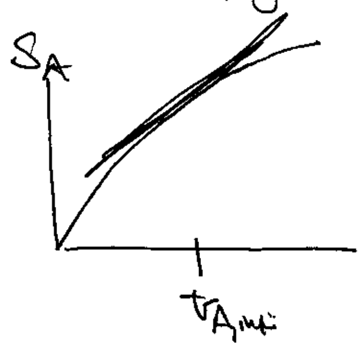
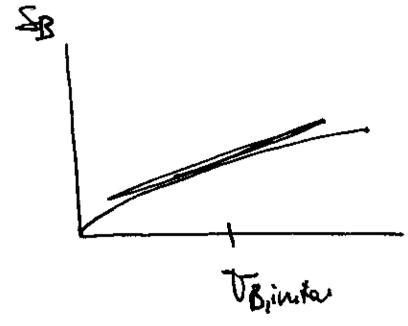
(Prob 3.3)

$$\frac{1}{T} \equiv \frac{\partial S}{\partial U}$$

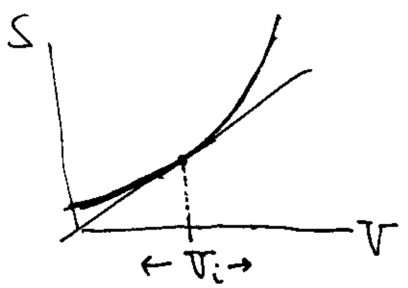



From the pictures

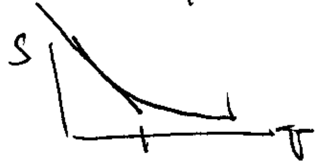
$\frac{\partial S_A}{\partial U_A} > \frac{\partial S_B}{\partial U_B}$ thus heat energy will flow from system B to system A. Since a fixed amount of heat flowing into system A will raise the total entropy of the entire system since the same amount flowing out of B will not be enough to decrease the total energy creation.

(Prob 3.4)




If I were to add energy to this system. the amount of entropy increase in the system would increase, to the point that any amount of ~~heat~~ ~~flow~~ thermal energy flowing into this solid would create much more entropy then that lost by the other body. Thus the entropy of the universe would increase.

I would think that you could put a misanty system up on that looked like




... Not sure

Prob 2.8

$N_A = 10 \quad N_B = 10 \quad q = q_A + q_B = 20.$

(a) $q_A$ can be taking from 0 to 20 → 21 different macrostates

(b) # microstates $\Omega(N,q) = \binom{20+20-1}{20} = \binom{39}{20} = 6.89 \cdot 10^{10}$

(c) To find all the energy in system A would mean

$q_A = 20 \qquad q_B = 0.$

$\Omega_A = \binom{N_A+q_A-1}{q_A} \quad \Omega_B = 1$

$= \binom{29}{20} = 1.00 \cdot 10^7.$

$\therefore$ Prob All energy is in system A $= \dfrac{\Omega_A \cdot \Omega_B \text{(This instance)}}{\sum_{\text{All instances}} \Omega_A \Omega_B}$

$= \dfrac{1.0 \cdot 10^7}{6.89 \cdot 10^{10}} = 1.45 \cdot 10^{-4}.$

(d) The prob of finding ½ energy in sys A ⇒ $q_A = 10$ & $q_B = 10$

$\therefore$ Prob $\dfrac{\Omega_A \Omega_B \text{(This instance)}}{\sum_{\text{All inst}} \Omega_A \Omega_B} = \dfrac{\binom{N_A+q_A-1}{q_A}\binom{N_B+q_B-1}{q_B}}{\Sigma}$

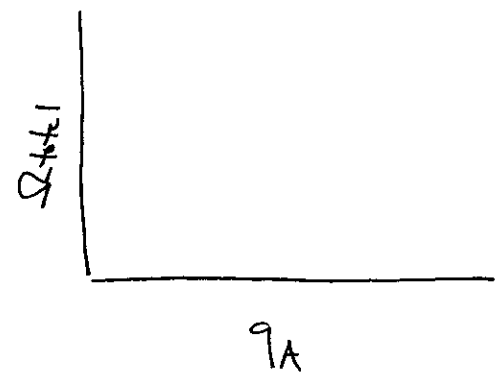
$= \dfrac{\binom{19}{10}^2}{6.89 \cdot 10^{10}} = .12$

(e) If the energy started entirely in one or the other solid one would have ~~to~~ ~~[illegible]~~ ~~[illegible]~~ ~~[illegible]~~ a $1/10^4$

$\frac{1}{10000}$ chance of finding All the energy in one or the other solid.

Prob 2.9 $q = q_A + q_B = 6$ $N_A = N_B = 3$.

| $q_A$ | $\Omega_A = \binom{q_A+N_A-1}{q_A}$ | $\Omega_B = \binom{q_B+N_B-1}{q_B}$ | $\Omega_{total} \equiv \Omega_A \Omega_B$ |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |
| $\vdots$ | | | |
| 6 | | | |




B Pseudo code

See Matlab code ...

Prob 2.10

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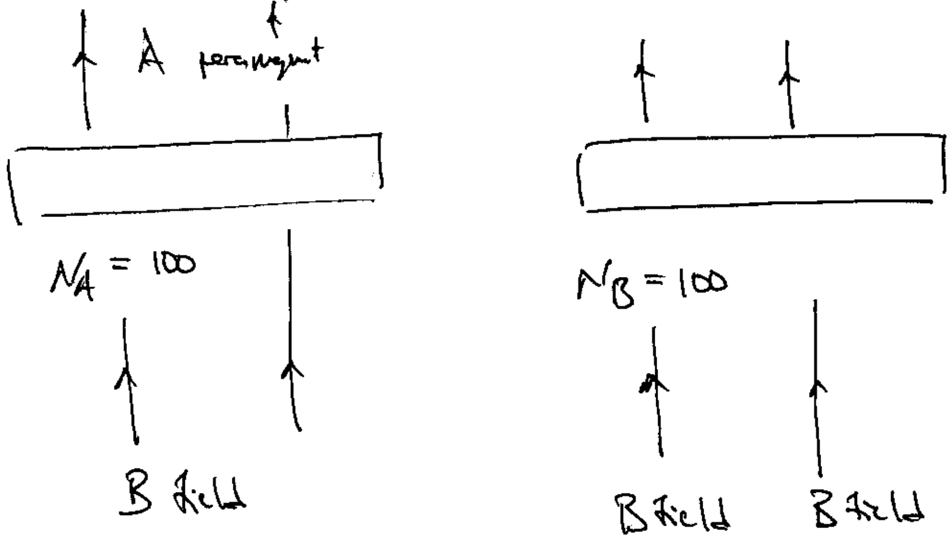



$q_{Total} = 80.$

See matlab ...

Prob 2.15 $N! \doteq N^N e^{-N}\sqrt{2\pi N}$

$\underline{\underline{N=50}}$

$50! = \cdots$

$(50)^{50} e^{-50}\sqrt{2\pi(50)} = \cdots$

$$\left.\frac{N^N e^{-N}\sqrt{2\pi N}}{N!}\right|_{N=50} = .99 \;\text{☺}$$

$\ln N! \cong N\ln N - N$

$\ln(N!) = \cdots$

$N\ln N - N = \cdots$

$$\frac{N\ln N - N}{\ln(N!)} \cong .98 .$$

Prob 2.16 1000 coins.

$$Pr = \frac{\binom{1000}{800}}{2^{1000}}$$

choose 800 ~~the~~ coins from the 1000 to be heads.

$2^{1000}$ total # of outcomes for 3 flip

this is not! it is ~~by~~ $\ln(N!)$ !!

$$Pr \neq \frac{1000!}{800!\,500!}$$

$$Pr = \frac{\frac{1000!}{800!\,500!}}{2^{1000}} \cong \frac{(1000\ln(1000) - 1000)}{(800\ln(800) - 800)(500(\ln 500) - 500)}\bigg/ 2^{1000}$$

$\cancel{P_r} \doteq \cancel{\ln P_r = \ln(1000!)}$

$$P_r = \frac{1000!}{2^{1000}\, 500!\, 500!} = \frac{1000!}{2^{1000}(500!)^2}$$

$$\ln P_r = \ln(1000!) - 2000\ln 2 - 2\ln(500!) \cancel{+ \ln(500!)}$$

$$\cong \cancel{1000\ln(1000) - 1000 - 2000\ln 2}$$

$$\cancel{- 500\ln(500) + 500 - 500\ln 500}$$

$$\cong 1000\ln(1000) - 1000 - 2000\ln 2$$

$$- 2(500\ln(500) - 500)$$

$$= 1000\ln(1000) - \cancel{1000} - 2000\ln 2$$

$$- 1000\ln(500) + \cancel{1000}$$

$$= 1000(\ln(1000) - \ln(500)) - 2000\ln 2$$

$$= -693.1$$

$P_r = 9.33 \cdot 10^{-302}$ !! Can this be correct??

(b) $$P_r = \frac{\binom{1000}{400}}{2^{1000}} = \frac{\frac{1000!}{600!\,400!}}{2^{1000}}$$

$$\ln P_r = \ln(1000!) - \ln(600!) - \ln(400!) - 1000\ln 2$$

$$\ln Pr \cong 1000 \ln(1000) - \cancel{1000}$$

$$- 600 \ln 600 + \cancel{600}$$

$$- 400 \ln(400) + \cancel{400}$$

$$- 1000 \ln 2$$

$$= - 20.1$$

$$Pr = 1.7 \cdot 10^{-9}.$$

Prob 2.17

In the low temperature limit. $q \ll N$.

$$\Omega(N,q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{q!\,(N-1)!} = \frac{(N+q)!\,(N)}{(N+q)\,q!\,N!}$$

$$= \left(\frac{N}{N+q}\right)\cdot\frac{(N+q)!}{q!\,N!} \cong \frac{(N+q)!}{q!\,N!}$$

Now

$$\ln\Omega = \ln(N+q)! - \ln q! - \ln N!$$

$$\cong (N+q)\ln(N+q) - \cancel{(N+q)} - q\ln q + \cancel{q} - N\ln N + \cancel{N}$$

$$= (N+q)\ln(N+q) - q\ln q - N\ln N.$$

$$= (N+q)\ln\left[N\left(1+\frac{q}{N}\right)\right] - q\ln q - N\ln N$$

$$= (N+q)\left[\ln N + \ln\left(1+\frac{q}{N}\right)\right] - q\ln q - N\ln N$$

$$\cong (N+q)\left[\ln N + \frac{q}{N}\right] - q\ln q - N\ln N$$

$$= N\cancel{\ln N} + q\ln N + q + \frac{q^2}{N} - q\ln q - N\cancel{\ln N}$$

$$= q + \frac{q^2}{N} + q\ln\left(\frac{N}{q}\right) \cong q + q\ln\left(\frac{N}{q}\right) = q - q\ln\left(\frac{q}{N}\right)$$

for $q \ll N$ (low temperature limit)

$$\Omega(N,q) = e^{q} e^{-q\ln\left(\frac{q}{N}\right)} = \left(\frac{N}{q}\right)^{q} e^{q} = \left(\frac{Ne}{q}\right)^{q}$$

Prob 2.18

$$\Omega(N,q) = \binom{N+q-1}{q} = \frac{(N+q-1)!}{(N-1)!\,q!}$$

$\left\{ N! \cong N^{N} e^{-N} \sqrt{2\pi N} \right\}$

$$= \frac{(N+q)!\,N}{N!\,(N+q)\,q!} = \left(\frac{N}{N+q}\right)\frac{(N+q)!}{N!\,q!}$$

$$\cong \begin{matrix}\text{large } N \\ \text{large } q\end{matrix} = \left(\frac{N}{N+q}\right)\frac{(N+q)^{N+q}\,\cancel{e^{-(N+q)}}\sqrt{2\pi(N+q)}}{N^{N}\,\cancel{e^{-N}}\sqrt{2\pi N}\;q^{q}\,\cancel{e^{-q}}\sqrt{2\pi q}}$$

$$= \left(\frac{N}{N+q}\right)\frac{(N+q)^{N+q}\sqrt{N+q}}{N^{N} q^{q}\sqrt{2\pi}\sqrt{Nq}}$$

$$= \frac{\sqrt{N}}{\sqrt{N+q}}\;\frac{(N+q)^{N}(N+q)^{q}}{N^{N}\quad q^{q}\quad\sqrt{2\pi}\sqrt{q}}$$

$$= \frac{\left(1+\frac{q}{N}\right)^{N}\left(1+\frac{N}{q}\right)^{q}}{\sqrt{2\pi q\left(\frac{N+q}{N}\right)}}$$

Prob 2.19

$\Omega(N_\uparrow) = \dfrac{N!}{N_\uparrow!\,N_\downarrow!}$ v.s. $\Omega(N,q) = \binom{q+N-1}{q}$

$N = N_\uparrow + N_\downarrow$

$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \binom{N_\uparrow + N_\downarrow}{N_\uparrow}$

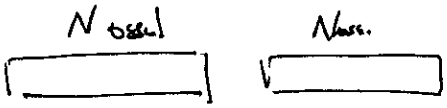
This is identical to $\Omega(N,q)$ where $N_\downarrow \equiv N-1$.

Thus in the limit where $N_\downarrow \ll N \iff N \ll q$ ?.

But basically the expressions are equivalent in some limit & obtaining $\Omega \approx \left(\frac{Ne}{N_\downarrow}\right)^{N_\downarrow}$ should not be surprising.

Prob 2.22

(a)




total energy = $2N$.

the energy in the 1st solid can be $0, 1, 2, \cdots, 2N$.

or $2N+1$ possible macrostates

(b) Prob 2.18 gives $\Omega(N,q) \cong \dfrac{\left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N}{\sqrt{2\pi q(q+N)/N}}$ for each solid + for the combined system:

For the combined system one gets

$\cancel{\Omega_A(N,} \quad \cancel{\Omega(2N, 2N)}$

$$\Omega(2N, 2N) = \cancel{\frac{\left(\frac{3N}{2N}\right)^{2N}\left(\frac{3N}{2N}\right)^{2N}}{\sqrt{2\pi(2N)(3N/2N)}}} \quad \frac{\left(\frac{4N}{2N}\right)^{2N}\left(\frac{4N}{2N}\right)^{2N}}{\sqrt{2\pi(2N)\left(\frac{4N}{2N}\right)}}$$

$$= \frac{2^{2N} \cdot 2^{2N}}{\sqrt{2\pi \cdot 4N}} = \frac{2^{4N}}{\sqrt{8\pi N}} \quad \checkmark$$

(c) From Prob 2.18 w/ $\Omega_1(N,N) \cdot \Omega_2(N,N) = \Omega(N,N)$

$$= \left(\frac{\left(\frac{2N}{N}\right)^N \left(\frac{2N}{N}\right)^N}{\sqrt{2\pi N\left(\frac{2N}{N}\right)}}\right)^2 = \left(\frac{2^{2N}}{\sqrt{2\pi \cdot 2 \cdot N}}\right)^2$$

$$= \frac{2^{4N}}{4\pi N} \quad \checkmark$$

**Prob 2.20** $x = \frac{q}{2\sqrt{N}}$

$\therefore \Delta x = \frac{q}{\sqrt{N}}$ so if $N \triangleq 10^{20}$.

$\Delta x \cong \frac{q}{10^{10}}$. What about the $q$?

If $10^{20}$ were to fit into 10 cm (width of a page)

our peak would have to be $\propto \frac{1}{10^{10}}$ of that thus.

peak width would be $\frac{10\,cm}{10^{10}}$.

**Prob 2.21** For

$$\Omega = \left(\frac{eq_A}{N}\right)^N \left(\frac{eq_B}{N}\right)^N = \left(\frac{e}{N}\right)^{2N} (q_A q_B)^N$$

$$Z = \frac{q_A}{q} \qquad 1-Z = \frac{q-q_A}{q} = \frac{q_B}{q}.$$

$$\therefore \quad \Omega = \left(\frac{e}{N}\right)^{2N} q^{2N} \left(Z(1-Z)\right)^N = \left(\frac{e}{N}\right)^{2N} \frac{q^{2N}}{4^N} \left(4Z(1-Z)\right)^N$$

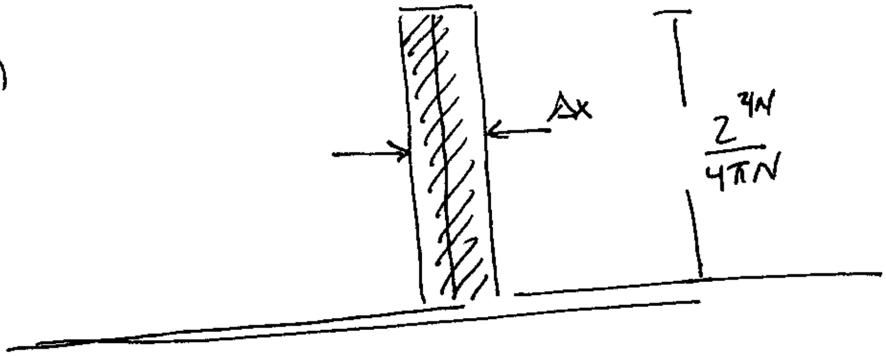
$$Z \in (0,1) \qquad = C_{N,q} \left(4Z(1-Z)\right)^N$$

Pseudocode: $Z =$ linspace(0,1,1000); ns = [1, 100 ....]

for N: 1: length(ns)

$yz_{n4} = (4Z(1-Z))^N$ figure

(d)




$$A_{total} = \frac{2^{4N}}{\sqrt{8\pi N}} = \Delta x \cdot \frac{2^{4N}}{4\pi N}$$

$$\Delta x = \frac{4\pi N}{\sqrt{8\pi N}} = \frac{\sqrt{4\pi N}}{\sqrt{2}}$$

For $N \cong 10^{20}$ $\quad \Delta x \propto 10^{10}$

$$\therefore \quad \frac{\Delta x}{A_{total}} \propto \frac{\left(\frac{\sqrt{4\pi N}}{\sqrt{2}}\right)}{\left(\frac{2^{4N}}{\sqrt{8\pi N}}\right)} = \cancel{\frac{\sqrt{4\pi N}}{\sqrt{2}}} \cdot \cancel{\frac{\sqrt{8\pi N}}{2^{4N}}}$$

$$= \frac{\sqrt{4\pi N}}{\sqrt{2}} \cdot \frac{\sqrt{8\pi N}}{2^{4N}}$$

? Don't see how this is extremely small.

Prob 2.23

(a) $N = 10^{23}$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow}$$

$$\Omega(N/2) = \binom{N}{N/2} = \frac{N!}{((\frac{N}{2})!)^2}$$

$$\ln \Omega = \ln N! - 2\ln(\tfrac{N}{2})!$$

$$= N\ln N - N - 2\left(\tfrac{N}{2}\ln(\tfrac{N}{2}) - \tfrac{N}{2}\right)$$

$$= N\ln N - \cancel{N} - N\ln(\tfrac{N}{2}) + \cancel{N}$$

$$= N\left(\ln \frac{N}{N/2}\right) = N\ln 2$$

$$\therefore \quad \Omega \cong e^{(\ln 2)N}$$

So $\ln \Omega = \underline{\underline{N\ln 2}}$

(b) $1\,yr = 3.15 \cdot 10^7 s$

$$\#_{\text{stats}} = (10^9 \cdot 1/s)(10 \cdot 10^9\,yr)(3.15 \cdot 10^7 s/yr) = 3.15 \cdot 10^{26}$$

$\log \#\text{states} = 61.0$

(c) No. If the age of Universe is $10^{10}$s one cannot expect to wait much longer than that.

Prob 2.24

(a) $$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,(N-N_\uparrow)!}$$

$$\ln \Omega(N_\uparrow) \cong N\ln N - N - (N_\uparrow \ln N_\uparrow - N_\uparrow) - \left((N-N_\uparrow)\ln(N-N_\uparrow) - (N-N_\uparrow)\right)$$

$$= N\ln N - \cancel{N} - N_\uparrow \ln N_\uparrow + \cancel{N_\uparrow} - (N-N_\uparrow)\ln(N-N_\uparrow) + \cancel{(N-N_\uparrow)}$$

~~Thus when~~ $\cancel{N_\uparrow} \cong \frac{N}{2}$

~~[illegible]~~

$$\ln \Omega = N\ln N - N_\uparrow \ln N_\uparrow - (N-N_\uparrow)\ln(N-N_\uparrow)$$

when $N_\uparrow = \frac{N}{2}$

$$\ln \Omega = N\ln N - \frac{N}{2}\ln\left(\frac{N}{2}\right) - \left(\frac{N}{2}\right)\ln\left(\frac{N}{2}\right) = N\ln N - N\ln\left(\frac{N}{2}\right)$$

$$= N\left(\ln \frac{N}{N/2}\right) = N\ln 2$$

(b)

let $x \equiv N_\uparrow - \frac{N}{2}$, then $N_\uparrow = x + \frac{N}{2}$

$$\ln \Omega = N\ln N - \left(x+\frac{N}{2}\right)\ln\left(x+\frac{N}{2}\right) - \left(N-x-\frac{N}{2}\right)\ln\left(N-x-\frac{N}{2}\right)$$

$$= N\ln N - \left(x+\frac{N}{2}\right)\ln\left(x+\frac{N}{2}\right) - \left(\frac{N}{2}-x\right)\ln\left(\frac{N}{2}-x\right)$$

. . . .

$$\Omega = N^N \, N_\uparrow^{-N_\uparrow} \, N_\downarrow^{-N_\downarrow}$$

$$= \frac{N^N}{N_\uparrow^{N_\uparrow} \, N_\downarrow^{N_\downarrow}}$$

let $N_\uparrow = x + \frac{N}{2}$ then $N_\downarrow = N - N_\uparrow = N - x - \frac{N}{2} = \frac{N}{2} - x.$

$$\Omega = \frac{N^N}{\left(x + \frac{N}{2}\right)^{\left(x + \frac{N}{2}\right)} \left(\frac{N}{2} - x\right)^{\left(\frac{N}{2} - x\right)}}$$

$$= \frac{N^{x + \frac{N}{2}} \cdot N^{-x + \frac{N}{2}}}{\left(x + \frac{N}{2}\right)^{x + N/2} \left(\frac{N}{2} - x\right)^{\frac{N}{2} - x}}$$

$$= \left[\frac{1}{\frac{x}{N} + \frac{1}{2}}\right]^{x + \frac{N}{2}} \left[\frac{1}{\frac{1}{2} - \frac{x}{N}}\right]^{\frac{N}{2} - x}$$

$$= \left[\frac{1}{\frac{1}{2} + \frac{x}{N}}\right]^{x} \cdot \left[\frac{1}{\frac{1}{2} + \frac{x}{N}}\right]^{\frac{N}{2}} \cdot \left[\frac{1}{\frac{1}{2} - \frac{x}{N}}\right]^{\frac{N}{2}} \cdot \left[\frac{1}{\frac{1}{2} - \frac{x}{N}}\right]^{-x}$$

$$= \left(\frac{\frac{1}{2} - \frac{x}{N}}{\frac{1}{2} + \frac{x}{N}}\right)^{x} \cdot \left(\frac{1}{\frac{1}{4} - \frac{x^2}{N^2}}\right)^{N/2}$$

If $x = 0$

$$\Omega(0) = (4)^{N/2} = 2^N. \; ?$$ Not the same.

(c)

$$P_1 = \frac{\binom{10^6}{510{,}000}}{2^{10^6}}$$

$$P_2 = \frac{\binom{10^6}{510.}}{2^{10^6}}$$

**Prob 2.25**

(a) By flipping coins let every head denote a step forward & a tail denote a step backwards.

Then a string of T's & H's denotes a path in space.

. . .

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Prob 2.25

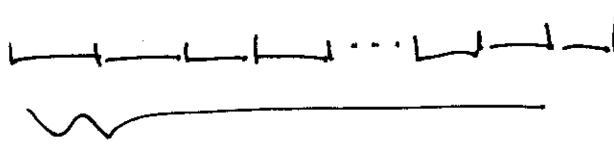
(a) Consider a string of L's + R's w/ L's representing steps to the left + R's representing steps to the right

Most likely location corresponds to the point w/ maximal probability.

$P_n$ = prob n units to the right =

$\Rightarrow$ must have n more R's in my string of N (R's + L's) than L's.

$$= \frac{X}{2^N}$$



N - spots

N - string is ÷ up into 2
(1) of length n. +
(2) of length $N-n$ ← has to be evenly ÷ into L's + R's

$\binom{N}{k}$ ← have k Rights + N-k lefts
$\Rightarrow$ total distance $+k-(N-k)$
$2k-N$.

$$P_n = \frac{\binom{N-n}{\frac{N-n}{2}}}{2^N} = \frac{(N-n)!}{\left(\left(\frac{N-n}{2}\right)!\right)^2 2^N}$$

ignoring issues of even/odd
N, n + if $\frac{N-n}{2}$ is fractional or not.

$$\ln P_n = \ln (N-n)! - 2\ln\left(\left(\tfrac{N-n}{2}\right)!\right) - N\ln 2$$

$$\overset{\uparrow}{\cong} (N-n)\ln(N-n) - \cancel{(N-n)} - 2\left(\tfrac{N-n}{2}\right)\ln\left(\tfrac{N-n}{2}\right) + \cancel{2\left(\tfrac{N-n}{2}\right)} - N\ln 2$$

Stirling's approximation

$$= (N-n)\left\{\ln(N-n) - \ln\left(\tfrac{N-n}{2}\right)\right\} - N\ln 2$$

$$= (N-n)\left\{\ln\left(\frac{N-n}{\frac{N-n}{2}}\right)\right\} - N\ln 2$$

$$= (\cancel{N}-n)\ln 2 - \cancel{N\ln 2} = -n\ln 2$$

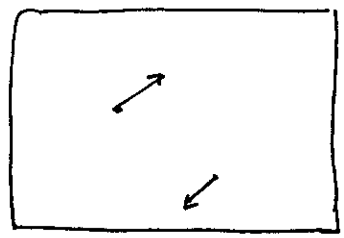
$P_n \cong e^{-n\ln 2}$ max of this fn is at $n \equiv 0$

$\Rightarrow$ your expected location is $\underline{0}$.

(b) would it not be 0?

(c) ?

(Prob 2.26)



$$\Omega_1 \propto A \cdot A_p$$

$$U = \frac{1}{2} m (v_x^2 + v_y^2) \Rightarrow p_x^2 + p_y^2 = 2mU. \qquad \Delta x \cdot \Delta p_x \cong h$$

$$\Omega_1 = \frac{A A_p}{h^2}.$$ for 1 gas molecule in flat land:

w/ 2 molecules

$$\Omega_2 = \frac{1}{2} \frac{A^2}{h^4} \times \text{(Area of 4 dimensional hyper sphere of radius } \sqrt{2mU}\text{)}$$

$\vdots$

$$\Omega_N = \frac{1}{N!} \frac{A^N}{h^{2N}} * \underbrace{\text{(Area of 2N hyper sphere of radius } \sqrt{2mU}\text{)}}_{\frac{2\pi^N r^{2N-1}}{(N-1)!}\Big|_{r=\sqrt{2mU}}}$$

$$\Omega_N = \frac{1}{N!}\frac{A^N}{h^{2N}} * \frac{2\pi^N (2mU)^{N-1/2}}{(N-1)!}$$

$$\Omega_N \cong \frac{1}{N!}\frac{A^N}{h^{2N}}\frac{2\pi^N}{N!}(2mU)^N \cdot \cong \cancel{\frac{A^N \pi^N}{(N!)^2 h^{2N}}}$$

$$\cong \frac{1}{(N!)^2}\frac{A^N}{h^{2N}}\pi^N (2$$

$$= \frac{(\pi A\, 2mU)^N}{(N!)^2\, h^{2N}} = \frac{(2\pi m A U)^N}{(N!)^2 h^{2N}}$$

Prob 2.27

$$\Omega_{total} = f(N)^2 (V_A V_B)^N (U_A U_B)^{3N/2}$$

$$\Omega(U,V,N) = f(N) V^N U^{3N/2}$$

$$\Omega(U, .99V, N) = (.99)^N \Omega(U,V,N)$$

For $N = 100,\ 10^4,\ 10^{23}$

$(.99)^N = .366\ ,\ 2.2 \cdot 10^{-44}\ ,\ 0.$

i.e. w/ 100 molecules 36 x's out of 100 one finds the state searched for
$10^4$ " $\frac{2.2}{10^{44}}$ 2 out of $10^{44}$ times one finds the

Prob 2.28 $\Omega = 52!$

# of arrangements $= 52!$

$$S = k \ln \Omega = k \ln 52! = k(156.36)$$

156.36 w/ $k = 1.381 \cdot 10^{-23}$ J/K $S = 2159 \cdot 10^{-21}$ J/K.

How determine it significant?

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$$\Omega_N \cong \frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}\left(\sqrt{2mU}\right)^{3N}$$

$$S = k\ln\Omega = k\Big[-\ln N! + N\ln V - 3N\ln h + \frac{3N}{2}\ln\pi - \ln\left(\frac{3N}{2}!\right) + \frac{3N}{2}\ln(2mU)\Big]$$

$$\cong k\Big[-N\ln N + N + N\ln V - 3N\ln h + \frac{3}{2}N\ln\pi - \left(\frac{3N}{2}\right)\ln\left(\frac{3N}{2}\right) + \frac{3N}{2} + \frac{3N}{2}\ln(2mU)\Big]$$

Stirling's relation

$\{\ln N! = N\ln N - N$

$$= Nk\Big[-\ln N + 1 + \ln V - 3\ln h + \frac{3}{2}\ln\pi - \frac{3}{2}\ln\left(\frac{3N}{2}\right) + \frac{3}{2} + \frac{3}{2}\ln(2mU)\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\frac{\pi^{3/2}}{h^3}\frac{1}{\left(\frac{3N}{2}\right)^{3/2}}\cdot(2mU)^{3/2}\right) + \frac{5}{2}\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\left(\frac{\pi}{h^2}\frac{2}{3N}\cdot 2mU\right)^{3/2}\right) + \frac{5}{2}\Big]$$

$$= Nk\Big[\ln\left(\frac{V}{N}\left(\frac{4\pi}{3N}\frac{mU}{h^2}\right)^{3/2}\right) + \frac{5}{2}\Big]$$ eq 2.49 ✓

Prob 2,29 $S = k\ln\Omega$

$N_A = 300$ $N_B = 200$ $q_{total} = 100$ ← this is the same system as on pg 59.

$$\Omega(N,q) = \binom{q+N-1}{q}$$

$\Omega_{min} = 2.8 \cdot 10^{81}$ $\Omega_{max} = 6.9 \cdot 10^{114}$

$S_{least} = 187.5$ $\Omega_{most} = 264.42$

For entropy over long time scales $\Omega = 9.3 \cdot 10^{115} \to S = 267.02$

Prob 2,30

(a) $\Omega_N = \dfrac{2^{4N}}{\sqrt{8\pi N}}$ $S = k\ln S = k(4N)\ln 2 - \frac{k}{2}\ln(8\pi N)$

$=$

If $N = 10^{23}$ $S = (1.381 \cdot 10^{-23}\,J/k)(4)(10^{23})\ln 2$

$- \dfrac{(1.381 \cdot 10^{-23}\,J/k)}{2}\ln(8\pi \cdot 10^{23})$

$= 3.8\ J/k$

(b) $\underbrace{\Omega_N}_{\text{most likely}} = \dfrac{2^{4N}}{4\pi N}$

$S = k(4N)\ln 2 - k\ln(4\pi N) = 3.8\ J/k.$

(c) One seems to get the same answer ~~as~~ ~~as~~ in ~~cat~~ ~~case~~ either case

(d) The ~~usual~~ usual argument is that we have created entropy by inserting the partition, This creation of entropy would more than compensate for the difference observed.

Prob 2.31 See notes for pg 77

Prob 2.32 From Prob 2.26 $\Omega_N = \dfrac{(2\pi m A U)^N}{(N!)^2 h^{2N}}$

$$S = k \ln \Omega = (N \ln(2\pi m A U) - 2\ln(N!) - 2N\ln(h))k$$

$$\cong (N\ln(2\pi m A U) - 2N\ln N + 2N - 2N\ln(h))k$$

$$= Nk\left\{ \ln(2\pi m A U) - 2\ln N + 2 - 2\ln(h)\right]$$

$$= Nk\left\{ \ln\left(\frac{2\pi m A U}{N^2 h^2}\right) + 2\right]$$

$$= Nk\left\{ \ln\left(\frac{A}{N}\left(\frac{2\pi m U}{N h^2}\right)\right) + 2\right].$$

$$U = \frac{3}{2} nRT$$

$$U = \frac{3}{2}(1)(8.314\ J/K\cdot mol)(300\ K) = 3741.0\ J.$$

$$\left\{ m = \frac{4}{6.022\cdot 10^{23}} = 6.64\cdot 10^{-27}\ kg \right.$$

$$V = \frac{nRT}{P} = \frac{(8.314\ J/K)(300\ K)}{10^5\ Pa} = 2.49\cdot 10^{-2}\ m^3$$

Then in Sackur-Tetrode eq:

$S = Nk \left[ \ln \right.$

$$\frac{2.49\cdot 10^{-2}}{6.022\cdot 10^{23}} \left( \frac{4\pi\cdot 6.64\cdot 10^{-27}\cdot (3741.0)}{3\cdot 6.022\cdot 10^{23}\cdot (6.626\cdot 10^{-34})^2} \right)^{3/2}$$

$$= 3.24\cdot 10^5 \xrightarrow[\ln]{} 12.68 +$$

$$\therefore S = kN \left[ \underbrace{\ln(\quad) + \frac{5}{2}}_{12.68 + 2.5 \ = \ 15.18} \right]$$

$$S = k(9.14\cdot 10^{24}) = 126.3\ J/K$$

(Prob 2.33) Argon is a noble gas $f=3$

$$U = N \cdot f\left(\tfrac{1}{2}kT\right) \qquad N = 6.022 \cdot 10^{23}$$

$$PV = kNT$$

$$V = \frac{kNT}{P}$$

$$m = \frac{39.948 \text{ g/mole}}{6.022 \cdot 10^{23} \text{ atoms/mole}} = 6.63 \cdot 10^{-23} \text{ g/atom.}$$

$$\equiv \frac{M}{N_A}$$ what is M called?

$$S(P,T,N) = Nk\left[\ln\left(\frac{k\cancel{N}T}{P\cancel{N}}\left(\frac{4\pi M}{3\cancel{N}N_A h^2}\,\frac{\cancel{N}\cdot f}{2}kT\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{kT}{P}\,\frac{2\pi M}{3N_A h^2}\,f\,kT\right)^{3/2} + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{2f\pi M}{3N_A h^2}\,\frac{(kT)^2}{P}\right)^{3/2} + \frac{5}{2}\right]$$

$$= 6.022 \cdot 10^{23} \cdot (1.381 \cdot 10^{-23} \text{ J/K})\left[\underbrace{\ln\left(\frac{2 \cdot 3 \cdot \pi(39.948 \cdot 10^{-3})}{3(6.022 \cdot 10^{23})(6.626 \cdot 10^{-34})^2}\,\frac{(1.381 \cdot 10^{-23} \cdot 300)^2}{10^5}\right)^{3/2}}_{-1.3} + \frac{5}{2}\right]$$

$S =$ ~~80~~ 9.97 J/K ? Should be larger .. Did you take the 3/2 power?

My guess would be that the molecular mass of Argon is about 10×s that of Helium.

Prob 2.34

quasi-static $\Rightarrow$ $\Delta W = p\Delta V$

isothermal expansion $\Rightarrow$ T does not change

$$\Delta U = \Delta Q + \Delta W$$

$$\therefore \Delta Q = \Delta U - \Delta W.$$
$$= \Delta U + p\Delta V.$$

$$\Delta S = \frac{\partial S}{\partial V}\Delta V + \frac{\partial S}{\partial U}\Delta U$$

$$= Nk \frac{1\left(\frac{1}{N}(\cdots)^{3/2}\right)}{\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right)}\Delta V + Nk\frac{\cancel{\frac{V}{N}}(\quad)^{1/2}\cdot\frac{3}{2}\left(\frac{4\pi m}{3Nh^2}\right)}{\left(\cancel{\frac{V}{N}}(\quad)^{3/2}\right)}\Delta U$$

$$= \frac{Nk}{V}\Delta V + \frac{Nk\,\frac{3}{2}\left(\frac{4\pi m}{3Nh^2}\right)}{\left(\frac{4\pi m U}{3Nh^2}\right)}\Delta U$$

$$= \frac{P}{T}\Delta V + \frac{3}{2}Nk\frac{1}{U}\Delta U. \qquad U = f\cdot N\cdot\tfrac{1}{2}kT$$

$$= \frac{P}{T}\Delta V + \frac{3}{2}\frac{\cancel{N}k}{\frac{f}{2}\cancel{N}kT}\Delta U \qquad f = 3 \text{ for a monatomic gas}$$

$$= \frac{P}{T}\Delta V + \frac{\Delta U}{T}$$

$$= \frac{P\Delta V + \Delta U}{T} = \frac{\Delta Q}{T} \checkmark.$$

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Prob 2.35

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$ Sackur-Tetrode eq

$T = 300K$ $\Rightarrow N = \frac{PV}{kT}$

$P = 10^5$ Pa

holding density fixed I will take as ~~mass~~ density, actually in stat. Physics ~~mass~~ density usually means molar/atom density i.e. $\left(\frac{V}{N}\right)$.

$U = N \cdot f \cdot \frac{1}{2}kT$

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m}{3Nh^2}\frac{Nf}{2}kT\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi f m}{3\cdot 2\cdot h^2}P\frac{V}{N}\right)\right) + \frac{5}{2}\right]$$

$PV = NkT$

$P\left(\frac{V}{N}\right) = kT$

$P =$

$$S = N\cdot k\left[\ln\left(\frac{V}{N}\left(\frac{2\pi m \cdot f k}{3h^2}\cdot T\right)^{3/2}\right) + \frac{5}{2}\right]$$

S is negative for a fixed $\frac{V}{N}$ when

$$\ln\left(\frac{V}{N}\left(\frac{2\pi}{3h^2}m\cdot f k\cdot T\right)^{3/2}\right) < -\frac{5}{2}$$

$$\Rightarrow T = \frac{\left(\frac{e^{-2.5}}{\frac{V}{N}}\right)^{2/3}}{\left(\frac{2\pi}{3h^2}m\cdot f\cdot k\right)}$$

$$\frac{V}{N} = \frac{kT}{P} = \frac{(1.381\cdot10^{-23}\ J/K)(300\ K)}{10^5\ Pa} = 4.14\cdot10^{-26}\ m^3$$

$$\frac{J}{Pa} = \frac{\cancel{kg}\ m^{\cancel{2}}/\cancel{s^2}}{\frac{\cancel{kg}\ m/\cancel{s^2}}{m^2}} = m^3$$

$$m_{He} = \frac{4\ g/mole}{6.022\cdot10^{23}\ atoms/mol} = 6.64\cdot10^{-24}\ g/atom$$

$$\cancel{\frac{V}{N}\left(\frac{2\pi(6.64\cdot10^{-24}\cdot10^{-3}\ kg/atom)}{3(6.626\cdot10^{-34}\ J\cdot s)^2}\right)}$$

$$\cancel{\frac{V}{N}\cdot\frac{2\pi}{3h^2}m\cdot T\cdot k = (4.14\cdot10^{-26}\ m^3)}\left(\frac{\cancel{2\pi(6.64\cdot10^{-24}\cdot10^{-3}\ kg/atom)}\cdot\cancel{3}(1.381\cdot10^{-23}\ J/K)}{\cancel{3}(6.626\cdot10^{-34}\ J\cdot s)^2}\right)$$

$$= \cancel{5.4\cdot10^{-8}}$$

$T =$ ~~$1.5\cdot10^6$. what am I doing wrong~~?

$$\frac{2\pi}{3h^2}m\cdot T\cdot k = 1.31\cdot10^{18}$$

Then $T = 1.20\cdot10^{-2}\ K$.

Prob 2.36

$$S_{monatomic\ gas} = Nk\left[\ln(\ \ ) + \frac{5}{2}\right]$$

$$S_{einst\ solid} = Nk\left[\ln(q/N) + 1\right]$$

$S_{Book}$

$1kg = 1000g$

$$N_{Book} = \frac{1000g}{12\ g/mol}(6.022 \cdot 10^{23}\ atoms/mol)$$

$$N_{moose} = \frac{400 \cdot 10^3 g}{18\ g/mol} \cdot 6.022 \cdot 10^{23}\ atoms/mol.$$

$$N_{sun} = \frac{2 \cdot 10^{30} \cdot 10^3 g}{2\ g/mol} \cdot 6.022 \cdot 10^{23}\ at/m$$

$$N_A \cdot k = 8.316\ J/K$$

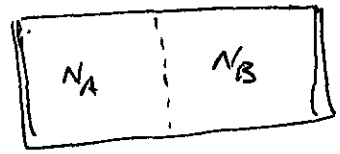
$\therefore S_{Book} = 693.0\ J/K$

$S_{moose} = 1.84 \cdot 10^5\ J/K$

$S_{sun} = \cancel{10^{33}}\ 8.31 \cdot 10^{33}\ J/K$

Prob 2.37

let: $x = \frac{N_A}{N}$, $1-x = \frac{N_B}{N}$




$\ni N_A + N_B = N.$

$$\Delta S_A = N_A k \ln\left(\frac{V_f}{V_i}\right) = N_A k \ln 2.$$

$$\Delta S_B = N_B k \ln 2$$

$$\Delta S = N_A k \ln 2 + N_B k \ln 2$$

$$= kN\left[x \ln 2 + (1-x)\ln 2\right]$$

... What you have calculated is for ... this is somehow for ? Don't see why this is wrong.

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$S_{total} = S_A + S_B$$

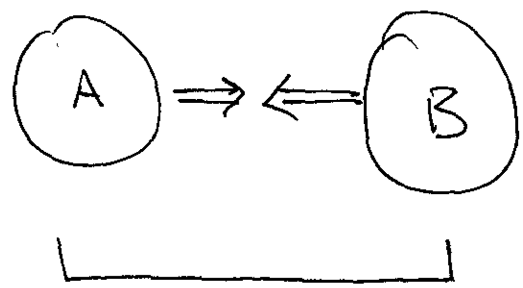
After adding $N_B$ molecules of molecule B.

$$= N_A k\left[\ln\left(\frac{V}{N_A}\left(\frac{4\pi m U}{3N_A h^2}\right)^{3/2}\right) + \frac{5}{2}\right] + N_B k\left[\ln\left(\frac{V}{N_B}\left(\frac{4\pi m U}{3N_B h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$\Rightarrow \Delta S_{total} = \ldots$$

How do this calculation?

Prob 2.37



N. total # of particles of type A + B

w/ $N_A$ + $N_B = N - N_A$.    $x \equiv \frac{N_B}{N}$ ; $\frac{N_A}{N} = 1-x$.

For an ideal gas the Sackur-Tetrode eq holds

$$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

Assuming that the Volume doubles $V_i$ w/o change in $U$.

So $$S_A^i = N_A k\left[\ln\left(\frac{V_i}{N_A}\left(\frac{4\pi m_A U}{3N_A h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$S_B^i = N_B k\left[\ln\left(\frac{V_i}{N_B}\left(\frac{4\pi m_B U}{3N_B h^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

For simplicity lets assume $m_A = m_B \equiv m$

Then $$S_A^f = N_A k\left[\ln\left(\frac{2V_i}{N_A}\left(\frac{4\pi m U}{3N_A h^2}\right)\right) + \frac{5}{2}\right]$$

$$S_B^f = N_B k \left[ \ln\left( \frac{2V_i}{N_B} \left( \frac{4\pi m U}{3N_B h^2} \right) \right) + \frac{5}{2} \right]$$

So $\Delta S = S_A^f + S_B^f - S_A^i - S_B^i$

$= \Delta S_A + \Delta S_B$

For $\Delta S_A = N_A k \ln 2$

---

Think volumes are $\propto$ to $\quad PV = kNT \quad V \propto N.$

$\frac{V}{N} =$

By Sackur - Tetrode eq

$S = Nk \left[ \ln \left( \quad \ldots \quad ? \right. \right.$

(Prob 2.38) The # of states available for a mixed solid is

$$\binom{N}{N_A} \; ?$$

$$\ln\binom{N}{N_A} = \ln\left(\frac{N!}{(N-N_A)!\,N_A!}\right) = \ln N! - \ln((N-N_A)!) - \ln N_A!$$

$$\cong N\ln N - \cancel{N} - (N-N_A)\ln(N-N_A) + (\cancel{N-N_A}) - N_A\ln N_A + \cancel{N_A}$$

$$= N\ln N - (N-N_A)\ln(N-N_A) - N_A\ln N_A$$

$N-N_A = N_B.$

$$= N\ln N - N_B\ln N_B - N_A\ln N_A$$

$= $ ~~[illegible]~~

$$= N\left[\ln N - \frac{N_B}{N}\ln N_B - \frac{N_A}{N}\ln N_A\right]$$

$$= N\left[\ln N - (1-x)\ln\left(\frac{N_B}{N}\cdot N\right) - x\ln\left(\frac{N_A}{N}\cdot N\right)\right]$$

$$= N\left[\ln N - (1-x)\ln(1-x) - (1-x)\ln N - x\ln x - x\ln N\right]$$

$$= -N\left[x\ln x + (1-x)\ln(1-x)\right]$$

$$\therefore \Delta S = k\ln\Omega = -Nk\left[x\ln x + (1-x)\ln(1-x)\right]$$

$S_A = k \ln \Omega(N_A)$

$S_B = k \ln \Omega(N_B)$

Assuming that all particles are of the same type the mixture would have a total of $N$

~~$\Omega(N_A)$~~ = I would argue that when combined v.s. separated the combined system would have more degrees of freedom & that these additional degrees of freedom could be categorized as

... ?

$$\Delta S_{mixing} = k \ln \binom{N}{N_A} = k \ln \left( \frac{N!}{N_A!\, N_B!} \right)$$

$$\cong k \left[ N \ln N - \cancel{N} - N_A \ln N_A + \cancel{N_A} - N_B \ln N_B + \cancel{N_B} \right]$$

$$= Nk \left[ \ln N - \frac{N_A}{N} \ln N_A - \frac{N_B}{N} \ln N_B \right]$$

$$= Nk \left[ \left( \frac{N_A}{N} + \frac{N_B}{N} \right) \ln N - \frac{N_A}{N} \ln N_A - \frac{N_B}{N} \ln N_B \right]$$

$$= Nk \left[ \frac{N_A}{N} \ln \frac{N}{N_A} + \frac{N_B}{N} \ln \left( \frac{N}{N_B} \right) \right]$$

$$= Nk \left[ x \ln x + (1-x) \ln (1-x) \right]$$

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Prob 2.39

$PV = nRT \qquad PV = NkT.$

$$U = N \cdot 7 \cdot \tfrac{1}{2} kT \qquad (T,P)$$

$$= \tfrac{7}{2} Nk \cdot T = \tfrac{7}{2} nRT = 3700\,J \text{ (same as before)}$$

$$\& \; V = .025\,m^3.$$

From pg 78 $S_{Helium} = 126\ J/K$ $\qquad m = \frac{4}{6.022 \cdot 10^{23}} = 6.64 \cdot 10^{-27}\,kg$

Assuming distinguishable molecules we get:

$$S = Nk \left[ \ln \left( V \left( \frac{4\pi m U}{3Nh^2} \right)^{3/2} \right) + \frac{3}{2} \right]$$

$$= 1(8.314\ J/K) \left[ \ln \left( .025\,m \left( \frac{4\pi \cdot 6.64 \cdot 10^{-27} \cdot 3700}{3 \cdot 6.02 \cdot 10^{23} (6.626 \cdot 10^{-34}\,J \cdot s)^2} \right)^{3/2} \right) + \frac{3}{2} \right]$$

$$= 8.314\ J/K \cdot 68.92$$

$= \cancel{687.2}\ J/K$    this is larger? What did I do wrong? ...

573.06 J/K    maybe that is correct since distinguishable molecules would have more states available to them. & thus

I would think more micro states.

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Problem 2.39

indist molecules:

(1) $$S = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$\left\{U = N f \cdot \frac{1}{2}kT = \frac{3}{2}NkT\right\}$$

$$PV = NkT$$

$$\frac{V}{N} = \frac{kT}{P}$$

$N = 6.022 \cdot 10^{23}$

$T = 300° K$

$P = 10^5$ Pa

distinguishable molecules

(2) $$S = Nk\left[\ln\left(V\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

w/ $U = \frac{3}{2}NkT$

$$S_{ind} = Nk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m \frac{3}{2}NkT}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right]$$

$$S_{dist} = Nk\left[\ln\left(V\left(4\pi m\right.\right.\right.$$

— — —

$$S_{ind} = Nk\left[\ln\left(V\left(\quad\right)^{3/2}\right) - \ln N + \frac{3}{2} + 1\right]$$

$$S_{ind} = Nk\left[\ln\left(V\left(\quad\right)^{3/2}\right) + \frac{3}{2}\right]$$

$$\cancel{\quad} \quad Nk\left[1 - \ln N\right]$$

$$\Rightarrow S_{ind} = S_{dist} + Nk\left[1 - \ln N\right]$$

Now $$S_{dist} = Nk\left[\ln\left(V\left(\frac{4\pi m \cdot \cancel{N} \frac{3}{2} kT}{3\cancel{N} h^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

$$\Rightarrow S_{dist} = Nk\left[\ln\left(V\left(\frac{4\pi m \, 3 \cdot kT}{6h^2}\right)^{3/2}\right) + \frac{3}{2}\right]$$

Now for He

$$m = \frac{4.002\ \text{g/mol}\ (1\,\text{mol})}{6.022 \cdot 10^{23}\ \text{atoms}} =$$

Prob 2,40

(a) The once orderly salt crystals are now a solute in the soup.

(b)

⋮

In all cases one can view the state before as being more orderly, in that the atoms were arranged to have a distinctive purpose before the process & afterwards did not.

Except for ~~t~~ cutting down a tree which might be more orderly than before, ...

Prob 2,41 With any building man creates or organization that he does the break down of molecular energy in the food he eats more than compensates for the organization that comes about from the building that he does.

Prob 2.40

In all the microscopic everyday processes the amount of disorder measured by the # of available states has increased.

Prob 2.41

The same sort of things that come into place in 2.40 show here

Prob 2.42

(a) Assuming the black hole has mass M

constants involved M, c, Gravitational const

$$[F] = N = \frac{kg \cdot m}{s^2} \qquad F = -\frac{G M_1 M_2}{r^2}$$

Unit wise

$$\frac{kg \cdot m}{s^2} = \frac{(kg)^2 [G]}{m^2} \rightarrow [G] = \frac{m^3}{(kg) \cdot s^2}$$

Now given M, c, & G form a unit of length

$$\cancel{M^{P_1} c^{P_2} G^{P_3} = m} \qquad [M]^{P_1} [c]^{P_2} [G]^{P_3} = m$$

$$(kg)^{p_1}\left(\frac{m^{p_2}}{s^{p_2}}\right)\frac{m^{3p_3}}{(kg)^{p_3}\,s^{2p_3}} = m$$

$$\Rightarrow (kg)^{p_1-p_3}\cdot\frac{1}{s^{(p_2+2p_3)}}\cdot m^{3p_3+p_2} = m$$

$$p_1 - p_3 = 0 \quad , \quad p_2 + 2p_3 = 0 \quad , \quad 3p_3 + p_2 = 1$$

$$p_1 = p_3 \qquad p_2 + 2p_1 = 0 \quad , \quad 3p_1 + p_2 = 1$$

$$3p_1 + p_2 = 1$$

$$2p_1 + p_2 = 0.$$

$$p_1 = \frac{\begin{vmatrix}1 & 1\\ 0 & 1\end{vmatrix}}{\begin{vmatrix}3 & 1\\ 2 & 1\end{vmatrix}} \qquad p_2 = \frac{\begin{vmatrix}3 & 1\\ 2 & 0\end{vmatrix}}{\begin{vmatrix}3 & 1\\ 2 & 1\end{vmatrix}}$$

$$p_1 = \frac{1}{(3-2)} \qquad p_2 = \frac{-2}{1} = -2$$

$$= +1$$

$\therefore \quad \frac{MG}{c^2} \approx r$

$$\frac{kg \cdot m}{s^2}\frac{m^2}{kg^2} = \frac{m^3}{s^2 kg}$$

$$r = \frac{(2 \cdot 10^{30}\,kg)(6.673 \cdot 10^{-11}\frac{N \cdot m^2}{kg^2})}{(3 \cdot 10^{8}\,m/s)^2}$$

$$= 1.4 \cdot 10^{3}\,m$$

(b) ~~if~~

if $\ln N$ is simply a large # while $N$ is a very large # or [illegible]

the calculation of $\frac{S}{k} = N \ln N$.

(c) To maximize $S$ one would need to increase $N$ to as large an extent as possible, this would involve using (to keep the mass constant) as small ~~#~~ ~~per~~ w.r.t. mass particles as possible using near massless photons

$$Mc^2 = \text{total energy} = N \cdot e_{photon} \qquad e_{photon} = \cancel{h\nu} = \cancel{\frac{hc}{\lambda}} = hf = \frac{hc}{\lambda}$$

$$= N \cdot \frac{hc}{\lambda} \cong N \cdot \frac{hc}{r_{B.H.}}$$

$$\Rightarrow \quad N = \frac{Mc\,r_{B.H.}}{h}$$

Then $\quad S_{BH} \cong k \cdot N_{BH} = k\,\frac{Mc}{h}\frac{MG}{c^2}$

$$S_{BH} = \frac{kGM^2}{hc}$$

d) $$\frac{(1{,}38\cdot10^{-23}\,J/K)(2\cdot10^{30}\,kg)^2(6{,}673\cdot10^{-11}\,N\cdot m^2/kg^2)}{(6{,}626\cdot10^{-34}\,J\cdot s)(3\cdot10^{8}\,m/s)}$$

$$= 1{,}85\cdot10^{52}\ J/K$$

Prob 3.1

$$T = \frac{\partial U}{\partial S}\Big|_{N,V} \cong \frac{U_{i+1} - U_{i-1}}{S_{i+1} - S_{i-1}}$$

when $q_A = 1$

$$T_A = \frac{2\epsilon - 0\cdot\epsilon}{10.7\cdot k - 0\cdot k} = \frac{2}{10.7}\frac{\epsilon}{k} = 1.8\cdot 10^{-1}\ \epsilon/k$$

$$T_B = \frac{98\epsilon - 100\epsilon}{185.3k - 187.5k} = \frac{-2\epsilon}{-2.2k} = \frac{2}{2.2}\ .909\ \epsilon/k$$

when $q_A = 60$

$$T_A = \cancel{\frac{7.7\cdot 10^{69}\epsilon - 2.2\cdot 10^{68}\epsilon}{160.9k - 157.4k}} = \cancel{\frac{7.4\cdot 10^{69}\epsilon}{3.5k}} = 2.1\cdot 10^{69}\frac{\epsilon}{k}\ ?$$

$$= \frac{59\epsilon - 61\epsilon}{157.4k - 160.9k} = \frac{-2\epsilon}{-3.5k} = \frac{2}{3.5}\frac{\epsilon}{k} = .5714\ \epsilon/k$$

$$T_B = \frac{39\epsilon - 40\epsilon}{103.5k - 107k} = \frac{-\epsilon}{-3.5k} = .285\ \epsilon/k$$

Assuming $\epsilon = .1\,eV$

$k = 8.617\cdot 10^{-5}\ eV/k$

so $\frac{\epsilon}{k} = \frac{.1\ eV}{8.617\cdot 10^{-5}\ eV/k} = 1.16\cdot 10^{3}\ k$

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Prob 3,2

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{N,V}$$

$A \cong B \qquad B \cong C \Rightarrow A \cong C.$

$\parallel \qquad \parallel$

$$\frac{1}{T_A} = \frac{1}{T_B} \qquad \frac{\partial S_B}{\partial U_B} = \frac{\partial S_C}{\partial U_C}$$

$\parallel$

$$\frac{\partial S_A}{\partial U_A} = \frac{\partial S_B}{\partial U_B}$$

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(Prob 3.5) Prob 2.17 gives

$$\Omega(N,q) = \left(\frac{Ne}{q}\right)^q \qquad \text{w/ } U = \epsilon q$$

$$\rightarrow q = \frac{U}{\epsilon}$$

$$\therefore \Omega(N,U) = \left(\frac{Ne}{U/\epsilon}\right)^{U/\epsilon}$$

$$= \left(\frac{Ne\epsilon}{U}\right)^{U/\epsilon}$$

$$S = S(N,U) = k \ln \Omega(N,U) = \frac{kU}{\epsilon} \ln\left(\frac{Ne\epsilon}{U}\right)$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N} = \frac{k}{\epsilon} \ln\left(\frac{Ne\epsilon}{U}\right) + \frac{kU}{\epsilon} \cdot \frac{1}{\left(\frac{Ne\epsilon}{U}\right)} \cdot \frac{Ne\epsilon}{U^2}(-1)$$

$$= \frac{k}{\epsilon} \ln\left(\frac{Ne\epsilon}{U}\right) + -\frac{k}{\epsilon}$$

$$\left(\frac{1}{T} + \frac{k}{\epsilon}\right)\frac{\epsilon}{k} = \ln\left(\frac{Ne\epsilon}{U}\right)$$

---

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N} = \left.\frac{\partial S}{\partial q}\right|_{V,N} \cdot \frac{dq}{dU}$$

$$= \frac{1}{\epsilon} \left.\frac{\partial S}{\partial q}\right|_{V,N.}$$

$$\therefore \frac{\epsilon}{T} = \left.\frac{\partial S}{\partial q}\right|_{V,N}$$

$$U = \epsilon q$$

$$\frac{dq}{dU} = ?$$

$$1 = \epsilon \cdot \frac{dq}{dU}$$

$$\frac{dq}{dU} = \frac{1}{\epsilon}$$

$$S(N,q) = k \ln\left(\frac{Ne}{q}\right)^q$$

$$= qk \ln\left(\frac{Ne}{q}\right)$$

$$= qk\left[\ln(Ne) - \ln q\right]$$

$$\frac{\partial S}{\partial q} = k\left[\ln(Ne) - \ln q\right] + qk\left(-\frac{1}{q}\right)$$

$$= k\left[\ln(Ne) - \ln q\right] - k$$

$$\frac{\epsilon}{kT} = \ln(Ne) - \ln q - 1$$

$$\frac{\epsilon}{kT} + 1 - \ln(Ne) = -\ln q$$

$$q = \exp\left\{\ln(Ne) - \frac{\epsilon}{kT} - 1\right\}$$

$$= Ne \cdot e^{-1} e^{-\epsilon/kT} = N e^{-\epsilon/kT}$$

$$\Rightarrow U = N\epsilon e^{-\epsilon/kT}$$

**Prob 3.6**

Assuming $\Omega = U^{Nf/2}$

Then $S = k \ln \Omega = k \frac{Nf}{2} \ln U$.

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{V,N} = \frac{kNf}{2U}$$

$$\Rightarrow \quad U = kT \cdot \frac{Nf}{2}$$

$$U = f \cdot N \cdot \frac{kT}{2}$$

if $U \ll 1$ $\quad T \ll 1$ ... ?

**Prob 3.7**

$$S_{BH} = \frac{8\pi^2 G M^2 k}{hc} \qquad \text{if} \quad Mc^2 = U. \quad M^2 = \frac{U^2}{c^4}$$


a miserly system

$$\therefore \quad S_{BH} = \frac{8\pi^2 G U^2 k}{hc^5}$$

$$\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{V,N} = \frac{8\pi^2 G \cdot 2U \cdot k}{hc^5} = \frac{16\pi^2 G U \cdot k}{hc^5}$$

$$\frac{1}{T} = \frac{16\pi^2 G (Mc^2) k}{hc^5} = \frac{16\pi^2 G M k}{hc^3}$$

$$= \frac{16\pi^2 (6.673\cdot 10^{-11}\ N\cdot m^2/kg^2)(2\cdot 10^{30}\ kg)(1.381\cdot 10^{-23}\ J/k)}{(6.626\cdot 10^{-34}\ J\cdot s)(3\cdot 10^{8}\ m/s)^3} = 1.6\cdot 10^{7}\ k^{-1}$$

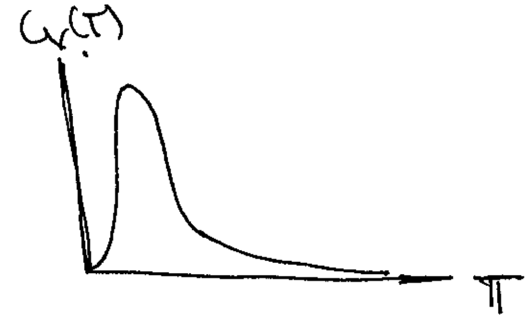
$T = 6.14 \cdot 10^{-8}\ k.$ ? seems low.

Prob 3.8

$$U = N\epsilon e^{-\epsilon/kT}$$

$$C_V \equiv \left.\frac{\partial U}{\partial T}\right|_{V,N} = N\epsilon e^{-\epsilon/kT}\left(-\frac{\epsilon}{kT^2}(-1)\right)$$

$$= \frac{N\epsilon^2}{kT^2} e^{-\epsilon/kT}$$




Prob 3.9 each atom can the have 2 states CO + OC

thus we have a 2 state paramagnetic

so $\Omega(N) = \binom{N}{2} = \frac{N(N-1)}{2}$

so $S = k\ln\Omega = k\ln\left(\frac{N(N-1)}{2}\right)$ w/ $N \approx 10^{23}$

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Prob 3.10

$T_c = 0°C$ $\qquad T_L = 25°C$ $\qquad T = 0 + 273 = 273\,K$

(a) $$\Delta S = +\frac{Q}{T} = \frac{L_{ice \to H_2O} \cdot m}{T}$$

$L = 333\ J/g$ melting ice.

$$\Delta S = \frac{(333\,J)(30\,g)}{273\,K} = 36.59\ J/K.$$

(b) Heat capacity of $H_2O$ is $\qquad dS = \frac{C_V\,dT}{T}$

$1\ cal/K$ so for $30\,g$

$1\,cal = 4.186\,J$

$$dS = \frac{(30\ cal/K)\,dT}{T} =$$

$$dS = \frac{30(4.186)\ J/K\ dT}{T}$$

So $$\Delta S = \int_{T_i}^{T_f} C\,\frac{dT}{T} = 30(4.186)\ln\left(\frac{T_f}{T_i}\right)$$

$$= 30(4.186)\ln\left(\frac{273+25}{273+0}\right)$$

$$= 11.0\ J/K$$

(c) $\Delta S = -\dfrac{L \cdot m}{T_{kitchen}}$

$= -\dfrac{(333\ J/g)(30g)}{(273+25K)} = -33.52\ J/K$

For the 2nd step of the heating process the $H_2O$ warming up. I would assume that the kitchen can be taken as a heat bath

So

$$\Delta S_{Kitch} = -\int_{T_i}^{T_f} \frac{C_v\,dT}{T} \cong \frac{-1}{T_{kit}} \int_{0°C}^{25°C} 30(4.186)\ J/K \cdot dT$$

$$= -\frac{30 \cdot 4.186 \cdot 25}{273+25} = -10.5\ J/K.$$

So in units of J/K

$\Delta S_{total} = +36.59 + 11 - 33.52 - 10.5 = 3.53\ J/K$ increased

Which is what I would expect

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(Prob 3.11)

A $\Leftrightarrow$ B

50 liters  25 liters

55°C  10°C

For this problem we will Ignore the entropy of mixing + only consider the entropy of heat flow

The $H_2O$ at 55°C will give up heat to the 10°C $H_2O$.

The equilibrium temperature will be when w/ $=$ $C = 1 \, cal/gK$

1 liter = 1000 cc = 1000 g $H_2O$.

$$C \cdot m_1 \Delta T_1 + C m_2 \Delta T_2 = 0$$

$$\Rightarrow \quad (50 \cdot 1000 \cdot 10^{-3}\,kg)(T_f - 55°C) + (25 \cdot 1000 \cdot 10^{-3}\,kg)(T_f - 10°C) = 0$$

$$\Leftrightarrow \quad 50(T_f - 55) + 25(T_f - 10) = 0$$

$$2T_f - 2 \cdot 55 + T_f - 10 = 0$$

$$3T_f - 100 = 0$$

$$T_f = 33.3°C.$$

Then the amount of entropy that flows from Hot to cold would be

$$\Delta S_A = + \int_{55°C}^{33.3°C} \frac{C_V \cdot dT}{T} = + \int_{55°C}^{33.3°C} \frac{m C_V \, dT}{T} = m C_V \ln T_F / T_I$$

$$= 50 \cdot (1000\,g)(1 \, cal/g \cdot k)\left(\frac{4.186\,J}{1\,cal}\right) \ln\left(\frac{273 + 33.3}{273 + 55}\right) = -1.432 \cdot 10^4 \, J/k$$

$$\Delta S_B = 25(1000g)(1\ cal/g\cdot K)\left(\frac{4{,}186\ J}{1\ cal}\right)\ln\left(\frac{273+33{,}3}{273+10}\right)$$

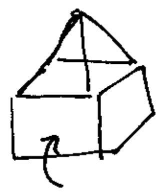
$$= 8{,}27\cdot 10^3\ J/K$$

$$\Delta S_{total} = \Delta S_A + \Delta S_B = -6{,}04\cdot 10^3\ J/K$$

How can this be negative? It I am to calculate the entropy of mixing How am I to do it?

Prob 3.12

$T_{outside} = 0°C = 273 k$



$T_{inside} = 20°C = 293 k$

$\Delta S_{inside} = -\frac{Q}{T_{inside}} =$ w/ $\Delta S$ + $Q$ in units of per unit time.

$\Delta S_{outside} = +\frac{Q}{T_{outside}}$

? What is Q? Heating cost in winter = \$300/month.

at rate of 50 cents per therm = $10^5$ Btu = $1054 \cdot 10^5$ J.

So at 50 cents per therm I am using

$\frac{300}{.5} = 600$ therms $= 6.32 \cdot 10^{10}$ J.

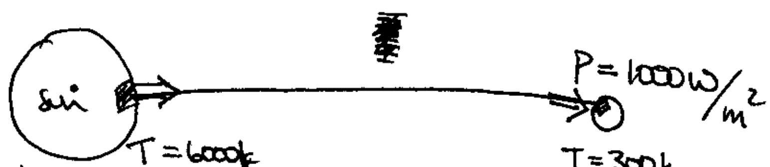
Then $\Delta S_{inside} = -\frac{6.32 \cdot 10^{10}\ J/month}{293\ k} = -2.15 \cdot 10^{8}\ J/k\ month$

$\Delta S_{outside} = +\frac{6.32 \cdot 10^{10}\ J}{273\ k} = +2.316 \cdot 10^{8}\ J/k\ month$

$\therefore$

$\Delta S_{total} = +1.66 \cdot 10^{7}\ J/k\ month$

Prob 3,13




(a) Assuming heat flows in a st line

The amount of heat leaving at temp $T_s$ + arriving at temp $T_e$ per unit time per unit area should be

$$P = 1000 \, W/m^2$$

$$= + \quad \Delta S_{created} = \frac{P(1 year)\left(\frac{365 \, days}{1 year}\right)\left(\frac{24 hr}{1 day}\right)\left(\frac{3600 s}{1 hr}\right)}{300 k}$$

$$= 1.05 \cdot 10^8 \, J/km^2 .$$

(b) so $(1.05 \cdot 10^8 \, J/k/m^2)(1 m^2)$

$\Rightarrow$ $1.05 \cdot 10^8 \, J/k$ of entropy is created in 1 year.

For a significant loss of entropy to occur due to plant growth one must have a plant w/ $S \cong kN$ $N$ molecules per year

$$\Rightarrow \quad N = \frac{1.05 \cdot 10^8 \, J/k}{1.381 \cdot 10^{-23} \, J/k} = 7.6 \cdot 10^{30} \text{ atoms or mole}$$

$\cong (1.26 \cdot 10^7)$ moles of "plant" growth

per year. What a hell of a plant!

Prob 3.14

$$C_V = aT + bT^3$$

$a = \cdots$ , $b = \cdots$

$$dS = \frac{dQ}{T} = \frac{C_V\,dT}{T} = \frac{(aT + bT^3)\,dT}{T} = (a + bT^2)\,dT$$

$$S(T) = S_0 + aT + \frac{bT^3}{3}$$

By the 3d law of thermodynamics $S(0) = 0 \Rightarrow S_0 = 0$.

$$\Rightarrow S(T) = aT + \frac{bT^3}{3}$$

Prob 3.15 Prob 1.55 . . .

Prob 3.16

(a) To remove a gigabyte of memory & have no record of what was there would result in a loss of $2^{30}$.

~~gigabyte~~

$1 \text{ kilobyte} = 2^{10}$

$1 \text{ Mbyte} = (2^{10})^2 = 2^{20}$

$1 \text{ Gbyte} = 2^{30}$

Guess $S = k \ln \Omega(N)$ w/ $N = 2^{30}$.

& $\Omega(N) = 2^{30}$.

$\therefore \Delta S = k(30) \ln 2$

$\Delta Q = T\Delta S = (300\,K)(1.381 \cdot 10^{-23}\,J/K)(30) \ln 2$ Not significant.

Prob 3.18

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| $N_\uparrow$ |
|---|
| 100 |
| 99 |
| 98 |
| ⋮ |

$-\ U = \mu B(N - 2N_\uparrow)$

$M = -\dfrac{U}{B}$

$\Omega = \dfrac{N!}{N_\uparrow!\,(N-N_\uparrow)!}$

$-\ S/k = \ln \Omega(N_\uparrow)$

$\dfrac{1}{T} = \dfrac{\partial S}{\partial U}$

$C_V = \dfrac{\partial U}{\partial T}\Big|_V$

At $N_\uparrow = 98 \quad N_\downarrow = 100 - 98 = 2 \quad U/\mu B = 100 - 2(98) = N_\downarrow - N_\uparrow = 2 - 98$
$= -96$

$$\frac{M}{\mu N} = \frac{N_\uparrow - N_\downarrow}{N} = \frac{98 - 2}{100} = .96$$

$$\Omega(N_\uparrow) = \binom{N}{N_\uparrow} = \frac{N!}{N_\uparrow!\,N_\downarrow!} = \frac{100!}{98!\,2!} = \frac{100 \cdot 99}{2} = 50 \cdot 99 = 4950$$

$$S/k = \ln \Omega(N_\uparrow) = \ln(4910) = \cdots$$

$$\frac{1}{T} = \frac{\partial S}{\partial U} = k\frac{\partial}{\partial U}(S/k) = \left(\frac{k}{\mu B}\right)\frac{\partial(S/k)}{\partial(U/\mu B)}$$

$$\Rightarrow T = \frac{\mu B}{k}\frac{\partial(U/\mu B)}{\partial(S/k)} \Rightarrow \frac{kT}{\mu B} = \frac{\partial(U/\mu B)}{\partial(S/k)} \cong \frac{\left(\frac{U}{\mu B}\right)_{i+1} - \left(\frac{U}{\mu B}\right)_{i-1}}{\left(\frac{S}{k}\right)_{i+1} - \left(\frac{S}{k}\right)_{i-1}}$$

$$\frac{-94-(-98)}{11.99-4.61} \cong \frac{4}{7.39} = 5.4\cdot 10^{-1}$$

**e** $$C_V = \frac{\partial U}{\partial T}\Big|_V = k\frac{\partial(U/\mu B)}{\partial(kT/\mu B)} \cong k \cdots$$

$$\Rightarrow \frac{C_V}{k} \cong \frac{\left(\frac{U}{\mu B}\right)_{i+1} - \left(\frac{U}{\mu B}\right)_{i-1}}{\left(\frac{kT}{\mu B}\right)_{i+1} - \left(\frac{kT}{\mu B}\right)_{i-1}} = \frac{-94-(-98)}{.6-.47} = \frac{4}{.13}$$

$$\frac{C_V}{k} = 30.7$$

$$\frac{C_V}{Nk} = .307$$

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Prob 3.19

$$\frac{1}{T} = \frac{\partial S}{\partial U}\bigg|_{N,B} = \frac{\partial S}{\partial N_\uparrow}\frac{\partial N_\uparrow}{\partial U}$$

$$\frac{S}{k} = N\ln N - N_\uparrow \ln N_\uparrow - (N-N_\uparrow)\ln(N-N_\uparrow)$$

$$U = \mu B(N-2N_\uparrow) \Rightarrow \qquad U/\mu B = N - 2N_\uparrow$$

$$\frac{\partial U}{\partial N_\uparrow} = -2\mu B \qquad \hookrightarrow N_\uparrow = \frac{N - U/\mu B}{2}$$

$$\frac{\partial S}{\partial N_\uparrow} = -\ln N_\uparrow - \frac{N_\uparrow}{N_\uparrow} + \ln(N-N_\uparrow) - \frac{(N-N_\uparrow)}{N-N_\uparrow}(-1)$$

$$= -\ln N_\uparrow - \cancel{1} + \ln(N-N_\uparrow) + \cancel{1}$$

$$= -\ln N_\uparrow + \ln(N-N_\uparrow)$$

The

$$\frac{1}{T} = \left(\frac{-k}{2\mu B}\right)\left[\ln\left(\frac{N-N_\uparrow}{N_\uparrow}\right)\right]$$

$$\Rightarrow \frac{1}{T} = \frac{-k}{2\mu B}\ln\left(\frac{N - \frac{1}{2}(N-U/\mu B)}{\frac{N-U/\mu B}{2}}\right) = \frac{-k}{2\mu B}\ln\left(\frac{2N - N + U/\mu B}{N - U/\mu B}\right)$$

$$\frac{1}{T} = \frac{-k}{2\mu B}\ln\left(\frac{N+U/\mu B}{N-U/\mu B}\right) = \frac{k}{2\mu B}\ln\left(\frac{N-U/\mu B}{N+U/\mu B}\right) \quad \text{eq } 3.30$$

$$\frac{2\mu B}{Tk} = \ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right)$$

$$(N + U/\mu B)\, e^{\frac{2\mu B}{kT}} = N - U/\mu B$$

$$N\left(e^{\frac{2\mu B}{kT}} - 1\right) = -\frac{U}{\mu B} e^{\frac{2\mu B}{kT}} - \frac{U}{\mu B}$$

$$N = \frac{-\frac{U}{\mu B}\left(e^{\frac{2\mu B}{kT}} + 1\right)}{e^{\frac{2\mu B}{kT}} - 1} \Rightarrow N = -\frac{U}{\mu B}\left(\frac{e^{\frac{\mu B}{kT}} + e^{-\frac{\mu B}{kT}}}{e^{\frac{\mu B}{kT}} - e^{-\frac{\mu B}{kT}}}\right)$$

$$\Rightarrow N = -\frac{U}{\mu B}\frac{\cosh\left(\frac{\mu B}{kT}\right)}{\sinh\left(\frac{\mu B}{kT}\right)} \Rightarrow U = -\mu B N \tanh\left(\frac{\mu B}{kT}\right) \quad \text{eq } 3.31$$

$$M = -\frac{U}{B} = \mu N \tanh\left(\frac{\mu B}{kT}\right) \quad \text{eq } 3.32$$

$$C_B = \left.\frac{\partial U}{\partial T}\right|_{N,B} = \frac{-\mu B N}{\cosh^2\left(\frac{\mu B}{kT}\right)}\left(-\frac{\mu B}{kT^2}\right) = \frac{Nk\left(\frac{\mu B}{kT}\right)^2}{\cosh^2\left(\frac{\mu B}{kT}\right)} \quad \text{eq } 3.33$$

$$\frac{d}{dx}\tanh(x) = \frac{d}{dx}\frac{\sinh(x)}{\cosh(x)} = \frac{\cosh(x)}{\cosh(x)} - \frac{\cancel{\sinh}\, \sinh(x)\sinh(x)}{\cosh^2(x)}$$

$$= 1 - \left(\frac{\sinh x}{\cosh x}\right)^2 \qquad \cosh^2 - \sinh^2 = 1$$

$$= \frac{\cosh^2 - \sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

Prob 3.20 $B = 2.06\,T$ $T = 2.2\,K.$

$\mu = \mu_B = 5.788\cdot 10^{-5}\ eV/T$ $k = 8.617\cdot 10^{-5}\ eV/K$

$$U = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

$$\frac{U}{(N\mu B)} = -\tanh\left(\frac{\mu B}{kT}\right) = -\tanh\left(\frac{5.788\cdot 10^{-5}\cdot 2.06}{8.617\cdot 10^{-5}\cdot 2.2}\right)$$

$$\frac{M}{N\mu} = \tanh\left(\frac{\mu B}{kT}\right) = \quad \ldots$$

$$\frac{S}{k} = ? \qquad \frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{N,B}$$

$$\frac{k}{2\mu B}\ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right) = \left.\frac{\partial S}{\partial U}\right|_{N,B}$$

$\rightarrow$ $S = S(U)$ ...

$\rightarrow$ $\frac{N_\uparrow}{N} = .99 \Rightarrow$ $\frac{S}{k} = N\ln N - N_\uparrow \ln N_\uparrow - (N - N_\uparrow)\ln(N - N_\uparrow)$

$$\frac{S}{k} = N\ln N - N\frac{N_\uparrow}{N}\ln\left(N\frac{N_\uparrow}{N}\right) - N\left(1 - \frac{N_\uparrow}{N}\right)\ln\left(N\left(1 - \frac{N_\uparrow}{N}\right)\right)$$

$$\frac{S}{k} = N\ln N - N\left(\frac{N_\uparrow}{N}\right)\left[\ln N + \ln\frac{N_\uparrow}{N}\right]$$

$$- N\left(1 - \frac{N_\uparrow}{N}\right)\left[\ln N + \ln\left(1 - \frac{N_\uparrow}{N}\right)\right]$$

... To obtain 99% magnetization

$$\frac{M}{N\mu} = .99 \quad = \tanh\left(\frac{\mu B}{kT}\right)$$

⇒ Make argument of $\tanh(x)$ closer to $+\infty$

⇒ increase $B$ or lower the temp. or both

Prob 3,21

$$\frac{M}{N} = \mu \tanh\left(\frac{\mu B}{kT}\right)$$

$$= (5\cdot10^{-8}\ \tfrac{eV}{T}) \tanh\left(\frac{(5\cdot10^{-8}\ \frac{eV}{T})(.63T)}{(8.617\cdot10^{-5}\ \frac{eV}{K})(300K)}\right)$$

$$= 6.09\cdot10^{-14}\ \frac{eV}{T}$$

$$\cancel{R_B}\cdot hf = E_{photon} = \mu B =$$

$$\frac{hc}{\lambda} = \mu B$$

$$\lambda = \frac{hc}{\mu B} = \frac{(4.136\cdot10^{-15}\ eV\cdot s)(2.998\cdot10^{8}\ m/s)}{(5\cdot10^{-8}\ \frac{eV}{T})(.63T)}$$

$$= 39.3\ m.$$

Prob 3,22 combine prob 3,23

Prob 3,23

$$\frac{1}{T} = \frac{\partial S}{\partial U}\bigg|_{N,B}$$

see next pg.

$$\cancel{\frac{k}{2\mu B}\ln\left(\frac{N - U/\mu B}{N + U/\mu B}\right) = \partial S/\partial U\Big|_{N,B}}$$

$$\frac{\partial(S/k)}{\partial(U/2\mu B)} = \ln(N - U/\mu B) - \ln(N + U/\mu B)$$

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Prob 3,23

Show

$$S = Nk\left[\ln(2\cosh x) - x\tanh x\right] \qquad x = \frac{\mu B}{kT}$$

from $\frac{1}{T} = \frac{\partial S}{\partial U}\Big|_{N,B} = \frac{\partial S}{\partial T}\Big|_{N,B} \cdot \frac{dT}{dU}$.

$$U = -N\mu B \tanh\left(\frac{\mu B}{kT}\right)$$

$$\frac{dU}{dT} = \frac{Nk\,(\mu B/kT)^2}{\cosh^2(\mu B/kT)}$$

$$\frac{\partial S}{\partial T}\Big|_{N,B} = \frac{1}{T}\frac{dU}{dT} = \frac{Nk}{T}\frac{(\mu B/kT)^2}{\cosh^2(\mu B/kT)}$$

define $x = \frac{\mu B}{kT}$

$$dx = -\frac{\mu B}{kT^2}\,dT$$

$$dx = -\frac{\mu B}{kT}\frac{dT}{T}$$

$$dx = -x\,\frac{dT}{T}$$

$$\frac{1}{T} = -\frac{1}{x}\frac{dx}{dT} \checkmark$$

$$\Rightarrow \frac{\partial S}{\partial T}\Big|_{N,B} = Nk\,\frac{x^2}{\cosh^2(x)}\left(-\frac{1}{x}\right)\frac{dx}{dT}$$

$$= Nk\,\frac{x}{\cosh^2(x)}\,\frac{dx}{dT}$$

$$S(T) = Nk\left[-\ln(\cosh(x)) + x\tanh(x)\right] + S_0.$$

$$S(0) = \cancel{Nk}\ \xi\ ?$$

$$\lim_{x\to+\infty}\left(x\tanh(x) - \ln(\cosh(x))\right) = \text{“}\infty - \infty\text{”}$$

$$= \ln\left[\lim_{x\to+\infty} e^{x\tanh(x) - \ln(\cosh(x))}\right]$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x\tanh(x)}}{\cosh(x)}\right] \qquad \frac{+\infty}{+\infty}$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x}}{\cosh(x)}\right] \qquad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$= \ln\left[\lim_{x\to+\infty} \frac{e^{x}}{\frac{1}{2}(e^x + e^{-x})}\right] = \ln(2)$$

$\therefore \; S(0) = Nk\ln 2 + S_0 \equiv 0$ by 3rd law thermo $\Rightarrow S_0 = -Nk\ln 2$

$$\therefore \; S(T) = Nk\left[-\ln(\cosh(x)) + x\tanh(x) - \ln 2\right]$$

$$= -Nk\left[\ln(2\cosh(x)) - x\tanh(x)\right] \quad \text{w/ } x =$$

Prob 3,24 For an einstein solid $q \gg N$ $U = q\epsilon$.

$$\frac{dU}{dq} = \epsilon$$

$$\Omega(N,q) = \left(\frac{eq}{N}\right)^N \qquad \frac{S}{k} = \ln \Omega(N,q)$$

$$\frac{1}{T} = T^{-1} \equiv \frac{\partial S}{\partial U} = \frac{dq}{dU}\frac{\partial S}{\partial q} = \frac{1}{\epsilon}\frac{\partial S}{\partial q}$$

$$\Rightarrow \frac{\epsilon}{T} = \frac{\partial S}{\partial q}$$

$$\Rightarrow \frac{T}{\epsilon} = \frac{\partial q}{\partial S} = \frac{\partial q}{\partial (S/k)\cdot k} \Rightarrow \frac{kT}{\epsilon} = \frac{\partial q}{\partial (S/k)} \cong \frac{q^{n+1} - q^{n-1}}{(\frac{S}{k})^{n+1} - (\frac{S}{k})^{n-1}}$$

$$C_B = \frac{\partial U}{\partial T} = \frac{\partial (q\epsilon)}{\partial (kT/\epsilon)\cdot(\frac{\epsilon}{k})} = \frac{\partial q}{\partial (kT/\epsilon)} k$$

$$\Rightarrow \frac{1}{N}\frac{C_B}{k} = \frac{1}{N}\frac{\partial q}{\partial (kT/\epsilon)}$$

entropy v.s. energy

+ C v.s. T

(Prob 3.25)

$$\Omega(N,q) \cong \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$$

(a) $$S = k\ln\Omega(N,q) = k\left[q\ln(q+N) - q\ln q + N\ln(q+N) - N\ln N\right]$$

$$U = \epsilon q \qquad \frac{dU}{dq} = \epsilon$$

(b) $$\frac{1}{T} = \frac{\partial S}{\partial U} =$$

$$= \frac{\partial S}{\partial q}\cdot\frac{dq}{dU} = \frac{1}{\epsilon}\frac{\partial S}{\partial q} = \frac{k}{\epsilon}\left[\ln(q+N) + \frac{q}{q+N} - \ln q - 1 + \frac{N}{q+N}\right]$$

$$\Rightarrow \frac{1}{T} = \frac{k}{\epsilon}\left[\ln\left(\frac{q+N}{q}\right) \cancel{-1} + \cancel{1}\right]$$

$$\frac{1}{T} = \frac{k}{\epsilon}\ln\left(1+\frac{N}{q}\right) \Rightarrow T = \frac{\epsilon}{k}\frac{1}{\ln(1+\frac{N}{q})}$$

(c) $$\frac{kT}{\epsilon} = \frac{1}{\ln(1+\frac{N}{q})} \Rightarrow \ln\left(1+\frac{N}{q}\right) = \frac{\epsilon}{kT}$$

$$1+\frac{N}{q} = e^{\epsilon/kT}$$

$$1-e^{\epsilon/kT} = +\frac{N}{q}$$

9-8-02 ?

$$q = \frac{N}{1-e^{\epsilon/kT}}$$

$$U = \frac{N\epsilon}{1-e^{\epsilon/kT}}$$

$$C = \frac{\partial U}{\partial T} = \frac{N\epsilon}{(1-e^{\epsilon/kT})^2}(-e^{\epsilon/kT})(-\epsilon/kT^2)$$

$$= \frac{Nk\left(\frac{\epsilon^2}{k^2T^2}\right)e^{\epsilon/kT}}{(1-e^{\frac{\epsilon}{kT}})^2}$$

(d) $\lim T \to +\infty$ $\quad \frac{0}{0}$ $\quad e^x \cong 1 + x + \frac{x^2}{2}$

$Nk$

$$\lim_{T\to+\infty} C = \lim_{T\to+\infty} Nk\frac{\left(\frac{\epsilon}{kT}\right)^2}{\left(1-1-\left(\frac{\epsilon}{kT}\right)\right)^2} = Nk$$

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Prob 3,26

~~[illegible]~~ From prob 3,25

$$C = Nk\left(1 - \frac{1}{12}\left(\frac{\epsilon}{kT}\right)^2\right)$$

$$\left\{ Nk = nR \right\}$$

$$= nR\left(1 - \frac{1}{12}\left(\frac{\epsilon}{kT}\right)^2\right)$$

At $T = 1000\,k$

$$C = \frac{5}{2} R$$

$$\frac{5}{2} R = 1 \cdot R\left(1 - \frac{1}{12}\left(\frac{\epsilon}{(8.617 \cdot 10^{-5}\,eV/k)(1000)}\right)\right) \Rightarrow \epsilon = \cdots$$

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Prob 3,27

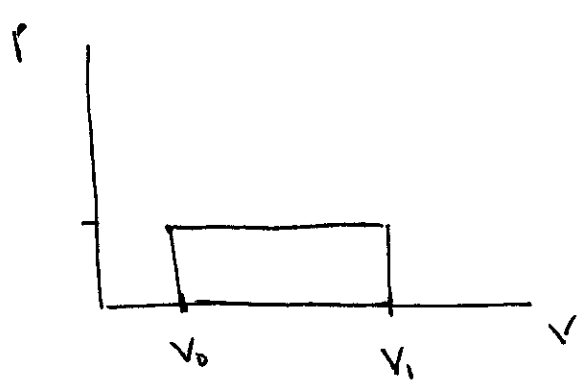
$dU = Q + W$

$dU = TdS - pdV$ at const entropy

$T = \frac{\partial U}{\partial V}\Big|_S$

$p = -\frac{\partial U}{\partial V}\Big|_S$ did I already know this ? I don't see how...

Prob 3,28





$W = P(V_1 - V_0)$ $\Delta U = Q + W$

For constant pressure processes like we have here

$$(\Delta S)_p = \int_{T_i}^{T_f} \frac{C_p\,dT}{T} \cong C_p \ln\left(\frac{T_f}{T_i}\right)$$

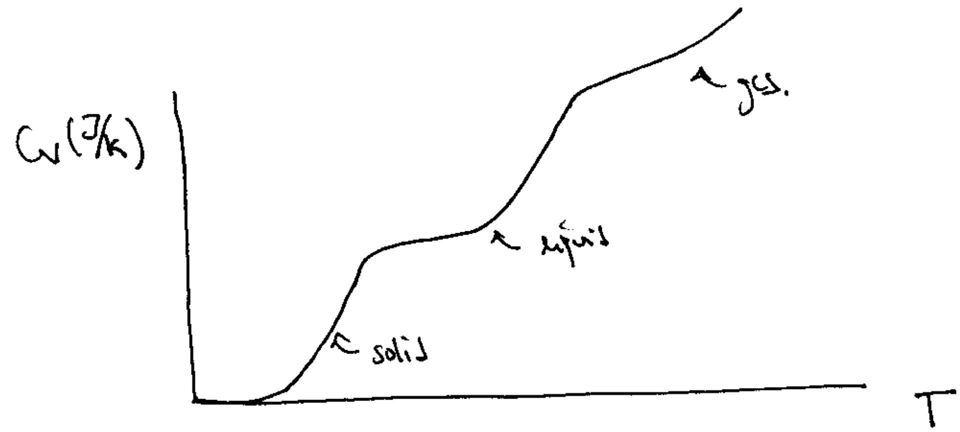
Assuming ~~[illegible]~~ $PV = nRT$

$$\frac{T_f}{T_i} = \frac{\cancel{P_f} V_f}{\cancel{P_i} V_i} = 2$$

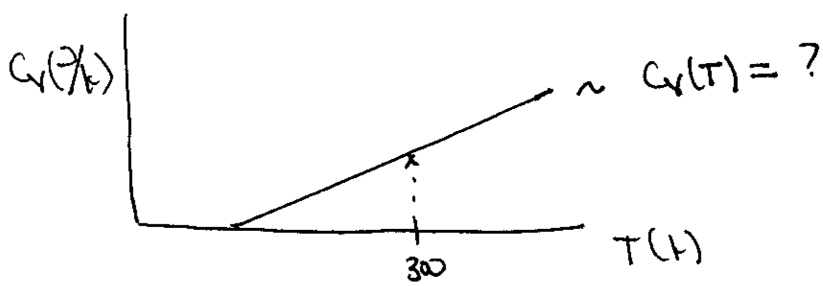
$(\Delta S)_p \cong C_p \cdot \ln 2$.

What is $C_p = ?$

**Prob 3,29** I would assume it has a shape like that seen on pg 30




**Prob 3,30**




$$\left.\begin{aligned} C_V(300K) &= 7.5 \text{ J/K} \\ C_V(400K) &= 10 \text{ J/K.} \end{aligned}\right\} \rightarrow C_V(T) = 7.5 + \frac{2.5}{(100)}(T-300)$$

$$\Delta S = \int_{298}^{500} \frac{C_V\,dT}{T} = \int_{298}^{500} \frac{(7.5 + 2.5\cdot10^{-2}(T-300))}{T}\,dT = 7.5\ln\left(\frac{500}{298}\right)$$

$$+ 2.5\cdot10^{-2}\left[(500-298) - 300\ln\left(\frac{500}{298}\right)\right]$$

$$= 3.88 + 5.05 - 3.88 = 5.05 \text{ J/K}$$

$$S(500K) = \Delta S_{298\to500} + S$$

$$2.38 \text{ J/K}$$

Prob 3,31 $\quad C_p = a + bT - \frac{c}{T^2} \qquad a = \cdots,\ b = \cdots,\ c = \cdots$

$$\Delta S_{298 \to 800} = \int_{298}^{800} \frac{C_p\,dT}{T} = \cancel{a(800-298) + \frac{b}{2}(800^2 - 298^2) + \frac{c}{T}} \Big|_{298}^{800}$$

$$= \cancel{3.4 \cdot 10^3\ J/K + 3.89 \cdot 10^2\ J/K}$$

$$= \int_{T_i}^{T_f} \left(\frac{a}{T} + b - \frac{c}{T^3}\right) dT = a \ln T + b(T) + \frac{c}{2T^2} \Big|_{T_i}^{T_f}$$

$$= a \ln\left(\frac{T_f}{T_i}\right) + b(T_f - T_i) + \frac{c}{2}\left(\frac{1}{T_f^2} - \frac{1}{T_i^2}\right)$$

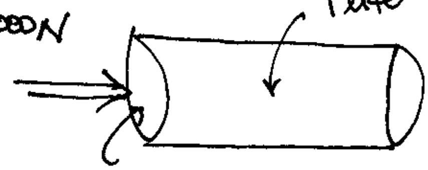
$$= 8.72 + 9.6 \cdot 10^{-1} - 3.1 \cdot 10^{0} = 6.57\ J/K$$

$$\Delta S_{0 \to 298} = 8.53\ J/K$$

$S(800\,K) \cong$ 15 J/K.

Prob 3,32

F = 2000 N



1 liter = 1000 cm$^3$ = V
T = 300 K
P = $10^5$ N/m$^2$ = 1 atm.

A = .01 m$^2$
$\Delta x = 10^{-3}$ m

Work applied by

(a) $W \cong \cancel{-}\,\Delta P \cdot A \cdot \Delta x$ done on gas

$= \cancel{-}\,(PA - F)\Delta x$

$= \cancel{-}\,(10^5 \cdot (.01) - 2000)(10^{-3})$

$= +1\,J.$

(b) Since the process occurred very quickly ... I am going to assume that it is ~~quasistatic~~ adiabatic + $\therefore$ no heat was added to the gas.

(c) $\Delta U = W + \cancel{Q}^{\,0}$

$\Delta U = 1\,J.$

(d) $dU = TdS - PdV \Rightarrow dS = \frac{1}{T}dU + \frac{P}{T}dV.$

Assuming again the adiabatic assumption ... + this not affecting temperature ... ??

How calculate the change in entropy?

Prob 3.33 Pr:

$$C_V = T \frac{\partial S}{\partial T}\Big|_V$$

The thermodynamic identity is:

$$dU = TdS - pdV + \cancel{\mu dN}^{\,0}$$ Assuming no loss of particles

B. $C_V \equiv \frac{\partial U}{\partial T}\Big|_V$.

$$\therefore \quad dU\Big|_V = TdS\Big|_V$$

$$\Rightarrow \quad \frac{dU}{dT}\Big|_V = T\frac{dS}{dT}\Big|_V = C_V.$$

$$H = U + pV.$$

$$dH = dU + pdV + Vdp \qquad C_p \equiv \frac{\partial U}{\partial T}\Big|_p$$

$$dH\Big|_p = dU\Big|_p + p\,dV\Big|_p.$$

$$\frac{\partial H}{\partial T}\Big|_p = \frac{\partial U}{\partial T}\Big|_p + p\frac{\partial V}{\partial T}\Big|_p \Rightarrow C_p = \frac{\partial H}{\partial T}\Big|_p - p\frac{\partial V}{\partial T}\Big|_p$$ Is this not wrong?

$$dH = TdS - \cancel{pdV} + \cancel{pdV} + Vdp$$

$dH = TdS + Vdp$ $\quad \div\ dT$ at constant $V$

$$\left.\frac{\partial H}{\partial T}\right|_V = T\left.\frac{\partial S}{\partial T}\right|_V + V\left.\frac{\partial p}{\partial T}\right|_V$$

$$\parallel \qquad \parallel$$

$$C_p = C_V + V\left.\frac{\partial p}{\partial T}\right|_V$$

from choice ... ?

Prob 3.34

(a) links

$\boxed{1}\ \boxed{2}\ \cdots\ \overset{k}{\boxed{k}}\ \overset{k+1}{\boxed{k+1}}\ \cdots\ \boxed{N}$

each link can point to the left or ~~too~~ the right, thus if I specify the # pointing left ~~there is still an~~ (and by doing so define a macro state of the system) the # pointing right is determined but not its distribution of the . $N_+$ = # pointing right

$$\Omega(N_+) = \binom{N}{N_+}$$

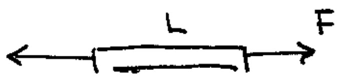
$$S = k\ln\Omega(N_+) = k\ln\left(\frac{N!}{(N-N_+)!\,N_+!}\right) = k\Big[N\ln N - \cancel{N} - (N-N_+)\ln(N-N_+) + (\cancel{N}-N_+) - N_+\ln N_+ + \cancel{N_+}\Big]$$

$$S = k\left[N\ln N - (N-N_+)\ln(N-N_+) - N_+\ln N_+\right]$$

(b) $L = +\ell N_R - \ell N_L = \ell(N_R - N_L) = \ell(N_R - N + N_R)$

$= \ell(2N_R - N)$

(c)




w/ a tension force of F one would have a spring constant k?

$F = -kx \qquad U = \frac{1}{2}kx^2$

$x = -\frac{F}{k} \qquad U = \frac{1}{2}k\left(\frac{F}{k}\right)^2 = \frac{1}{2}\frac{1}{k}F^2$

$dU = \frac{F}{k}\,dF$

... I don't see how to derive this from 1st principles but only by analogy.

$dU = T\,dS - p\,dV$

$\cong \quad dU = T\,dS - F\,dL$

(d) → $F = T\left.\frac{\partial S}{\partial L}\right|_U = T\frac{\partial S}{\partial N_R}\cdot\frac{dN_R}{dL}$

$\frac{dL}{dN_R} = 2\ell \qquad \frac{\partial S}{\partial N_R} = k\left[\ln(N-N_R) + 1 - 1 - \ln N_R\right]$

$= k\left[\ln(N-N_R) - \ln N_R\right]$

$$\therefore \quad F = \frac{Tk}{2\ell}\left[\ln(N-N_R) - \ln N_R\right]$$

By

$$\frac{L}{\ell} = 2N_R - N \Rightarrow \frac{1}{2}\left(\frac{L}{\ell}+N\right) = N_R$$

$$\cancel{1 = 2\frac{N_R}{N} - \frac{L}{\ell}}$$

$$2\left(\frac{N_R}{N}\right) - 1 = \frac{L}{\ell N}$$

$$\therefore \quad F = \frac{kT}{2\ell}\left[\ln\left(\frac{N}{N_R} - 1\right)\right]$$

$$\frac{N_R}{N} = \frac{1}{2}\left(\frac{L}{\ell N}+1\right)$$

$$\frac{N}{N_R} = \frac{2}{\left(1+\frac{L}{\ell N}\right)}$$

$\therefore$

$$F = \frac{kT}{2\ell}\ln\left(\frac{2}{1+\frac{L}{\ell N}}\right) \quad \text{if} \quad \frac{L}{\ell N} \ll 1$$

(e)

$$\frac{2}{1+\frac{L}{\ell N}} \cong 2\left(1-\frac{L}{\ell N}\right)$$

$$\cong \frac{kT}{2\ell}\ln\left(2\left(1-\frac{L}{\ell N}\right)\right)$$

$$= \frac{kT}{2\ell}\left(\ln\left(1-\frac{L}{\ell N}\right) + \ln 2\right)$$

$x \ll 1$

$\ln(1+x) \cong x$

$$\cong \frac{kT}{2\ell}\cdot\left(-\frac{L}{\ell N}\right) + \frac{kT}{2\ell}\ln 2$$

$$\therefore \quad F = -\frac{kT}{2\ell^2 N}\cdot L + F_0$$

9-9-02

(f) By $T \nearrow$ ~~Ther~~ the ~~Band~~ Force in the ~~B~~ band increases

$\therefore$ it contracts,

This makes sence since the entropy $\left\{ \frac{1}{T} \equiv \frac{\partial S}{\partial U}\Big|_{N,V} \right\}$

has to get ....

(g) By stretching I make the tension in the band increase.

Based on $F = -\frac{kT}{2l^2N} \cdot L + F_0$ I would expect that the

tempeture would increase. ...

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~~q = 100~~

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$$\Omega(N,q) = \frac{(q+N-1)!}{q!(N-1)!}$$

$\{ N = $ # of particles

$q$ = total energy shared among all particles.

If $N=3$ $q=3$

$$\Omega(3,3) = \frac{5!}{3!\,2!} = \frac{5\cdot 4}{2} = 10$$

$$\Omega(4,q) = \frac{(q+3)!}{q!\,3!} = \frac{(q+3)(q+2)(q+1)}{3\cdot 2} = 10$$

$$(q+3)(q+2)(q+1) = 60$$

If $q=2$

$5\cdot 4\cdot 3 = 60$ ✓.

$$\cancel{\Omega(N+1,q)}) = \frac{(q+N)!}{q!\,N!} = \Omega(N,q) \cancel{=} \frac{(q\cancel{+N-1})!}{q!\,(N-1)!}$$

$\Omega(N,q) = \ldots$

$$\Omega(N+1,q) = \frac{(q+N+1-1)!}{q!\,N!} = \frac{(q+1+N-1)!}{q!\,(N-1+1)!}$$

$$= \frac{(q+1+N-1)}{N}\frac{(q+N-1)!}{q!\,(N-1)!} = \frac{q+N}{N}\,\Omega(N,q)$$

$$\Omega(N+1, q') = \Omega(N,q)$$

$$\|$$

$$\frac{(q'+N)!}{q'!\,N!} = \frac{(q+N-1)!}{q!\,(N-1)!}$$

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$$S = Nk\left[\ln\left(V\left(\frac{4\pi m U}{3h^2}\right)\frac{1}{N\cdot N^{3/2}}\right) + \frac{5}{2}\right]$$

$$= Nk\left[\ln\left(V\left(\frac{4\pi m U}{3h^2}\right)\right) - \frac{5}{2}\ln N + \frac{5}{2}\right]$$

$$\frac{\partial S}{\partial N} = k\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right) + \frac{5}{2}\right] + Nk\left[-\frac{5}{2}\frac{1}{N}\right]$$

$$= k\left[\ln\left(\quad\right) + \cancel{\frac{5}{2}}\right] - \cancel{\frac{5}{2}}k$$

$$\mu = -T\left.\frac{\partial S}{\partial N}\right|_{U,V} = -Tk\left[\ln\left(\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right)\right] \quad \text{eq } 3.63$$

Prob 3,35

3 ossilators & 4 units of energy

$$\mu \cong \left.\frac{(\Delta U)}{\Delta N}\right|_S = \frac{\quad}{1}$$

$\Omega(N,q) =$ ~~[illegible]~~ $\frac{(q+N-1)!}{q!\,(N-1)!}$ From this expression Adding 1 to N would require decreasing q. I would think it would require decresing q by more then 1 since w/ q=3 decresing q by 1 was all that was required.

P. 3.36 $N \gg 1 \quad q \gg 1$

(a) $\mu \equiv -T \left.\frac{\partial S}{\partial N}\right|_{U,N}$.

From problem 2.18 $\Omega(N,q) \cong \left(\frac{q+N}{q}\right)^q \left(\frac{q+N}{N}\right)^N$

$$S = kq\ln\left(\frac{q+N}{q}\right) + kN\ln\left(\frac{q+N}{N}\right)$$

$$\frac{\partial S}{\partial N} = kq\,\frac{1}{\frac{q+N}{q}}\left(\frac{1}{q}\right) + k\ln\left(\frac{q+N}{N}\right) + kN\,\frac{1}{\frac{q+N}{N}}\cdot\left(\frac{-q}{N^2}\right)$$

$$\frac{\partial S}{\partial N} = k\frac{\cancel{q}}{\cancel{q}+N} + k\ln\left(\frac{q+N}{N}\right) + \frac{k(\cancel{-q})}{\cancel{q+N}}$$

$$= k\ln\left(\frac{q+N}{N}\right)$$

$$\mu = -Tk\ln\left(\frac{q+N}{N}\right) = -kT\ln\left(1+\frac{q}{N}\right)$$

$$= \cancel{kT\ln\left(\frac{N}{N+q}\right)} = kT\ln\left(\frac{N}{N+q}\right)$$

(b) $N \gg q$

~~$\mu \approx -kT\ell$~~

$$\mu \cong -kT\frac{q}{N}$$

$N \ll q$

$$= kT\ln\left(\frac{N/q}{1+N/q}\right)$$

$$\cong kT\ln\left(\frac{N}{q}\left(1-\frac{N}{q}\right)\right)$$

$$\mu \cong -kT\ln\left(\frac{q}{N}\right)$$

$$= kT\ln\left(\frac{N}{q}\right)$$

Prob 3,37

$$\mu = -T \frac{\partial S}{\partial N}\Big|_{U,V} \qquad \mu = \frac{\partial U}{\partial N}\Big|_{S,V}$$

If $U = N \cdot f \cdot \frac{1}{2}kT + N \cdot mg \cdot z$

$= N\left(\frac{f}{2}kT + mgz\right)$ For a monatomic gas $f=3$

$$U = N\left(\frac{3}{2}kT + mgz\right)$$

Deriving from $\mu = -T \frac{\partial S}{\partial N}\Big|_{U,V}$

expect that Sackur-Tetrode eq does not change:

$S = \quad \ldots \qquad$ eq 3,62

so

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{4\pi m U}{3Nh^2}\right)^{3/2}\right] \qquad \text{eq 3.63 still holds}$$

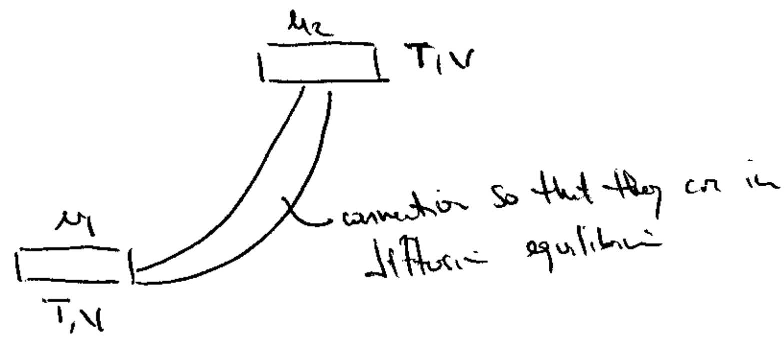
$\Rightarrow$

$$\mu = -kT \ln\left[\frac{V}{N}\left(\frac{4\pi m}{3h^2}\cdot\left(\frac{3}{2}kT + mgz\right)\right)^{3/2}\right]$$

$$= -kT \ln\left[\frac{V}{N}\left(\frac{2\pi m kT}{h^2} + \frac{4\pi m}{3h^2}\cdot mgz\right)^{3/2}\right]$$

(a) ... Don't see how to get.

(b)




$\Rightarrow \quad \mu_1 = \mu_2$

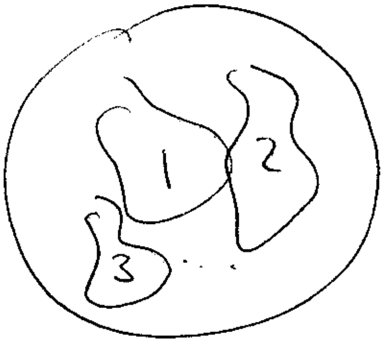
$\quad \quad ||$

$$-kT \ln\left[\frac{V}{N_0}\left(\frac{2\pi m k T}{h^2}\right)^{3/2}\right] = -kT \ln\left[\frac{V}{N(z)}\left(\frac{2\pi m k T}{h^2}\right)^{3/2}\right] + mgz$$

$$\frac{\cancel{V}}{N(0)}\left(\cancel{\phantom{x}}\right)^{3/2} = \frac{\cancel{V}}{N(z)}\left(\cancel{\phantom{x}}\right)^{3/2} \cdot \exp\left\{-\frac{mgz}{kT}\right\}$$

$$\Rightarrow \quad N(z) = N_0 \exp\left\{-\frac{mgz}{kT}\right\}$$

3.38




$$\mu_1 = -T\frac{\partial S}{\partial N_1}\Big|_{U,V,N_2,\dots} \qquad \mu_2 = -T\frac{\partial S}{\partial N_2}\Big|_{U,V,}$$

By an ideal gas we assume no ~~no~~ interactions among the particles. $\Rightarrow$ $\Omega(N) = \Omega(N_1)\,\Omega(N_2)\cdots\Omega(N_n)$ but they are distinguishable & thus for ~~no~~ no division is needed.

$$\rightarrow \quad S = \sum_{k=1}^{n} S_k .$$

w/ each gas obeying the Sack - Tetrode eq.

$$\Rightarrow \quad \mu = \sum_k \mu_k \qquad \text{w/} \quad \mu_k = -kT\ln\left[\frac{V}{N_k}\left(\frac{2\pi m kT}{h^2}\right)^{3/2}\right]$$

$$P_k V = k N_k T$$

Ph 121 Sol

(3.39) Prob 2.32 I got

$$S = Nk\left\{\ln\left(\frac{A}{N}\left(\frac{2\pi m U}{Nh^2}\right)\right) + 2\right\}$$

$$dU = T\,dS - p\,dV + \mu\,dN \; ; \; dU = T\,dS - \tau\,dA + \mu\,dN$$

$$T = \left.\frac{\partial U}{\partial S}\right|_{V,N} \quad \frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{V,N}$$

$$p = -\left.\frac{\partial U}{\partial V}\right|_{S,N}$$

$$dS = \frac{dU}{T} + \frac{\tau}{T}\,dA - \frac{\mu}{T}\,dN$$

$$\frac{1}{T} = \left.\frac{\partial S}{\partial U}\right|_{A,N} \; ; \; \frac{\tau}{T} = \left.\frac{\partial S}{\partial A}\right|_{U,N} \; ; \; -\frac{\mu}{T} = \left.\frac{\partial S}{\partial N}\right|_{T,A}$$