frac{n}{2}+1}$$

— — — — This is Not what I want — — —

$x_1 = \frac{1+\epsilon}{\frac{n}{2}+1} - \{1 - 0\}\epsilon$ requirement that $x_1 \geq 0$

$$= \frac{1+\epsilon}{\frac{n}{2}+1} - \epsilon$$

$$\frac{1+\epsilon}{\frac{n}{2}+1} \geq \epsilon$$

$$1+\cancel{\epsilon} \geq \epsilon\left(\frac{n}{2}+\cancel{1}\right)$$

$$\boxed{\epsilon \leq \frac{2}{n}}$$

Thus for 8 experiments $\epsilon < \frac{1}{4} = .25$ →←.

Assume problem meant $\epsilon = .03$ + work from there.

Now with an even # of experiments the best locations for a simultaneous search plan are given by eq 2-3

w/

$$x_k = \frac{(1+\epsilon)\left\lfloor \frac{k+1}{2} \right\rfloor}{\left(\frac{n}{2}+1\right)} - \left\{\left\lfloor \frac{k+1}{2} \right\rfloor - \left\lfloor \frac{k}{2} \right\rfloor\right\}\epsilon \qquad 1 \leq k \leq n$$

This gives

$x_1 = \ldots$

Note these $x_k \in (0,1)$ Thus the actual exp should be taken at $x_k = a + (b-a)\hat{x}_k$ w/ $\hat{x}_k \in (0,1)$

Then the length of the max interval of uncertainty is

$$L_8^* = \frac{1+\epsilon}{\frac{n}{2}+1} = \frac{1+\epsilon}{5} = \frac{1.03}{5} = \ldots$$

H57 Wilde

5

your choice	Quiz	No Quiz
Reed	+10	+5
Don't Reed / Movies	0	+3

his choice

~~Bt~~ ~~P~~ By Assumption I go 1st + ~~Assuming~~ that the professor is out to get me, what ever I do ~~with~~ ~~to~~ he will do what he can to make me have the lowest score. Thus if I pick Reed ~~two~~ he will have no quiz. If I pick Movies ~~thee~~ he will pick Quiz.

So my choice is a +5 or a zero. I had better read the assignment !!

Pg 57 Wilde

(6) $y = 3 + 6x - 4x^2$ w/ 4 simultaneous experiments

$\epsilon = .05$

$$x_k = \left(\frac{1+\epsilon}{\frac{\Delta}{2}+1}\right)\left(\left\lfloor \frac{k+1}{2} \right\rfloor\right) - \left\{\left\lfloor \frac{k+1}{2} \right\rfloor - \left\lfloor \frac{k}{2} \right\rfloor\right\}\epsilon$$

$$\frac{1+\epsilon}{\frac{\Delta}{2}+1} = \frac{1.05}{3} = .35$$

		Y
$x_1 = .35(1) - .05$	$= .3$	4.44
$x_2 = .35(1) - 0$	$= .35$	4.61
$x_3 = .35(2) - (2-1)(.05)$	$= .65$	5.21
$x_4 = .35(2) - 0$	$= .7$	5.24 ←
	1.0	—

$\Delta x = .35$

$$L_4^* = \frac{1+\epsilon}{\frac{\Delta}{2}+1} = \frac{1.05}{3} = .35$$

(7) $y = 3 + 6x - 4x^2$

Choosing the same locations as in problem 6. (We have no reason to choose different ones) we get

$F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3, F_4 = 5, F_5 = 8$

	x_1		x_2	x_3	
0	.3	.35	.65	.7	1

Then k = # of points $= 4 + 2 = 6$ ← including 0 + 1

so $k = 6 = F_4 + 1 \quad \therefore$ # of cells is F_4

Test fn at $\quad F_4 = F_3 + F_2 = 3 + 2$

x: $\quad x_1 \quad x_2 \quad x_3 \quad x_4$

y: $\quad 4.61 \quad 5.21$

Take as the new interval $(x_2 \; x_3 \; x_4)$ 1

$4 = F_3 + 1$
$\quad = F_2 + F_1$

y: $\quad 5.21 \quad 5.24$

Thus the maximum is at x_4 of 5.24.

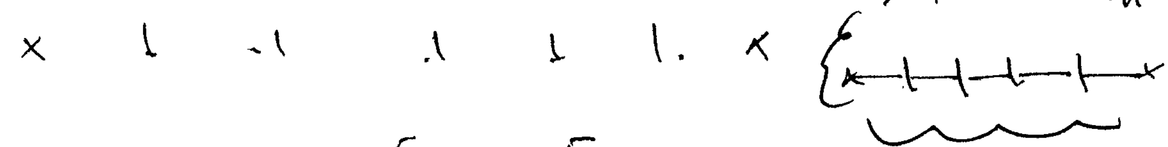
& $\Delta x = 1 - x_3 = 1 - .65 = .35$

pg 57 Wilde

(B) 5 experiments would not help in problem 6 since no improvement is made in final interval of uncertainty when conducting an odd # of experiments.

For the lattice search we would like $k = F_n - 1$

$\left\{ \begin{array}{l} 1+2 = F_n + 1 \\ k = F_n - 1 \end{array} \right\}$



we would require $5 + x = F_n - 1$

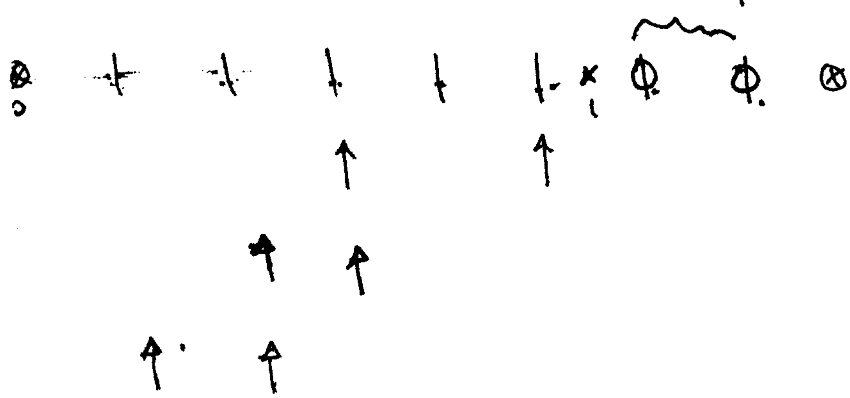
$$\parallel$$

$$6 + x = F_n$$

$F_0 = 1,\ F_1 = 1,\ F_2 = 2,$
$F_3 = 3,\ F_4 = 5,$
$F_5 = 8,\ F_6 = 13, \ldots$

Pick $F_n = 8$ then $x = 2$ fictitious points

Additional fictitious points.



$F_5 = 8 = 5 + 3$

Assuming the min/max is the left most point.

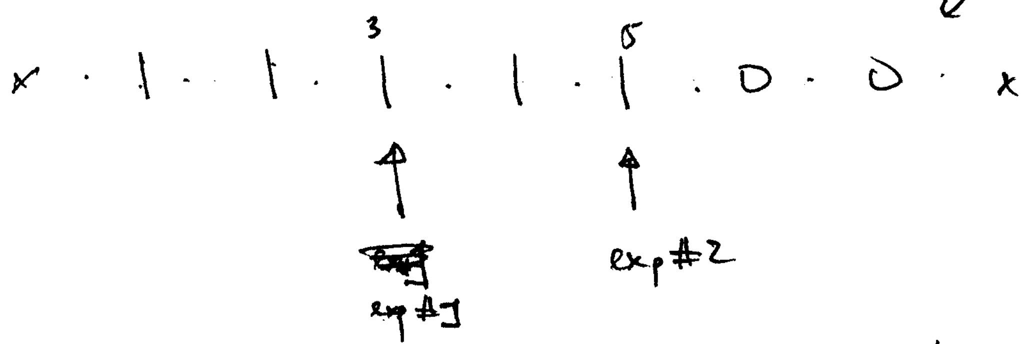
$5 = 3 + 2$

Note: The method of fictitious points assumes that the min/max of the fn is not at the end points where the fictitious points are added.

Note. 1) Whenever we put the fictitious points they will Not be searched over on the 1st iteration (the fact that they are used to fill up a Fibonacci # is what guarantees this

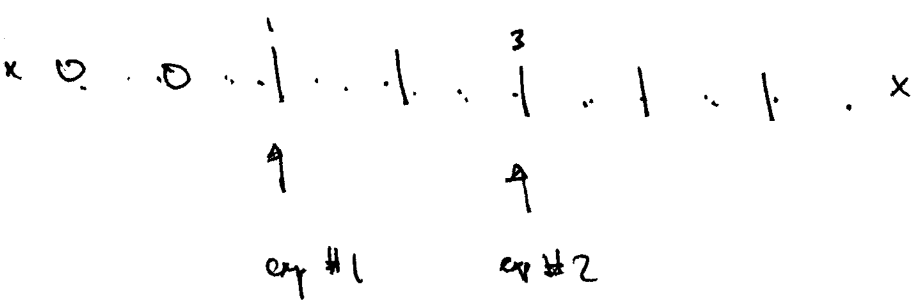
2) After the 1st two experiments if ~~val~~ we have the rare case that the max/min is at the endpoints to which we have ~~attached~~ the fictitious points we know then that we have just placed the fictitious points at the wrong side of the points. Consider the following

Maximum is at right end of interval of J points
Add fictitious points to right end



New interval → took fictitious points on wrong end

If we had placed them on the left, we would have the following situation



New interval. + we can continue the lattice search

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(9) Lucas relationship is

$$F_n = \frac{\tau^{n+1} - (-\tau)^{-(n+1)}}{\sqrt{5}} \quad \text{w/} \quad \tau = \frac{1+\sqrt{5}}{2} = 1.61803\ldots$$

$$F_0 = \frac{\tau - (-\tau)^{-1}}{\sqrt{5}} = \frac{\frac{1+\sqrt{5}}{2} + \frac{2}{1+\sqrt{5}}}{\sqrt{5}} = \frac{\frac{(1+\sqrt{5})(1+\sqrt{5})}{2\,(1+\sqrt{5})} + \frac{2^2}{2(1+\sqrt{5})}}{\sqrt{5}}$$

$$= \frac{1+2\sqrt{5}+5+4}{2\sqrt{5}(1+\sqrt{5})} = \frac{10+2\sqrt{5}}{2\sqrt{5}(1+\sqrt{5})} = \frac{\cancel{2}\cdot(5+\sqrt{5})}{\cancel{2}\sqrt{5}(1+\sqrt{5})}$$

$$= \frac{\sqrt{5}(\sqrt{5}+1)}{\sqrt{5}(1+\sqrt{5})} = 1 \quad ✓$$

$$F_1 = \frac{\tau^2 - (-\tau)^{-2}}{\sqrt{5}} \underset{\text{w/ MMA}}{=} \frac{1}{\sqrt{5}}\left(\frac{5+3\sqrt{5}}{3+\sqrt{5}}\right) = \frac{\sqrt{5}(\sqrt{5}+3)}{\sqrt{5}(3+\sqrt{5})} = 1 \quad ✓$$

$$F_2 = \frac{\tau^3 - (-\tau)^{-3}}{\sqrt{5}} = \frac{1}{\sqrt{5}}\left(\frac{2(5+2\sqrt{5})}{2+\sqrt{5}}\right) = 2 \quad ✓$$

(10) $F_n = \dfrac{\tau^{n+1} - (-\tau)^{-(n+1)}}{\sqrt{5}}$ pg 57 with By Luc

Sin $\tau = 1.618 > 1$ $\lim_{n \to +\infty} \tau^{-(n+1)} = 0$

$\therefore \quad F_n \sim \dfrac{\tau^{n+1}}{\sqrt{5}}$ eq 2-17

pg 51 Wilde

(11) $y = 4 - 3x + 5x^2$ $\qquad y' = -3 + 10x = 0 \qquad x = 3/10 =$

$y(3/10) = 3.55 \qquad y'' = 10 > 0$ y has min.

Since we desire to bracket the minimum & don't know how many iterations to take I would suggest a golden ratio search.

$\tau = 1.618$

$\frac{1}{\tau} = .61803 \qquad 1 - \frac{1}{\tau} = .3819$

One would continue the golden ratio search method as long as needed until a fn value is less than 3.56. I'll just do 2 iterations to show the idea

$I_0 = 1$

		x_1	x_2	
x:	0	.3819	.61803	1
y:		3.5835	4.0887	

new interval

$x_3 = 0 + (.3819)(.61803) = .236 \qquad y_3 = 3.57$

$x_3 \overset{?}{=} 0 + (.61803)^2 = .38196$ already tested.

X:	0	.236	.3819	.61803
Y:	—	3.57	3.5835	—

New interval. continue pattern

pg 52 Wilde

(12)

$E = E(BD)$

Assuming the days are discrete (they are to some degree) we wish to locate with in 365 days the max/min of the fn $E(BD)$. I think this is the correct assumption of the problem

Looking at table 2-1 for a ~~Dicho~~ Lattice search w/ $k_n \cong 365$ we pick $k_n = 376$ requires 12 experiments !!

For a Dichotomous search $l_0 = 365$ & $L_n = 1$

$\Rightarrow$ between 17 + 18 experiments.

19 SR Wilde

(13)

	a	x_1	x_2	x_3	x_4	b
x:		$-.5$	1.3	2.0	2.9	
					8.95	
y:			$\uparrow$	$\uparrow$		

$\{F_0 = 1, F_1 = 1, F_2 = 2, F_3 = 3$
$\{F_4 = 5, F_5 = 8, F_6 = 13, \ldots$

$\#$ points $= F_n - 1$ $\qquad k+2-1 = F_n$

4 pts $= F_4 - 1$ ✓

$$F_4 = \underset{\substack{\parallel \\ 2}}{F_2} + \underset{\substack{\parallel \\ 3}}{F_3}$$

That next two experiments should be at x_2 + x_3. I don't see how the knowledge of y_4 helps the situation any.

pg 82 Wilde

(14)

O x x x x x x O

I would think it has to be best in the minmax sense for wherever one places the maximum. The procedure always finds it to an accuracy of one point.

pg 54 Wilde

15 ~~Watching the experiments at the [illegible]~~

	Quit	No Quit	# pts if professor picks the min
Reed	+10	+5	+5
Don't Reed: Movies	0	+3	+0
Don't Reed: Study other course	+4	+8	+4

∴ to maximize the # of points returned to ~~use~~ us pick +5, or reed

optimization Hwk.

pg 52 wilde

(16) I don't understand this problem.

pg 52 Wilde

(17) Using $F_n = \dfrac{\tau^{n+1} - (-\tau)^{-(n+1)}}{\sqrt{5}}$ $\quad \tau = \dfrac{1+\sqrt{5}}{2} = 1.618\ldots$

Pr: $\quad F_{n-2}^2 - F_{n-1}F_{n-3} = (-1)^n$

$$F_{n-2} = \frac{\tau^{n-1} - (-\tau)^{-(n-1)}}{\sqrt{5}} \quad \therefore \quad F_{n-2}^2 = \frac{\left(\tau^{n-1} - (-\tau)^{-(n-1)}\right)^2}{5}$$

$$= \frac{\tau^{2(n-1)} - 2\tau^{n-1}\cdot(-\tau)^{-(n-1)} + \tau^{-2(n-1)}}{5}$$

$$= \frac{\tau^{2(n-1)} - (-1)^{n-1}(2) + \tau^{-2(n-1)}}{5} =$$

$$F_{n-1}F_{n-3} = \left(\frac{\tau^{n} - (-\tau)^{-n}}{\sqrt{5}}\right)\left(\frac{\tau^{n-2} - (-\tau)^{-(n-2)}}{\sqrt{5}}\right)$$

$$= \frac{1}{5}\left(\tau^{2n-2} - \tau^{n}(-\tau)^{-n-2} - \tau^{n-2}(-\tau)^{-n} + (-1)^n(-1)^{n-2}\cdot \tau^{-n}\tau^{-(n-2)}\right)$$

$$= \frac{1}{5}\left(\tau^{2(n-1)} + \tau^{2}(-1)^{n+1} + \tau^{-2}(-1)^{n+1} + \tau^{-2(n-1)}\right)$$

~~-2n-2)~~

Thus

$$F_{n-2}^2 - F_{n-1}F_{n-3} = \frac{-(-1)^{n-1}(2)}{5} + \frac{(-1)^n\tau^2}{5} + \frac{(-1)^n\tau^{-2}}{5}$$

$$= \frac{(-1)^n}{5}(2+\tau^2+\tau^{-2})$$

$\tau^2+\tau^{-2} = 3$. $\therefore$ RHS $= \frac{(-1)^n}{5}(5) = (-1)^n$ ✓

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(18) eq 2-14

$$\tau = \frac{1+\sqrt{5}}{2} = 1.61303\cdots$$

$$\cancel{\tau^2} \quad \tau^2 = \tau + 1$$

$$\tau^2 - \tau - 1 = 0$$

$$\tau = \frac{1 \pm \sqrt{1-4(1)(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

Neg root $\frac{1-\sqrt{5}}{2}$

But $\frac{1}{\tau} = \frac{2}{1+\sqrt{5}} \cdot \left(\frac{1-\sqrt{5}}{1-\sqrt{5}}\right) = \frac{2(1-\sqrt{5})}{1-5} = \frac{2(1-\sqrt{5})}{-4}$

$$= -\frac{(1-\sqrt{5})}{2} \Rightarrow \frac{1-\sqrt{5}}{2} = -\frac{1}{\tau}$$