An Automated Algorithm For High Resolution Single Frequency Imaging

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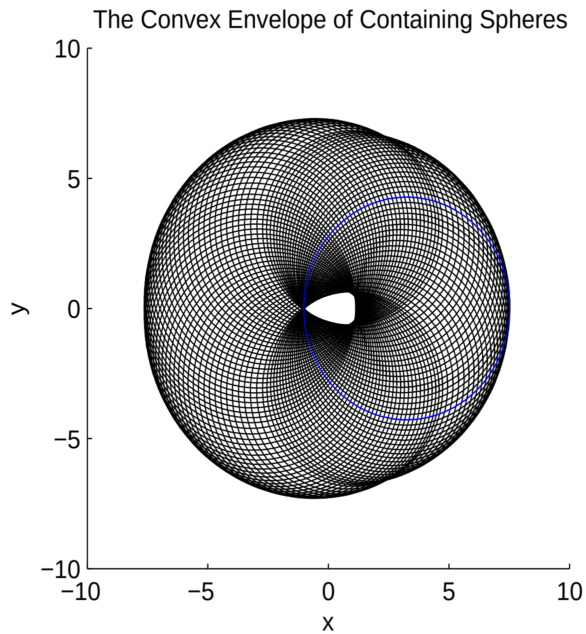
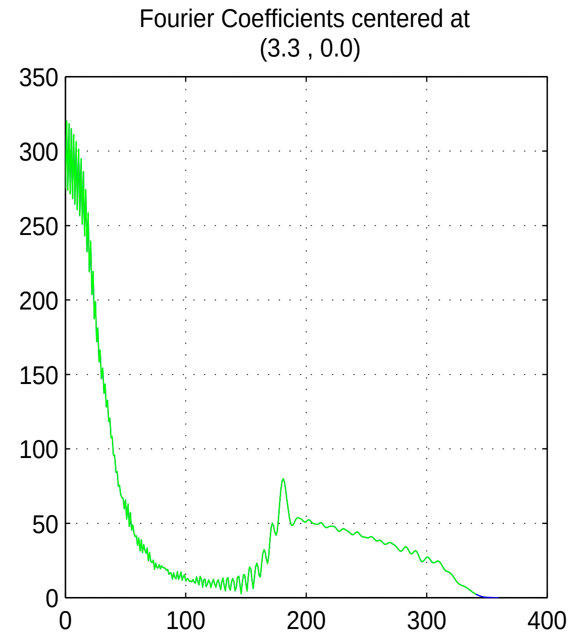
June 21, 2005

Outline

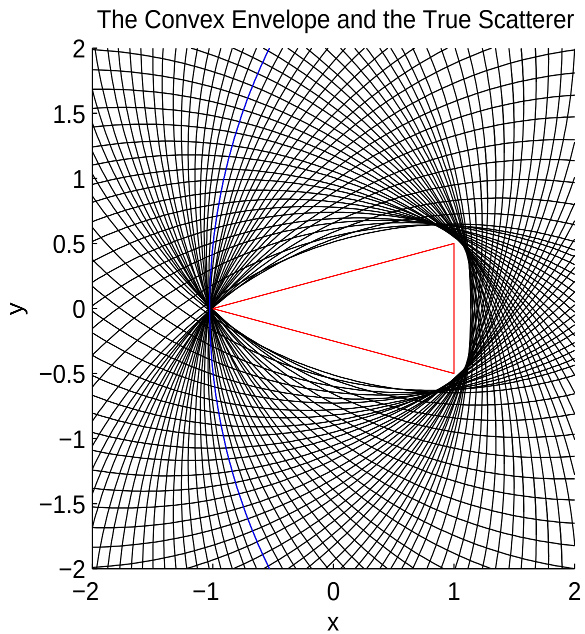
- Background for this algorithm (what problem does it solve?)
- Algorithmic details (how does it solve it?)
- Algorithm results (how well does it solve it?)
- Possible extensions...

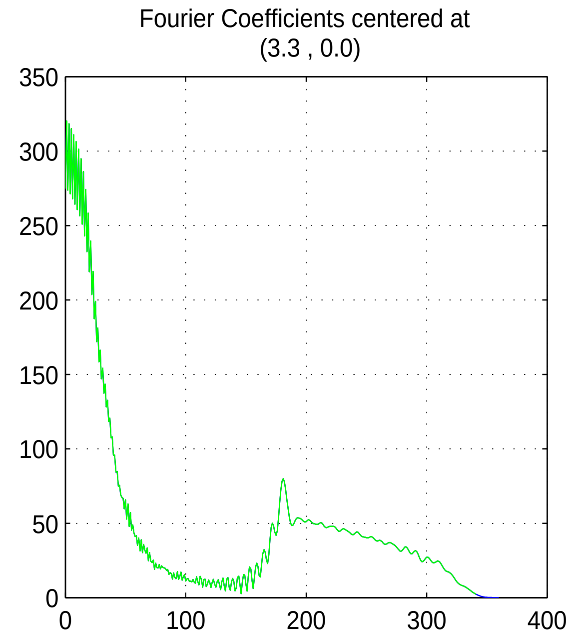
Gross Visualization of Range Characterization





A Close Up View





Some Theorem Details

First, for each $c \in \mathcal{C}$ we seek n^*, so that $R_c = n^*/2k$, such that

$$\left(|\alpha_0^{(c)}|^2 + \sum_{n=1}^{n^*} |\alpha_{-n}^{(c)}|^2 + |\alpha_n^{(c)}|^2 \right) / \sum_{n=-\infty}^{\infty} |\alpha_n^{(c)}|^2 = \tau(c) < 1$$

where

$$\alpha_n^{(c)} = \int_0^{2\pi} e^{-in\phi} e^{ik|c|\cos(\phi - \phi_c)} u_\infty(\phi) d\phi$$

Then, our approximation of the boundary of the scatterer is given by

$$\partial D \approx \bigcap_{c \in \mathcal{C}} B_{R_c(c)}$$

Some Issues With Achieving High Resolution

- Imaging flat surfaces requires moving centers moving to infinity
  - Farther centers require more and more signal data
  - Numerical calculation of the generalized Fourier coeffients becomes difficult

- Range characterization provides a very good “Initial Guess”

- Question: Can we develop a Newton-like method that would converge to the object of interest at a much higher resolution with the same data, and can we reconstruct non-convex objects?

- Answer: Yes, we can both!

Newton's Method in a Nutshell

Recall, the root ξ of some function f such that

$$f(\xi) = v$$

may be found with the aid of the scheme

$$x_{n+1} = x_n - \frac{f(x_n) - v}{f'(x_n)} \quad \Leftrightarrow \quad f'(x_n)(x_{n+1} - x_n) = v - f(x_n)$$

for $n = 1, 2, \cdots$, provided dist(ξ, x_1) is "small."

Newton's Method in Inverse Scattering

Now, we seek the object ∂D associated with some a nonlinear function F such that

$$F(\partial D) = u_\infty$$

Hence, we formulate

$$\left(F'_{\partial D_n} q_n\right)(\hat{x}) = u_\infty(\hat{x}) - \tilde{u}_\infty^{(n)}(\hat{x}), \quad \hat{x} \in \Gamma$$

where

$$\partial D_{n+1} = \partial D_n + q_n \quad \text{and} \quad \lim_{n\to\infty} \|\partial D - \partial D_n\| = 0$$

provided now that $\text{dist}(\partial D, \partial D_0)$ is "small."

An Explanation of this Version of NM

$$\left(F'_{\partial D_n} q_n\right)(\hat{x}) \quad = \quad \text{Fréchet derivative of } F \text{ in the direction } q_n.$$

$$F \quad = \quad \text{Nonlinear forward model.}$$

$$u_\infty(\hat{x}) \quad = \quad \text{Measured data over angle.}$$

$$\tilde{u}_\infty^{(n)}(\hat{x}) \quad = \quad \text{Predicted forward data correspindong to } \partial D_n.$$

Some Further Explanations

The mapping

$$F : \partial D \to u_\infty$$

is highly nonlinear in the objects ∂D, and becomes more so as k increases, but is **injective**!

$$F(\partial D_1 + \partial D_2) \neq F(\partial D_1) + F(\partial D_2)$$

The Point: F may be locally inverted, i.e. our Newton scheme can find the root we seek (the boundary of the sought scatterer) by locally inverting the data we have in hand. (Arbitrarily well for a single frequency and any open aperture!!)

Return to Newton's Method

$$\left(F'_{\partial D_n} q_n\right)(\hat{x}) = u_\infty(\hat{x}) - \tilde{u}_\infty^{(n)}(\hat{x}), \quad \hat{x} \in \Gamma$$

is equivalent to

$$\sum_{m=-M}^{M} B(m, \phi_j) a_m^{(n)} = u_\infty(\phi_j) - \tilde{u}_\infty^{(n)}(\phi_j), \quad j = 1, \cdots, N$$

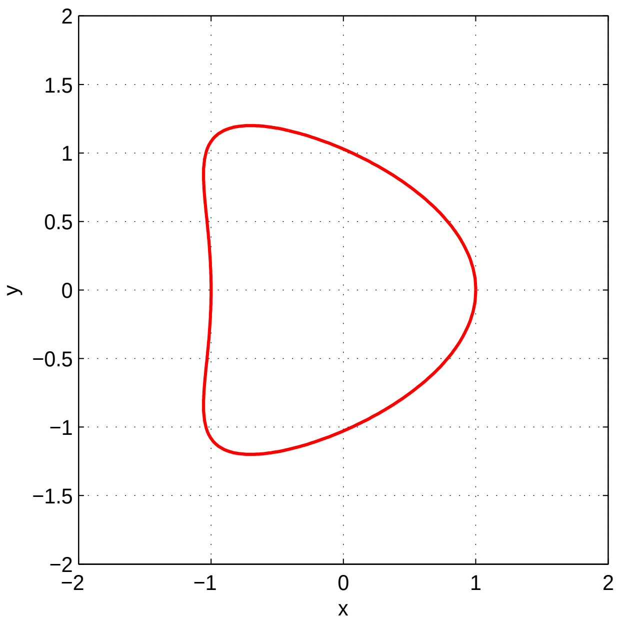
where

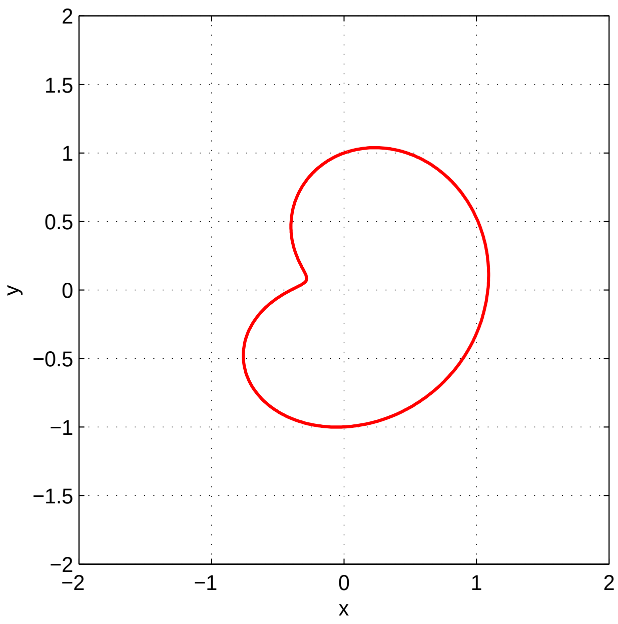
$$q_n(\phi) = \sum_{m=-\infty}^{\infty} a_m^{(n)} e^{im\phi}, \quad a_{-m}^{(n)} = \overline{a}_{-m}^{(n)}, \quad \text{for each } n, m \geq 1$$

Some Important Issues

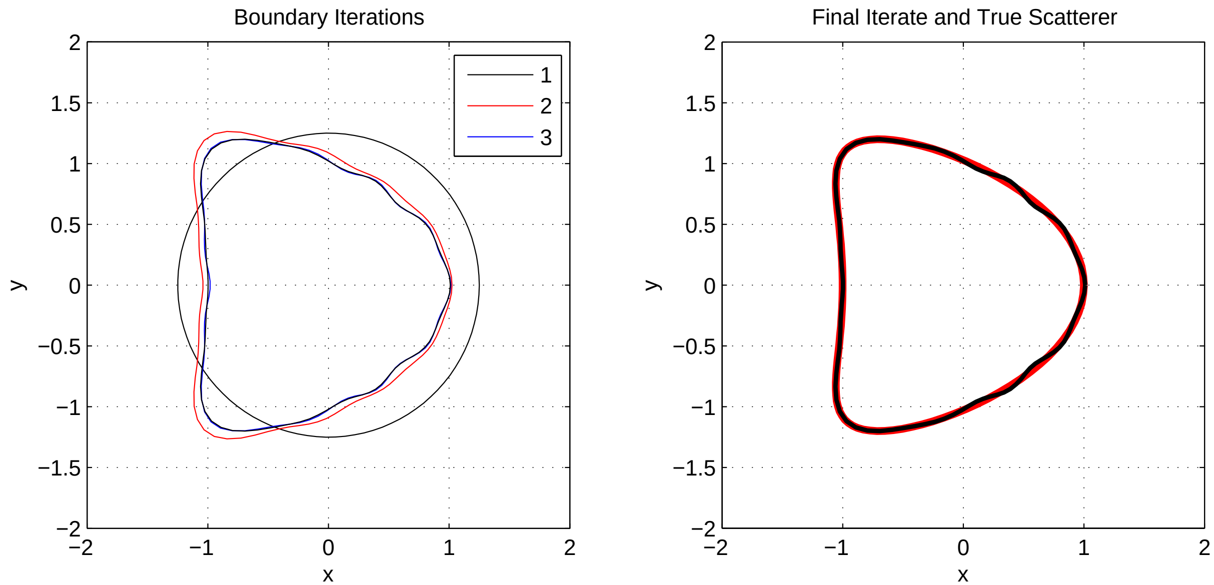
- The secant method is fast at computing derivatives, but results in slower convergence to the root.
- The inversion presented is ill-posed and must be regularized correctly.
- The maximum number of modes one can hope to reconstruct (i.e. the resolution) is dependent upon the wave number k.
  - At large k we get high resolution, but suffer more ill-conditioning in the matrix A
- We *can* do what we have espoused on an arbitrary partial aperture, we need not "see" the target from *all* angles, up to the fundamental ill-posed nature of their determination.

The Test Scatterers: The Kite and Bean

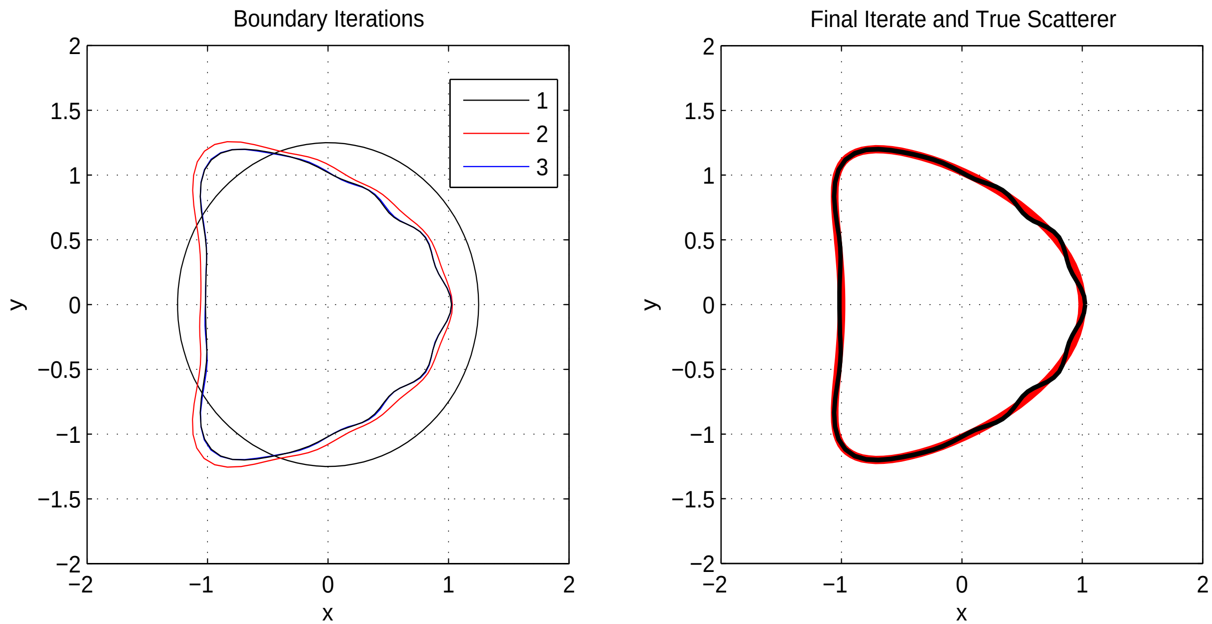




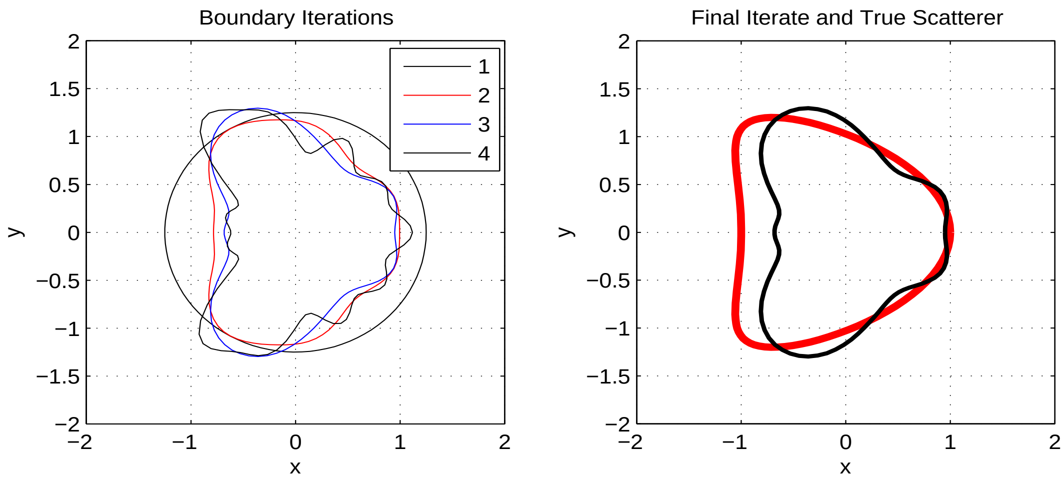
Kite: $k = 1$, $\Gamma = [0, 2\pi]$



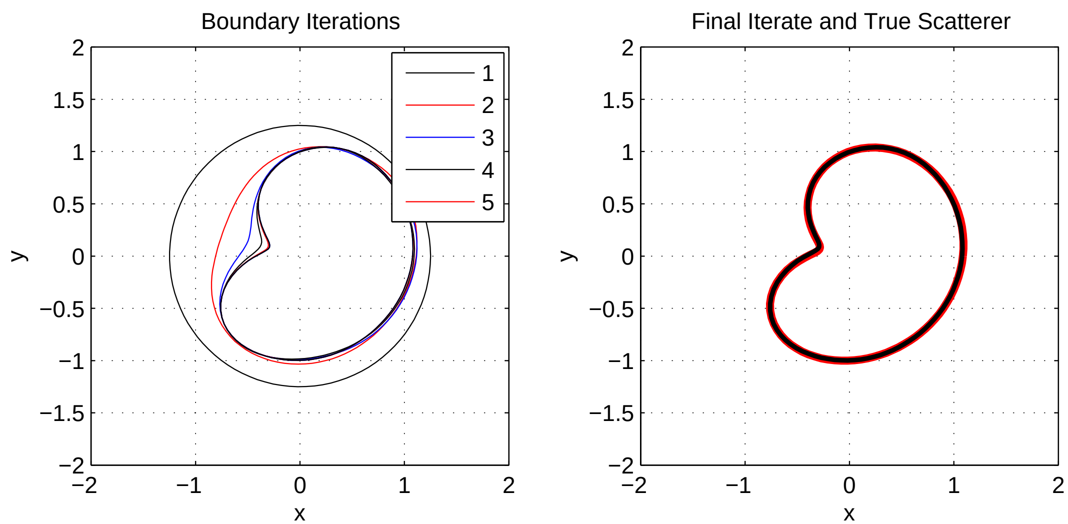
The Kite: $k = 1$, $\Gamma = [0, \pi]$



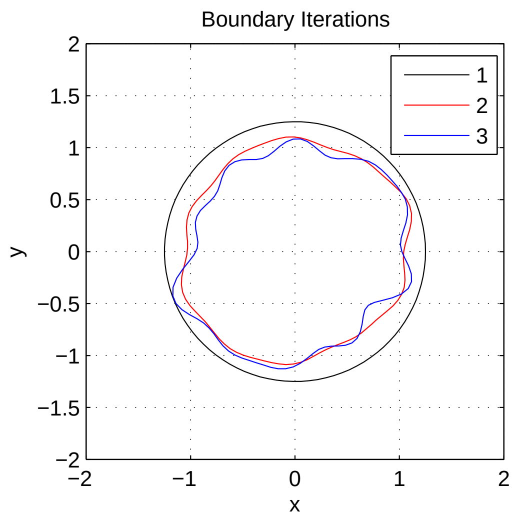
The Kite: $k = 1$, $\Gamma = [0, \pi/16] = [0, 11.25°]$

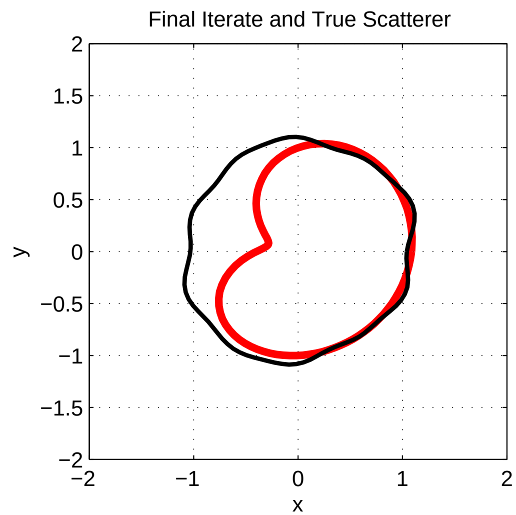


The Bean: $k = 1$, $\Gamma = [3\pi/4, \pi]$

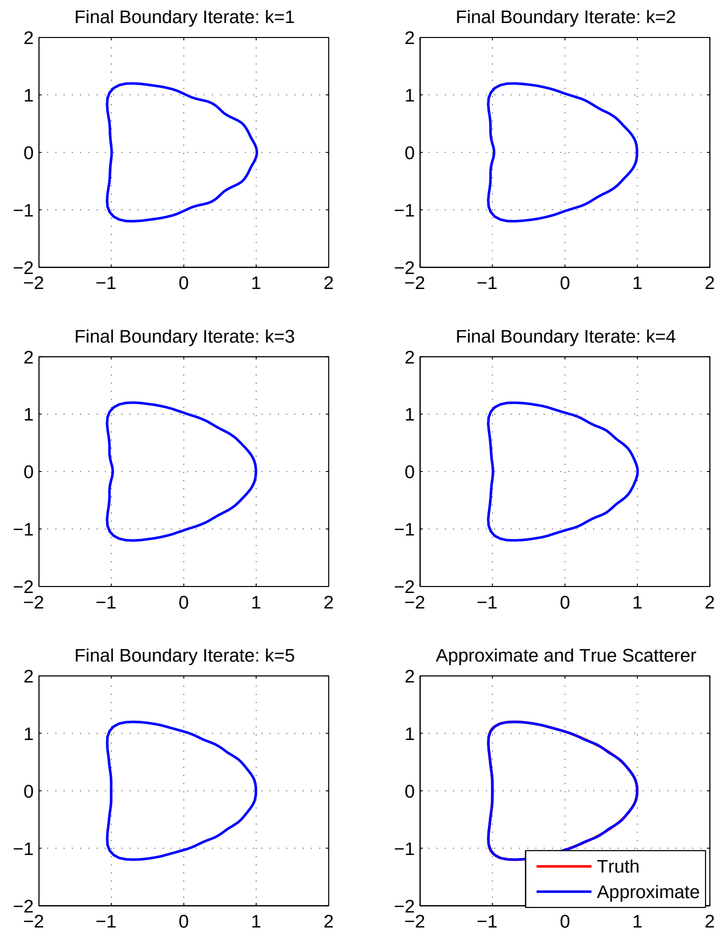


The Bean: $k = 1$, $\Gamma = [0, \pi/8] = [0, 22.5°]$





Muiltiple Sparse Frequencies



Conclusions

- High resolution reconstrutions are achievable with single freqeuncy data.
- The mathematical framework provides for partial aperature imaging.
- Imaging of *non-convex* objects possible!
  - **Only known non-convex imaging method viable in all frequency regimes!!**
- In its present version, this technique is sensitive to noise.
- Boundary roots can be more and more difficult to find as $k \to \infty$.
  - Potential solution: stepped k iterations
- This method provides for prinicipled approaches to data fusion problems.
  - Pure sensor autonomy – data fusion without fusing data!!
  - An architecture with a single transmitter and multiple recievers is a simple extension.