(5.83) Equilibrium conditions: equality of gibbs free energies

(a) $2\mu_H = \mu_{H_2}$

(b) $2\mu_{CO} + \mu_{O_2} = 2\mu_{CO_2}$

(c) $\mu_{CH_4} + 2\mu_{O_2} = 2\mu_{H_2O} + \mu_{CO_2}$

(d) $\mu_{H_2SO_4} = 2\mu_{H^+} + \mu_{SO_4}$

(e) $2\mu_p + 2\mu_n = \mu_{^4He}$

(5.84) eq ~~5.108~~ 5.108

$$\frac{P_{NH_3}^2 (P^0)^2}{P_{N_2} P_{H_2}^3} = k \equiv 6.9 \cdot 10^{-5}$$

$$\frac{\left(\frac{P_{NH_3}}{P^0}\right)^2}{\left(\frac{P_{N_2}}{P_0}\right)\left(\frac{P_{H_2}}{P_0}\right)^3} = K$$

?

(5.85) $k \equiv e^{-\Delta G/RT}$

$$\ln k = -\frac{\Delta G}{RT}$$

$$\frac{d}{dT}(\ln k) = +\frac{\Delta G}{RT^2} - \frac{1}{RT}\frac{d\Delta G}{dT}$$

$dU = -PdV + SdT$

$G = U + PV - ST$

$\Delta G = -\cancel{PdV} + \cancel{SdT} + \cancel{PdV} + VdP - \cancel{SdT} - TdS$

$= VdP - TdS$

$$\frac{d}{dT}(\ln k) = \frac{1}{RT^2}\left(\Delta G - T\frac{d\Delta G}{dT}\right)$$

$$= \cancel{\quad}$$

Is $\frac{d\Delta G}{dT} = -S$ yes.

Then $\Delta G - T\frac{d\Delta G}{dT} = \Delta G + ST = \Delta H$

$\therefore$

$$\frac{d}{dT}(\ln k) = \frac{\Delta H}{RT^2}$$

5.86 $N_2 + 3H_2 \leftrightarrow 2NH_3$

$T_1 = 373\ k$

$T_2 = 273 + 500 = 773\ k$

$$\Delta H^\circ = 2\Delta_f H(NH_3) - 3\Delta_f \overset{0}{\cancel{H}}(H_2) - \Delta_f \overset{0}{\cancel{H}}(N_2)$$

$$= 2(-46.11) \approx -90\ kJ$$

eq 5.69 H_2O

$$\mu_A = \mu_0(T,P) - \frac{N_B}{N_A} kT$$

eq 5.72 for H^+ + OH^-

$$\mu_B = \mu^\circ(T,P) + kT \ln m_B \qquad m = \frac{\text{moles solute}}{\text{kilogram solvent}}$$

$$\Rightarrow \mu^\circ_{H_2O}(T,P) = \mu^\circ_{H^+} + kT \ln m_{H^+} + \mu^\circ_{OH^-} + kT \ln m_{OH^-}$$

$$\Rightarrow -N_A(\mu^\circ_{H^+} + \mu^\circ_{OH} - \mu^\circ_{H_2O}) = N_A kT \ln (m_{H^+} m_{OH^-})$$

$$-\Delta G^\circ = RT \ln(m_{H^+} m_{OH^-})$$

$$pH \equiv -\log_{10} m_{H^+}$$

(5.87)

$$H_2SO_4 \rightarrow H^+ + HSO_4^- \qquad k = 10^2$$

$$HSO_4^- \longleftrightarrow H^+ + SO_4^{2-} \qquad k = 10^{-1.9}$$

(a) Based on eq 5.114

$$m_{H^+} m_{HSO_4^-} = 10^2 \Rightarrow m_{H^+} = 10 = m_{HSO_4^-}$$

$$\parallel$$

$= \dfrac{\text{moles } \{H^+, HSO_4^-\}}{\text{kilograms solvent.}}$ quite a ~~few~~ few moles !!

I would assume at a pH equivalent to this

$$pH \cong -\log_{10} m_{H^+} \cong -\log_{10}(10) = -1.$$

(b) If $m_{HSO_4^-} = 5 \cdot 10^{-5}$ mol/kg

Then ?

(c) k for that reaction is 10^{-14} $\Rightarrow m_{H^+} = 10^{-7} \ll m_{H^+}$ due to sulfuric acid.

(d) ?

5.88 $\frac{\partial(\Delta G^\circ)}{\partial P}$

$$\left.\frac{\partial(\Delta G)}{\partial P}\right|_T = V$$

Show that for volumes of liquids in solution this # V is small.

$$G = U + PV - ST$$

$$\Delta G = \Delta U + \Delta P \cdot V + V\Delta P - \Delta S \cdot T - S\Delta T$$

$$= -P\cancel{\Delta V} + S\cancel{\Delta T} + \Delta P\cancel{V} + V\Delta P - \Delta S \cdot T - S\cancel{\Delta T}$$

$$= V\Delta P - \Delta S \cdot T$$

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$$\mu_{gas} = \mu_{solute}$$

$$\parallel$$

$$\mu^\circ_{gas}(T) + kT\ln(P/P^\circ) = \mu^\circ_{solute}(T,P) + kT\ln m_B$$

$$N_A(\mu^\circ_{solute} - \mu^\circ_{gas}) = N_A kT\left(\ln(P/P^\circ) - \ln(m_B)\right)$$

$$\underbrace{\qquad\qquad}_{\Delta G^\circ} = RT\ln\left(\frac{P/P^\circ}{m}\right)$$

$$\frac{m}{P/P^\circ} = e^{-\Delta G^\circ/RT}$$

Prob 5.89

$\Delta H^\circ = -11.7\,kJ$

Van't Hoff equation:

$$\frac{d(\ln K)}{dT} = \frac{\Delta H^\circ}{RT^2}$$

$\Rightarrow$ if ΔH° is independent of temperature

~~Ideal~~ $$\ln K(T_2) - \ln K(T_1) = \frac{\Delta H^\circ}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\Rightarrow \ln(K(372K)) - \ln(1/780) = \frac{(-11.7\cdot 10^3 J)}{(8.314\ J/K)}\left(\frac{1}{273} - \frac{1}{373}\right)$$

$$\Rightarrow K(372K) = \ldots$$

How calculate $K(272K)$? Can I not just use 5.124?

5.90 $\Delta G^\circ = -1307.67 - (-856.64) - 2(-237.13)$ kJ

(a) $T = 273 + 25$ K

$$\exp\left(-\frac{\Delta G^\circ}{RT}\right) = \ldots$$

Then $\frac{m}{P/P^\circ} = \ldots$ what is the corresponding P expression?

think $P/P^\circ \cong 1$, but why?

(b) compute $K(372K)$ by ~~van't~~ van't Hoff eq.

$+ \Delta H^\circ$

5.91 Using eq 5.123 calculate m, i.e.:

$$\frac{m}{(P/P^0)} = e^{-\Delta G^0/RT}$$

~~ES~~ $P = 3.4 \cdot 10^{-4}$ bar

$m = \ldots$ molality of carbonic acid

Given that "neutral" H_2O has this much.

$$\Delta G^0 = -623.08 - 1(-237.13) - 1(-394.36) \text{ kJ}$$

$$= 8.41 \text{ kJ}$$

$$k = e^{-\Delta G^0/RT} = .03167$$

$$\Rightarrow m = 1.07 \cdot 10^{-5} \ \frac{\text{moles}}{\text{kg solvent}}$$

Then the entirely aqueous transformation is governed by:

$$\cancel{m_{H^+} \cdot m_{HCO_3^-} = e^{-\Delta G^0/RT}}$$

w/ ~~ΔG~~ $\mu_{H_2CO_3} = \mu_{H^+} + \mu_{HCO_3^-}$

$$\mu^0_{H_2CO_3} + kT\ln(m_{H_2CO_3}) = \mu^0_{H^+} + kT\ln(m_{H^+}) + \mu^0_{HCO_3^-} + kT\ln(m_{HCO_3^-})$$

$$N_A\left(\mu^\circ_{H_2CO_3} - \mu^\circ_{H^+} - \mu^\circ_{HCO_3^-}\right) =$$

$$= N_A kT \ln\left[\frac{m_{H^+}\, m_{HCO_3^-}}{m_{H_2CO_3}}\right]$$

$$\Rightarrow \exp\left(-\frac{\Delta G^\circ}{RT}\right) = \frac{m_{H_2CO_3}}{m_{H^+}\, m_{HCO_3^-}}$$

$$\|$$

Assuming that $m_{H^+} = m_{HCO_3^-}$ ..

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$$\mu_H = \mu_p + \mu_e$$

$$\|$$

$$\mu_H^\circ + kT\ln\left(\frac{P_H}{P_0}\right) = \mu_p^\circ + kT\ln\left(\frac{P_p}{P_0}\right) + \mu_e^\circ + kT\ln\left(\frac{P_e}{P_0}\right)$$

$$\Rightarrow \mu_p^\circ + \mu_e^\circ - \mu_H^\circ = kT\ln\left[\frac{P_H/P_0}{\left(\frac{P_p}{P_0}\right)\left(\frac{P_e}{P_0}\right)}\right]$$

$$= kT\ln\left[\frac{P_0\,P_H}{P_p\;P_e}\right] \quad \text{eq } 5.126 \ \checkmark$$

$$\mu = -kT\ln\left[\frac{V}{N}\left(\frac{2\pi mkT}{h^2}\right)^{3/2}\right]$$

$$PV = NkT$$

$$\frac{V}{N} = \frac{kT}{P}$$

5.92

(a) $\frac{N_p}{N_H^\circ}$

$N_H^\circ = N_H(T) + N_p(T)$

$PV = kNT$ $\qquad P = \frac{kNT}{V}$

$N = \frac{PV}{kT}$

P_{H° ?

(b) ?

(c) ?

(d)

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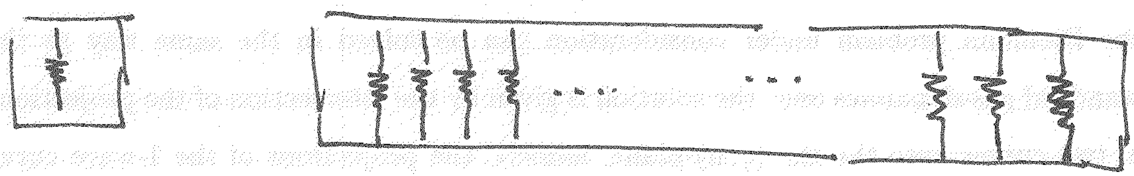
(6.1)



$\Omega(N,q) = \binom{q+N-1}{q}$ Multiplicity of an einstein solid w/ N oscillators & q units of energy

$\Omega_{AB}^{q} = \Omega\binom{q+N_A+N_B-1}{q}$

$N_A = 1$; $N_B = 100$; $q = 500$; ? I should be able to do this problem...

(6.2) $P(s) = \frac{1}{Z} e^{-E(s)/kT}$

$P(E) = ?$

since there are $\Omega(E)$ multiplicities of states w/ energy E one gets

$P(E) = \frac{\Omega(E)}{Z} e^{-E/kT}$ ~~$S(E) = k\ln$~~

$S = k\ln\Omega(E)$

w/ E = energy of system in state S. $\Omega = \exp\{S/k\}$

so $P(E) = \frac{1}{Z} e^{\frac{S}{k}} e^{-\frac{E}{kT}} = \frac{1}{Z}\exp\left\{-\frac{(E-TS)}{kT}\right\}$

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$$P(E) = \frac{1}{Z} \exp\left\{ \frac{-E}{kT} \right\}$$

$$P(E) = \frac{1}{Z} e^{-E/kT}$$

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(6.3) 2 grand states

$E_0 = 0\ eV$

$E_1 = 2 eV$

$$Z = \sum_s e^{-E(s)/kT}$$

$$= e^{-0} + e^{-2eV/kT} = 1 + e^{-\frac{2}{kT}}$$

~~(kT...)~~

$[kT] = eV$

(6.4) $Z \cong \sum e^{-E(s)/kT}$ $E(s) =$

$$= \oint \sum_s P(s) = \int_0^\infty P(s)\,ds = \int_0^\infty e^{-s/kT} ds = \dots$$

(6.5) (a)

$$Z = \sum_s e^{-\frac{E(s)}{kT}}$$

$$= e^{\frac{-.05}{kT}} + 1 + e^{\frac{.05}{kT}} = \dots$$

T=300k

(b)

$$P(s_1; E_1 = -.05\ eV) = \frac{e^{-.05/kT}}{Z} \qquad P(s_3; E_3 = +.05\ eV) = \frac{e^{+\frac{.05}{kT}}}{Z}$$

$$P(s_2; E_2 = 0) = \frac{1}{Z}$$

~~2~~) (c) Z will change since E has changed

The prob. will not change.

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Prob 6.6

$$\frac{P(s_2)}{P(s_1)} = \frac{e^{-E(s_2)/kT}}{e^{-E(s_1)/kT}} = \exp\left\{-\frac{(E(s_2)-E(s_1))}{kT}\right\}$$

$$= \exp\left\{\frac{-(10.2\,eV)}{(8.62\cdot 10^{-5}\,eV/k)(293\,k)}\right\} = \ldots$$

Then since 1st excited state has 4 levels all w/ the same energy we get

$4 \times$ (this #)

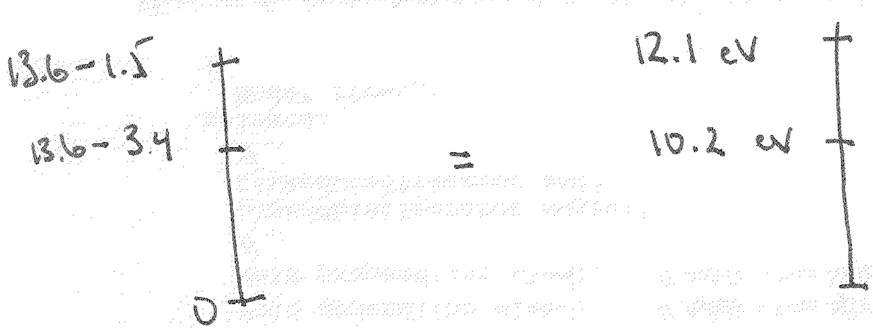
Same w/ $T = 9500\,k$.

6.7 Think Both $p(s_2)$ + $p(s_1)$ would get multiplied by a constant # 2

6.8 The methods of this section apply to a given system emersed in a "universe" at ~~another~~ a temperature T.

Don't know?

6.9 $$Z = \sum_s e^{-E(s)/kT} =$$



(a)

$$Z = 1 + e^{\frac{-10.2}{kT}} + e^{\frac{-12.1}{kT}}$$

$T = 5800\,k$

is there a multiplicity coefficient here?

From Appendix A: $E(n) = -\frac{13.6\ eV}{n^2} + 13.6\ eV$

$= 13.6\ eV\left[1 - \frac{1}{n^2}\right]$

(b)

How do I know

$$\sum_n \exp\left\{\frac{13.6}{kT}\left(1 - \frac{1}{n^2}\right)\right\}$$ diverges?

$$= \exp\left\{\frac{13.6}{kT}\right\}\sum_n \exp\left\{-\frac{13.6}{kTn^2}\right\}$$

$\lim_{n\to\infty} \exp\left\{\frac{-13.6}{kTn^2}\right\} \neq 0$

$\therefore$ series diverges

(c) $R_n = a_0 n^2$

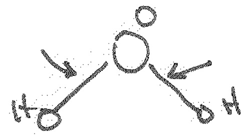
dV for large n's then becomes

$$dV = a_0((n+1)^2 - n^2) = a_0(\cancel{n^2} + 2n + 1 - \cancel{n^2}) = a_0(2n+1) = \Theta(n)$$

$\therefore$ $PdV = \Theta(n)$ for large n

$\therefore$ P-V work ~~associated~~ that the atmosphere does when an atom transfers levels is not negligible for high energy level transfers. Don't see where this P-V work effects $E(s)$?

(6.10)



$f_0 = 4.8 \cdot 10^{13}$ Hz $\qquad E(s) = \frac{(2s+1)hf}{2}$

(a) $$Z = \sum_{s} e^{-\frac{E(s)}{kT}} = \sum_{s=0}^{\infty} \exp\left\{\frac{\left(\frac{2s+1}{2}\cdot hf_0\right)}{kT}\right\} = \cdots$$

$$P(s_1) = \frac{\exp\left\{\frac{hf_0}{kT}\right\}}{Z}$$

Think in the partition formulation if we had degeneracy at a given energy level then we must add in that # of $\exp\left\{-\frac{E(s)}{kT}\right\}$ factors.

(b) ...

(6.11) $E = m\mu B \qquad \mu = 1.03 \cdot 10^{-7} \frac{eV}{T} \qquad B = 0.63\,T$

$$Z = \sum_{s} e^{-E(s)/kT} = e^{+\frac{3}{2}\mu B/kT} + e^{\frac{1}{2}\mu B/kT} + e^{-\frac{1}{2}\mu B/kT} + e^{-\frac{3}{2}\mu B/kT}$$

$$P\left(-\tfrac{3}{2}\right) = \frac{e^{+\frac{3}{2}\frac{\mu B}{kT}}}{Z}, \ldots$$

reversing $B \leftrightarrow -B \Leftrightarrow -T$

6.12

$$\frac{P(s^*)}{P(s_0)} = \frac{3}{10}$$ By experimental observation.

$$= \frac{3e^{-\frac{E(s^*)}{kT}}}{e^{-\frac{E(s_0)}{kT}}} = .3$$ By Boltzmann statistics. + knowledge of degeneracy of 1st excited state

$$\rightarrow \exp\left\{-(E(s^*)-E(s_0))/kT\right\} = .1$$

$$\Rightarrow \exp\left\{\frac{-4.7\cdot 10^{-4}\,eV}{kT}\right\} = .1 \rightarrow t = \ldots$$

6.13

~~[illegible]~~ $E_n - E_p = (2.3\cdot 10^{-30}\,kg)\,c^2$

$T = 10^{11}\,k.$

$$\frac{P(n)}{P(p)} = \exp\left\{-\frac{(E_n-E_p)}{kT}\right\} = \ldots$$ = fraction of nucleons that were neutrons.

w/ no degeneracy.

6.14

- - o - - - molecule of air at z

- - o - - molecule of air at $z=0$ (see level)

$$\frac{N(z)}{N(0)} = \frac{P(z)}{P(0)} = \exp\left\{-\frac{(E(z)-E(0))}{kT}\right\}$$

Then w/ $E(z) = mgz$ potential energy

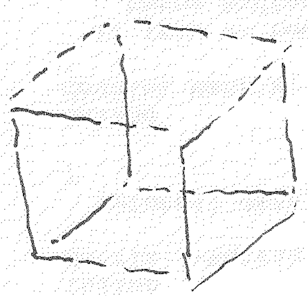
$$N(z) = N_0 \exp\left\{-\frac{mgz}{kT}\right\}$$

to relate to pressure

$$PV = kNT$$

$$\Rightarrow N = \frac{PV}{kT}$$

What does V represent? A fixed volume that one moves up to height z



then

$$\frac{P(z)V}{kT} = \frac{P(0)V}{kT} \exp\left\{-\frac{mgz}{kT}\right\}$$

same as eq 1.16 —

6.15

(a) $$\bar{E} = \frac{4\cdot 0eV + 3\cdot 1eV + 2\cdot 4eV + 1\cdot 6eV}{10}$$

b) $P(0eV) = \frac{4}{10}$ $P(4eV) = \frac{2}{10}$

$P(1eV) = \frac{3}{10}$ $P(6eV) = \frac{1}{10}$

(c) $\bar{E} = (\frac{4}{10})\cdot 0eV + (\frac{3}{10})\cdot 1eV + (\frac{2}{10})\cdot 4eV + (\frac{1}{10})\cdot 6eV$

6.16 $$Z = \sum_s e^{-\frac{E(s)}{kT}} = \sum_s e^{-E(s)\beta}$$

$$\frac{\partial Z}{\partial \beta} = \sum_s -E(s)e^{-E(s)\beta}$$

since $$\bar{E} = \frac{1}{Z}\sum_s E(s)e^{-E(s)\beta}$$

we see that $$\bar{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta}$$

6.17 (a) $$E_i - \bar{E} = \left\{\begin{array}{l} 7-3 = 4 \\ 4-3 = 1 \\ 4-3 = 1 \\ 0-3 = -3 \\ 0-3 = -3 \end{array}\right\} \Delta E_i \; ; \; \Delta E_i^2 = \left\{\begin{array}{l} 16 \\ 1 \\ 1 \\ 9 \\ 9 \end{array}\right.$$

$$\overline{(\Delta E_i)^2} = \frac{1}{5}(16+1+1+9+9) = \frac{36}{5} = 7\tfrac{1}{5} \equiv \sigma_E^2$$

$$\sigma_E = \sqrt{7\tfrac{1}{5}}$$

$$\sigma^2 = \operatorname{Avg}(E_i - \overline{E})^2$$

$$= \operatorname*{Avg}_i\left(E_i^2 - 2E_i\overline{E} + \overline{E}^2\right)$$

$$= \overline{E^2} - 2\overline{E}^2 + \overline{E}^2 = \overline{E^2} - \overline{E}^2$$

(d) $\sigma^2 = \ldots$

Prob 6.18

$$Z \equiv \sum_s \exp(-E(s)\beta) \qquad \overline{E^2} = \sum_s E^2(s)\frac{e^{-E(s)\beta}}{Z}$$

$$= \frac{1}{Z}\sum_s E(s)^2 e^{-E(s)\beta}$$

$$\frac{\partial Z}{\partial \beta} = \sum_s -E(s)e^{-E(s)\beta}$$

$$\frac{\partial^2 Z}{\partial \beta^2} = \sum_s E^2(s)\, e^{-E(s)\beta}$$

$$\therefore \quad \overline{E^2} = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2}$$

Since $\sigma^2 = \overline{E^2} - \overline{E}^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial \beta^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial \beta}\right)^2$

Since $\overline{E} = \frac{1}{Z}\frac{\partial Z}{\partial B}$

$$\sigma^2 = \frac{1}{Z}\frac{\partial}{\partial B}(\overline{E}) - \overline{E}^2$$

Something is fishy... try again

$$\sigma_E^2 = \overline{E^2} - (\overline{E})^2 = \frac{1}{Z}\frac{\partial^2 Z}{\partial B^2} - \left(-\frac{1}{Z}\frac{\partial Z}{\partial B}\right)^2$$

$$= \frac{1}{Z}\frac{\partial^2 Z}{\partial B^2} - \frac{1}{Z^2}\left(\frac{\partial Z}{\partial B}\right)^2$$

$$= \frac{\partial}{\partial B}\left(\frac{1}{Z}\frac{\partial Z}{\partial B}\right) = \frac{\partial}{\partial B}(-\overline{E})$$

w/ $B = \frac{1}{kT}$

$$\frac{\partial}{\partial B} = \frac{\partial T}{\partial B}\frac{\partial}{\partial T} \qquad \frac{\partial B}{\partial T} = \frac{-1}{kT^2}$$

$$\therefore \quad \sigma_E^2 = \left(\frac{1}{kT^2}\right)^{-1}\frac{\partial E}{\partial T} = \left(\frac{1}{kT^2}\right)^{-1} C = kT^2 C$$

$$\sigma_E = T\sqrt{kC} = kT\sqrt{C/k} \quad \checkmark$$

(6.19) $\frac{\sigma_E}{E}$ = fractional fluctuation in Energy

Prob 6.19 Einstein Solid high temp limit $\Omega(N,q) \cong \left(\frac{eq}{N}\right)^N$

$$S = Nk\left[\ln(q/N) + 1\right]$$

$$U = NkT$$

$$C_V = Nk$$

Based on prob 6.18

$$\sigma_E = kT\sqrt{\frac{C}{k}} = kT\sqrt{N} =$$ ~~standard deviation of~~ standard deviation of the energy of system

Fractional fluctuation in energy $= \frac{\sigma_E}{U} = \frac{\cancel{kT}\sqrt{N}}{N\cancel{kT}} = \frac{1}{\sqrt{N}}$

$f(N=1) = 1$

$f(N=10^4) = 10^{-2}$

$f(N=10^{20}) = 10^{-10}$.

6.20

(a) $|x|<1$

$$\begin{array}{r|l} & 1+x+x^2+\dots \\ \hline 1-x & 1+0x+0x^2+0x^3+0x^4+\dots \\ & \underline{1-x} \\ & \quad x+0x^2+0x^3+\dots \\ & \quad \underline{x-x^2} \\ & \qquad +x^2+0x^3+0x^4+\dots \\ & \qquad \underline{+x^2-x^3} \\ & \qquad\qquad x^3 \end{array}$$

(b) $Z=\sum_s e^{-E(s)\beta}$ $\qquad E(s)=shf \qquad s=0,1,2,\dots$

$$=\sum_{s=0}^{\infty} e^{-shf\beta}=\sum_{s=0}^{\infty}(e^{-hf\beta})^s=\frac{1}{1-e^{-hf\beta}}\cdot\frac{e^{+hf\beta/2}}{e^{+hf\beta/2}}$$

$$=\frac{e^{hf\beta/2}}{2\sinh(hf\beta)}$$

(c) $$\bar{E}=-\frac{1}{Z}\frac{\partial Z}{\partial\beta}=-\frac{1}{Z}\frac{\partial}{\partial\beta}\left(\frac{1}{1-e^{-hf\beta}}\right)$$

$$=-\frac{1}{Z}\frac{1}{(1-e^{-hf\beta})^2}(+hf)=-\frac{1}{Z}Z^2hf$$

$$=-Zhf=\frac{-hf}{1-e^{-hf\beta}}$$

(d) $U = N\bar{E} = \dfrac{-hfN}{1-e^{-hf\beta}}$

Prob ~~3.25~~ 3.25

(e) $C = \dfrac{\partial \bar{E}}{\partial T} = \dfrac{\partial}{\partial T}\left(\dfrac{-hfN}{1-e^{-hf\beta}}\right) = \ldots$

(6.21) $E_n \approx \epsilon(1.03n - 0.03n^2) \qquad n = 0, 1, 2 \ldots$

$$Z = \sum_s e^{-E(s)/kT} = \sum_{n=0}^{\infty} e^{\frac{-\epsilon(1.03n - 0.03n^2)}{kT}}$$ partition fn

$$\bar{E} = \frac{-1}{Z}\frac{\partial Z}{\partial \beta} + \frac{\partial Z}{\partial \beta} = \sum_{n=0}^{\infty} e^{-\epsilon(1.03n-0.03n^2)\beta}\left(-\epsilon(1.03n - 0.03n^2)\right)$$

$$= -\epsilon \sum_{n=0}^{\infty} (1.03n - 0.03n^2)\, e^{-\epsilon(1.03n-0.03n^2)\beta}$$

$$C = \frac{\partial \bar{E}}{\partial T} = \ldots$$

do summations only up to $n = 15$.

(6.22)

$\mu_z = -j\delta_\mu, (-j+1)\delta_\mu, (-j+2)\delta_\mu, \ldots, (j-1)\delta_\mu, j\delta_\mu$

δ_μ constant $= \Delta\mu_z$

$\{-5, 4\}$

$-2, 1$

$\underbrace{-2, -1, 0, 1,}_{4}$

(a)

$$\begin{array}{r} 1 + x + x^2 + \ldots \quad x^{n-1} + x^n \\ 1-x \,\overline{)\, 1 + 0x + 0x^2 + 0x^3 + \cdots + 0x^{n-1} + 0x^n + 0x^{n+1}} \end{array}$$

$1 - x$

x

$x - x^2$

$x^2 + 0x^3 + \ldots \quad + 0x^n + x^{n+1}$

$\vdots$

$x^3 + 0x^4 + \ldots$

$x^n + x^{n+1}$

$x^n - x^{n+1}$

$2x^{n+1}$?

Σ do of ∞ series ...

(b)

$$Z = \sum e^{-E(s)\beta} = \sum_{s=-j}^{j-1} e^{-s\delta_\mu\beta} = \sum_{s=0}^{2j-1} e^{-\delta_\mu\beta(s-j)}$$

$$= e^{+\delta_\mu\beta j} \sum_{s=0}^{2j-1} e^{-\delta_\mu\beta s} = e^{\delta_\mu\beta j} \frac{1 - e^{-\delta_\mu\beta(2j)}}{1 - e^{-\delta_\mu\beta}}$$

$$= \frac{e^{\delta\beta j} - e^{-\delta\beta j}}{1 - e^{-\delta\beta}} \cdot \frac{e^{\delta\beta/2}}{e^{\delta\beta/2}}$$

$$= e^{d\beta(j+1/2)} - e^{-d\beta(\bar{j}+1/2)}$$

6.22

$$\mu_z = -j\delta_\mu, (-j+1)\delta_\mu, \ldots, (j-1)\delta_\mu, j\delta_\mu$$

(a) ✓

(b) $$Z = \sum_s e^{-E(s)\beta}$$

$$E(s) = -\mu_z(s)B = -s\delta_\mu B$$

$$\therefore \quad Z = \sum_{s=-j}^{+j} \exp\{-s\delta_\mu B\beta\} \qquad \text{let } b = \beta\delta_\mu B$$

$$= \sum_{s=-j}^{+j} \exp\{-sb\} = \sum_{s=0}^{2j} \exp\{-(s-j)b\}$$

$$= \sum_{s=0}^{2j} e^{-sb} \cdot e^{jb} = e^{jb} \sum_{s=0}^{2j} e^{-sb}$$

$$= e^{jb}\left(\frac{1-e^{-b(2j+1)}}{1-e^{-b}}\right) = \frac{e^{jb} - e^{-b(j+1)}}{1-e^{-b}} \cdot \frac{e^{b/2}}{e^{b/2}}$$

$$= \frac{e^{b(j+1/2)} - e^{-b(j+1/2)}}{e^{b/2} - e^{-b/2}} = \frac{\sinh(b(j+1/2))}{\sinh(b/2)}$$

(c) ~~[illegible]~~

$$\overline{\mu_z} = \sum_s \mu_z(s) P(s) = \frac{1}{Z} \sum_s \mu_z(s) e^{-E(s)\beta}$$

$$\mu_z(s) = s\delta_\mu \qquad s = -j, -j+1, -j+2, \ldots, j-1, j$$

$$E(s) = -\mu_z(s)B = -s\delta_\mu B$$

$$\overline{\mu_z} = \frac{1}{Z} \sum_{s=-j}^{+j} (s\delta_\mu) e^{+s\delta_\mu B\beta} = \frac{\delta_\mu}{Z} \sum_{s=-j}^{+j} s e^{sb} \qquad b = \delta_\mu B\beta$$

$$= \cancel{\frac{\delta_\mu}{Z} \sum_{s=-j}^{+j} \frac{\partial}{\partial b} e^{sb}}$$

$$= \frac{\delta_\mu}{Z} \sum_{s=-j}^{+j} \frac{\partial}{\partial b} e^{sb} = \frac{\delta_\mu}{Z} \frac{\partial}{\partial b} \sum_{s=-j}^{+j} e^{sb}$$

$$= \cancel{\frac{\delta_\mu}{Z} \frac{\partial}{\partial b} \left[\frac{1-e^{+b(2j+1)}}{1-e^{b}} \right]}$$

$$= \frac{\delta_\mu}{Z} \frac{\partial}{\partial b} \left[\sum_{s=0}^{2j} e^{(s-j)b} \right] = \frac{\delta_\mu}{Z} \frac{\partial}{\partial b} \left[e^{-jb} \sum_{s=0}^{2j} e^{sb} \right]$$

$$= \frac{\delta_\mu}{Z} \frac{\partial}{\partial b} \left[e^{-jb} \left(\frac{1-e^{b(2j+1)}}{1-e^{b}} \right) \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[\frac{e^{-jb} - e^{b(j+1)}}{1-e^{b}} \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\Big[$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left[\frac{e^{-b(j+1/2)} - e^{b(j+1/2)}}{e^{-b/2} - e^{b/2}} \right]$$

$$= \frac{\delta\mu}{Z}\frac{\partial}{\partial b}\left(\frac{\sinh(b(j+1/2))}{\sinh(b/2)} \right)$$

$$= \frac{\delta\mu}{Z}\cdot\left[\frac{(j+1/2)\cosh(b(j+1/2))}{\sinh(b/2)} - \frac{\sinh(b(j+1/2))\cosh(b/2)\cdot 1/2}{(\sinh(b/2))^2} \right]$$

$$= \delta\mu\left[\frac{(j+1/2)\,\cancel{\sinh(b/2)}}{\sinh(b(j+1/2))}\,\frac{\cosh(b(j+1/2))}{\cancel{\sinh(b/2)}} - \frac{1}{2}\frac{\cancel{\sinh(b/2)}}{\cancel{\sinh(b(j+1/2))}}\,\frac{\cancel{\sinh(b(j+1/2))}\cosh(b/2)}{(\sinh(b/2))^{\cancel{2}}} \right]$$

$$= \delta\mu\left[(j+1/2)\coth(b(j+1/2)) - \frac{1}{2}\coth(b/2) \right]$$

Then $M = N\overline{\mu_z}$

$$= N\delta\mu\left[(j+1/2)\coth(b(j+1/2)) \quad \ldots \right]$$

(d) $T \to 0$ $M \to 0$ why is this expected...? 01-15-03 4

(e) $M \sim \frac{1}{T}$ does this not contradict the plots obtained?

$$\coth(x) = \frac{\cosh(x)}{\sinh(x)} \cong$$

$$\cancel{\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!}}$$

$$\cosh(x) = 1 + \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2} = \frac{1}{2}\sum_{k=0}^{\infty} \frac{x^k}{k!} - \frac{1}{2}\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{k!}$$

$$= \sum_{k=0}^{\infty} \frac{1}{2}(1-(-1)^k)\frac{x^k}{k!}$$

$$= \sum_{k=1,3,5,\ldots}^{\infty} \frac{x^k}{k!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

Thus

$$\coth(x) \approx \frac{1 + x^2/2 + x^4/4!}{x + x^3/3! + \cdots} = \frac{1}{x + x^3/3!} + \frac{x^2/2}{x + x^3/3!} + \cdots$$

$$\cong \cancel{\frac{1}{x} + \frac{x}{2} + O(x^2)}$$

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$$= \frac{1}{x(1+x^2/6)} + \frac{x}{2}\frac{1}{(1+x^2/6)} + O(x^3)$$

$$= \frac{1}{x}\left[\sum_{k=0}^{\infty}(-1)^k \frac{x^{2k}}{6^k}\right] + \frac{x}{2}\left[\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{6^k}\right] + O(x^3)$$

$$= \frac{1}{x}\left[1 - \frac{x^2}{6} + O(x^4)\right] + \frac{x}{2}\left[1 - O(x^2)\right]$$

$$= \frac{1}{x} - \frac{x}{6} + \frac{x}{2} + O(x^2)$$

$$= \frac{1}{x} + x\left[\frac{1}{2} - \frac{1}{6}\right] + O(x^2)$$

$$= \frac{1}{x} + x\left[\frac{2}{6}\right] + O(x^2)$$

$$= \frac{1}{x} + \frac{x}{3} + O(x^2)$$

Then $T \to \infty \Rightarrow \beta \to 0 \Rightarrow b \to 0$

$$M \sim N\delta_\mu\left[(j+1/2)\cdot\left[\frac{1}{b(j+1/2)} + \frac{b(j+1/2)}{3}\right] - \frac{1}{2}\left[\frac{1}{b/2} + \frac{b/2}{3}\right]\right]$$

$$M \sim N\delta_\mu \left[\cancel{\frac{1}{b}} + \frac{b}{3}(j+1/2)^2 - \cancel{\frac{1}{b}} - \frac{b}{12} \right]$$

$$= \frac{N\delta_\mu}{3} \left[b(j+1/2)^2 - \frac{b}{4} \right]$$

$$= \frac{N\delta_\mu}{3} b \left[(j+1/2)^2 - (1/2)^2 \right] \sim \frac{1}{T} \quad \text{curies law.}$$

(7) $M = N\delta_\mu \left[\cosh(b) - \frac{1}{2}\cosh(1/4) \right]$ . .

(6.23) $\epsilon = .00024\ eV$

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$Z_{rot}(300k) = \sum_{j=0}^{\infty} (2j+1) e^{\frac{-j(j+1)(2.4 \cdot 10^{-4})\,eV}{(8.617 \cdot 10^{-5}\,eV/k)(300k)}}$$

$$=$$

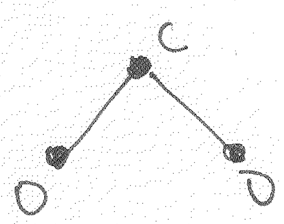
$$Z_{rot} = \frac{kT}{\epsilon}$$ Approx...

(6.24)

$\cancel{Z_{rot} \approx kT}$ O_2 ~~is~~ has 2 indistinguishable atoms

$$Z_{rot} \sim \frac{kT}{2\epsilon}$$

(6.25)



$$Z_{rot} \approx \frac{kT}{\epsilon}$$ if analysis holds ...

6.26 $kT \ll \epsilon$

$$z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

$$\cong 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\bar{E} = -\frac{1}{z}\frac{\partial z}{\partial \beta} = -\frac{1}{z}\frac{\partial}{\partial \beta}(1+3e^{-2\epsilon\beta}) = -\frac{1}{z}(3(-2\epsilon)e^{-2\epsilon\beta})$$

$$= -\frac{1}{z}(-2\epsilon)(3e^{-2\epsilon\beta})$$

$$= -\frac{1}{z}(-2\epsilon)(z-1) \doteq \cdots + 2\epsilon\left(1 - \frac{1}{z(\beta)}\right)$$

$$\bar{C}_V = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \bar{E}}{\partial \beta} \qquad \beta = \frac{1}{kT}$$

$$= -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta} \qquad \frac{\partial \beta}{\partial T} = \frac{-1}{kT^2} = \frac{-1}{T}\beta$$

$$\therefore \quad \bar{C}_V = -\frac{\beta}{T}\left(\frac{1}{z^2}\frac{\partial z}{\partial \beta}\right)$$

$$\bar{C}_V(T \to 0) \stackrel{?}{=} 0$$

$$\bar{C}_V =$$

(6.26)

$$Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$$

low temp limit $\frac{\epsilon}{kT} \gg 1$

$$\cong 1 + 3e^{-2\epsilon/kT} = 1 + 3e^{-2\epsilon\beta}$$

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{Z}\frac{\partial}{\partial \beta}(1+3e^{-2\epsilon\beta}) = -\frac{1}{Z}(3(-2\epsilon)e^{-2\epsilon\beta})$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}}$$

$$\bar{C}_V = \frac{\partial \bar{E}}{\partial T} \quad ; \quad \bar{C} = \frac{\partial \bar{E}}{\partial T} = \frac{\partial \beta}{\partial T}\frac{\partial \bar{E}}{\partial \beta}$$

$$= -\frac{\beta}{T}\frac{\partial \bar{E}}{\partial \beta}$$

$$\beta = \frac{1}{kT}$$

$$\frac{\partial \beta}{\partial T} = -\frac{1}{kT^2}$$

$$= -\frac{1}{T}\beta$$

$$k\left(\frac{6\epsilon}{kT}\right)^2 = C.$$

So

$$\frac{\partial \bar{E}}{\partial \beta} = \frac{-12\epsilon^2 e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} - \frac{6\epsilon e^{-2\epsilon\beta}\cdot(-6\epsilon e^{-2\epsilon\beta})}{(1+3e^{-2\epsilon\beta})^2}$$

$$= \frac{-12\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} + \frac{36\epsilon^2 e^{-4\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left[-2(1+3e^{-2\epsilon\beta}) + 6\epsilon e^{-2\epsilon\beta}\right]$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(\cancel{-2-6+6\epsilon}\ -2-6e^{-2\epsilon\beta}+6\epsilon e^{-2\epsilon\beta}\right)$$

$$= \frac{6\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(-2-6(1+\epsilon)e^{-2\epsilon\beta}\right)$$

$$= -\frac{12\epsilon e^{-2\epsilon\beta}}{(1+3e^{-2\epsilon\beta})^2}\left(1+3(1+\epsilon)e^{-2\epsilon\beta}\right)$$

to simplify keeping only the largest T dependent term $\Rightarrow \beta \ll 1$

$$\Rightarrow \bar{E} = \frac{6\epsilon e^{-2\epsilon\beta}}{1+3e^{-2\epsilon\beta}} \simeq 6\epsilon \cancel{e^{-2\epsilon\beta}}(1-3e^{-2\epsilon\beta})$$

$$= 6\epsilon(\cancel{e^{-2\epsilon\beta}} - 3e^{-4\epsilon\beta})$$

$$C \equiv \frac{\partial E}{\partial T} = -\frac{\beta}{T}\frac{\partial E}{\partial \beta} = -\frac{\beta}{T}6\epsilon(-2\epsilon e^{-2\epsilon\beta} + 12\epsilon e^{-4\epsilon\beta})$$

$\cancel{*}$

$$\cancel{E} \simeq 6\epsilon \cancel{e}^{-2\epsilon\beta}$$

Using the full expression:

(6.27) $Z_{rot} = \sum_{j=0}^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT}$ v.s.

$$Z_{rot} = \int_0^{\infty} (2j+1) e^{-j(j+1)\epsilon/kT} \, dj = \frac{kT}{\epsilon}$$

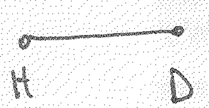
...

(6.28) $Z_{rot} \cong \sum_{j=0}^{6} (2j+1) e^{-j(j+1)\epsilon/kT}$

$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$ $\quad 0 \le \frac{kT}{\epsilon} \le 3 \quad \Rightarrow \quad \frac{1}{3} \le \frac{\epsilon}{kT} \le \infty$

. .

(6.29) $\epsilon = 0.0057$ eV



Assume $\frac{kT}{\epsilon} \ll 1$ $\qquad \frac{\epsilon}{kT} \gg 1$

$$\Rightarrow T \ll \frac{\epsilon}{k} = \frac{0.0057 \text{ eV}}{(8.617 \cdot 10^{-5} \text{ eV/K})}$$

(6.30) $\epsilon = .0076$ eV

singlet (1) triplet (3)

parahydrogen orthohydrogen.

(a) j even

$$Z_{rot} = \sum_{j=0}^{\infty} (4j+1)\, e^{-2j(2j+1)\epsilon/kT}$$

ϵ known

$\frac{\epsilon/k}{T}$

...

(b) $$Z_{rot} = \sum_{j=0}^{\infty} (2(2j+1)+1)\, e^{-(2j+1)((2j+1)+1)\epsilon/kT}$$

...

(c) $\frac{1}{4}$(parahy) + $\frac{3}{4}$(ortho hy)

$$Z_{rot\,mix} \overset{?}{=} \frac{1}{4} Z_{rot_{para}} + \frac{3}{4} Z_{rot_{ortho.}}$$

(d) $$Z = \sum_{j=0}^{\infty} (4j+1)\, e^{-2j(2j+1)\epsilon/kT} + \sum_{j=0}^{\infty} 3\cdot(2(2j+1)+1)\, e^{-(2j+1)(2j+2)\epsilon/kT}$$

...

(e)

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det 9 tot

3 anti 6 symm

(6.3) $E = c|q|$

Steps in Equip thm

1) Assume energy levels are discrete q_i
2) compute the partition function for a system w/ this energy

$$Z = \sum_q e^{-\beta E(q)} = \sum_q e^{-\beta c|q|} = \frac{1}{\Delta q}\sum_q e^{-\beta c|q|}\Delta q$$

$$\cong \frac{1}{\Delta q}\int_{-\infty}^{+\infty} e^{-\beta c|q|}\,dq = \frac{2}{\Delta q}\int_0^{\infty} e^{-\beta c q}\,dq = \frac{2}{\Delta q}\left.\frac{e^{-\beta c q}}{(-\beta c)}\right|_0^{\infty}$$

$$= \frac{2}{\Delta q(-\beta c)}(0-1) = \frac{2}{\Delta q \beta c}$$

Now:

$$\bar{E} = -\frac{1}{Z}\frac{\partial Z}{\partial \beta} = -\frac{1}{\left(\frac{2}{\Delta q\beta c}\right)}\frac{\partial}{\partial\beta}\left(\frac{2}{\Delta q\beta c}\right)$$

$$= -\frac{\Delta q\beta c}{2}\left(\frac{2}{\Delta q c}\right)\left(-1\beta^{-2}\right) = \beta\cdot\beta^{-2} = \beta^{-1} = \underline{kT}$$

(6.32)

(a) $Z = \sum_q e^{-\beta q} = \int_{-\infty}^{\infty} e^{-\beta q(x)} dx$

This is the definition of

$\bar{x} = \sum_i x_i P(x_i)$ or if the variable x is given continuously

w/ $P(x_i) = \dfrac{e^{-\beta U(x_i)}}{Z}$

$\bar{x} = \int_{\mathbb{R}} x P(x)\, dx = \int x \dfrac{e^{-\beta U(x)}}{Z} dx = \ldots$

(b) For stability $F = -\dfrac{dU}{dx}$

~~B~~ ~~must~~ be this If a linear term was not zero

$F = -\dfrac{d}{dx}\left[(x-x_0)\dfrac{dU}{dx}\Big|_{x_0} + \dfrac{(x-x_0)^2}{2}\dfrac{d^2U}{dx^2}\Big|_{x_0} + \cdots \right]$

$\approx -\dfrac{dU}{dx}\Big|_{x_0} \mp (x-x_0)\dfrac{d^2U}{dx^2}\Big|_{x_0} + O((x-x_0)^2)$

↑

gives constant force →← to point x_0 being a stationary point.

If $\left.\frac{du}{dx}\right|_{x_0} = 0$ ~~[illegible]~~ & $\left.\frac{d^2u}{dx^2}\right|_{x} > 0$ then

~~[illegible]~~.

w/ $F \approx -(x-x_0)\left.\frac{d^2u}{dx^2}\right|_{x_0}$

$F < 0 \quad x > x_0$ } point x_0 is a stable equilibrium point.
$F > 0 \quad x < x_0$

Assume $u(x) \approx u(x_0) + \frac{1}{2}(x-x_0)^2 \left.\frac{d^2u}{dx^2}\right|_{x_0}$

Then

$$\overline{x} = \frac{\int e^{-\beta\left(u(x_0) + \frac{1}{2}(x-x_0)^2 u''(x_0)\right)} \cdot x \, dx}{\int e^{-\beta\left(u(x_0) + \frac{1}{2}(x-x_0)^2 u''(x_0)\right)} dx}$$

$$= \frac{\int_{-\infty}^{\infty} x e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)} dx}{\int e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)} dx} = \frac{\int_{-\infty}^{\infty} (x-x_0) e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)} dx + x_0 \int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 u''(x_0)} dx}{\int e^{-\beta}}$$

$$= \frac{\displaystyle\int_{-\infty}^{+\infty} (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, dx}{\displaystyle\int_{-\infty}^{\infty} e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, dx} + x_0$$

$$= x_0 \quad \checkmark$$

(c) Ass $U(x) \approx U(x_0) + \frac{1}{2}(x-x_0)^2 U''(x_0) + \frac{1}{6}(x-x_0)^3 U'''(x_0)$

$$\bar{X} = \frac{\displaystyle\int x e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}{\displaystyle\int e^{-\beta (x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}$$

$$= x_0 + \frac{\displaystyle\int (x-x_0)\, e^{-\beta \frac{1}{2}(x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}{\displaystyle\int e^{-\beta (x-x_0)^2 U''(x_0)}\, e^{-\beta \frac{1}{6}(x-x_0)^3 U'''(x_0)}\, dx}$$

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$$= x_0 + \frac{\displaystyle\int x e^{-\frac{\beta}{2}x^2 U''(x_0)} \cdot \left[1 - \frac{\beta}{6} x^3 U'''(x_0) + O(x^6)\right] dx}{\displaystyle\int e^{-\frac{\beta}{2}x^2 U''(x_0)} \left[1 - \frac{\beta}{6} x^3 U''(x_0) + \frac{\beta^2}{36} x^6 U'''(x_0)^2 + O(x^9)\right]}$$

$$e^x = \sum \frac{x^k}{k!}$$

$$= x_0 + \frac{\displaystyle\cancel{\int x e^{-\left(\frac{\beta}{2}\right)x^2 U''(x_0)}}^{\,0} - \frac{\beta}{6}\int x^4 e^{-\frac{\beta}{2}x^2 U''(x_0)} dx}{\displaystyle\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx}$$

Now

$$\int e^{-\frac{\beta}{2}x^2 U''(x_0)} dx$$

let $v^2 = \frac{\beta}{2} U''(x_0) x^2$ $\qquad v = \sqrt{\frac{\beta U''(x_0)}{2}}\, x$

$$\int e^{-v^2} \frac{\sqrt{2}}{\sqrt{\beta U''(x_0)}} dv \qquad dv = \sqrt{}\, dx$$

$$= \frac{\sqrt{2\pi}}{\sqrt{\beta U''(x_0)}}$$

Now

$$\int x^4 e^{-\frac{\beta}{2} U''(x_0) x^2} dx$$

let $V = \sqrt{\frac{\beta U''(x_0)}{2}}\, x \qquad x = \sqrt{\frac{2}{\beta U''(x_0)}}\, V$

$$= \int \left(\frac{2}{\beta U''(x_0)}\right)^2 V^4 e^{-V^2} \sqrt{\frac{2}{\beta U''(x_0)}}\, dV$$

$$= \left(\frac{2}{\beta U''(x_0)}\right)^{5/2} \underbrace{\int V^4 e^{-V^2} dV}_{\frac{3\sqrt{\pi}}{4}}$$

$$\therefore \bar{x} = x_0 + \frac{-\frac{\beta}{6}\left(\frac{3\sqrt{\pi}}{4}\right)\left(\frac{2}{\beta U''(x_0)}\right)^{5/2}}{\sqrt{\pi}\left(\frac{2}{\beta U''(x_0)}\right)^{1/2}}$$

$$= x_0 - \frac{\beta}{6}\,\frac{3}{4}\left(\frac{2}{\beta U''(x_0)}\right)^2 = x_0 - \frac{1}{8}\cdot\frac{4}{\beta\,(U''(x_0))^2}$$

$$= x_0 - C_1 \cdot kT$$

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B (d) compute $U''(x_0) = \dots$

6.33

V_{mp} = velocity at which $D(v)$ is largest.

$$= \sqrt{\frac{2kT}{m}}$$

$$m_{O_2} = \frac{2.16\ g}{6.022 \cdot 10^{23}} = \frac{2.16 \cdot 10^{-3}\ kg}{6.022 \cdot 10^{23}}$$

$$kT = (1.381 \cdot 10^{-23}\ J/K)(300\ K) =$$

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = \cdots$$

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

6.34 $$D(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}}$$

$$m_{N_2} = \frac{2.14 \cdot 10^{-3}\ kg}{6.022 \cdot 10^{23}} \cong \cdots \frac{2.14}{6} \cdot 10^{-26}\ kg$$

$$kT = (1.381 \cdot 10^{-23}\ J/K)\ T =$$

$\frac{m}{kT}$ = spaced in units of $\hat{v} = \sqrt{\frac{kT}{m}}$ say

6.35

$$D(v)\,dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}}\,dv$$

$$\frac{dD}{dv} = \left(\frac{m}{2\pi kT}\right)^{3/2}\cdot 4\pi\left[2v\,e^{-\frac{mv^2}{2}\beta} + v^2\left(\cancel{\frac{mv^2}{2}} - \frac{2mv\beta}{2}\right)e^{-\frac{mv^2}{2}\beta}\right] \underset{\text{set}}{=} 0$$

$$= 2v - v^3 m\beta = 0$$

$$v = 0 \quad \text{or} \quad 2 = v^2 m\beta$$

$$v = \sqrt{\frac{2}{m\beta}} = \sqrt{\frac{2kT}{m}}$$

6.36 eq 6.51

$$\bar{v} = \sum_{\text{all } v} v\,D(v)\,dv = \int_0^{\infty} v\,C\,4\pi v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

$$= 4\pi C\int_0^{\infty} v^3 e^{-\frac{mv^2}{2}\beta}\,dv \qquad \text{let } u = \sqrt{\frac{m\beta}{2}}\,v$$

$$dv = \sqrt{\frac{2}{m\beta}}\,du$$

$$= 4\pi C\int_0^{\infty}\left(\frac{2}{m\beta}\right)^{3/2} e^{-u^2} u^3 \left(\frac{2}{m\beta}\right)^{1/2} du$$

$$= 4\pi G \left(\frac{2}{m\beta}\right)^2 \int_0^\infty v^3 e^{-v^2} \, dv$$

(6.37)

$$\overline{v^2} = \int_0^\infty v^2 D(v)\,dv = \int_0^\infty v^2 \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^4 e^{-\frac{mv^2}{2}\beta}\,dv$$

let $u = \sqrt{\frac{m}{2}\beta}\; v$ $\qquad v = \left(\frac{2}{m\beta}\right)^{1/2} u$

$$dv = \left(\frac{2}{m\beta}\right)^{1/2} du$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty \left(\frac{2}{m\beta}\right)^{4/2} \left(\frac{2}{m\beta}\right)^{1/2} u^4 e^{-u^2}\,du$$

$$= \cancel{4\pi \left(\frac{1}{2\pi}\right)^{3/2} \left(\frac{m}{\beta}\right)}$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \left(\frac{2}{m\beta}\right)^{5/2} \int_0^\infty u^4 e^{-u^2}\,du$$

(6.38) $$f = \int_0^{300\,m/s} D(v)\,dv = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{300\,m/s} v^2 e^{-\frac{mv^2}{2}\beta}\,dv$$

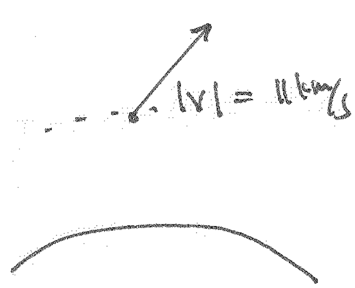
~~Set~~ ~~v~~ $u = \left(\frac{m\beta}{2}\right)^{1/2} v \qquad v =$

$$f = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{\left(\frac{m\beta}{2}\right)^{1/2}\cdot 300} \left(\frac{2}{m\beta}\right)^{2/2} \cdot \left(\frac{2}{m\beta}\right)^{1/2} e^{-u^2} u^2\,du$$

$$= \cancel{\left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \int_0^{\frac{300}{\left(\frac{2}{m\beta}\right)^{1/2}}}}$$

$$= \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi \left(\frac{2}{m\beta}\right)^{5/2} \int_0^{\frac{300}{\left(\frac{2}{m\beta}\right)^{1/2}}} u^2 e^{-u^2}\,du$$

(6.39)



$N_2 =$

$$m_{N_2} = \frac{2\cdot 14\cdot 10^{-3}\,kg}{6.022\cdot 10^{23}}$$

$T = 1000\,K.$

$$P = \int_{11\cdot 10^3\,m/s}^{\infty} D(v)\,dv$$

But "several" of these particles will not be pointing in the outward direction

$$P = \int_{11\cdot 10^3 \, m/s}^{\infty} \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 e^{-\frac{mv^2}{2kT}} \, dv = \ldots$$

(b) $m_{H_2} = \dfrac{2\cdot 10^{-3}}{6.022\ldots}$

$m_{He} = \underline{\qquad}$

Based on $P_{N_2} / P_{H_2} : P_{He}$ one should find proportionally the same # of molecules.

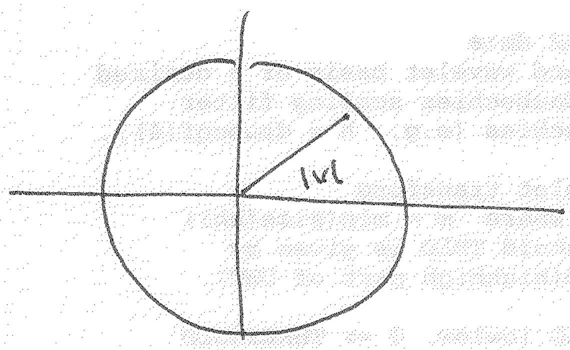
(c)

$$P = \int_{2.4\cdot 10^3 \, m/s}^{\infty} D(v)\, dv \approx 1.$$

it is very probable ~~that~~ that molecule will be able to escape

(6.40) Don't see how? Must be inelastic collisions

(6.41)



$$D(v) \propto \begin{pmatrix} \text{prob molec} \\ \text{have vector } \vec{v} \end{pmatrix} \times \begin{pmatrix} \text{\# of vectors } \vec{v} \\ \text{w/ } |\vec{v}| = v \end{pmatrix}$$

$$\propto e^{-\frac{mv^2}{2kT}} \cdot 2\pi v$$

$$D(v) = 2\pi C v e^{-\frac{mv^2}{2kT}}$$

$$1 = 2\pi C \int_0^\infty v e^{-\frac{mv^2}{2kT}} dv$$ solve for C

most likely vector ~~$\frac{dD}{d\vec{v}}$~~ $= 0$ one w/ maximum D(v)

$$\vec{v} = 0$$

most likely speed ~~$D(v)$~~ $\frac{dD(v)}{dv} = 0 \Rightarrow v = \ldots$

(6.42) $Z_{h.o.} = \frac{1}{1-e^{-\beta\epsilon}}$

(a) $F = -kT \ln Z$

$F = -kT \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$ is there a power of N + a Normalization factor of $N!$?

Think yes

$Z_{total} = \frac{Z_{ho}^{N}}{N!}$

$F = -kTN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right)$

(b) $S = -\frac{\partial F}{\partial T}\Big|_{V,N} = -kN \ln\left(\frac{1}{1-e^{-\beta\epsilon}}\right) - kTN\left(1-e^{-\beta\epsilon}\right)$.

$\frac{\partial}{\partial T}\left(1-e^{-\beta\epsilon}\right)$

$= -kN \ln\left(\frac{1}{\quad}\right) - kTN\left(1-e^{-\beta\epsilon}\right) \cdot \frac{-1}{kT^2}\frac{\partial}{\partial \beta}\left(1-e^{-\beta\epsilon}\right)$

$\left\{ \frac{\partial \beta}{\partial T} = -\frac{1}{kT^2} \right\}$

6.43

$$S = -k \sum_s P(s) \ln P(s)$$

(a) Isolated system $p(s) = \frac{1}{\Omega}$? Don't see

$$S = -k \sum_s \frac{1}{\Omega} \ln(1/\Omega)$$ know $\sum_s p(s) = 1$

$$= \cancel{k \sum_s \frac{\ln(\Omega)}{\Omega}}$$ can I factor out $\ln(1/\Omega)$ to get

$$S = -k \ln(1/\Omega) \sum_s 1/\Omega = +k \ln \Omega \checkmark.$$

(b) $p(s) = \frac{e^{-E(s)\beta}}{Z}$ $\sum_s p(s) = 1$

$$S = -k \sum \frac{e^{-E(s)\beta}}{Z} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

$$= -\frac{k}{Z} \sum e^{-E(s)\beta} \ln\left(\frac{e^{-E(s)\beta}}{Z}\right)$$

How show ?

(6.44)

$$Z_{total} = \frac{Z_1^N}{N!}$$

$$\left\{ \ln N! \approx N \ln N - N \right\}$$

$$F = -kT \ln(Z_{total}) = \cancel{\ldots}$$

$$-kTN \ln Z_1 + kT \ln N!$$

$$\approx -kTN \ln Z_1 + kTN \ln N - kTN$$

$$= kTN \left[-\ln Z_1 + \ln N - 1 \right]$$

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{V,T} = kT \left[-\ln Z_1 + \ln N - 1 \right] + kTN \left[\frac{1}{N} \right]$$

$$= kT + kT \left[-\ln Z_1 + \ln N - 1 \right]$$

$$= kT \left[-\ln Z_1 + \ln N \right]$$

(6.45)

$$S = -\frac{\partial F}{\partial T}\bigg|_{V,N}$$

w/ $F = -NkT\left[\ln V - \ln N - \ln v_Q + 1\right] + F_{int}$

$$\frac{\partial F}{\partial T}\bigg|_{V,N} = -Nk\left[\ln V - \ln N - \ln V_Q + 1\right] - NkT\left[\cancel{\ } - \frac{1}{V_Q}\frac{\partial v_Q}{\partial T}\right] + \frac{\partial F_{int}}{\partial T}\bigg|_{V,N}$$

$$* \quad \frac{1}{V_Q}\frac{\partial v_Q}{\partial T} = \left(-\frac{3}{2}\frac{V_Q}{T}\right)\frac{1}{V_Q} = -\frac{3}{2}\cdot\frac{1}{T}$$

$$V_Q = \left(\frac{h}{\sqrt{2\pi mkT}}\right)^3 = \left(\frac{h}{\sqrt{2\pi mk}}\right)^3 \cdot T^{-3/2}$$

$$f = x^p$$

$$\frac{df}{dx} = p\frac{f}{x}$$

$$\therefore \quad S = -\frac{\partial F}{\partial T}\bigg|_{V,N} = +Nk\left[\ln V - \ln N - \ln V_Q + 1\right]$$

$$+ NkT\left(\frac{3}{2}\cdot\frac{1}{T}\right) + -\frac{\partial F_{int}}{\partial T}\bigg|_{V,N}.$$

$$\Rightarrow \quad S = +Nk\left[\ln V - \ln N - \ln V_Q + 1 + \frac{3}{2}\right] - \frac{\partial F_{int}}{\partial T}\bigg|_{V,N}$$

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$$S = +Nk\left[\ln\left(\frac{V}{V_Q \cdot N}\right) + \frac{5}{2}\right] - \left.\frac{\partial F_{int}}{\partial T}\right|_{V,N} \quad \text{eq 6.92} \checkmark$$

$$= Nk\left[\cancel{\ln(}\right.$$

$$\mu = \left.\frac{\partial F}{\partial N}\right|_{T,V} = \left.\frac{\partial}{\partial N}\right|_{T,V}\left[-NkT\left[\ln V - \ln N - \ln v_Q + 1\right] + F_{int}\right]$$

$$= -kT\left[\ln V - \ln N - \ln V_Q \cancel{+1}\right] - NkT\left[\cancel{-\frac{1}{N}}\right] + \left.\frac{\partial F_{int}}{\partial N}\right|_{T,V}$$

$$= kT\left[\ln\left(\frac{N V_Q}{V}\right)\right] + \left.\frac{\partial F_{int}}{\partial N}\right|_{T,V}$$

$$F_{int} = -kT \ln Z$$

$$\frac{\partial F_{int}}{\partial N} = -\frac{kT}{Z}\frac{\partial Z_{int}}{\partial N} \qquad \cancel{Z_{int}} = \frac{\cancel{Z_{int}^N}}{N!}$$

$$= -kT\left[\ln\left(\frac{V}{N v_Q}\right)\right] + \frac{\partial}{\partial N}(-kT \ln Z_{int})$$

$$= -kT\left[\ln\left(\frac{V}{N v_Q}\right) + \frac{\partial (N \ln Z_{int})}{\partial N}\right]$$

w/ Z_{int} independent of N.

$$\Rightarrow \mu = -kT\left[\ln\left(\frac{VZ_{int}}{NV_Q}\right)\right] \qquad \text{eq } 6.93 \checkmark$$

6.46 $\frac{VZ_{int}}{NV_Q}$

$V \sim 1$ liter $= 10^{-3}\ m^3$

$N \sim 6.23 \cdot 10^{23}$

$V_Q \sim (2\cdot 10^{-11})^3 \sim 8\cdot 10^{-33}$

Nitrogen

$Z_{int} \sim$ Don't know. what values this has? $\gg 1$

$$\frac{(10^{-3})(?)}{(10^{23})(10^{-33})} \gg 1$$

6.47 $Z_{1d} = \frac{L}{l_Q}$ $\qquad l_Q = \sqrt{\frac{2\pi mkT}{h^2}}$

Assume we have freeze out of translational degrees of freedom would be when

$Z_{1d} \sim 1$ or $Z_{1d} \ll 1$

$$\Rightarrow L \sim l_Q \Rightarrow \sqrt{\frac{2\pi mkT}{h^2}} = L$$

$$\frac{2\pi m k T}{h^2} = L^2$$

$$\Rightarrow T = \frac{h^2 L^2}{2\pi m k} \Rightarrow T = \frac{(6.626\cdot 10^{-34}\,J\cdot s)^2 (10^{-2}\,m)^2}{2\pi\left(\frac{14\cdot 10^{-3}\,kg}{6.022\cdot 10^{23}}\right)(1.381\cdot 10^{-23}\,J/K)}$$

$$\Rightarrow T = 2.1\cdot 10^{-23}\,K.$$

(6.48)  $Z_{int} = Z_e \cdot Z_{rot}$

$$F_{int} = -kTN \ln Z_{int} = -kTN \ln(Z_e Z_{rot})$$

From eq 6.92 we get

$$S = Nk\left[\ln\left(\frac{V}{Nv_Q}\right) + \frac{5}{2}\right] - \frac{\partial}{\partial T}\left(-kTN\ln(Z_e Z_{rot})\right)$$

$$\cancel{kN\ln(Z_e Z_{rot})}$$

$$= \cancel{Nk\left[\ln\left(\frac{V Z_e Z_{rot}}{N v_Q}\right)\right.}$$ w/ $Z_{rot} \cong \frac{kT}{\epsilon}$

$$\Rightarrow S = Nk\left[\ln\left(\frac{V}{Nv_Q}\right) + \frac{5}{2}\right] + kN\ln(Z_e Z_{rot})$$

$$+ \frac{kN\cancel{T}}{Z_e \cancel{Z_{rot}}}\cdot Z_e \cdot \frac{\cancel{k}}{\epsilon}$$

$$\Rightarrow S = Nk\left[\ln\left(\frac{V Z_e Z_{rot}}{N V_Q}\right) + \frac{5}{2}\right] + kN$$

$$= Nk\left[\ln\left(\frac{V Z_e Z_{rot}}{N V_Q}\right) + \frac{7}{2}\right]$$

~~En~~ $\cancel{PV = NkT}$ $\cancel{\frac{V}{N} = \frac{kT}{P}}$ $\cancel{Nk = \frac{PV}{T}}$

1 mole $\Rightarrow N = N_A = 6.022 \cdot 10^{23}$

Also $PV = NkT \Rightarrow \frac{V}{N} = \frac{kT}{P}$

$Z_{rot} = \frac{kT}{\epsilon}$ $\epsilon_{O_2} \cong 0.00018\ \text{eV}$ (pg 236 Schroeder)

(b) $\mu = -kT \ln\left(\frac{V Z_e Z_{rot}}{N V_Q}\right)$ ...

(6.49) U, H, F, G, S, μ $\quad \epsilon_{N_2} = .00025\ eV$

$T = 273 k$

$P = 1 atm$

... Not sure

(6.50) $G = U - TS + PV$ $\quad dU = TdS - pdV$

so $G = U_{int} + \frac{3}{2}NkT - T\left(Nk\left[\ln\left(\frac{V}{Nv_Q}\right) + \frac{5}{2}\right] - \frac{\partial F_{int}}{\partial T}\right) + PV$

$\overset{?}{=} N\left(-kT \ln\left(\frac{V Z_{int}}{N v_Q}\right)\right)$

what is Z_{int}?

?

(6.51) $Z_{tr} = \frac{1}{h^3}\int d^3r\, d^3p\, e^{-E_{tr}/kT}$ $\quad E_{tr} = +\left(\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}\right)$

$= \frac{1}{2m}(p_x^2 + p_y^2 + p_z^2)$

$$= \frac{1}{h^3}\underbrace{\int d^3r}_{= V} \cdot \int d^3p\, e^{-E_{tr}/kT}$$

$$= \frac{V}{h^3}\int d^3p \exp\left\{-\frac{1}{2m(kT)}(p_x^2 + p_y^2 + p_z^2)\right\}$$

$$= \frac{V}{h^3}\prod_{i=1}^{3}\int dp\, e^{-\frac{p^2}{2mkT}}$$

let $v^2 = \frac{p^2}{2mkT} \Rightarrow v = \frac{p}{\sqrt{2mkT}}$

$\Rightarrow p = \sqrt{2mkT}\, v.$

$dp = \sqrt{2mkT}\, dv$

so $$Z_{tr} = \frac{V}{h^3}\left(\sqrt{2mkT}\int_{-\infty}^{\infty} dv\, e^{-v^2}\right)^3$$

$$= \frac{V}{h^3}(2mkT)^{3/2}\left(\sqrt{\pi}\right)^3$$

$$= V \cdot \left(\frac{2\pi mkT}{h^2}\right)^{3/2} = \frac{V}{\left(\frac{h^2}{2\pi mkT}\right)^{3/2}}$$

define $v_Q = \left(\frac{h^2}{2\pi mkT}\right)^{3/2}$ ✓.

6.52 $E = pc$ vs. $E = \frac{p^2}{2m}$

$$Z_1 = \frac{1}{h}\int dx\,dp\, e^{-E/kT} = \frac{1}{h}\cdot L \int_{-\infty}^{+\infty} dp\, e^{-p/ckT}$$

$v = \frac{p}{ckT}$

$dp = ckT\,dv$

$$= \frac{L}{h}\cdot ckT \int_{-\infty}^{\infty} dv\, e^{-v}.$$

$$= \frac{L\cdot ckT}{h}\left(-e^{-v}\right|_{-\infty}^{+\infty} \quad \times$$

in $E = pc$ p is $|p|$

$$\therefore Z_1 = \frac{L}{h}\int_0^{\infty} \cdots = \frac{L\cdot ckT}{h}\left(-e^{-v}\right|_0^{\infty}$$

$$= -\frac{LckT}{h}(0-1) = \frac{LckT}{h}$$

$$\Rightarrow \frac{L}{\left(\frac{h}{ckT}\right)} \; ; \quad l_Q = \frac{h}{ckT}$$

Check units $[l_Q] = \dfrac{J\cdot s}{\left(\frac{m}{s}\right)\left(\frac{J}{\cancel{K}}\right)(\cancel{K})} = \dfrac{s^2}{m}$? error somewhere

(6.53) $H_2 \longleftrightarrow 2H$

$$k = \frac{[H]^2}{[H_2]} \cong \frac{P_H^2}{P_{H_2} \cdot P^0}$$

P's = partial press

... Not sure.

7.1

$Z = \ldots ?$

$$Z = 1 + e^{-(\epsilon-\mu)/kT}$$

$$P = \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}} \quad \text{w/ } \mu = \mu(P^\circ_{O_2})$$

$$\epsilon = -.7\,eV.$$

$PV = NkT$ so $\mu(P^\circ) = -kT \ln\left(\frac{Z_{int}\, kT}{v_Q P^\circ}\right)$

plot $P(P^\circ) = \ldots$

7.2

$$Z = 1 + 2e^{-(-.55eV - \mu(P_{O_2}))/kT} + e^{-(-1.3eV - \mu(P_{O_2}))/kT}$$

Then

$$P_{\text{zero sites occupied}}(P_{O_2}) = \frac{1}{Z(P_{O_2})}$$

$$P_{\text{one site occupied}}(P_{O_2}) = \frac{2e^{-(-.55eV - \cdots)/kT}}{Z(P_{O_2})}$$

$$P_{\text{two sites occ}} = \frac{e^{-(-1.3eV \cdots)/kT}}{Z(P_{O_2})}$$

7.3 $$Z = 1 + e^{-(\epsilon-\mu)/kT}$$

ϵ = ionization energy for H atom

μ = chemical potential of ideal ~~gas~~ gas

7.4 $$Z = 1 + e^{-(\epsilon'-\mu)/kT} + e^{-(\epsilon''-\mu)/kT}$$

How change chemical potential of gas?

7.5

(a) $$P_{\text{ionized}} = 1 - P_{\substack{\text{original donor}\\ \text{site occupied}}} = 1 - \frac{2e^{-(I-\mu)/kT}}{1+2e^{-(I-\mu)/kT}}$$

2 for 2 electron spin states

(doesn't change the ionization energy)

(b) $$\mu = -kT\ln\left(\frac{Z_{int}}{V_Q}\cdot\frac{V}{N}\right)$$

(c) ?

(d) P ..

7.6 $$p(s) = \frac{e^{-(E(s)-\mu N(s))/kT}}{Z}$$

$$\frac{\partial P}{\partial N} = 0$$ solve for N

$$p(s)\cdot\frac{\mu}{kT} - \frac{p(s)}{Z}\frac{\partial Z}{\partial N} = 0$$

$$\Rightarrow \frac{\mu}{kT} - \frac{1}{Z}\frac{\partial Z}{\partial N} = 0$$

$$\sigma_N^2 = \langle N^2\rangle - \langle N\rangle^2 = \frac{(kT)^2}{Z}\frac{\partial^2 Z}{\partial \mu^2} - \frac{kT}{Z}\frac{\partial Z}{\partial \mu}$$

$$=$$

...

7.7 Prob 5.23 is $\Phi = U - TS - \mu N$

$F = -kT \ln Z$ $\qquad \Phi = -kT \ln \mathcal{Z}$

show $\Phi = U - TS - \mu N$ + $\Phi = -kT\ln\mathcal{Z}$ satisfy same

D.E. w/ initial condition.

7.8

$N_s = 10$.

(a) $Z \equiv \sum_{s} e^{-\beta E(s)} = \sum_{s=1}^{10} e^{-\beta E_s} = 10$ since each term is 1

(b) ~~Z~~ $Z = 10^2$

(c) Bosons: can occupy the same state & identical

$\tilde{A}$ $\tilde{B}$
x x

$$Z = \frac{(Z_1)(Z_1)}{2} = \frac{Z_1^2}{2}$$

(d) Fermions: cannot occupy the same state.

$$Z = \frac{Z_1(Z_1 - 1)}{2}$$

(e) $Z = \frac{1}{2!} Z_1^2$

(f) $P_{\text{Fermions}} = 0$ can't happen

$$P_{\text{Bosons}} = \frac{?}{\left(\frac{Z_1^2}{2}\right)}$$

7.9 N_2

$$V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^{3}$$

$$m_{N_2} = \frac{2\ g/mole\,(14)}{6.022 \cdot 10^{23}\ \text{molecules/mole}}$$

$PV = N \cdot kT$

$$\frac{V}{N} = \frac{kT}{P}$$

$\frac{V}{N} \gg V_Q$ Boltzmann statistics regime.

$$\frac{kT}{P} \gg \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

$$(2\pi m k T)^{3/2} \cdot kT \gg h^3 P$$

Density constant $\Rightarrow \frac{1}{V} = \rho_N$

$$\Rightarrow \left(\frac{N}{V}\right) = \frac{P}{kT}$$

$$(2\pi m k T)^{3/2} \gg h^3 \left(\frac{P}{kT}\right)$$

$\uparrow$

7.10

(a)	Identical Fermions	Identical Bosons	Distinguishable Part
E_{ground}	$\epsilon_1+\epsilon_2+\epsilon_3$	$3\epsilon_1$	$\epsilon_1+\epsilon_2+\epsilon_3$ vs. $3\epsilon_1$

(b) 4. ?

(c) ?

(d) ?

7.11

$$\bar{n}_{FD} = \frac{1}{e^{(\epsilon-\mu)/kT}+1}$$

(a) $\epsilon - \mu = -.1\ eV$

$kT = (8.617\cdot 10^{-5}\ eV/K)(300K)$

$\cong 24 \cdot 10^{-3}\ eV$

$\cong 2.4\cdot 10^{-2}\ eV$

(b) $\epsilon - \mu = -.01\ eV$

(c) $\epsilon - \mu = 0$

$$\bar{n}_{FD} = \frac{1}{2}$$

(d) $\epsilon - \mu = .01\ eV$

(e) $\epsilon - \mu = 1\ eV$

7.12

$\mu + x$ ——•—— $= \epsilon_B$

μ ————

$\mu - x$ ———— $= \epsilon_A$

$$P_{B\ \text{occupied}} = \frac{e^{-(\epsilon_B-\mu)/kT}}{e^{-(\epsilon_B-\mu)/kT} + 1 + e^{-(\epsilon_A-\mu)/kT}} = \frac{e^{-x/kT}}{1+e^{-x/kT}+e^{+x/kT}}$$

$$P_{A\ \text{unoccupied}} = \frac{e^{-(\epsilon_B-\mu)/kT}+1}{e^{-x/kT}+e^{+x/kT}+1} = \frac{e^{-x/kT}+1}{e^{-x/kT}+e^{+x/kT}+1} \quad ?$$

(7.13) $$\bar{n}_{BE} = \frac{1}{e^{(\epsilon-\mu)/kT}-1} \qquad Z = \frac{1}{1-e^{-(\epsilon-\mu)/kT}}$$ Bosons

(a) $\epsilon-\mu = .001$ eV

(b) $\epsilon-\mu = 0.01$ eV

(c) $\epsilon-\mu = .1$ eV

(d) $\epsilon-\mu = 1$ eV

$$P = \frac{1}{Z}e^{-(n\epsilon-n\mu)/kT} = \frac{e^{-n(\epsilon-\mu)/kT}}{Z} \qquad n=0,1,2,3.$$

(7.14) For large energy $\{\epsilon-\mu\}$ all statistics should obey Boltzmann statistics $\Rightarrow$ expect each that the Boltzmann distribution

$$\bar{n}_{Boltzmann} = e^{-(\epsilon-\mu)/kT}$$

$$\bar{n}_{BE} = \frac{e^{-(\epsilon-\mu)/kT}}{1-e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT}\sum_{k\geq 0} e^{-((\epsilon-\mu)/kT)\cdot k}$$

$$\bar{n}_{BE} \cong e^{-(\epsilon-\mu)/kT}\left[1+e^{-(\epsilon-\mu)/kT}+\text{H.O.T.}\right]$$

$$\bar{n}_{FD} = \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}} = e^{-(\epsilon-\mu)/kT}\sum_{n\geq 0}(-1)^n e^{-(\epsilon-\mu)n/kT}$$

$$\cong e^{-(\epsilon-\mu)/kT}\left[1-e^{-(\epsilon-\mu)/kT}\right]$$

$\therefore$ ~~BH~~ Both statistics have relative error of

$$\left|e^{-(\epsilon-\mu)/kT}\right| \ll .01$$

$$\Rightarrow \quad -\frac{(\epsilon-\mu)}{kT} \ll \ln(10^{-2})$$

$$(\epsilon-\mu) \gg \underline{2kT\ln(10)}$$

How tell if violated?

7.15

7.31 $\bar{n}_{Boltzmann} = e^{-(\epsilon-\mu)/kT}$

$\cancel{\sum \bar{n}_{Boltz}}$ $\quad \sum_{n\geq 0} e^{-(\epsilon-\mu)n/kT} = N$ why? don't see?

$$= \sum_{n\geq 0} e^{-\frac{\epsilon n}{kT}} \cdot \sum_{n\geq 0} e^{+\mu n/kT} = N$$

$$\underbrace{\sum_{n\geq 0} e^{-\frac{\epsilon n}{kT}}}_{Z_1} \qquad \sum_{n\geq 0} e^{+\mu n/kT} = \frac{1}{1-e^{\mu/kT}}$$

$$\rightarrow \frac{Z_1}{N} = 1 - e^{\mu/kT}$$

$$e^{\mu/kT} = 1 - \frac{Z_1}{N}$$

$$\mu = kT \ln\left(1 - \frac{Z_1}{N}\right) \quad ?$$

7.16

(a) ?

(b) ?

(c) ?

(d)

7.17 ? $q=0$ $\qquad q=1$

Energy

All in zero state

(7.18)

$$Z = e^{-0(\epsilon-\mu)/kT} + e^{-1(\epsilon-\mu)/kT} + e^{-2(\epsilon-\mu)/kT}$$

$$\bar{n} = \sum_n n P(n) = 0 \cdot P(0) + 1 \cdot P(1) + 2P(2)$$

$$= \frac{e^{-(\epsilon-\mu)/kT}}{Z} + \frac{e^{-2(\epsilon-\mu)/kT}}{Z}$$

plot picture like Fig. 7.6.

(7.19)

$\rho_{Cu} = ?$

$m_{Cu} = 63.546 \ g/mol$ $\qquad V = 7.12 \ cm^3/mol$

$$\rho_{Cu} = \frac{63.546 \ g/mol}{7.12 \ cm^3/mol} \cong 9 \ g/cm^3$$

$$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3} = \cdots$$

$$m = \frac{63.546 \ g/\cancel{mol}}{6.022 \cdot 10^{23}} = \cdots$$

$$T_F = \frac{\varepsilon_F}{k} = \cdots$$

$$P = -\frac{\partial}{\partial V}\left[\frac{3}{5}N \cdot \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \frac{2N\varepsilon_F}{5V} = \cdots$$ degeneracy pressure

$$B = -V\left.\frac{\partial P}{\partial V}\right|_T = \frac{10}{9}\frac{U}{V} =$$

$$= -V\frac{\partial}{\partial V}\left[\frac{3}{5}N \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}\right] = \cdots$$

(7.20) ~~[illegible]~~ $\nu = 10^{32} /m^3$

To determine the statistics to use in dealing w/ this system

consider $\frac{V}{N} = \nu^{-1} = 10^{-32}\ m$

v.s. $V_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3 = \left(\frac{(6.626\cdot 10^{-34}\ J\cdot s)}{\sqrt{2\pi(9.109\cdot 10^{-31}\ kg)(1.381\cdot 10^{-23}\ J/k)(\ )10^7\ k}}\right)^3$

~~[illegible]~~

$V_Q = 1.309\cdot 10^{-32}$

since $V_Q \sim \frac{V}{N}$ I would argue that $\Rightarrow$ use degenerate Fermi gas.

(7.21) $\frac{N}{V} = .18\ (10^{-15}\ m)^{-3} = (.18)\ 10^{45}\ m^{-3}$

$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3}{\pi}\frac{N}{V}\right)^{2/3}$

Since we can hold 4 nucleons

$N = \underline{4} \times (\text{vol of } 1/8 \text{ sphere}) = 4\cdot\left(\frac{1}{8}\right)\frac{4}{3}\pi\, n_{max}^3 = \frac{2}{3}\pi\, n_{max}^3$

$\downarrow$

Also $\varepsilon_F = \frac{h^2 n_{max}^2}{8mL^2}$

$n_{max} = \left(\frac{3}{2}\frac{N}{\pi}\right)^{1/3}$

so that

$$\varepsilon_F = \frac{h^2}{8mL^2}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$= \frac{h^2}{8mV^{2/3}}\left(\frac{3}{2}\frac{N}{\pi}\right)^{2/3}$$

$$T_F = \frac{\varepsilon_F}{k}$$

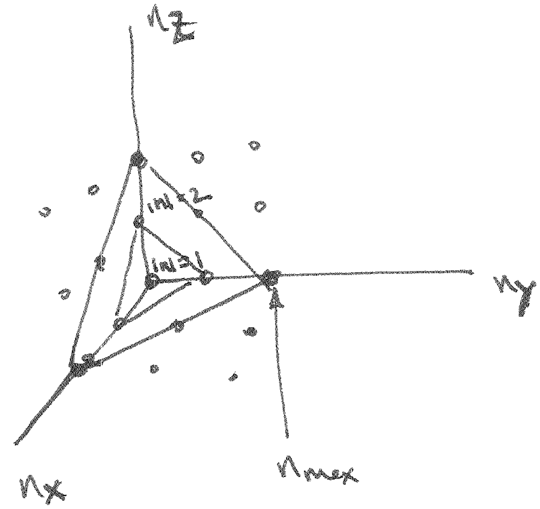
7.22 $\epsilon = pc = \frac{hc}{\lambda}$ $\lambda_n = \frac{2L}{n}$

$\therefore \epsilon_n = c\frac{h}{2L}n$ $p_n = \frac{h}{\lambda_n} = \frac{h \cdot n}{2L}$

w/ 3 degrees of mot freedom

$$\epsilon_{\vec{n}} = \frac{ch}{2L}(n_x + n_y + n_z)$$

$$\therefore \varepsilon_F = \frac{ch}{2L}(n_{max})$$



N = Vol of pyramid w/ side of length n_{max}

= Vol of tetrahedron w/ side of length n_{max} $\sim \left(\frac{1}{2}hb\right)\cdot h \sim O(n_{max}^3)$

$= C_T n_{max}^3$

$$\varepsilon_F = \frac{ch}{2L}\left(\frac{N}{c_f}\right)^{1/3} = \frac{hc}{2LV}$$

$$= \frac{hc}{c_f^{1/3}}\left(\frac{N}{V}\right)^{1/3} \quad \text{iff.} \quad c_f = \left(\frac{3}{8\pi}\right)$$

(b)
$$U = 1 \sum_{n_x}\sum_{n_y}\sum_{n_z} \varepsilon(\bar{n}) = \sum_{n_x}\sum_{n_y}\sum_{n_z} \frac{ch}{2L}(n_x+n_y+n_z)$$

$$= \int_0^{n_{max}} \int_0^{n_{max}-n_x} \int_0^{n_{max}-n_x-n_y} \frac{ch}{2L}(n_x+n_y+n_z)\, dn_z\, dn_y\, dn_x$$

$$\cancel{= \frac{ch}{2L} \int_0^{n_{max}} \int_0^{n_{max}-n_x} \int_0^{n_{max}-n_x-n_y} n_x n_z + n_y n_z + \frac{n_z^2}{2}}$$

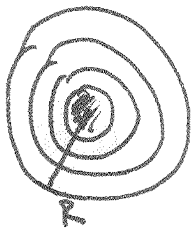
$$= \frac{ch}{2L} \int_0^{n_{max}} \int_0^{n_{max}-n_x} \left(n_x n_z + n_y n_z + \frac{n_z^2}{2} \right|_0^{n_{max}-n_x-n_y} dn_y\, dn_x$$

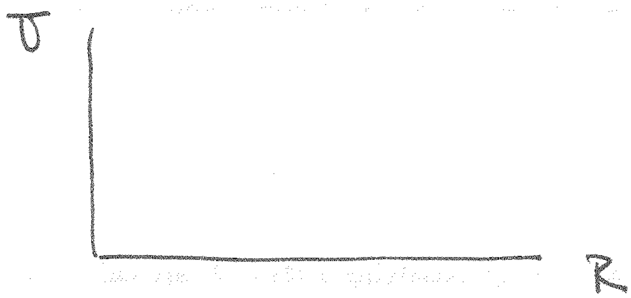
$$= \frac{ch}{2L} \int_0^{n_{max}} \int_0^{n_{max}-n_x} \Big[n_x(n_{max}-n_x-n_y) + n_y(n_{max}-n_x-n_y) + \frac{1}{2}(n_{max}-n_x-n_y)^2 \Big]\, dn_y\, dn_x$$

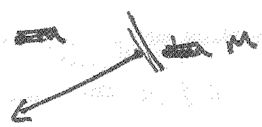
$$= \frac{ch}{2L} \int_0^{n_{max}} \int_0^{n_{max}-n_x} \cdots$$

7.23

(a) $U_{grav} = -C\dfrac{GM^2}{R}$

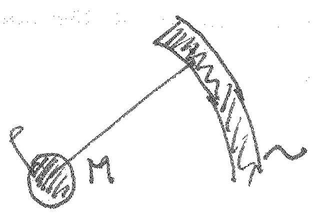






to bring dm to mass m from ∞ requires $W = \int F \cdot dr$

$F = GM \cdot dm$ $\qquad F = \dfrac{GMm}{r^2}$



$dM = 4\pi r^2 \rho$

$M = \frac{4}{3}\pi r^3 \rho$

(b) ?

(c) $U_{total} = U_{grav} + U_{K.E} = -C_1\dfrac{GM^2}{R} + \dfrac{C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^2}$

$U_{total}(R)$

$\dfrac{dU_{total}}{dR} = C_1\dfrac{GM^2}{R^2} - \dfrac{2C_2 h^2 M^{5/3}}{m_e m_p^{5/3} R^3} = 0 \rightarrow R = \cdots$

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(d) $R = \ldots$

$$\rho = \frac{M}{\frac{4\pi}{3}R^3} = \ldots$$

(e) $$\varepsilon_F = \frac{U}{\left(\frac{5}{3}N\right)}$$

$$\varepsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} \qquad kT \ll \varepsilon_F \;?$$

(f) ?

(g) $\langle KE \rangle \cong mc^2$

$\langle KE \rangle =$

?

7.24 one-solar-mass all neutron star.

Follow results of the class

(7.25)

$$C_V = \frac{\pi^2}{2} \frac{N k^2 T}{\epsilon_F}$$

$N = 6.022 \cdot 10^{23}$

$k = \ldots$

$T = 300K$

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$m_{cu} = (63.54 \; g / 6.022 \cdot 10^{23})$

$V = 7.12 \; cm^3$

What is the contributions due to lattice vibrations?

(7.26) ^3He

(a) $$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

Assuming 1) e^- has neg mass comp w/ proton

$V = 37 \; cm^3$ $\quad m_{^3He} = \dfrac{4.002 + 1.007}{6.022 \cdot 10^{23}}$ 2) proton + neutron have the same mass.

$N = 6.022 \cdot 10^{23}$

$$T_F = \frac{\epsilon_F}{k}$$

(b) $C_V = \frac{\pi^2}{2}\frac{Nk^2T}{\epsilon_F}$

(c) 2 states ∀ neuclon. $\Omega(N) = \binom{2N}{N}$

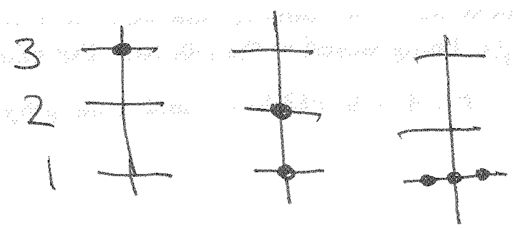
$=$

$S = k\ln\Omega(N)$

? Not sure

(7.27)

(a) $q = 3$
$= 2+1$
$= 1+1+1$



?

$p(q)$ = unrestricted partitions

$p(1) = 1$ $p(3) = 3$
$p(2) = 2$ $p(4) = 4$
$p(5) = 7$

(b) $p(7) = ?$

$$p(n) = 1 + \sum_{k=1}^{n-1}(k + p(k))$$

$= 7$

$= 6+1$

$= 5+2,\ 5+1+1$

$= 4+3,\ 4+2+1,\ 4+1+1+1$

$= 3+$

4
3+1
2+1+1
1+1+1+1

5, 4+1, 3+2, 3+1+1
2+2+1, 1+1+1+1+1
2+1+1+1

$p(7) = ?$

7

6+1

5+2, 5+1+1

4+3, 4+2+1, 4+1+1+1

$\underset{x}{3+4}$, 3+1+3, 3+2+2, 3+1+1+2, 3+1+1+1+1

$\underset{x}{2+5}$, $\underset{x}{2+1+4}$, $\underset{x}{2+2+3}$,

1	2	3	4	5	6	7
0	0	0	0	0	0	1.
1	0	0	0	0	1	0.
0	1	0	0	1	0	0.
2	0	0	0	1	0	0
0	0	1	1	0	0	0
1	1	0	1	0	0	0
3	0	0	1	0	0	0
0	2	1	0	0	0	0
2	1	1	0	0	0	0
42	0	1	0	0	0	0.
1	0	2	0	0	0	0.
5	1	0	0	0	0	0
3	2	0	0	0	0	0
1	3	0	0	0	0	0
7	0	0	0	0	0	0.

$p(7) = 15$.

(c) ?

(d) $p(q) \approx \dfrac{e^{\pi\sqrt{2q/3}}}{4\sqrt{3}\,q}$

$p(10) =$

$S = k\ln\Omega = k\ln p(q) = \cdots$

Where does temperature come in?

7.28

(a) 2D:

$$\epsilon = \frac{|p|^2}{2m} \qquad p_x = \frac{h}{\lambda_x} \quad \cancel{p_y} = \frac{h}{\lambda_y}$$

$$\text{+ } \lambda_x = \frac{2L_x}{n} \quad \lambda_y = \frac{2L_y}{n}$$

$$\therefore \epsilon = \frac{(p_x^2 + p_y^2)}{2m}$$

$$= \frac{h^2\left(\frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}\right)}{2m} = \frac{h^2}{2m}\left(\frac{n_x^2}{4L^2} + \frac{n_y^2}{4L^2}\right) = \frac{h^2}{8mL^2}\vec{n}^2$$

$N = 2 \times$ Area of $\frac{1}{4}$ circle of radius n_{max} $= 2 \cdot \frac{\pi n_{max}^2}{4} = \frac{\pi}{2} n_{max}^2$

$$\text{+ } \epsilon_F = \frac{h^2}{8mA} n_{max}^2 \qquad n_{max} = \left(\frac{2N}{\pi}\right)^{1/2}$$

$$\Rightarrow \epsilon_F = \left(\frac{h^2}{8mA}\right)\left(\frac{2N}{\pi}\right) = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$$

~~Avg energy~~

$$U = 2 \iint \epsilon(\vec{n})\, dn_x\, dn_y = 2\int_0^{n_{max}}\int_0^{\pi/2}\left(\frac{h^2}{8mA}\right) r^2\, r\, dr\, d\theta$$

$$\epsilon(\vec{n}) = \frac{h^2}{8mA}|\vec{n}|^2$$

$$= 2\left(\frac{h^2}{8mA}\right)\left(\frac{\pi}{2}\right)\int_0^{n_{max}} r^3\, dr$$

$$= \pi\left(\frac{h^2}{8mA}\right)\frac{n_{max}^4}{4} = \frac{\pi}{4}\left(\frac{h^2}{8mA}\right) n_{max}^4$$

$$\Rightarrow \cancel{E_{tn}}\ U = \frac{\pi}{4}\underbrace{\left(\frac{h^2}{8mA}\right) n_{max}^2}_{\epsilon_F} \cdot n_{max}^2$$

$$= \frac{\cancel{\pi}}{4} \cdot \epsilon_F \left(\frac{2}{\cancel{\pi}} N\right) = \frac{\epsilon_F \cdot N}{2}$$

$$\Rightarrow \frac{U}{N} = \frac{\epsilon_F}{2} \qquad \checkmark.$$

(b) consider

$$U = 2\int_0^{n_{max}}\int_0^{\pi/2} \epsilon(n)\, n\, dn\, d\theta = \pi \int_0^{n_{max}} \epsilon(n)\, n\, dn$$

But $\epsilon(n) = \frac{h^2}{8mA} n^2 \Rightarrow n = \left(\frac{8mA}{h^2}\right)^{1/2} \epsilon^{1/2}$

$$dn = \left(\frac{8mA}{h^2}\right)^{1/2} \frac{1}{2} \epsilon^{-1/2}\, d\epsilon$$

Thus

$$U = \pi \int_0^{\epsilon_F} \epsilon \left(\frac{8mA}{h^2}\right)^{1/2} \cancel{\epsilon^{1/2}} \cdot \left(\frac{8mA}{h^2}\right)^{1/2} \cdot \frac{1}{2} \cdot \cancel{\epsilon^{-1/2}}\, d\epsilon$$

$$= \frac{\pi}{2}\left(\frac{8mA}{h^2}\right) \int_0^{\epsilon_F} \epsilon\, d\epsilon = \frac{\pi}{2}\left(\frac{8mA}{h^2}\right) \frac{\epsilon_F^2}{2}$$

$$U = \frac{\epsilon_F}{2} \cdot \left(\pi \left(\frac{8mA}{h^2} \right) \epsilon_F \right)$$

From $N = \frac{\pi}{2} n_{max}^2 = \frac{\pi}{2} \cdot \left(\frac{8mA}{h^2} \right) \epsilon_F$

$\therefore U = \frac{\epsilon_F}{2} (2N)$ ... off by factor of 2?

Now consider

$$U = \pi \int_0^{n_{max}} \epsilon(n) n \, dn = \pi \int_0^{\epsilon_F} \epsilon \cdot \left[\frac{1}{2} \left(\frac{8mA}{h^2} \right) \right] d\epsilon$$

$g(\epsilon) = \frac{1}{2} \left(\frac{8mA}{h^2} \right)$ ind. of ϵ ✓.

(c) for Non zero T

$$N = \int_0^\infty g(\epsilon) \frac{1}{\left(e^{(\epsilon-\mu)/kT} + 1\right)} d\epsilon = \frac{1}{2} \left(\frac{8mA}{h^2} \right) \int_0^\infty \frac{1}{e^{(\epsilon-\mu)/kT} + 1} d\epsilon$$

$$= \frac{1}{2} \left(\frac{8mA}{h^2} \right) \int_0^\infty \left(\sum_{k \geq 0} (-1)^k e^{k(\epsilon-\mu)/kT} \right) d\epsilon$$ X Not convergent series

$$= \frac{1}{2} \left(\frac{8mA}{h^2} \right) \sum_{k \geq 0} (-1)^k \int_0^\infty e^{k(\epsilon-\mu)/kT} d\epsilon$$

$$N = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty \frac{e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}}\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty e^{-(\epsilon-\mu)\beta}\sum_{n\geq 0} e^{-n(\epsilon-\mu)/kT}\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\int_0^\infty e^{-(n+1)(\epsilon-\mu)\beta}\,d\epsilon$$

$$= \cancel{\frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\left.\frac{e^{(n+1)(\epsilon-\mu)\beta}}{(n+1)}\right|_0^\infty}$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{+(n+1)\mu\beta}\int_0^\infty e^{-(n+1)\epsilon\beta}\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{+(n+1)\mu\beta}\left.\frac{e^{-(n+1)\epsilon\beta}}{-(n+1)\beta}\right|_0^\infty$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0}\frac{e^{(n+1)\mu\beta}}{-(n+1)\beta}(0-1)$$

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$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} \frac{e^{(n+1)\mu\beta}}{(n+1)\beta} \equiv F(\mu)$$

$$= \cancel{\frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} \int e^{(n+1)\mu\beta}}$$

Then $$F'(\mu) = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\sum_{n\geq 0} e^{(n+1)\mu\beta}$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right) e^{\mu\beta} \sum_{n\geq 0} e^{n\mu\beta} \qquad e^{\mu\beta} < 1$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right) e^{\mu\beta}\left(\frac{1}{1-e^{\mu\beta}}\right) =$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right) \frac{1}{(e^{-\mu\beta}-1)}$$

$$\therefore \quad F'(\mu) = \frac{C'}{(e^{-\mu\beta}-1)}$$

$$F(\mu) = C' \int \frac{d\mu}{e^{-\mu\beta}-1} = C' \int \frac{e^{\mu\beta}}{1-e^{\mu\beta}}\, d\mu$$

$$= -\frac{C'}{\beta} \ln(1-e^{\mu\beta})$$

$$= -\frac{C'}{\beta} \ln(1-e^{+\mu\beta})$$

$$N = -\frac{1}{2}\left(\frac{8mA}{h^2}\right) kT \ln(1-e^{\mu B})$$

$$\ln(1-e^{\mu B}) = -2\frac{h^2}{8m}\left(\frac{N}{A}\right)\frac{1}{kT} = -\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}$$

$$\Rightarrow 1-e^{\mu B} = \exp\left\{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}\right\}$$

$$e^{\mu B} = 1 - e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}$$

$$\Rightarrow \mu = \frac{1}{\beta}\ln\left(1-e^{-\frac{h^2}{4m}\left(\frac{N}{A}\right)\frac{1}{kT}}\right) = kT\ln(1- \cdots)$$

(e) $kT \gg \epsilon_F$ $\quad \epsilon_F = \frac{h^2}{4\pi m}\left(\frac{N}{A}\right)$

$$\therefore \mu = \frac{1}{\beta}\ln(1-e^{-\pi\epsilon_F/kT})$$

Assuming $\epsilon_F \ll kT$

$\cancel{\ln(1-x) \cong x - \frac{x^2}{2} + \frac{x^3}{3}}$

$\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \cdots$

$$\mu = \frac{1}{\beta}\left[\ln\left(1-\left(1-\frac{\pi\epsilon_F}{kT}\right)\right)\right]$$

$$= \frac{1}{\beta}\ln\left(\frac{\pi\epsilon_F}{kT}\right)$$

$$\mu = -kT \ln\left(\frac{kT}{\cancel{A}\left(\frac{h^2}{4\cancel{\pi} m}\left(\frac{N}{A}\right)\right)}\right)$$

$$= -kT \ln\left(\frac{A}{N}\cdot\left(\frac{4mkT}{h^2}\right)^{1}\right)$$ similar to an ideal gas.

7.29

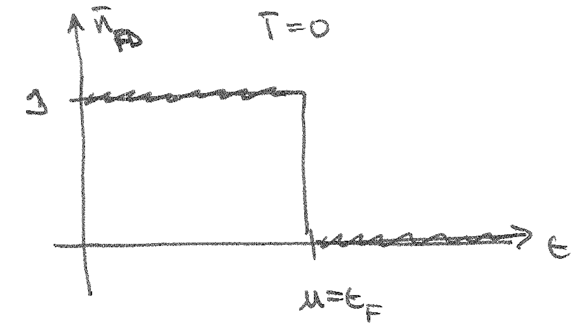
eq 7.68 is

$$U = \frac{3}{5} N \frac{\mu^{5/2}}{\epsilon_F^{3/2}} + \frac{3\pi^2}{8} N \frac{(kT)^2}{\epsilon_F} + \cdots$$

eq 7.54 is

$$U = \int_0^\infty \epsilon\, g(\epsilon)\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$

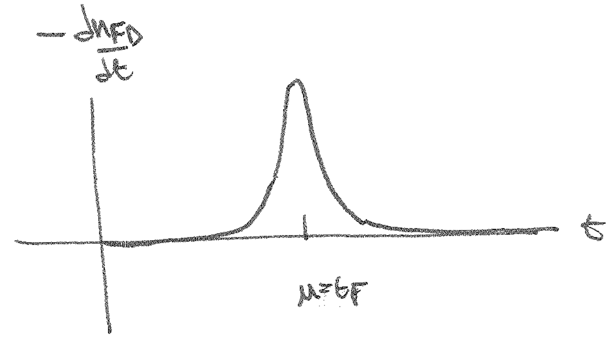
let $g(\epsilon) = g_0 \epsilon^{1/2}$



$$= g_0 \int_0^\infty \epsilon^{3/2}\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$

$$= g_0\, \bar{n}_{FD}(\epsilon)\, \frac{\epsilon^{5/2}}{5/2}\Bigg|_0^\infty \to 0$$

$$- \frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$



$$= + \frac{2}{5} g_0 \int_0^\infty \epsilon^{5/2} \left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$

$$\approx \frac{2}{5} g_0 \int_{-\infty}^{+\infty} \epsilon^{5/2} \left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right) d\epsilon$$

$$\bar{n}_{FD}(\epsilon) = \frac{1}{e^{+(\epsilon-\mu)/kT} + 1}$$

let $x \equiv \frac{\epsilon - \mu}{kT}$ $\Rightarrow$ $\epsilon = \cancel{(x+\mu kT)}$
$= kTx + \mu$

$$\bar{n}_{FD} = \frac{1}{e^x + 1}$$

$$\frac{d\bar{n}_{FD}}{dx} = \frac{-e^x}{(e^x+1)^2}$$

$$= \frac{2}{5} g_0 \int_{-\infty}^{+\infty} (kTx+\mu)^{5/2} \frac{e^x}{(e^x+1)^2}\, dx\,(kT)$$

$$= \frac{2}{5} g_0\, kT \int_{-\infty}^{\infty} (kT)^{5/2}\left(x+\frac{\mu}{kT}\right)^{5/2} \frac{e^x}{(e^x+1)^2}\, dx$$

peeks only at $x=0$.

$$\left(x+\frac{\mu}{kT}\right)^{5/2} = \left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(x+\frac{\mu}{kT}\right)^{3/2}\Big|_{x=0}(x) + \frac{5}{2}\cdot\frac{3}{2}\left(x+\frac{\mu}{kT}\right)^{1/2}\Big|_{x=0}\frac{x^2}{2} + O(x^3)$$

$$\approx \frac{2}{5} g_0 (kT)^{7/2} \int_{-\infty}^{+\infty} \left(\left(\frac{\mu}{kT}\right)^{5/2} + \frac{5}{2}\left(\frac{\mu}{kT}\right)^{3/2} x + \frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2}\frac{x^2}{2} + O(x^3)\right) \frac{e^x}{(e^x+1)^2}\, dx$$

$$= \frac{2}{5} g_0 (kT)^{7/2}\left(\frac{\mu}{kT}\right)^{5/2} \underbrace{\int_{-\infty}^{\infty} \frac{e^x}{(e^x+1)^2}\, dx}_{+1} + 0$$

$$+ \frac{\cancel{2}}{5} g_0 (kT)^{7/2}\cdot\frac{15}{4}\left(\frac{\mu}{kT}\right)^{1/2}\cdot\frac{1}{\cancel{2}} \int_{-\infty}^{+\infty} x^2 \frac{e^x}{(e^x+1)^2}\, dx$$

$$= \frac{2}{5} g_0 \frac{\mu^{5/2}}{\cancel{kT}}(kT) + \frac{\cancel{3}}{4} g_0\, \mu^{1/2}(kT)^3\cdot\frac{\pi^2}{\cancel{3}} + \ldots$$

$$\therefore\ U = \frac{2}{5} g_0\, \mu^{5/2}(kT) + \frac{\pi^2}{4} g_0\, \mu^{1/2}(kT)^3 + \ldots$$

w/ $g_0 = \frac{3N}{2\epsilon_F^{3/2}}$ we get

$\Rightarrow$ ~~[illegible]~~

$$\Rightarrow U = \frac{2}{5} \cdot \frac{3N}{2\epsilon_F^{3/2}} \cdot \mu^{5/2}(kT) + \frac{\pi^2}{4} \cdot \frac{3N}{2\epsilon_F^{3/2}} \cdot \mu^{1/2}(kT)^3 + \dots$$

$$= \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{3/2}}\mu^{1/2}(kT)^3 + \dots \quad \text{eq 7.67} \checkmark$$

$$\Rightarrow U = \frac{3}{5}\frac{N}{\epsilon_F^{3/2}}\mu^{5/2}(kT) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F^{2}}(kT)^{3}$$

$$\frac{\mu}{\epsilon_F} = 1 - O((kT)^2)$$

different exponent ...

~~[illegible]~~

$$U = \frac{3}{5}N\epsilon_F\left(\frac{\mu}{\epsilon_F}\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^3 + \dots$$

Assume square

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{\pi^2}{12}\left(\frac{kT}{\epsilon_F}\right)^2\right)^{5/2} + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3}{5}N\epsilon_F\left(1 - \frac{5\pi^2}{24}\left(\frac{kT}{\epsilon_F}\right)^2\right) + \frac{3\pi^2}{8}\frac{N}{\epsilon_F}(kT)^2$$

$$= \frac{3N}{5}\epsilon_F + \left(\underbrace{-\frac{\pi^2}{8} + \frac{3\pi^2}{8}}_{\frac{\pi^2}{4}}\right)\frac{N}{\epsilon_F}(kT)^2 \quad \text{eq 7.68} \checkmark$$

Prob 7.30

~~[illegible]~~ All odd terms would be zero since the integrals on odd fns from ~~[illegible]~~ $(-\infty, +\infty)$

The T^4 term could be evaluated easily enough

Prob 7.31 From problem 7.28

$$g(\epsilon) = \frac{1}{2}\left(\frac{8mA}{h^2}\right) \quad \text{independent of } \epsilon.$$

Then

$$U = \int_0^\infty g(\epsilon)\,\epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon = \frac{1}{2}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \frac{1}{2}\left(\frac{8mA}{h^2}\right)\left[\bar{n}_{FD}(\epsilon)\frac{\epsilon^2}{2}\Big|_0^\infty + \frac{1}{2}\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon \right]$$

$$= \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon^2\left(-\frac{d\bar{n}_{FD}}{d\epsilon}\right)d\epsilon \qquad \bar{n}_{FD} = \frac{1}{\left(e^{(\epsilon-\mu)/kT}+1\right)}$$

$$= \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_0^\infty \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon \qquad x \equiv \frac{(\epsilon-\mu)}{kT} \qquad \epsilon = \mu + kTx$$

$$\cong \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_{-\infty}^{+\infty} \epsilon^2\,\frac{e^x}{(e^x+1)^2}\,d\epsilon = \frac{1}{4}\left(\frac{8mA}{h^2}\right)\int_{-\infty}^{+\infty}(\mu+kTx)^2\,\frac{e^x}{(e^x+1)^2}\cdot(kT)\,dx$$

$$= \frac{(kT)}{4}\left(\frac{8mA}{h^2}\right)(kT)^2\int_{-\infty}^{\infty}\left(x+\frac{\mu}{kT}\right)^2\frac{e^x}{(e^x+1)^2}\,dx$$

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$$= \left(\frac{2\pi m A}{h^2}\right)(kT)^3 \int_{-\infty}^{+\infty} \left(\left(\frac{\mu}{kT}\right)^2 + 2\left(\frac{\mu}{kT}\right)x + 2\frac{x^2}{2} \right) \frac{e^x}{(e^x+1)^2}\, dx$$

$$= \left(\frac{2\pi m A}{h^2}\right)(kT)^3 \left[\left(\frac{\mu}{kT}\right)^2 \cdot 1 + 0 + \underbrace{\int_{-\infty}^{+\infty} x^2 \frac{e^x}{(e^x+1)^2}\, dx}_{\pi^2/3} \right]$$

$$= \frac{2\pi m A}{h^2}\left[\mu^2 (kT) + \frac{\pi^2}{3}(kT)^3 \right]$$

(I think this is good ✓ kT)

How Big in the factor of ϵ_F = ?

P. 232

(7.53) $$N = \int_0^\infty g(\epsilon) \frac{1}{(e^{(\epsilon-\mu)/kT}+1)} d\epsilon$$

(7.54) $$U = \int_0^\infty g(\epsilon)\, \epsilon \frac{1}{(e^{(\epsilon-\mu)/kT}+1)} d\epsilon$$

(a) If $kT \equiv \epsilon_F$

(7.53) $$N = \int_0^\infty g_0 \epsilon^{1/2} \frac{1}{(e^{(\epsilon-\mu)/\epsilon_F}+1)} d\epsilon = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{(e^{\epsilon/\epsilon_F}+1)}$$

Question is can I exclude the integrals chance New

$$N = g_0 \int_0^\infty \epsilon^{1/2} \frac{d\epsilon}{e^{(\epsilon-\mu)/kT}+1} = g_0 \int_0^\infty \cancel{\frac{\epsilon^{1/2}\, d\epsilon}{e^{(x-\mu)/t}+1}}$$

let $x = \frac{\epsilon-\mu}{kT}$ $\qquad \epsilon = \mu + kTx$

$$N = g_0 \int_{-\mu/kT}^\infty (\mu + kTx)^{1/2} \frac{(kT)\,dx}{e^x+1} = g_0 (kT)^{3/2} \int_{-\mu/kT}^\infty \left(x + \frac{\mu}{kT}\right)^{1/2} \frac{dx}{e^x+1}$$

$$N(kT=\epsilon_F;\, \mu=0) = g_0 \epsilon_F^{3/2} \int_0^\infty x^{1/2} \frac{dx}{e^x+1}$$

(b) ... ?

P. 7.33

(a) $g(\epsilon) = \begin{cases} g_0\sqrt{\epsilon - \epsilon_c} & \epsilon > \epsilon_c \\ g_0\sqrt{\epsilon_v - \epsilon} & \epsilon < \epsilon_v \\ 0 & \text{otherwise } \epsilon_v < \epsilon < \epsilon_c \end{cases}$

$$N = \int_0^\infty g(\epsilon)\,\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \int_0^{\epsilon_v} g_0\sqrt{\epsilon_v - \epsilon}\;\bar{n}_{FD}(\epsilon)\,d\epsilon + 0 + \int_{\epsilon_c}^\infty g_0\sqrt{\epsilon - \epsilon_c}\;\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$N = \int_0^{\epsilon_v} \frac{g_0\sqrt{\epsilon_v - \epsilon}\;d\epsilon}{e^{(\epsilon-\mu)/kT}+1} + \int_{\epsilon_c}^\infty \frac{g_0\sqrt{\epsilon - \epsilon_c}}{e^{(\epsilon-\mu)/kT}+1}\,d\epsilon$$

Don't know why μ ~~is~~ must be between $\epsilon_v + \epsilon_c$ for a solution to this problem.

Assume $\epsilon_c - \epsilon_v \gg kT$

(b) Am I supposed to actually evaluate these integrals?

(c) Si

$1\,cm^3$ of Si is approx sam size as ~~Si~~O_2 which is $22.69\ cm^3$

$\therefore \frac{1\,cm^3}{22.69\,cm^3} = moles\,*$

$= \cdot N_A = $ # of Si atoms = has "4" available e^-

=

For Cu w/ ρ $7.12\ cm^3/mole$

$\frac{1\,cm^3}{7.12}$ ~ 3 ~~##~~ times larger than Si

But w/ How many ~~####~~ conduction electrons? ≈ 11.

~~##~~ ∴ Cu has about 33 times as many conduction e^-.

(d) Don't know how to get #'s

(e) I would guess $E_C - E_V \gg kT$

say $E_C - E_V = \Delta E \approx 10kT$

$kT = (8.617 \cdot 10^{-5}\ eV/K)(300K) = \cancel{24.617}\ \ 24 \cdot 10^{-3}\ eV$

$= .024\ eV$

P. 7.34

(a) ?

(b) ~~[illegible]~~ for

$$N_c = \int_{\epsilon_c}^{\infty} \frac{g_{0c}\sqrt{\epsilon-\epsilon_c}}{e^{(\epsilon-\mu)/kT}+1}\,d\epsilon$$

Assum $\epsilon_c - \mu \gg kT$

$$\frac{(\epsilon_c-\mu)}{kT} \gg 1$$

$$N_c = g_{0c}\int_{\epsilon_c}^{\infty} \frac{\sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}}{1+e^{-(\epsilon-\mu)/kT}}\,d\epsilon$$

$$= g_{0c}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}\left[1 - e^{-(\epsilon-\mu)/kT} + \cdots\right]d\epsilon$$

$$= g_{0c}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-(\epsilon-\mu)/kT}\,d\epsilon$$

$$- g_{0c}\int_{\epsilon_c}^{\infty} \sqrt{\epsilon-\epsilon_c}\; e^{-2(\epsilon-\mu)/kT}\,d\epsilon + \cdots$$

Just to get a "feel" for what things look like

lets use the 1st term

$$N_c \approx g_\alpha \int_{\epsilon_c}^{\infty} \sqrt{\epsilon - \epsilon_c}\, e^{-(\epsilon-\mu)/kT}\, d\epsilon$$

let $x = \frac{\epsilon - \mu}{kT}$ $\qquad dx = \frac{1}{kT} d\epsilon$

$$\epsilon = kTx + \mu$$

$$N_c \approx g_\alpha \int_{x_c}^{\infty} \sqrt{kTx + \mu - \epsilon_c}\, e^{-x}\, d\epsilon$$

$$= g_\alpha (kT)^{1/2} \int_{x_c}^{\infty} \sqrt{x + \left(\frac{\mu - \epsilon_c}{kT}\right)}\, e^{-x}\, (kT)\, dx$$

$$= g_\alpha (kT)^{3/2} \int_{x_c}^{\infty} \sqrt{x - x_c}\, e^{-x}\, dx$$

let $\cancel{\xi}$ $\xi = x - x_c$ Then $x = \xi + x_c$

$$N_c = g_\alpha (kT)^{3/2} \int_0^{\infty} \xi^{1/2} e^{-\xi - x_c}\, d\xi$$

$$= g_\alpha (kT)^{3/2} e^{-x_c} \underbrace{\int_0^{\infty} \xi^{1/2} e^{-\xi}\, d\xi}_{\#}$$

(c) $$N_v = \int_0^{\epsilon_v} g_{0v}\sqrt{\epsilon_v-\epsilon}\;\bar{n}_{FD}(\epsilon)\,d\epsilon$$

$$= \int_0^{\epsilon_v} g_{0v}\sqrt{\epsilon_v-\epsilon}\cdot\frac{1}{\left(e^{+(\epsilon-\mu)/kT}+1\right)}\,d\epsilon$$

$$\left\{\begin{array}{ll}
(\mu-\epsilon_v) \gg kT & \Rightarrow \quad -\frac{(\epsilon_v-\mu)}{kT} \gg 1 \\
-(\epsilon_v-\mu) \gg kT & \Rightarrow \quad e^{-\frac{(\epsilon_v-\mu)}{kT}} \gg e^1 = \mathcal{O}(1) \\
\epsilon_v-\mu \ll -kT & \\
\frac{\epsilon_v-\mu}{kT} \ll -1 & \Rightarrow \quad e^{(\epsilon_v-\mu)/kT} \ll 1
\end{array}\right.$$

N_v = # of valence e^-.

$N_{holes} = N_v(T=0) - N_v(T\neq 0)$ $\quad\left\{ n_{FD}(T=0) = \text{step fn at } \epsilon=\mu.\right.$

$$\cancel{N_v \approx \int_{-\mu/kT}^{\frac{\epsilon_v-\mu}{kT}} g_{0v}\sqrt{\epsilon_v-\epsilon}} \qquad x \equiv \frac{\epsilon-\mu}{kT}$$

$$N_v \approx g_{0v}\int_0^{\epsilon_v}\sqrt{\epsilon_v-\epsilon}\left[1 - e^{(\epsilon-\mu)/kT} + \dots\right]d\epsilon$$

1st term is third.

2nd term

$$-g_{0v}\int_0^{\epsilon_v}\sqrt{\epsilon_v-\epsilon}\; e^{(\epsilon-\mu)/kT}\, d\epsilon$$

$$x = \frac{\epsilon-\mu}{kT} \qquad dx = \frac{d\epsilon}{kT}$$

$$\cancel{kT}\quad \epsilon = \mu + kTx$$

$$= -g_{0v}\int_{\frac{0-\mu}{kT}}^{\frac{\epsilon_v-\mu}{kT}}\sqrt{\epsilon_v-\mu-kTx}\; e^{x}\; kT\,dx$$

$$= -g_{0v}(kT) \quad \ldots$$

(d) $N_{total} = \cancel{N_v +}\; N_v(\mu) + N_c(\mu)$

invert $\mu = \mu(N)$

(e) . . .

P. 7.35

(a) ? What should I use for

(b) ?

(c) ?

P. 7.36

(a) Due to their thermal motion I would think that the spin states would be in constant flux i.e. always changing and ∴ the total magnetic moments would tend to be somewhere between $+\mu B$ & $-\mu B$ ≈ 0.

(b) $$P_{\uparrow}(\ ) = \frac{e^{-\mu B/kT}}{\sum e^{-\mu B/kT} + \sum e^{+\mu B/kT}}.$$

(c) ?

7.37 integrand in Planck spectrum is

$$f(x) = \frac{x^3}{e^x - 1}$$

$$f'(x) = \frac{3x^2}{e^x-1} - \frac{x^3(e^x)}{(e^x-1)^2} = 0$$

set

$x = 0$ $\quad x \neq 0$ $\quad \dfrac{3}{(e^x-1)} - \dfrac{xe^x}{(e^x-1)^2} = 0$

multiply by $(e^x-1)^2$

$$\Rightarrow \quad 3(e^x-1) - xe^x = 0$$

$$3e^x - 3 - xe^x = 0$$

$$(3-x)e^x - 3 = 0$$

7.38 $\quad u(\epsilon;T) = \dfrac{8\pi}{(hc)^3} \dfrac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$

$k =$

$8.617 \cdot 10^{-5}$ eV/K

...

7.39

$$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_0^\infty \frac{\epsilon^3 d\epsilon}{e^{\epsilon/kT}-1}$$

$f\lambda = c$

$\epsilon = h\nu = hf = \frac{hc}{\lambda}$ $\qquad d\epsilon = \frac{-hc}{\lambda^2}d\lambda$

$\lambda = \frac{hc}{\epsilon}$

$$\frac{U}{V} = \frac{8\pi}{(hc)^3}\int_\infty^0 \frac{(hc/\lambda)^3(-hc/\lambda^2)\,d\lambda}{e^{hc/\lambda kT}-1} = \frac{8\pi}{(hc)^3}\cdot(hc)^3(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

$$= 8\pi(hc)\int_0^\infty \frac{d\lambda}{\lambda^5(e^{hc/\lambda kT}-1)}$$

spectrum $\quad u(\lambda) = \dfrac{1}{\lambda^5\left(e^{\frac{hc/kT}{\lambda}}-1\right)}$

why does the peak not occur at $1/x_c$

Assume $F(x)$ has max at $x_c \Rightarrow F'(x_c)=0$

$G(y) \equiv F(x(y))$ $\qquad x(y) = \frac{1}{y}$

$G'(y) = F'(x)$

$$F(x) = F_1(x)F_2(x)$$

$$F'(x_c) = 0$$

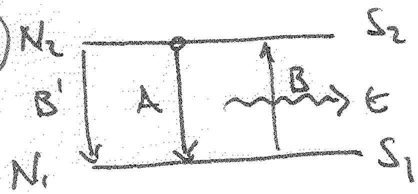
$$G(y) \equiv F(x(y)) \qquad y = \frac{c}{x}$$

$$G'(y) = \frac{dF_1}{dx}\cdot\frac{dx}{dy}F_2 + F_1\frac{dF_2}{dy}\cdot\frac{dx}{dy} \underset{\text{set}}{=} 0$$

$$\frac{dx}{dx}\left[\frac{dF_1}{dx}\cdot F_2 + F_1\frac{dF_2}{dx}\right] = 0$$

7.40 ?

7.41



P

$$B = \frac{\text{prob ct cbs}}{u(f)}$$

(a) $\cancel{\frac{dN_1}{dt} = +AN_1 - BN_1 + B'N_2}$

$$\cancel{\frac{dN_2}{dt}} = \quad \frac{dN_1}{dt} = AN_1 - Bu(f)N_1 + B'u(f)N_2$$

$$\frac{N_1}{N_2} \approx C$$

$$\frac{dW_1}{dt} = -\frac{dW_1}{dt}$$

How show related as given?

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7.42

(a) $U = V\left(\frac{8\pi^5}{15}\frac{(kT)^4}{(hc)^3}\right)$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$ All

(c) $1 = \int$

$\lambda_{min} \approx 400\,nm \rightarrow \epsilon_{max} = \frac{hc}{\lambda} = \cdots$

$\lambda_{max} \cong 700\,nm \rightarrow \epsilon_{min} = \frac{hc}{\lambda_{max}} = \cdots$

$$F = \int_{\epsilon_{min}}^{\epsilon_{max}} \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}\,d\epsilon$$

7.43 $T = 5800\,k$

(a) $U = V\cdot\left(\frac{8\pi^5}{15}\right)\frac{(kT)^4}{(hc)^3}$

(b) $u(\epsilon) = \frac{8\pi}{(hc)^3}\frac{\epsilon^3}{(e^{\epsilon/kT}-1)}$

(c) - —

P.7.44

(a) $N = 2\sum_{n_x}\sum_{n_y}\sum_{n_z} \bar{n}_{pl}(\epsilon) = 2\sum_{n_x,n_y,n_z} \frac{1}{(e^{hcn/2LkT}-1)}$

$$= 2\int_0^\infty dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi\, n^2 \sin\theta \frac{1}{(e^{hcn/2LkT}-1)}$$

$$= 2\left(\frac{\pi}{2}\right) \int_0^{\pi/2} \sin\theta\, d\theta \int_0^\infty \frac{n^2\, dn}{(e^{hcn/2LkT}-1)}$$

$$= \pi\left[-\cos\theta\Big|_0^{\pi/2}\right] \int_0^\infty \frac{n^2\, dn}{(e^{hcn/2LkT}-1)} \qquad \epsilon = \frac{hcn}{2L}$$

$$d\epsilon = \frac{hc}{2L}\, dn$$

$$= \pi[+1] \cdot \int_0^\infty \frac{\frac{4L^2}{h^2c^2}\epsilon^2 \left(\frac{2L}{hc}\right) d\epsilon}{e^{\epsilon/kT}-1}$$

$$= \pi \frac{8L^3}{(hc)^3} \int_0^\infty \frac{\epsilon^2\, d\epsilon}{e^{\epsilon/kT}-1} \qquad \text{let } x = \frac{\epsilon}{kT}$$

$$dx = \frac{1}{kT}\, d\epsilon$$

$$= \frac{\cancel{8\pi V}}{(hc)^3}$$

$$= \frac{8\pi V}{(hc)^3} \int_0^\infty \frac{(kT)^2 x^2\, d(kT)\, dx}{e^x - 1}$$

$$N = \frac{8\pi (kT)^3 V}{(hc)^3} \int_0^\infty \frac{x^2\,dx}{e^x - 1} = \frac{8\pi (kT)^3 V}{(hc)^3} I$$

(b) $S(T) = \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 k$

So $s = \frac{S}{N} = \frac{\frac{32}{45}\pi^5 \cancel{V} \left(\cancel{\frac{kT}{hc}}\right)^3 k}{8\pi \cancel{\frac{(kT)^3}{(hc)^3}} \cancel{V} \cdot I}$

$= \frac{4}{45}\pi^4 \left(\frac{1}{I}\right) \cdot k$

entropy per photon is constant.

(c) $\frac{N}{V} = 8\pi \left(\frac{kT}{hc}\right)^3 \cdot I$

P.7.45

$$P = -\frac{\partial U}{\partial V}\bigg|_{S,N}$$

since from P.7.44 $N = 8\pi V \left(\frac{kT}{hc}\right)^3 \cdot I \rightarrow \left(\frac{kT}{hc}\right) = \left(\frac{N}{8\pi V I}\right)^{1/3}$

$$U = \cancel{V \frac{8\pi^5}{15}\frac{1}{(hc)^3}} \quad V \cdot \frac{8\pi^5}{15} \frac{(kT)^4}{(hc)^4} \cdot (hc)$$

$$= V \cdot \frac{8\pi^5}{15}(hc)\left(\frac{N}{8\pi V I}\right)^{4/3}$$

$$= V^{1-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$\frac{\partial U}{\partial V}\bigg|_{S,N} = \left(1-\frac{4}{3}\right)V^{-4/3} \cdot \frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}$$

$$= \left(1-\frac{4}{3}\right)\frac{V^{1-4/3}}{V} \cdot \left(\frac{8\pi^5}{15}(hc)\frac{N^{4/3}}{(8\pi I)^{4/3}}\right)$$

$$= \left(1-\frac{4}{3}\right)\frac{U}{V} = -\frac{1}{3}\frac{U}{V}$$

So $P = -\frac{\partial U}{\partial V}\bigg|_{S,N} = \frac{1}{3}\frac{U}{V}$

P.7.46

(a) $F \equiv U - ST$

$$= V \cdot \frac{8\pi^5 (kT)^4}{15(hc)^3} - \frac{32\pi^5}{45} V \left(\frac{kT}{hc}\right)^3 kT$$

$$= \frac{8}{15} V \pi^5 \frac{(kT)^4}{(hc)^3} \left[\underbrace{1 - \frac{4}{3}}_{-\frac{1}{3}}\right]$$

$$F = -\frac{8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3}$$

(b) $S = -\frac{\partial F}{\partial T}\Big|_V = -\frac{\partial}{\partial T}\left[\;\cdots\;\right]$

$$= +\frac{32}{45} V \pi^5 \left(\left(\frac{kT}{hc}\right)\right)^3 k.$$

(c) $\frac{\partial F}{\partial V}$

$F \equiv U - ST$

$dF = dU - TdS - SdT$

$$dU = TdS - pdV$$

$$dF = \cancel{TdS} - pdV - \cancel{TdS} - SdT = -pdV - SdT$$

$$\therefore \quad p = -\frac{\partial F}{\partial V}\Big|_S \quad + \quad S = -\frac{\partial F}{\partial T}\Big|_V$$

given $S = \frac{32}{45} V \pi^5 \left(\frac{kT}{hc}\right)^3 k \rightarrow \left(\frac{kT}{hc}\right) = \left(\frac{45}{32} \frac{1}{\pi^5} \frac{1}{V} \cdot \frac{1}{k} \cdot S\right)^{1/3}$

$$p = -\frac{\partial}{\partial V}\Big|_S \left[-\frac{8}{45} V \pi^5 \frac{(kT)^4}{(hc)^3} \right]$$

$$= -\frac{\partial}{\partial V}\Big|_S \left[-\frac{8}{45} V \cdot \pi^5 (hc) \left(\frac{45}{32} \cdot \frac{1}{\pi^5} \frac{S}{V \cdot k} \right)^{4/3} \right]$$

$$= \frac{\partial}{\partial V}\Big|_S \left[\frac{8}{45} V^{1-\frac{4}{3}} \pi^5 (hc) \left(\frac{45}{32} \frac{1}{\pi^5} \frac{S}{k} \right)^{4/3} \right]$$

$$= \frac{8}{45} \frac{V^{1-\frac{4}{3}}}{V} \cdot \left(1 - \frac{4}{3}\right) \ldots$$

$$= \frac{\left(1-\frac{4}{3}\right)}{V} \cdot \frac{8}{45} V \pi^5 (hc) \left(\ldots \right)^{4/3}$$

$$= \frac{1}{3\cdot V}\cdot\frac{8}{45} V\pi^5 (hc)\left(\frac{kT}{hc}\right)^4$$

$$= \frac{1}{3\cdot V}$$ ... check ...

(d) $F = -kT \ln Z$

$$Z = \sum_i e^{-\epsilon_i/kT} \qquad \epsilon_n = \frac{hc\,n}{2L}$$

$$= \sum_n \exp\left\{-\frac{hc\,n}{2L\cdot kT}\right\} = \sum_n \left(e^{-\frac{hc}{2LkT}}\right)^n$$

$$= \frac{1}{1 - e^{-\frac{hc}{2LkT}}}$$

$$F \overset{?}{=} kT \ln\left(1 - e^{-\frac{hc}{2LkT}}\right)$$ What's wrong?

P. 7.47

$\cancel{\text{}}$ $r_{pH} = 10^9$

? How do?

P. 7.48

ν, $\bar{\nu}$ $T_{eff} = 1.95\,k$

(a) $\mu_\nu = \mu_{\bar{\nu}}$ $\nu + \bar{\nu} \longleftrightarrow 2\gamma$

By equating chemical potentials for a reaction of this type

$$\mu_\nu + \mu_{\bar{\nu}} = \mu_\gamma = 0$$ chemical potential of light is zero

$\Rightarrow$ w/ fact that $\mu_\nu = \mu_{\bar{\nu}}$ $\Rightarrow$ $\mu_\nu = 0$

(b) Fermion $\bar{n}_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} - 1}$

$$\bar{U} = \cancel{2}^{\,3} \int \epsilon\, \bar{n}_{FD}(\epsilon)\, d\epsilon$$ How set up?

(c) $N = 3 \int_\epsilon \bar{n}_{FD}(\epsilon) \cdot n(\epsilon)\, d\epsilon$

prob. state w/ energy ϵ is occupied. degeneracy of state!

(d) ?

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7.49 $\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$ $p = [0, \infty)$

(a) $U = 2^2 \int_0^\infty \epsilon \, n_{F.D}(\epsilon) \, d\epsilon$ $n_{FD}(\epsilon) = \dfrac{1}{e^{\epsilon/kT} + 1}$

$= 4 \int_0^\infty \dfrac{\epsilon \, d\epsilon}{e^{\epsilon/kT} + 1}$ $\epsilon = \sqrt{(pc)^2 + (mc^2)^2}$

$\epsilon^2 = (pc)^2 + (mc^2)^2$

$= 4 \int_0^\infty \cdots$

(b) $u(T) = \int_0^\infty \dfrac{x^2 \sqrt{x^2 + (mc^2/kT)^2}}{e^{\sqrt{x^2 + (mc^2/kT)^2}} + 1} \, dx$

(d) $F = -kT \ln Z$

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P. 7.50 $\frac{kT}{c^2} \gg m$

(a) $V(t)$ T $F = U - ST$

$$\Rightarrow S = \frac{F - U}{T}$$

$$S = -\frac{16\pi(kT)^4}{(hc)^3} f(T) \cdot V - \frac{16\pi(kT)^4}{(hc)^3} u(T) \cdot V$$

(b) ?

(c) ?

(7.57) $T = 3000\ K$

$e = 1/3$

(a) $P = 100\ W$.

Using the $\text{power} = \sigma e A T^4$

$$100\ W = \left(5.67\cdot 10^{-8}\ \frac{W}{m^2 K^4}\right)\left(\frac{1}{3}\right)(A)(3000\ K)^4$$

$$100\ W = \frac{3^4\cdot 10^{12}}{3}(A)\ 5.67\cdot 10^{-8}\ \frac{W}{m^2}$$

$$100\ W = 3^3\cdot 10^{12}\cdot A\cdot 5.67\cdot 10^{-8}\ W/m^2$$

$$A = \frac{100}{3^3\cdot 10^{12}(5.67\cdot 10^{-8})}\ m^2 = 6.532\cdot 10^{-5}\ m^2$$

$$(mm)^2 = 10^{-6}\ m$$

$$= .6532\cdot (mm)^2$$

(b) $\epsilon = 2.82\ kT$

$$= 2.82(1.381\cdot 10^{-23}\ J/K)(300\ K)$$

$$= 1.1683\cdot 10^{-20}\ J = 0.0729\ eV$$

$$\epsilon = h\nu = hf = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{\epsilon}$$

$$\lambda = \frac{(6.626 \cdot 10^{-34}\ J \cdot s)(3 \cdot 10^{8}\ m/s)}{1.1683 \cdot 10^{-20}\ J}$$

$$= 1.701 \cdot 10^{-5}\ m$$

(c) Spectrum of light $\quad u(\epsilon) = \frac{8\pi}{(hc)^3} \frac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$ energy density per unit photon energy

$\cancel{u(\epsilon)} =$ Now

$$\frac{8\pi}{(hc)^3} = \frac{8 \cdot \pi}{[(6.626 \cdot 10^{-34}\ J \cdot s)(3 \cdot 10^{8}\ m/s)]^3} = 3.1998 \cdot 10^{75}$$

$$= \cancel{3.1998 \cdot 10^{75}} \quad \cancel{\epsilon^3}$$

$$kT = (1.381 \cdot 10^{-23}\ J/k)(3000\ k) = 4.143 \cdot 10^{-20}\ J$$

$$\therefore \quad u(\epsilon) = (3.1998 \cdot 10^{75}) \frac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$$

let $x \equiv \epsilon/kT$

$$U(x) = (3.1998 \cdot 10^{75}) \frac{(kT)^3 x^3}{(e^{x} - 1)}$$

$$U(x) = (2.2755 \cdot 10^{17}) \frac{x^3}{(e^x - 1)}$$

— — —

Spectrum of light $\quad u(\epsilon) = \dfrac{8\pi}{(hc)^3} \dfrac{\epsilon^3}{(e^{\epsilon/kT} - 1)}$

$$\epsilon = h\nu = \frac{hc}{\lambda}$$

$$u(\lambda) = \frac{8\pi}{\cancel{(hc)^3}} \frac{\cancel{(hc)^3}(\lambda^{-3})}{e^{\frac{hc}{kT}\cdot\frac{1}{\lambda}} - 1} = \frac{8\pi}{\lambda^3} \frac{1}{\left(e^{\left(\frac{hc}{kT}\right)\cdot\frac{1}{\lambda}} - 1\right)}$$

$$\left.\frac{hc}{kT}\right|_{T=3000K} = \frac{(6.626 \cdot 10^{-34} J \cdot s)(3 \cdot 10^8 \; m/s)}{(1.381 \cdot 10^{-23} \; J/K)(3000 \; K)} = 4.798 \cdot 10^{-6} \; m$$

$$= 4.798 \; \mu m \equiv \lambda_0$$

$$u(\lambda) = \frac{8\pi}{\lambda^3} \frac{1}{(e^{\lambda_0/\lambda} - 1)} \qquad \text{plot in units of}$$

$\lambda \sim 0 \quad \dfrac{\infty}{} \cdot \dfrac{1}{\infty} = \dfrac{\infty}{\infty}$ undetermined guess $\rightarrow 0$.

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$$I(T) = \int_{400\,nm}^{700\,nm} U(t)\,dt = \ldots$$ Calculated radiant energy per unit time? in the visible spectrum.

(e)

(f) Find Maximum of $I_{vis}(T)$ intensity of radiated energy per unit time.

P.7.52

(a) $P = \sigma e A T^4$ $T \simeq 300\,k$

$A_{Body} \cong 1\,m^2$

$e = 1$

$\sigma = 5.67 \cdot 10^{-8} \frac{W}{m^2 k^4}$

$\Rightarrow P = (5.67 \cdot 10^{-8} \frac{W}{m^2 k^4})(1)(1 m^2)(300\,k)^4 = 459.27\,W$

What is surface area of Body?

(b) $E/_{day} = P \cdot (3600)(24) = 3.968092 \cdot 10^7\,J$ radiated in photons.

$Cal/_{day} = \cancel{3} \cdot 2000$ kilocal $= 8.3736 \cdot 10^6\,J$ ingested as calories.

~~So~~ to look for humans get T_H exactly & then max return from a black body at T_H is $\epsilon \approx 2.8 k T_H = \cancel{\frac{hc}{e}} \frac{hc}{\lambda}$ $\Rightarrow$ tells one how to build a sensor to look for people.

$f = \frac{c}{\lambda}$

Don't know?

(c) $\left(\frac{P}{M}\right)_{sun} = \frac{3.9 \cdot 10^{26}\,W}{(2 \cdot 10^{30}\,kg)} = 1.98 \cdot 10^{-4}\ W/kg$

$\left(\frac{P}{M}\right)_{Hum} = \frac{459.27 W}{\cancel{180 lbs}\ 81.64\,kg} = 5.62\ W/kg$

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7.53

(a) $T_{B.H.} =$

$$\epsilon_{peak} = 2.8\, kT_{BH} = 2.8(1.381\cdot 10^{-23}\, J/k)(\ldots\, h$$

$$\epsilon_{peak} = 2.8 \cancel{k} \frac{hc^3}{16\pi^2 \cancel{k} GM} = 2.8 \frac{hc^3}{16\pi^2 GM}$$

$$= \frac{2.8(6.626\cdot 10^{-34}\, J\cdot s)(3\cdot 10^8\, m/s)^3}{16\pi^2(6.673\cdot 10^{-11}\, N\cdot m^2/kg^2)(2\cdot 10^{30}\, kg)}$$

$$= 2.3769\cdot 10^{-30}\ \frac{J\cdot \cancel{s}}{N\cdot \cancel{m^2}}\ \frac{m^{\cancel{3}}}{s^{\cancel{3}2}}\, kg$$

$$= \quad \text{''} \quad \frac{J\, m\cdot kg}{N\cdot s^2} = J. \qquad J = F\cdot L = N\cdot m$$

$$N = kg\cdot \frac{m}{s^2}$$

$$\lambda = \qquad \epsilon = h\nu = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\epsilon}$$

$$\lambda_{peak} = \frac{hc}{\frac{2.8\, hc^3}{16\pi^2 GM}} = \frac{16\pi^2}{2.8}\ \frac{G\cdot M}{c^2} = 8.36\cdot 10^4\, m$$

$$SA = 4\pi R^2 \qquad R = \left(\frac{SA}{4\pi}\right)^{1/2} = 4\pi^{1/2}\frac{GM}{c^2}$$

$$= 1.05\cdot 10^4\, m$$

(b) $P = \sigma e A T^4$

$$= \sigma e \left(\frac{\cancel{16}\pi G^2 M^2}{c^4} \right) \left(\frac{hc^3}{\cancel{16}\pi^2 kGM} \right)$$

$$= \frac{\sigma e}{\pi c} \frac{GM}{k} = \cancel{5.8} \; 5.8 \cdot 10^{26} \; W$$

(c) $E = Mc^2$

$$\frac{dE}{dt} = c^2 \frac{dM}{dt} = P = \left(\frac{\sigma e}{\pi c} \frac{G}{k} \right) \cdot M$$

$$\frac{dM}{dt} = \left(\frac{\sigma e}{\pi c^3} \frac{G}{k} \right) M$$

$$M = M_0 \exp \left\{ 8 \, t \Big/ \frac{\pi c^3 k}{\sigma e G} \right\}$$

$$t_0 \equiv \frac{\pi c^3 k}{\sigma e G} = 3.09 \cdot 10^{20} \; s = 9.7 \cdot 10^{12} \text{ years}$$

(e) $M_0 = ?$ $\quad M \ldots$

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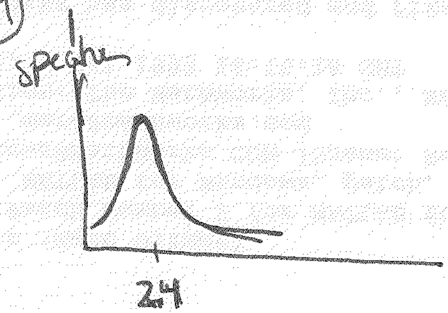
P.7.54

(a)



$$E_{peak} = 2.8kT \Rightarrow T = \frac{2.4\ eV}{2.8(8.617\cdot 10^{-5}\ eV/K)} = 9.94\cdot 10^3\ K$$

$$P = \sigma e A T^4 \qquad \cancel{P \Leftarrow \frac{(24\cdot 3.9\cdot 10^{26}\ W)}{(5.67\cdot 10^{-8}\ \frac{W}{m^2K^4})(1)(A)}}$$

$$24\cdot 3.9\cdot 10^{26}\ W = (5.67\cdot 10^{-8}\ \tfrac{W}{m^2K^4})(1)A\ (9.94\cdot 10^3\ K)^4$$

$$A = 1.69\cdot 10^{19}$$

$$4\pi R^2 = \text{"}$$

$$R = 1.16\cdot 10^9\ m = 1.16\cdot 10^6\ km$$

$$r_{sun} = 7\cdot 10^8\ m \qquad \text{about } 10\times \text{ larger}$$

(b) $P = (.03)(3.9\cdot 10^{26}\ W)$

$$E_{peak} = 2.8kT \underset{set}{=} 7eV$$

$$\Rightarrow T = \frac{7\ eV}{2.8(8.617\cdot 10^{-5}\ eV/K)} = 29\,000\ K$$

$$P = \sigma e A T^4$$

$$\Rightarrow A = \frac{P}{\sigma e T^4} = \frac{(.03)(3.9\cdot 10^{26}\ W)}{(5.67\cdot 10^{-8}\ \frac{W}{m^2K^4})(1)(29\cdot 10^3\ K)^4} =$$

$A = 2.9 \cdot 10^{14}\ m^2$

$\Rightarrow R = 4 \cdot 10^6\ m$. About ~~that~~ $\frac{1}{100}$ radius of our sun

(c) $\epsilon = 2.8kT = 0.8eV$

$$T = \frac{0.8\,eV}{(2.8)(8.617 \cdot 10^{-5}\,eV/K)} = \cancel{3000\,k}.\ 3300\,k.$$

$$\epsilon = 0.8eV = h\nu = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{\epsilon} = \frac{(4.136 \cdot 10^{-15}\,eV \cdot s)(3 \cdot 10^8\,m/s)}{.8\,eV} = 1.5\,\mu m$$

Visible light $\sim$ 400 nm – 700 nm $\Rightarrow$ .4 μm – .7 μm

$$P = (10^4 \cdot 3.9 \cdot 10^{26}\,w) = \sigma e A T^4$$

$$\Rightarrow A = \frac{P}{\sigma e T^4} = 5.8 \cdot 10^{23}\ m^2$$

$\Rightarrow R = 2.1 \cdot 10^{11}\ m$ $\qquad R_{sun} = 7 \cdot 10^8$

$\approx$ 1000 times greater than our sun.

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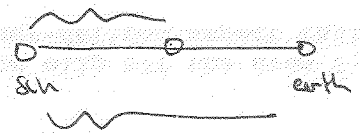
Prob 7.55 ?

P.7.56 Infrared light responsible for

$$R = .7(150 \cdot 10^6 \text{ km}) = 105 \cdot 10^6 \text{ km}$$

(a)



$$R = 150 \cdot 10^6 \text{ km}$$

$$P = 3.9 \cdot 10^{26} \text{ W}$$

Solar constant for Venus $= \dfrac{P}{4\pi(R^2)} = \dfrac{3.9 \cdot 10^{26} \text{ W}}{4\pi(105 \cdot 10^6 \cdot 10^3)^2 m^2}$

$$= 2.8 \cdot 10^3 \text{ W/m}^2$$

(a) $\Rightarrow$?

Amt heat absorbed by

$$(\text{solar constant})\cancel{\pi}\cancel{R}^2 = \sigma e \cdot \underset{\pi R^2}{A} \cdot T^4 = \sigma e\, 4\cancel{\pi}\cancel{R}^2 \cdot T^4$$

$$\Rightarrow \quad T = \left(\frac{2800 \text{ W/m}^2}{4 (5.67 \cdot 10^{-8} \frac{W}{m^2 K^4})} \right)^{1/4} = 333.3 \text{ K}$$

(b) Assuming on 30% of incident flux gets through

$$T = \left(\frac{(.3)(2800\ W/m^2)}{\ldots}\right)^{1/4} = \cdots \quad 246.69$$

(c) ?

P.7.57 eq 7.112 $U =$

eq 7.108 is

$$U = 3\int_0^{n_{max}} dn \int_0^{\pi/2} d\theta \int_0^{\pi/2} d\phi\, n^2 \sin\theta \, \frac{\epsilon}{e^{\epsilon/kT}-1}$$

$$= 3\left(\frac{\pi}{2}\right)\left(-\cos\theta\right)\Big|_0^{\pi/2} = \frac{3\pi}{2}(1) = \frac{3\pi}{2}$$

$$= \frac{3\pi}{2}\int_0^{n_{max}} dn\, n^2 \left(\frac{hc_s n}{2L}\right)\frac{1}{\left(e^{hc_s n/2LkT}-1\right)}$$

let $x = \dfrac{hc_s n}{2LkT} \Rightarrow n = \dfrac{2LkT}{hc_s}\cdot x$

$$dn = \frac{2LkT}{hc_s}\,dx$$

$$= \frac{3\pi}{2}\int_0^{x_{max}} \left(\frac{2LkT}{hc_s}\right)\cdot dx \left(\frac{2LkT}{hc_s}\right)^2 x^2 (xkT)\frac{1}{(e^x-1)}$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc_s}\right)^3 \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}\cdot kT$$

$$= \frac{3\pi}{2}\left(\frac{2LkT}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$x_{max} = \frac{hc_s}{2LkT}\cdot\left(\frac{6N}{\pi}\right)^{1/3} = \frac{hc_s}{2kT}\left(\frac{6N}{\pi V}\right)^{1/3} \equiv \frac{T_D}{T}$$

Then $T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3}$ $\left(\frac{2k}{hc_s}\right) = \frac{1}{T_D}\left(\frac{6N}{\pi V}\right)^{1/3}$

$U =$

$$= \frac{3\pi}{2}\cdot L^3 T^3 \left(\frac{2k}{hc_s}\right)^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{3^2\cancel{\pi}}{\cancel{2}} \frac{1}{T_D^3}\left(\frac{\cancel{6}N}{\cancel{\pi}\cancel{V}}\right) \cancel{L^3} T^3 kT \int_0^{x_{max}} \frac{x^3\,dx}{e^x-1}$$

$$= 3^2 \left(\frac{T}{T_D}\right)^3 NkT \int_0^{T_D/T} \frac{x^3\,dx}{e^x-1}$$

$$= \frac{9NkT^4}{T_D^3} \int_0^{T_D/T} \frac{x^3\,dx}{e^x-1} \qquad \text{eq } 7.112$$

$$C_V = \frac{\partial U}{\partial T}$$

$$U = \frac{3\pi}{2}\int_0^{n_{max}} \frac{hc_s}{2L}\frac{n^3}{(e^{hc_s n/2LkT}-1)}\,dn$$

$$\frac{\partial U}{\partial T} = \frac{3\pi}{2}\int_0^{n_{max}} \frac{hc_s}{2L}\frac{n^3}{(e^{hc_s n/2LkT}-1)^2}\,e^{\frac{hc_s n}{2LkT}}\left(\frac{hc_s n}{2LkT^2}\right)(-1)\,dn$$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2}\frac{hc_s}{2L}\left(\frac{hc_s}{2LkT^2}\right)\int_0^{n_{max}} \frac{n^4\,e^{\frac{hc_s n}{2LkT}}}{(e^{hc_s n/2LkT}-1)^2}\,dn$$

let $x = \frac{hc_s n}{2LkT}$ $\quad n = \frac{2LkT}{hc_s}x$ $\quad dn = \left(\frac{2LkT}{hc_s}\right)dx$

$$\frac{\partial U}{\partial T} = -\frac{3\pi}{2}\left(\frac{hc_s}{2L}\right)^2\frac{1}{kT^2}\int_0^{x_{max}}$$

. . .

P.7.58 $c_s = 3560 \, m/s$

$$T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3} \qquad V = 7.12 \, cm^3$$

$$N = N_A = 6.02 \cdot 10^{23}$$

$$= \frac{(6.626 \cdot 10^{-34} \, J \cdot s)(3560 \, m/s)}{2(1.381 \cdot 10^{-23} \, J/k)}\left(\frac{6 \cdot 6.02 \cdot 10^{23}}{\pi (7.12 \cdot 10^{-6} \, m^3)}\right)^{1/3}$$

$$= 465.0686 \, k.$$

Experimentally:

How do I measure T_D from this plot?

$$\text{Slope of curve} = \frac{12\pi^4 Nk}{5T_D^3}$$

$$\Rightarrow \quad = \frac{1.8 - .9}{16 - 2} = \frac{.9}{14} = .0643 \, \frac{mJ}{k^4}$$

$$= .0643 \cdot 10^{-3} \, \frac{J}{k^4}$$

So that

$$\left(\frac{1}{T_D}\right)^3 = \frac{5}{12\pi^4 N_A k}(.0643 \cdot 10^{-3} \, J/k^4)$$

$$T_D = 311.5 \, k$$

P.7.59 According to the free electron model of specific heat w/ ...

$$\gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$$

If ϵ_F is approximately the same $\forall$ species then the y-intercept will be also.

P.7.60 $C_V = \gamma T + \frac{12\pi^4 N k}{5 T_D^3} T^3$ $T \ll T_D$

w/ $\gamma = \frac{\pi^2 N k^2}{2\epsilon_F}$ $T_D = 465.06\ k$

$$\epsilon_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} = \cdots$$

$$\gamma T = \frac{12\pi^4 N k}{5 T_D^3} T^3$$

$$T^2 = (\quad)\gamma$$

P.7.61

P.7.61

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$^4He = 4.0 + 2$ neutrons

$\approx 6.0\ g/mole = M$ atomic weight

$C_V = Nk\left(\frac{T}{4.67\,k}\right)^3 \quad T < 0.6\ k$

$c_s = 238\ m/s \qquad \rho = 0.145\ g/cm^3 \qquad V = \frac{M}{\rho}$

$\frac{N}{V} = \frac{nN_A}{V} =$

$V =$ (1) take 1 mole of ^4He

(2) $N = N_A$

(3) $V = \frac{M}{\rho} \doteq \frac{[M]}{[\rho]} = \frac{g}{(g/cm^3)}$

$\therefore \frac{N}{V} = \frac{N_A}{M}\rho$

n moles $= nM$ grams $= \frac{N}{N_A}M$

$C_V = ? \qquad C_V = \frac{12\pi^4}{5}\left(\frac{T}{T_D}\right)^3 Nk$

$T_D = \frac{hc_s}{2k}\left(\frac{6N}{\pi V}\right)^{1/3}$

$\frac{\Delta}{\Delta V} =$ (1) compute $T_D = \ldots$

(2) compute C_V

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P. 262 7.112

$$U = \frac{9NkT^4}{T_D^3} \int_0^{T_D/T} \frac{x^3\,dx}{e^x-1}$$

$$\frac{x^3}{e^x-1} = x^3 \cdot \frac{1}{(e^x-1)}$$

~~Let~~ $f(x) = \dfrac{x^3}{e^x-1}$ $\cancel{f(0) = 0}$

$$f(0) = \frac{3x^2}{e^x}\Bigg|_{x=0} = 0 \quad \frac{\text{"}0\text{"}}{\text{"}0\text{"}}$$

$$f'(x) = \frac{3x^2}{e^x-1} + \frac{x^3(-1)e^x}{(e^x-1)^2} = \frac{x^2}{(e^x-1)^2}\left[3(e^x-1) + x(-1)e^x\right]$$

$$= \frac{x^2}{(e^x-1)^2}\left(3e^x - 3 - xe^x\right) = \frac{x^2}{(e^x-1)^2} \cdots$$

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P. 7.63 ?

P. 7.64 $f \propto \left(\frac{1}{\lambda}\right)^2$

$E = hf$ + $p = \frac{h}{\lambda}$ for particles

$E \propto p^2$ for magnons

$E = \frac{p^2}{2m^*}$ $m^* = \text{constant} = 1.24 \cdot 10^{-29}$ kg

$= 14 \cdot m_e$

(a) ?

(b) ?

P. 7.64

$\epsilon = hf \qquad p = \frac{h}{\lambda}$

$$\epsilon^2 = \frac{p^2}{2m^*} = \frac{h^2}{2m^*}\frac{1}{\lambda^2} \qquad \lambda = \frac{2L}{n}$$ same as for photons

(a) $N = g\sum_n n\,\bar{n}_{PL}(\epsilon) = \int_0^\infty n \quad \ldots$ Not entirely sure

(b) $$\frac{M(0) - M(T)}{M(0)} = \frac{2\mu_B N - 2\mu_B N_m}{2\mu_B N}$$

$$= 1 - \frac{N_m}{N} = \quad \ldots$$

(c)

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P. 7.65   eq 7.124

$$\int_0^\infty \frac{\sqrt{x}\,dx}{e^x-1} = \cdots$$

P. 7.66

(a) $\epsilon_0 =$

(b) $kT_c = (0.527)\left(\frac{h^2}{2\pi m}\right)\left(\frac{N}{V}\right)^{2/3}$

?

(d) ?

P. 7.67 ?

P. 7.68 ?

P. 7.69

(a) $N = \int_0^\infty g(\epsilon)\,\frac{d\epsilon}{e^{(\epsilon-\mu)/kT}-1}$ let $t = T/T_c$ + $x = \frac{\epsilon}{kT_c}$, $c = \frac{\mu}{kT_c}$

$$= \int_0^\infty g(x\,kT_c)\,\frac{(dx/kT_c)}{e^{(xkT_c - ckT_c)/tkT_c}-1} \qquad d\epsilon = \frac{dx}{kT_c}$$

$$= \int_0^\infty \frac{g(xkT_c)}{kT_c}\cdot\frac{dx}{e^{(x-c)/t}-1}$$

...

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7.122 is

P.7.70

(a) ?

P.7.71 ?

P.7.72 ?

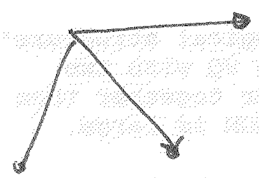
P.7.73 (a) ?

P.7.74 ?

P.7.75 ?

$$Z_{ideal} = \frac{1}{N!}\left(\frac{V Z_{int}}{v_Q}\right)^N \qquad v_Q = \left(\frac{h}{\sqrt{2\pi m k T}}\right)^3$$

n(



$\binom{n}{2} = \frac{n(n-1)}{2}$ connections for a network of n nodes

$$Z_c = \frac{1}{V^N}\int d^3r_1 \cdots d^3r_N \prod_{\text{pairs}} e^{-\beta u_{ij}}$$

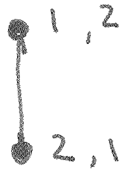
$$e^{-\beta u_{ij}} = 1 + f_{ij}$$

$$\prod_{\text{pairs}} e^{-\beta u_{ij}} = \prod_{\text{pairs}} (1 + f_{ij}) = (1+f_{12})(1+f_{13})(1+f_{14})\cdots(1+f_{1N})(1+f_{23})(1+f_{24})\cdots$$

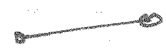
$$= 1 + \sum_{\substack{\text{pairs} \\ \text{(connections)}}} f_{ij} + \sum_{\substack{\text{distinct} \\ \text{pairs}}} f_{ij} f_{kl} + \cdots$$

$$\frac{1}{V^N}\int d^3r_1 \cdots d^3r_N\, f_{12} = \frac{1}{V^N}\, V^{N-2}\int d^3r_1\, d^3r_2\, f_{12}$$

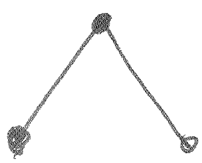
$$= \frac{1}{V^2}$$



$$\frac{1}{V^2}\int d^3r_1\, d^3r_2\, f_{12} \quad \frac{N(N-1)}{2}$$



$\cancel{N}$ $\cancel{N(N-1)}$



$$\frac{N(N-1)(N-2)}{\boxed{2}}$$

$2!$

$$\frac{N(N-1)(N-2)(N-3)}{8}$$

$$Z = Z_{ideal} \cdot Z_c$$

$$F = -kT \ln Z = -kT \ln Z_{ideal} - kT \ln Z_c$$

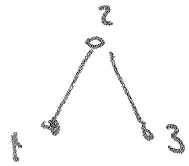
$$= -NkT \ln\left(\frac{V}{N v_Q}\right) - kT\left(\; + \; + \; + \cdots \right)$$

$$P = -\frac{\partial F}{\partial V}\bigg|_{N,T} =$$

$$F = U - TS$$

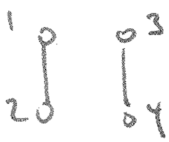
P.8.1

$$= \frac{N(N-1)}{2V^2}\int d^3r_1\, d^3r_2\, f_{12}$$

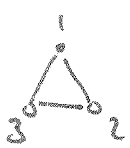


$$= \frac{N(N-1)(N-2)}{2V^3}\int d^3r_1\, d^3r_2\, d^3r_3\, f_{12} f_{23}$$

Don't follow the symmetry factor very well



$$= \frac{N(N-1)(N-2)(N-3)}{2V^4}\int d^3r_1 d^3r_2 d^3r_3 d^3r_4\, f_{12} f_{34}$$



$$= \frac{N(N-1)(N-2)}{4\cancel{6}V^3}\int d^3r_1 d^3r_2\, d^3r_3\, f_{12} f_{23} f_{31}$$

$2 \cdot 2 = 4$

P.8.2 ...

P.8.3 ..

P.8.4 ...